CARNEGIE INSTITUTE OF TECHNOLOGY

SCHOOL OF APPLIED SCIENCE

THESIS

SUBJECT Secondary Stresses in Framed Structures

PRESENTED BY Thornspn Eason Mao

DEPARTMENT OF Civil Engineering CLASS OF

CANDIDATE FOR DEGREE OF Doctor of Engineering

ACCEPTED BY F. M. McCullough DATE June 1921

DEPARTMENT OF Civil Engineering

APPROVED BY THE FACULTY Wm. E. Mott
-6-'21 DEAN.

P R E F A C E .

The purpose of this paper is two-fold: First, to present an adequate treatment of secondary stresses, and second, to introduce-three new methods evolved by the writer.

During recent years when the importance of secondary stresses has exhibited itself a large amount of literature has been written on the subject for advancing the science of structural design* & critical study made of these papers will at once reveal that they are subject to one or more defects which greatly impair their practical value. In the first place, too much attention has been paid to the mathematical theories, which could be greatly shortened for the benefit of practical engineers. Secondly, the treatment of the subject is generally limited to a narrow field, in which only a few methods are applied to only a certain class of structure. Thirdly, the treatment of the four existing methods is entirely too individual, in that the methods are generally considered as being separate from each other^notwithstanding the fact that they are more or less equivalent. Lastly, but not the least, the effect of secondary stresses on the design is not adequately considered and sometimes sadly neglected.

With a view to avoid the above defects it has been the writer's endeavor that the subject be treated in an entirely different way---that it be more practical, more comprehensive, and more logical; so that it could be easily understood and appreciated by those for whom the secondary stress has the most direct bearing---the practical engineers. The writer is aware that not all of these objects are attained in this paper, on account of limited amount of time, but it is believed that the scope of the work and the arrangement of materials are sufficiently effective so as to produce the desired results,

The methods for computing secondary stresses have been greatly improved in recent years. Two objections, however, still stand in the way: First, the amount of time involved is often ~~exorbitant,~~ excessive and second, the lack of a checking device by which the correctness of the various steps of procedure may be ensured. While there are numerous other defects these two alone are generally sufficient to reduce their practical utility. Ever since the beginning of 1917, when the writer undertook the analysis of secondary stresses in a two hinged arch, the results of which have been published in the Transactions of the American Society of Civil Engineers, Vol. 82, p. 1104, it always occured to him that there must be some method which is not only shorter and less cumbersome than the current ones but which also admits of a unique check. For two and a half years he has worked on the subject almost incessantly, striving to find some new method that will accomplish both. At last, much to his satisfaction, the graphic method of deformation contour was obtained; which not only takes less time, furnishes a unique check, but also gives remarkably accurate results. Along with this method, almost contemporaneously, two more methods were evolved--the graphic method of successive deduction and the analytic solution of the graphic methods. All of the three methods are described in detail in Part III, page 38, which,being treated more or less independently, could be read without reference to other parts of the paper. A perusal of Chapter 10, page 105, is hereby recommended. As these methods are new in their field it is hoped that their usefulness may be actually tested by further investigators.

Besides the two purposes of the paper as heretofore mentioned the following points deserve special attention: (1) The method for the solution of simultaneous equations, p. 25. (2) The approximate methods in Chapters 2 and 3, pages 115-125. (3) The well digested principles of design in Chapters 1 to 6, pages 140-151.

In conclusion the writer wishes to express his indebtedness to Professor H. R. Thayer, under whosB direction the present work was undertaken.

Respectfully submitted,

Thomson E. Mao

October, 1919.

CONTENTS.

		Page
PREFACE		ii
CONTENTS		vi
INTRODUCTION.		
Chapter 1.	General and Historical Notes	1
2.	Nature of Problem	4
3.	Field of Application	6
PART I.	ANALYSIS OF SECONDARY STRESSES DUE TO RIGIDITY OF JOINTS.	
Chapter 1.	Introductory Notes	8
2.	Change of Length due to Primary Stresses	10
3.	Change in Angles due to Primary Stresses	10
4.	The Two Fundamental Equations	12
	Article 1. The Deflection Equation	12
	2. The Moment Equation	13
5.	Deflection Angles and the Rigidity of Joints	14
6.	Solution of the Problem	14
PART II.	EXISTING METHODS FOR COMPUTING SECONDARY STRESSES DUE TO RIGIDITY OF JOINTS.	
Chapter 1.	The Method of Joints	16
2.	The Method of Manderla	17
	Article 1. The Reference Deflection Angle	17
	2. Moment Equations in Terms of the Reference Angles	18
	3. Details of Procedure.	20

Page

Chapter'3. The Method of Mohr. 21

Article 1. The Rotation of Joints and the Slope Deflection 21

2. Moment Equations in "Terms of the Rotation of Joints 22

3. Details of Procedure 23

4« The Solution of Simultaneous Equations 24

5. The Method of Triangles 29

6. The Method of Members 32

7. Comparison of the Different Methods 35

Article 1. Features of the Method of Manderla . 36

2. Features of the Method of Mohr 36

PART III. NEW METHODS FOR COMPUTING SECONDARY STRESSES DUE TO RIGIDITY OF JOINTS,

Chapter 1. Fundamental Principles • 38

Article 1. The Fundamental Conception 38

2. The Fundamental Equation 41

2. Analytic Equations 45

3. Graphical Constructions 48

Article 1. Preliminary Considerations 48

2« Change of Angles 51

3. Deformation and Property Lines 54

4. The Equilibrium Contour 57

4. The Theory of Deformation Contour 61

5. Graphic Method by the Theory of Deformation Contour 66

Page

Article 1. Graphical Representation of a Linear Equation involving Three Variables 66

2. Construction of the Base and Vertex Lines 68

3. Solution of the Problem 75

4. Checks 76

5. Example. Details of Procedure 76

6. Construction of Influence Lines 77

Chapter 6. The Process of Successive Deduction 79

Article 1. Empirical Formulas for f_x and f_y . 8!

2. Mathematical Formulas for f_x and f_y 85

3. The Process of Characteristic Increments 83

4. The Method of Substitution 85

7. Graphic Method by the Process of Successive Deduction 87

Article 1. Checks 89

2. Example, Details of Proceed 3ure 90

5* Construction of Influence Lines 90

8. The Scientific Arrangement of Computations 91

Page

Chapter 9. Analytic Solution by the Scientific Arrangement of Computations 93

Article 1. General Considerations 93

2. General Procedures 94

3. Change of Angles 97

4. Example. Details of Procedures 98

5. Construction of Influence Lines 104

10. Features of the New Methods and Comparison 105

Article 1. Features Common to all Methods 105

2. Features of the Graphic Methods 106

3. Features of the Graphic Method of Deformation Contour 106

4. Features of the Graphic Method of Successive Deduction 107

5. Features of the Analytic Method with the Scientific Arrangement of Computations 107

6. Comparison of the Three Methods 108

Page

PART IV, .OTHER CONSIDERATIONS OF SECONDARY STRESSES DUE TO RIGIDITY OB' JOINTS

Chapter 1. Exact Methods 109

Article 1. The Exact Method of Manderla including the Effects of Primary Stresses 109

2. The Exact Method including the Weight of the .Member 114

2. Ap^bximate Methods 115

Article 1. The Approximate Method of Turneaure 116

2. The Approximate Method of Mohr 116

3. The Approximate Methods involving Empirical Formulas 118

3. The Approximate Methods by the Theory of Continuous Beams 118

Article 1. The Graphic Method of Muller-Breslau 120

2. The Graphic Method of Engresser and Landsberg 122

5. The Analytic Solution 125

4. Secondary Stresses in Special Forms of Bridge Trusses 126

5. Secondary Stresses in Pin Connected Trusses 127

6« Analysis of Secondary Stresses in Structures other than Bridge Trusses 128

Page

PART V. SECONDARY STRESSES DUE TO OTHER INFLUENCES THAN THE RIGIDITY OF JOINTS 132

Chapter 1. Bending Stresses in a Transverse Frame due to Primary Stresses in the Posts 133

2. Stresses in a Horizontal Plane due to Unequal Deformations of Chords and Stringers 134

3. Bending Stresses due to Variation of Axial Stresses in Different Elements of a Member 135

4. Torsional Stresses due to Various Combinations of Secondary Stresses 136

PART VI. SECONDARY STRESS: A FACTOR IN DESIGN

Chapter 1. Introduction 137

2. Selection of Trusses 140

3. Arrangement of Span 143

4. Design of Floor System 145

5. Proportioning of Members 146

6. Designing of Details 149

7. Correction of Secondary Stresses 151

I N T R O D U C T I O N .

CHAPTER I. GENERAL AND HISTORICAL NOTES.

Before proceeding *With* the subject of secondary stresses it is proper to define exactly what "secondary" means. According to German writers the stresses in a framed structure are divided into two classes—the primary and the secondary. By "primary" is meant all those stresses which pass through, the centre of gravity of the sections and act along the axes of the member, producing either an elongation or a shortening. The primary stresses caused by the dead ioad or live load are called the "principal" stresses while those due to impact, wind, centrifugal, yielding of support, temperature, etc.jare called the "additional" stresses. The "secondary" stresses include all. tfoose stresses that are not axial whatever their nature may be: bending, shearing, torsional, etc.

An English authority limits the secondary stresses to those which will be automatically reduced Mien an incipient failure of the parts stressed occurs.[1] This is very hypothetical and not so clear as the following definition evolved by an American writer.[2] "Secondary stresses are those which make up the difference between the ppimary stresses and the actual stresses which the assumed static load would produce". From practical point of view the following is representa-

*** *** *** *** ***

1. Londfton Engineering, Jan. 7, 1916.
2% Proceedings of Eng. Society of Western Penn., Vol. 25.

tivs:[1] "The secondary stress in any member at any section is equal to one half the difference of the two extreme fibre stresses measured at the same section. The primary stress is then equal to one half the sum of the same extreme fibre stresses"•

As a distinction between the secondary and additional stresses is advisable the present paper will define secondary stresses as those which arise from bending, twisting, and shearing^ whatever may be their source.

The subject of secondary stresses was largely developed by German writers. In the year of 1877 the folytechnic School in Munich offered a prize Hap the solution of the following problem, formulated by Asimont. "What stresses arise in the members of a bridge truss owing to the fact that the angles of the tnuss triangles do not undergo any change?". This prize was awarded to H. Manderla who gave his solution in a paper published in 1880, in Allgemeine Bauzeitung, under the title "The calculation o£ secondary stresses which occur in simple trusses as a consequence of rigid joints.

In 1879 Engresser published an approximate method in Zeitschrist fur Baukunde.

The first detailed computations of secondary stress are found in l'inkler's Theorie der Brucken published in 1881. In 1885 Landsberg contributed a graphical solution under the assumption that the chords aloneaare riveted; in 1886 muller Breslan made an analytic contribution. Ritter, in 1890, gave a graphical solution and Engresser in 1892-93 published a book on secondary and additional stresses. The last

*** *** *** *** ***

1. Transactions' of Am. Soc. of Eng., Vol. 82

analytic method was contributed by Mohr in 1892.

The first direct measurement of secondary stresses was made by Frankel in 1883. In 1899 Mesnager published an account of the measurement of stresses in a Pratt truss of 180 foot span on the Orleans Railway in France. In 1901 M.Rabut described a series of stress measurements which had been made on the bridges of the Orleans Railway. In 1905-06 W. Gehler conducted a series of tests and measurements on a railway bridge of 128 foot span at Elsterwerda, Saxony. In 1907-09 the American Railway Engineering Association conducted a series of tests on a large number of plate girders and truss bridges, ranging in span from 30 to 440 feet. The latest and most extensive measurement was made by Steinman on the Hell Gate Arch in New York in 1915.

CHAPTER II. NATURE OF PROBLEM

According to the committee report presented to the American Railway Engineering Association in 1914 and printed in Volume 15 of its Proceedings the secondary stresses in a bridge are divided into five classes, as follows: .

1, Bending stresses in the plane of the main truss due to rigidity of joints, eccentricity of joints and weight of members.

2 Bending stresses in members of a transverse frame due to the deflection of floor beams and to primary stresses in posts.

$. Stresses in horizontal plane due to longitudinal deformation of chords, especially the stresses in floor beams and in their connections.

4. Variation of axial stresses in different elements of a membsr.

5 Stresses due to vibration of individual members.

The stresses under Nos. 1, 2, and 3 can be analysed mors or less completely but those under Mos. 4 and 5 cannot be as accurately determined. It should be noticed that the 5th class does not conform t6 i "the definition for secondary stresses adopted in this papsr. Among the first thpee classes No. 1 is the most important and is the one which has receives most of the attention in the present paper.

By "rigidity of joints" is meant the incapability of the melabers meeting at a joint to rotate relatively one with respect to li the other. This occurs in structureawwhere the members are connected together by gusset plates and rivets and also to some extent in structures connected by pins. Now if a structure is under load the various members will undergo deformations as a result of the elasticity of the material of which the members are built. These deformations tend to change the angles between the various members but are prevented from doing so by the rigidity of joints. A restraint is therefore imposed on the members and must be relieved by bending the members. The stresses produced in the members by this bending form the principal source of secondary stresses and are designated by class No. 1 above. It will be noted that if the joint is made of frictionless pins so that any member can rotate freely irrespective of the others the deformations in the members could be taken care of by the change of angles and the so called secondary stresses would disappear. This is, of course, not true in practice but is the assumption upon which all the primary stresses are computed.

Here, it is necessary to differentiate between secondary stresses which are attributive and those which are essential. The former applies to all structures composed of triangular elements while the latter belongs to those structures with rectangular frames. In triangular structure the secondary stress is produced as a result of certain conditions imposed on the structures and is removed as soon as those conditions are relieved. It is not essential for the stability of the structure, i.e., the structure can stand without the existence of secondary stresses. In a rectangular frame, on the other hand, the secondary stress is not attributive but essential, without which the structure cannot stand. Here, the secondary stress cannot be removed without having the structure collapse.

Furthermore, even in a structure of triangular forms the presence of secondary stresses is very necessary although not essential. It has been found that rigidity of joints is the very thing that ~~stables~~ stiffens the structure and is highly desirable in an economic design. Hence it is important to note that the object in investigating secondary stresses is not to remove such stresses wherever possible but to reduce them only where they are objectionable.

CHAPTER III. FIELD OF APPLICATION.

Until recently the subject of secondary stresses has not been received with favor by practical engineers. Perhaps the strongest argument against the consideration of secondary stresses in structures was advanced by the Engish writers who contended that[1] "we have never heard or known of a case in which the failure of a roof or a bridge member not subject to altering forces has been traced to the existence of secondary stresses". They even went so far as to declare that it is "doubtful whether an exact analysis of secondary stresses in ordinary bridge structures will lead to any marked improvements in design." Opposed to this view is that of some American engineers who stated that[2] "the fact that these structures stand up does not warrant a total disregard of secondary stress. In structures with good details a much higher unit stresses could have been employed in the design if the secondary stresses had been considered". "For a design to be good it must be well balanced. Although a structure must be safe it also should be economical, .Good engineering is that best insurance against failure and wasted material is a tribute to ignorance,"[3] "Further in trusses designed for lighter loads thain actual it behooves one to know the dangerous limits"[4] While both views seem to approach the extremes the fact remains that actual observations of secondary stresses are relied upon as the best guidance. So far as the available data of past experimsnts are concerned the importance of secondary stresses

*** *** *** *** ***

1. London Engineering, Jan. 7, 1916.
2. Engineering News, Nov. 11, 1915
3. Proceedings of Eng. Soc. of Eng., Vol. 21.
4. Grimm, Secondary Stresses, P. 136

cannot be minimized and should be considered at least in the important and unusual structures. In buildings where the primary stresses are largely statically indeterminate and never in practice accurately computed it is not worth ~~the~~ while to consider the secondary stresses which depend on the primary. But in bridges, railway or highway, the primary stresses are generally computed from a method which is fairly correct and is altogether dependable. Based on this consideration it is entirely advisable to consider the secondary stresses in the design which would mean an increase in the safety of the structure and a decrease of waste in the material. It should be noted that the object is not so much to find the amount of secondary stresses in every individual structure as to determine the distribution of secondary stresses in every type of structure. It cannot be denied that such a knowledge would help considerably in choosing the type of the structure and also in improving the design.

Summarizing from what has been stated above it may be concluded that the secondary stresses should be considered in all types of bridges where the distribution of secondary stresses has not been known for any kind of loading. For the same type of bridge it is also advisable to investigate the effects of secondary stresses on the change of dimensions.

PART I. ANALYSIS 9F SECONDARY STRESSES DUE T0 RIGIDITY OP JOINTS.

CHAPTER I. INTRODUCTORY NOTES,

If a truss, the members of which are connected at the panel points by frictionless pins, be loaded in any manner, the various members will change in length slightly, the various panel points will deflect, and the angles made with each other by the various members meeting at each point will alter. The members will remain straight between panel points, however, as they can rotate freely oh the pins. If now a similar truss having rigid joints be consflered, the changes in length in the members and the deflections of the panel points will be substantially as before, but the angles between the various members meeting at a panel point will be forced to remain unchanged. As a result, each joint will rotate as a whole into some such positions that equilibrium throughout the truss will be maintained and each member will thereby be bent to some extent. Sending moments are thus produced iťn *the* members reaching a maximum nearing the joints. (This is not the general case in compression members where the maximum moment may occur somewhere between the two ends) The fibre stresses result- Therefrom constitute the secondary stresses due to rigidity of joints.

While .it is improper it is usual to consider the secondary stress as that due to rigidity of joints alone. This is in fact the only kind whfch T&aJ the current methods for secondary stresses apply and is the one *whtch has* received the greatest attention ever since the nature of theproblem of the problem was formulated by As&mont. There are at present four methods for computing this kind of secondary stresses and each has been received with irrork or less favor. The fundamental principles of these methods are exactly the same and it does not seem natural to consider each of the methods separately as this would invlsve much repetition of material* In the present paper, therefore, the treatment of the subject begins with the fundamental principles which apply to all of the different methods. The four existing methods are next taken up followed by the three new methods proposed by the writer."' The latfer treatment, however, is more or less independent so that its perusal does not require constant references t& the previous discussions. Lastly some paJpSs will be taken in the treatment of the so called approximate and exact methods for the solution of this kind of stresses.

Just as there are assumptions made for the computation of primary stresses so likewise are assumptions made for the secondary stresses. a#e aet exceptions tr© %-tee rule* The following is list of the more important assumptions which must be remembered.

(1), The axes of all the members are situated in the same plane and bent in the same plane. It will be understood that the expression "rigidity of joints" always applies to the joints in "the same plane.

(2), All the external loads are applied in the same plane.

(3). The primary stresses remain unchanged after the panel points are deflected.

(4), The dBflectinn of the truss with rigid joints is the same as if the joints were made of frictionless. pins.

(5). The effects of shear upon the flexure is neglected.

(6). All assumptions made in deriving the flexure formulas hold true for all the members of the truss.

CHAPTER II. CHANGE OF LENGTH DUE TO PRIMARY STRESSES.

The primary stresses to be used in computing the change in length of members are assumed to be the same as for the truss with frictionless pin joints.

Let P = primary stress per unit area of the section

L = the length of the member

E = the Modulus of Elasticity

δL = the change in length L, or the axial deformation,

Then,

$$\delta L = P \cdot L / E,$$

Or,

$$E \cdot \delta L = P \cdot L \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

This equation will be used in the method of Mohr.

CHAPTER III. CHANGE IN ANGLES DUE TO PRIMARY STRESSES.

By the "change in angles" is meant the alteration of the angle formed by any two members meeting at a point connected by a frictionless pin. Since the triangle is an elementary figure in a truss the change in angles in a triangle when the three sides are stressed will be first considered.

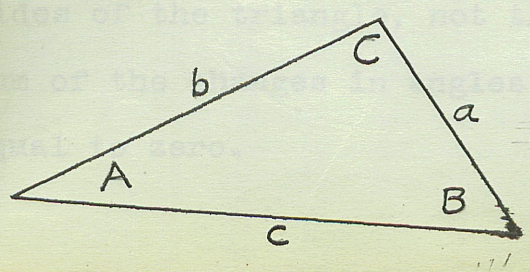


Fig. 1

Let P_a, P_b, P_c be the unit primary stresses in sides a, b, c and δA, δB, δC be the changes in angles of A, B, and C, Fig. 1.

From trigonometry,

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

$$A = \cos^{-1}\,(b^2 + c^2 - a^2)/\,2\,b\,c$$

Find the derivative of A with respect to a and substitute the expressions involving a, b, and c by *functions of* Cot B and cot C from trigonometry there is obtained

$$\frac{\partial A}{\partial a} = \frac{\operatorname{Cot} B + \operatorname{Cot} C}{a}$$

Similarly,

$$\frac{\partial A}{\partial b} = -\frac{\operatorname{Cot} C}{b}$$

$$\frac{\partial A}{\partial c} = -\frac{\cot B}{c}$$

now

$$\delta A = \frac{\partial A}{\partial a}\delta a + \frac{\partial A}{\partial b}\delta b + \frac{\partial A}{\partial c}\delta c$$

$$= (\operatorname{Cot} B + \cot C)\frac{\delta a}{a} - (\cot C)\frac{\delta b}{b} - (\cot B)\frac{\delta c}{c}$$

$$\delta A = \left(\frac{\delta a}{a} - \frac{\delta c}{c}\right)\cot B + \left(\frac{\delta a}{a} - \frac{\delta b}{b}\right)\cot C$$

But by (1), $\frac{\delta a}{a} = \frac{P_a}{E}$, $\frac{\delta b}{b} = \frac{P_b}{E}$, $\frac{\delta c}{c} = \frac{P_c}{E}$,

$$\left.\begin{aligned} \text{Therefore,}\quad E\,\delta A &= (P_a - P_c)\cot B + (P_a - P_b)\cot C \\ \text{Similarly,}\quad E\,\delta B &= (P_b - P_a)\cot C + (P_b - P_c)\cot A \\ E\,\delta C &= (P_c - P_b)\cot A + (P_c - P_a)\cot B \end{aligned}\right\} \quad (2)$$

The above method is due to Manderla. Other authorities like Muller Breslau, Land, Winkler, *and* Ritter have also derived the same formulas by either analytic or graphic methods.

Three features should be noted in the above Eq. (2): (1) The change in angles are linear functions of the primary stresses. (2) They are linear functions of the angles included between the sides of the triangle, not the actual lengths of the sides. (3) The sum of the changes in angles of all the angles of any triangle is equal to zero.

CHAPTER IV. THE TWO FUNDAMENTAL EQUATIONS

Article 1. The Deflection Equation.

Consider a beam 1-2, Fig.2, subjected to the moments M_1 and M_2 and shears V_1 and V_2, but sustaining no intermediate loads. Assume a counter clockwise moment to be positive.

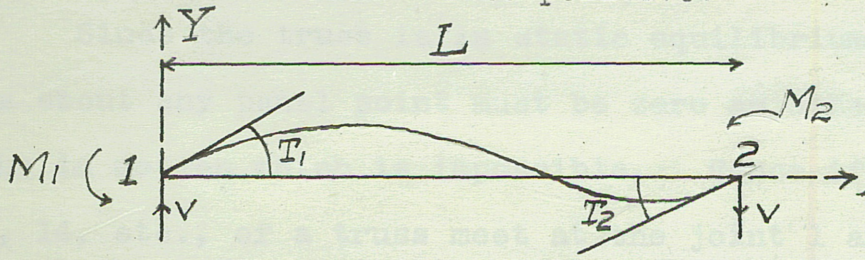


Fig. 2.

From equilibrium of the member as a whole,

$$V_1 = V_2 = V, \qquad M_1 + M_2 - V L = 0, \qquad V = (M_1 + M_2)/L$$

Moment at any section distant x from point 1 =

$$M_x = M_1 - V x = M_1 - (M_1 + M_2)\frac{x}{L},$$

Now the equation of the elastic curve referred to X and Y as axes is

$$E I \frac{d^2 y}{d x^2} = - M_x,$$

$$= - M_1 + (M_1 + M_2)\frac{x}{L},$$

$$E I \frac{d y}{d x} = - M_1 x + (M_1 + M_2)\frac{x^2}{2L} + C_1,$$

$$E I\, y = - M_1 \frac{x^2}{2} + (M_1 + M_2)\, x^3 / 6L + C_1 x + C_2,$$

Since $y = 0$ for $x = 0$, $\quad C_2 = 0,$

$y = 0$ for $x = L$, $\quad C_1 = \frac{2M_1 - M_2}{6} L,$

Therefore, $$E I \frac{d y}{d x} = -M_1 x + (M_1 + M_2)\frac{x^2}{2L} + \frac{2M_1 - M_2}{6} L,$$

Let T_1 be the value of $\frac{d y}{d x}$ at 1 = the deflection angle of the end tangent at 1 and T_2 be $\frac{d y}{d x}$ at 2, then T_1 is obtained from the above equation by making $x = 0$, and T_2 by making $x = L$. Hence,

$$E I T_1 = \frac{2 M_1 - M_2}{6} L, \qquad E I T_2 = \frac{2 M_2 - M_1}{6} L,$$

Or, $$M_1 = \frac{2 E I}{L}(2 T_1 + T_2); \qquad M_2 = \frac{2 E I}{L}(2 T_2 + T_1)$$

... ... (3)

This is the fundamental equation expressing the relations between the deflection angles and moments at the two ends of the beam and is applicable to any member of the truss.

Article 2« The Moment Equation*

Since the truss is in static equilibrium the resulting moments about any panel point must be zero as otherwise the truss would be in motion which is impossible. Hence if the members 12, 13, 14, etc., of a truss meet at the joint 1 and M_{12}, M_{13}, M_{14}, etc. are the moments of the respective members about joint 1, then,

$$M_{12} + M_{13} + M_{14} + M_{15} \cdots = 0$$

Or, in general,

$$\Sigma M \text{ around any joint} = 0 \quad \cdots \quad (4)$$

Since M is a function of T by Eq. (5) this equation would give the relations of T which must be satisfied by all the members meeting at the joint.

In case the axes of members do not meet at the same point there is introduced an eccentric moment M_e which may be taken care of in the above equation by expressing M_e under the summation sign, or,

$$\Sigma M + M_e \text{ around any joint} = 0 \quad \ldots\ldots \quad \ldots \quad (4a)$$

It is interesting to note here that the stresses of eccentricity due to primary and secondary stresses are usually opposite in sign and counter balance each other. Tnis fact was first noted by Hitter*

CHAPTER V. DEFLECTION ANGLES AND THE RIGIDITY OF JOINTS.

If two members meeting at a joint that is rigid the deflection angles of the two members at the joint will not be independent of each other but will be connected by an equation which ~~is formed with~~ includes expressions depending on the primary stresses and the properties of the truss. To put this into algebraic form let T_{1n} and T_{1p} be the deflection angles of two members 1n and 1p meeting at joint 1, then,

$$T_{1n} = F (T_{1p}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where F is a known function depending on the property of the truss and the conditions of the loading. This equation is very important in that it forms the basis for all the methods used for analyzing secondary stresses.

CHAPTER VI. SOLUTION OF THE PROBLEM.

To analyze secondary stresses in a structure amounts to nothing more than the solution of moments which the members of the structure must be subjected to as a result of the rigidity of the joints. In Eq. (3) it has been found that the moments at the ends of any member are dependent on the deflection angles at the same ends. If these deflection angles or their relations are known for every member of the truss the moments can then be derived by a simple procedure. In this connection use must be made of Eqs. (4) and (5) as they express the relations which the deflection angles of different members must hold. It will be found later that Eq. (4) and (5) are not only necessary conditions for the solution but they are also sufficient. To utilize Eq. (4) and (5) three methods have been in use at the present time: (1) The method of joints; (2) The method of triangles; and (3) The method of members. To the first me&hdd belong the jnethod of Manderla and the method of Mixbr; to the second, the *aethod of Muller isreslau; and to the last, the Sferthod of Ritter. These will be considerediy separately in Part II. In Part III will be given three new methods proposed by the writer in which a new ffi©fctee&~aR& principle is used differing slightly from that given before.

PART II. EXISTING METHODS FOR COMPUTING SECONDARY STRESSES DUE TO RIGIDITY OF JOINTS.

CHAPTER I. THE METHOD OF JOINTS.

By applying Eq. (5) of Part I successively to the members meeting at a joint it will be found that all the deflection angles at the joint are deducible one from the other and may be expressed as a function in terms of a quantity that is a constant for the joint. If this constant be C, then for any member m at the joint the deflection angle

$$T_m = F'_m(C) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where F'_m is a known function of the property of the truss and the condition of the loading. Apply Eq. (1) to every joint of the truss it is seen that the total number of unknowns is simply equal to the total number of joints, as each joint has only one unknown C. Now for every joint of the truss there is a moment equation (4) of Part I connecting the quantities T of the members meeting at the joint and consequently connecting the unknowns C. There are therefore as many moment equations for C as the number of C and the problem is always solvable. These equations, it should be remembered, are simultaneous.

The quantity C forms the basis of the two methods into which the method of joints is divided. In one method C is the deflection angle of one of the members meeting at the joint and in the other it is the rotation of the joint which is a constant for every member meeting at the joint on account of the joints being rigidly connected. This first method due to Manderla and improved analytically by Winkler, is known as ~~the~~ Manderla's Method while the second is known as ~~the~~ Mohr's Method. As the quantity C used in Mohr's method is a linear function of that used in Manderla's these two methods are essentially equivalent although they differ widely in procedure. For a combination of these two methods see Waddell's Bridge Engineering, page 181, and Thayer's Structural Design, Vol. 2, page 230.

CHAPTER II. THE METHOD OF MANDERLA.

Article 1. The Reference Deflection Angle

Consider any joint n of any structure and let the straight lines n-1, n-2, etc., represent the lines joining the several joints after distortion. Fig. 3. The heavy lines show the bent forms of the several members. The angles T_{n1}, T_{n2}, T_{n3}, and T_{n4} represent the the deflection angles of the several members at joint n. Let A_1 A_2, A_3 be the original angles between the members 1n2, 2n3, and 3n4. After distortion the angles between the straight lines joining the

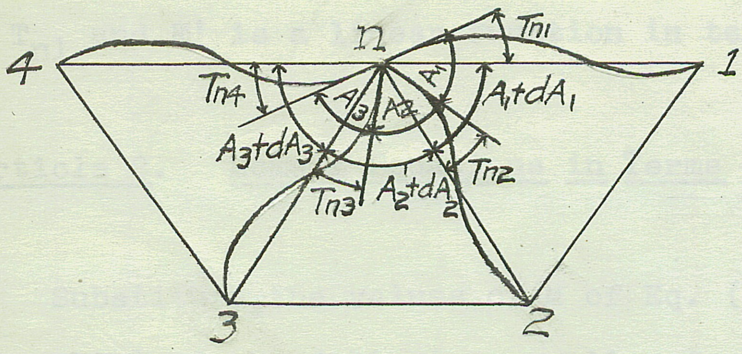


Fig. 3.

apexes will be respectively Aj -+ 4 $A_{1?}$ Ag -f- SAg, and A3 -f SAg as shown. Then if one of the deflection angles, T_n^ , be selected as a "Reference Angle" the other values of T at the joint n may be expressed in terms of the change of angles &A, as follows:

T_{n2} - T_{nl} -+ *ik*$_{l9}$

T_{n3} " T_{nl} + ^^{A}l +3.A_2,

Tn4 = T_nl + &Ai + *$A*$_2$ + 5A3,

and so on for any number of members. Or, in general, for any joint n,

nm nl ^; ' * ••• ^^

where T_{nm} represents any value of f, T_{nl} is the reference angle selected and ^T *&&* is tiie sum of a^1 angular changes tip to the member nm in consideration.

The reference angle may be selected at random b$tt for *the purpose of arrangement* systematic purpooc it is convenient to select it as th© deflection angle of the first member encountered in passing around a joint in a counter clockwise direction, beginning on the outside of the truss. This deflection angle being the base for all the deflection angles at the joint, will be denoted by a subscript *which is the* same as that for the joint, i.e., T_1 will be the reference angle for joint 1, etc.

Comparing Eq. (2) with Eq. (1) e£ Papk-£ it will be seen that C = T_n^ an< 3- F* is a linear function in terms of the angular changes.

Article 2, Moment Equations in Terms of tiie Reference Angles.

Substituting the values of M of Eq. (3) in Eq. (4) of Part I, there is obtained the following equation for the figure in the previous article:

$$\frac{2EI_{n1}}{L_{n1}}(2\,T_{n1} + T_{1n}) + \frac{2EI_{n2}}{L_{n2}}(2\,T_{n2} + T_{2n}) + \frac{2EI_{n3}}{L_{n3}}(2\,T_{n3} + T_{3n}) + \frac{2EI_{n4}}{L_{n4}}(2\,T_{n4} + T_{4n}) = 0$$

Placing I/L = D and dividing by 2E,

$$D_{n1}(2T_{n1} + T_{1n}) + D_{n2}(2T_{n2} + T_{2n}) + D_{n3}(2T_{n3} + T_{3n}) + D_{n4}(2T_{n4} + T_{4n}) = 0.$$

Substituting T in terms of the reference angles,

$$\begin{aligned} &2\,T_n\,(D_{n1} + D_{n2} + D_{n3} + D_{n4}) + \\ &2\left[D_{n2}\delta A_1 + D_{n3}(\delta A_1 + \delta A_2) + D_{n4}(\delta A_1 + \delta A_2 + \delta A_3)\right] + \\ &(D_{1n}T_1 + D_{1n}\textstyle\sum_{T_1}^{1n}\delta A) + (D_{2n}T_2 + D_{2n}\sum_{T_2}^{2n}\delta A) + \\ &(D_{3n}T_3 + D_{3n}\textstyle\sum_{T_3}^{3n}\delta A) + (D_{4n}T_4 + D_{4n}\sum_{T_4}^{4n}\delta A) = 0 \end{aligned}$$

in which $\sum_{T_2}^{2n} dA$ represents the sum of all angular changes from member 2n up to T_2, the reference member. Expressed in a general form the above may be written as

$$\begin{aligned} &2\left[(\Sigma D)\,T_n + \Sigma(D\,\Sigma\,\delta A)\right] + \\ &\left[(D_{mn}\,T_m + D_{mn}\textstyle\sum_{T_m}^{mn}\delta A) + (\text{similar terms for other members})\right] = 0 \end{aligned} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

The above equation may be written out for each of the joints of the structure and there are therefore as many equations as there are joints. Since the total number of unknowns, the reference angle, is also equal to the number of joints the number of the equations is just sufficient to solve all the unknowns. These equations will, of course, be simultaneous but the number of unknowns in each equation is far less than the total number of unknowns. A consideration of Eq. (3) will show that the number of terms in each equation is only one more than the number of members entering into the joint for which the equation is formed.

After the reference angles are obtained by solving the simultaneous equations the deflection angles for all the members may be obtained from Eq. (2) and the moments in the members from Eg. (3) of Part I. The fibre stresses are then computed from the flexure formula f = M y / I. Or mere directly from the formula

$$f_{mn} = \frac{2y}{L}(\wedge - \wedge_{mn} + \wedge_{mn}) \wedge \quad \dots \quad \dots \quad \wedge$$

Article 3. Details of Procedure.

By this method there are the following steps required in the complete solution of the secondary stresses: (1) Calculation of the changes of angles dA, from Eq, (2) of Part I, the primary stressss being assumed to have been known, (2) tabulation of the values of 2?&A for expressing T's in term of the reference angle, (SReformulation of equations, one for each joint, (4) solution of the equations, and finally (5) the calculation of the several individual values of T and of the secondarytstresses.

In a procedure such as here considered it is highly important to reduce the work to a mechanical basis and arrange the computations in a most systemmatic way. This has been largely accomplished by Turneaure in his book of "Modern Framed Structures", Fart II, and also in an article in Engineering News, Vol. 68, p. 438. As it is not the intent of this paper to advocate this method the details of the numerical computations will not be given here. Attention, however, should be directed to the example worked out by this method in the article in Engineering Hews just referred to as the same truss used there will be analysed by the new methods in Part III. For working details of Manderla's Method see Johnson, Bryan, and Turneaure's Modern Framed structures, Part II, p. 440, and also the Proceedings of the American Railway Engineering Association, Vol. 15.

CHAPTER III. THE METHOD OF MOHR.

Article 1. The Rotation of Joints and the Slope Deflection.

In Mohr's method the quantity C in Eq. (1) is made the angle through which the joint as a whole rotates as a result of the truss deformation. Let line 12, Fig. 4, be the original position of member 12 and 1' and 2' the displaced positions of the panel points 1 and 2, after the truss is under load. The bent form of the truss is shown by the curved line connecting 1' and 2'. Draw lines 1'2" and 2'1" parallel to the original positions of 12. From the figure,

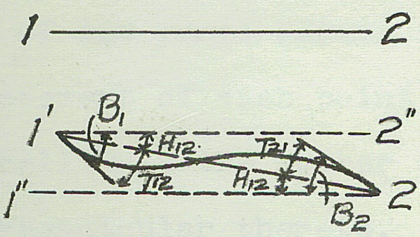


Fig. 4.

B_1, B_2 = the angles of rotation of the end tangents of the elastic lines from the original positions 1-2,

T_{12}, T_{21} = the deflection angles from the elastic lines from axis 1'2',

H_{12}, = the slope deflection of the axis 1'2' from the original position 12.

Also, $T_{12} = B_1 - H_{12}$, $T_{21} = B_2 - H_{12}$,

Since all the members at joint 1 are rigidly connected together, the end tangents of the elastic lines of these members are fixed at constant angles apart and the rotation of one tangent must bring about the same rotation of all the others. Hence B_1 for member 1-2 is the same for any member 1n entering into the same joint, and

$$T_{1n} = B_1 - H_{1n} \quad \dots \quad \dots \quad \dots \quad (5)$$

where Hi|_n is the slope deflection In from its original position. Comparing Eqs. (1) and (5) it will be seen that C = B and F^f is a linear function of H, the slope deflection.

The slope deflections H are geometrical functions of the axial deformations of the truss and can be found graphically as follows: From the changes in length computed by Eq. (l) of Part I a Williot or displacement diagram can be drawn from which the displacements of the panel points are obtained. It will be found that the displacement of each point is brought about in two motions, one is parallel to the member of which the point is one end and the other is perpendicular thereto. The last named is therefore approximately the arc described by the member during the distortsion and is a- measure of the rotation of the member. This quantity divided by the length of the member gives the slope deflection required- The Mohr movement of the Williot diagram is not necessary here as it is only the relative motion of the straight axis of the member that is required and not the absolute value. For each aaaiamed condition of fixity of the truss there corresponds a different set of B's in Eq. (5) but the value of T is not influenced by the asaumption as the variation in B is taken care of by the corresponding variation in H. In case the values of B are to be tested by experiment, however, the Mohr movemenet is necessary and the conditions of the fixity of the truss should be determined beforehand,

Article 2. Moment Equations in terms of the Hofeation of Joinfrs.

Subsitituti25 the values of T in (5) in Eq. (3) of Fart I there is obtained

$$M_{12} = \frac{2EI_{12}}{L_{12}} (2 B_1 + B_2 - 3 H_{12})$$

$$M_{21} = \frac{2EI_{12}}{L_{12}} (2 B_2 + B_1 - 3 H_{12})$$

Let the above equation be formed for all the members meeting at any joint 1, then by Eq. (4) of Part I,

$$\Sigma M_{1n} = 2 E (2 B_1 \Sigma D_{1n} + \Sigma D_{1n}B_n - 3 \Sigma D_{1n}H_{1n}) = 0$$

where D as before = I/L. Hence,

$$2 B_1 \Sigma D_{1n} + \Sigma D_{1n}B_n - 3 \Sigma D_{1n}H_{1n} = 0 \quad \ldots \quad (6)$$

where Σ includes all the members at the joint.

Since this equation could be formed for every joint of the truss and for each joint there is only one unknown B, the solution of the problem can always be effected by solving the simultaneous equations. After the values of B are obtained the secondary stress is then found from the following equations

$$f_{nm} = \frac{2y}{L} (2 B_n + B_m - 3 H_{nm}) E \quad \ldots \quad (7)$$

Article 5. Details of Procedure.

The necessary steps required in the method of Mohr are as follows: (1) Computation of the changes of length by Eq. (1) of Part I, (2) construction of the displacement diagram and the calculation of the slope deflection, (3) formulation of the equations, (4) solution of the equations and (5) the calculation of B and the secondary stresses.

Like the method of Manderla the systemmatic arrangement of computations is essential in this method. This is largely accomplished by Kunz as illustrated in an article in Engineering News, Vol. 66, p. 397. The example used in this article was taken later by Turneaure in illustrating the method of Manderla and also taken by the writer in illustrating the three new methods in Part III. A comparision of the results obtained by the various methods is not only instructive but also determines the relative merits of the different methods. It is hoped, therefore, that this article in Engineering News be familiar with the reader of this paper.

CHAPTER IV. THE SOLUTION OF SIMULTANEOUS EQUATIONS

It has been shown in the previous chapters that the method of joints requires the solution of a set of simultaneous equations involving as many unknowns as there are joints in the structure. This part of the work has long been considered the most laborious in the solution of secondary stresses and is the one that taxes ~~most of~~ the patience of the computer to the utmost. At the present there are no less than four methods which have been used for the solution: (1). The method of Gauss[1], proposed by Paez, formerly of Cornell University, (2) the method of elimination[2] proposed by Turneaure, (3) the method of approximation[3], proposed by Mohr and (4) the method of trial[4] proposed by Waddell. Among the four methods the first seems to be the most expedient and practical. The writer has used this method in the solution of 10 sets of 34 simultaneous equations each in connection with the secondary stresses in a 2 hinged

*** *** *** *** *** ***

1. Thesis No.75"of Cornell University by J. Paez.
2. Modern Framed Structures, Part II, p. 448.
3. Engineering News, Vol. 66, p. 379.
4. Bridge Engineering, By J. A. L. Waddell, p. 182.

arch and have found it entirely satisfactory. Two more improvements have been made by the writer[1] and they deserve special attention here: (1) As is evident from Eqs. (3) and (6) the coefficients of the unknowns T and B are functions of the truss dimensions and are therefore constant. As the truss dimensions are usually symmetrical about the centre line the unknowns for symmetrical joints will likewise have the same coefficients. This means that the reduction of the equations from one end of the truss toward the centre is the same as that from the other end of the truss toward the centre and the number of simultaneous equations could then be reduced to one half with one more set of constant terms. This saves much time for a truss having a great number of panels. (2) The solution of the equations is accomplished by means of a specially constructed table in which every operation is reduced to a mechanical basis and no memory work is required. To illustrate this method there are reproduced the tables which the writer used in computing the secondary stresses of the two hinged arch referred to above. In Plate I are shown the coefficients of the 34 unknowns which are characteristic of the arch. It will be seen that there are two symmetrical arrangements of these coefficients: (a) about the diagonal line AB, which fact, first noted by Paez, renders it possible to use Gauss' method for the solution of "Normal Equations", (b) about the two horizontal rows marked by two crossing dotted lines. This fact, noted by the writer, renders it possible to reduce the coefficients of only 18 equations instead of 34 . It should be remarked here that this result is obtained only by a special convention of notations, i.e., symmetrical joints should be denoted by symmetrical figures or the sum of the numerals denoting the symmetrical joints

1. Transactions of Am. So. of Civil Eng. Vol. 82, p. 1102.

should be one more than the total number of joints. By this arrangement one figure in the table serves to denote the coefficients of the two symmetfical joints referred to two "Remark " columns^ one at the top and the other at the bottom of the table. The constant or absolute terms of the equations of which there are ten sets are not shown in Plate I but are found in the next drav/ing, Plate:;II.

The solution of the 10 sets of equations is accomplished *6n* only one sheet of drawing, Plate II. In this drawing there is constructed a table which has as many vertical and horizontal columns as may be needed in the solution. These are bounded by heavy black lines. All the horizontal columns are next divided into longitudinal rectangles, the number of which in each column is to be determined as follows: For equations (1) and (34) one line; (2) and (33) two lines; (3) and (32) and all the remaining equations four lines except in those equations where the unknowns are not for uninterrupted successive joints as equation (8), in which case five lines should be allowed. The coefficients of unknowns, absolute terms and • check terms, marked by numerals without primes, are next entered into those rectangles which are next to the bottom rectangles in each of the horizontal columns. The check term is found by taking the sum of the coeffidients of the unkncnws and the absolute term and is to be treated just as an absolute term in the solution. This eheck term:: should be tested at every step of the solution by taking the same sum after some arithmetic operations havaing been perfommed.

A step *line,* shown by double black lines, is next constructed in the table beginning with the first horizontal column at the extreme left as shown. The figures in the first column to the right of the step lines will be hereafter known as the first column figure; those in the second column to the right, the second column figure, etc.

To begin with the solution of the equations multiply (1) by the ratio of $\frac{\text{2nd}}{\text{1st}}$ columns of (1) with reversed signs, and put ~~it~~ the product in the first line of the horizontal column below, in the same vertical columns. Mark this line with the numeral (1"). Multiply (1) by $\frac{\text{3rd}}{\text{1st}}$ columns of (1), calling it (1"'), and put it in the first line of the third horizontal column below, and also in the same vertical columns. The sign of (1") is always the opposite of (1) but that of (1"') is not yet determined. In the second horizontal column there will be only one line left at the bottom which is to be filled by the sum of (2) and (1"). This line will be designated by the letter (II). It is to be noticed that the first unknown of (2) is now eliminated in (II),

Multiply (II) by the ratio of $\frac{\text{2nd}}{\text{1st}}$ columns of (II) with signs reversed, and put ~~it~~ the product in the second line of the third horizontal column, in which the first line has already been occupied by (1"'). These figures, in the same vertical columns as (II), are to be denoted by the letter (II"). Multiply (II) by $\frac{\text{3rd}}{\text{1st}}$ columns of (II), calling it (II"'), and put it in the first line of the 4th horizontal column in the same vertical columns. Similarly, if there is a 4th column in (II), multiply it by $\frac{\text{4th}}{\text{1st}}$ columns of (II), calling it (II^{iv}), and put it in the first line of the 5th horizontal column, and also in the same vertical columns as (II).

Now in the third horizontal columns all the lines are filled up except the last one which is to be occupied by the sum of (r^{ff})> (II") and (3). The signs for (l^{tft}) should be so fixed that its first term would cancel out that of (3). The last line thus obtained will not contain the first two unknowns of (3), anctiiwill be designated by the letter (III). Equation (III) is to be treated in the same manner as (II) as described above.

It should be noted that the above statements apply equally well to equations (34), (XXXIII), etc.

The check term should be satisfied at every step of the process *$,o* as to ensure correctness at every stage of the solution.

Repeating the same process as described above the bottom line is eventually *rgocckod* when the 34 equtaions will be reduced to six equations involving six unknowns; i.e., Eqs. (XV), (XVI), (XVII), (XVIII), (XIX) and (XX) involving <f>*1$,* *<fil6>* *017** *<PlQ>* *<fi*19- and *<p20** t^^ie solution of these 6 equations, simultaneously, gives the values of the six unknowns *A* which when substituted successively in the equations marked by Roman capitals will give all the unknowns required. These six equations should also be treated systemmatically in arranging them in tabular form and reduci^the coefficients in some such manner as described before.

The above method of arranging the coefficients in symmetrical positions is very useful in structures having a large number of joints as Me one illustrated above. For ordinary structures having 10 or 12 joints its use *It* not advisable.

CHAPTER V, THE METHOD OF TRIANGLE.

This method is due to Muller Breslau. Before taking up the principle of this method a few words of explanation are necessary in regard to the signs. To secure uniformity in the treatment an assumption is made that all the members bend with a single curvature and that all the members composing a triangle have only one kind of curvature, either inward or outward, as shown in Fig. 5 below. This assumption, it should be noted, does not influence the final results of the solution as a reversed curvature could always be taken care of by a negative sign of the moment. As a matter of fact the bending of truss members is in most cases opposite to that assumed, being generally a double curvature in form.

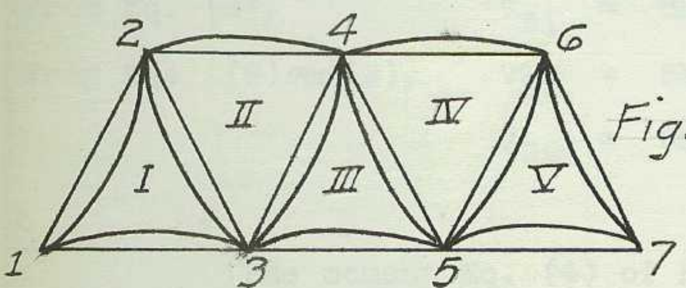


Fig.5.

Fig. 6.

Take a triangle as shown. 1, 2, and 3 are the displaced positions of the triangle 123 after loading. The original angles between the members are A_1, A_2, and A_3 which are preserved by the rigidity of joints. From the figure,

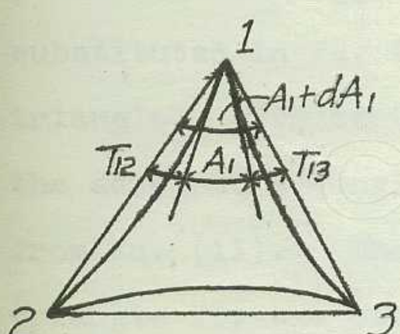


Fig. 6.

$$T_{12} + T_{13} = \delta A_1$$

$$T_{21} + T_{23} = \delta A_2$$

$$T_{32} + T_{31} = \delta A_3$$

Substitute the values of T from Eq. (3) Part I, noting that the moments at the two ends of the member are opposite in sign,

$$(2M_{12} + M_{21})\frac{L_{12}}{6EI_{12}} + (2M_{13} + M_{31})\frac{L_{13}}{6EI_{13}} = \delta A_1$$

Put [illegible] M L/I = V, there are then obtained for the triangle 123,

$$\left.\begin{aligned} V_{21} + 2\,(V_{12} + V_{13}) + V_{31} &= 6E\,\delta A_1 \\ V_{12} + 2\,(V_{21} + V_{23}) + V_{32} &= 6E\,\delta A_2 \\ V_{23} + 2\,(V_{32} + V_{31}) + V_{13} &= 6E\,\delta A_3 \end{aligned}\right\} \ldots \quad \ldots \quad (8)$$

Since the sum of dA's in a triangle must be zero ,

$$V_{12} + V_{21} + V_{23} + V_{32} + V_{31} + V_{13} = 0, \ldots \quad \ldots \quad (9)$$

If three of the V's are known, say V_{12}, V_{21}, and V_{13}, the remaining three may be found from Eqs. (8) and (9) as follows:

$$\left.\begin{aligned} &\text{From Eq. (8),} & V_{31} &= 6E\,dA_1 - 2\,(V_{13} + V_{12}) - V_{21} \\ &\text{From Eqs. (8) and (9),} & V_{23} &= 6E\,dA_2 + V_{13} - V_{21} + V_{31} \\ & & V_{32} &= 6E\,dA_3 + V_{12} + V_{21} - V_{31} \end{aligned}\right\} \ldots \quad \ldots \quad (10)$$

The moment Eq. (4) of Part I is here expressed in terms of V as follows:

$$M = \sum V \frac{I}{L} \text{ around any joint } = 0 \ldots \quad \ldots \quad (11)$$

The solution of secondary stresses by the above method applied to a truss as shown in the previous page is essentially as follows: In the first triangle I assume V_{12} and V_{21} to have been known and find V_{13} from V_{12} by Eq. (11). These three V's when substituted in Eq. (10) will give the three remaing V's in the triangle. Two of these V's so found, V_{23} and V_{32}, also belong to the adjacent triangle II and a third value of V, V_{35}, may be found from Eq. (11). There are therefore three values of V known in triangle II, the other V's may then be found from Eq. (10). This process is continued throughout the series of triangles until the last one is reached where two extra moment equations become available to determine the two assumed values V_{12} and V_{21}. After V_{12} and V_{21} are known all the other V's are known by substitution and the secondary stresses are found by the equation

$$f = V \frac{y}{L} \ldots \quad \ldots \quad \ldots \quad (12)$$

The solution of the whole problem is possible because for any framed structure composed of triangular elements if there are m members there will be $\frac{1}{2}$ (m - 1) triangles and $\frac{1}{2}$ (m + 3) joints. For each triangle there are three equations like Eq. (10) and for each joint there is one like Eq. (11) so that the total number of equations availkable is $\frac{3}{2}$ (m - 1) + $\frac{1}{2}$ (m + 3) = 2m which is just the number <Sf unknowns required.

In applying the above method to the solution of secondary stresses the following points should be noted:

(1). The directions ®f bending of members are assumed and should sbe corrected by the computed results which give both the magnitude and the sign.

(2). In computing the change of angles by Eq. (2), Part I, the signs of the <£A!s thus obtained should be reversed for those triangles whose members *are* assumed to have bent outward like triangles II and IV of the previous figure.

(3). The sign of the secondary stress and the fibre to which the stress belongs should be determined from the form of the bending.

(4). The eccentric connections of the joints should be taken care of in Eq. (11).

For details of procedure of the method of Muller Breslau see Molitor's Kinetic Theory of Engineering Structures, p. 235.

CHAPTER VI. THE METHOD OF MEMBERS.

This method is due to Ritter. Here a new system of notations is essential for further considerations. Consider any joint 5 of a truss as shown in Fig.7, where four members intersect forming three angles. Each of the angles included between two adjacent straight axes of the members will be denoted by the numerals indicating the opposite member in the triangle. Thus the angle 354 will be denoted by ~~by~~ A_{34}, angle 456 by A_{46}, etc. The two moments of any

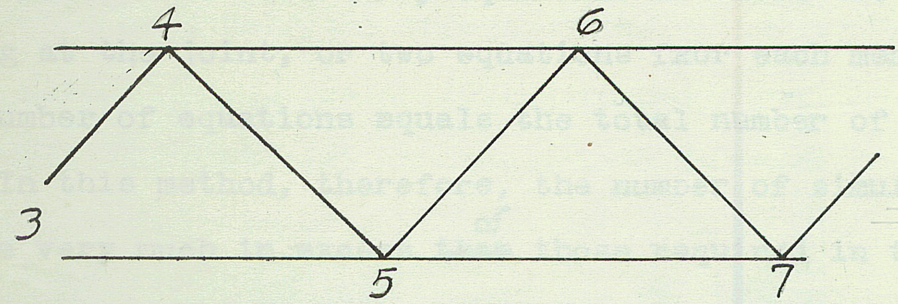


Fig. 7.

member like 35 will be designated by M_3 at joint 5 and $M_{3'}$ at joint 3. For member 45, the moments shall be M_4 at joint 5 and $M_{4'}$ at joint 4. But in case joint 4 is considered instead of 5 the same moments of the member 45 will be designated by M_5 at joint 4 and $M_{5'}$ at joint 5. That is,

$$M_4 \text{ for Joint } 5 = M_{5'} \text{ for Joint } 4$$

$$M_{4'} \text{ for Joint } 5 = M_5 \text{ for Joint } 4$$

The sign of the moments is made positive if the moment is counter clockwise.

Applying Eq. (2) in Manderla's method to members 53 and 54,

$$T_{53} - T_{54} = dA_{34},$$

Substituting the values of T from Eq. (3) of Part I,

$$(2M_3 - M_{3'})\frac{L_3}{I_3} - (2M_4 - M_{4'})\frac{L_4}{I_4} = 6E\,\delta A_{34}$$

Similarly for dA_{46} and dA_{67}.

Putting $M L/I = V$, there is obtained for joint 5, including the moment equation (11),

$$6E\,\delta A_{34} = (2V_3 - V_3') - (2V_4 - V_4')$$

$$6E\,\delta A_{46} = (2V_4 - V_4') - (2V_6 - V_6')$$

$$6E\,\delta A_{67} = (2V_6 - V_6') - (2V_7 - V_7')$$

$$V_3\,(I_3/L_3) + V_4\,(I_4/L_4) + V_6\,(I_6/L_6) + V_7\,(I_7/L_7) = 0 \qquad \ldots \quad \ldots \quad \ldots \quad (13)$$

For every joint there are as many equations as there are members intersecting at the joint, or two equations for each member, so that the total number of equations equals the total number of unknowns moments. In this method, therefore, the number of simultaneous equations is very much in excess ~~than~~ of those required in the method of joints and an expedient method for the solution is absolutely essential. The following graphic method has been used by Ritter.

Lay off 4 vertical lines spaced at $6E\,\delta A$ apart as shown, Fig. 8. This applies to the same joint as the above equation (13). Assuming

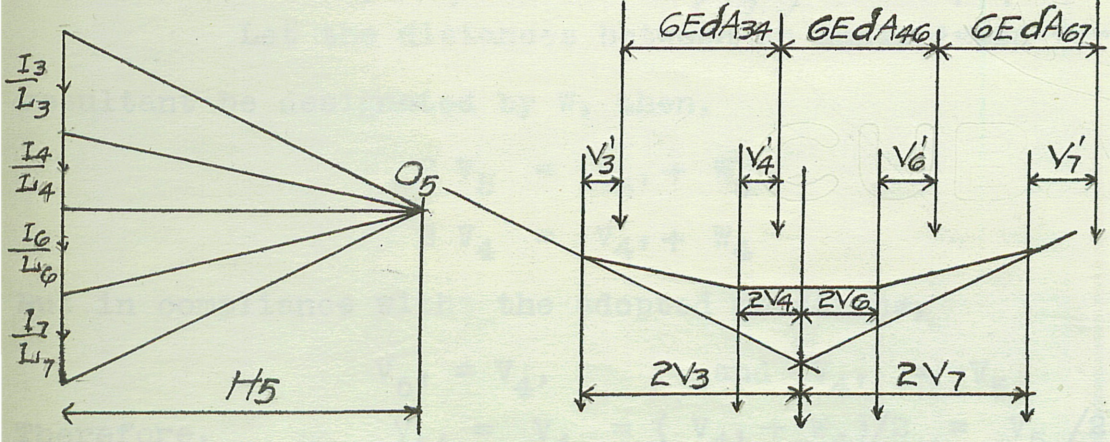


Fig. 8.

the values of V' as having been known draw the vertical lines spaced at V' apart from the lines already drawn. These lines will be ~~considered~~ assumed as the lines of action of the I/L's considered as forces. Lay off the values of I/L on a vertical load line and, with pole O_5, distance H_5, draw the rays as shown. Parallel to these rays construct an equilibrium polygon for the forces i/L. Find the the position of the resultant of these vertical forces and measure the distances of the forces I/L spaced therefrom. These will then give the corresponding values of 2V. The proof of this fact comes directly from Eq. (13).

Now in actual solutions V' is as unknown as V itself and must be found in some other way. For practical purposes it is obtained by trials. Considering the fact that any change in the values of V' has only half the effect on the values of V the first trial could make $V' = 0$. The following figure shows the positions of the undisplaced forces i/L for the joints (4) and (5), the equilibrium polygons being omitted.

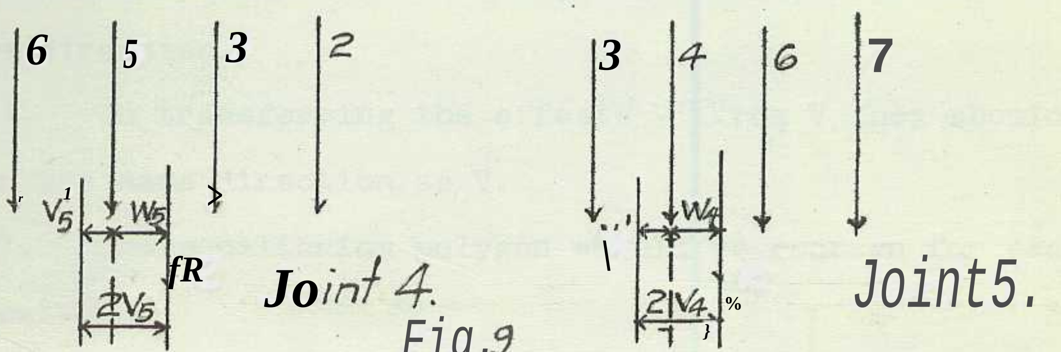


Fig.9

Let the distances between an undisplaced force and the resultant be designated by W, then,

$$2V_5 = V_{5'} + W_5$$

$$2V_4 = V_{4'} + W_4$$

But in compliance with the adopted notations,

$$V_{5'} = V_4, \quad \text{and} \quad V_{4'} = V_5$$

Therefore, $V_{5'} = V_4 = (V_{4'} + W_4)/2 = V_5/2 + W_4/2$

$$= (V_{5'} + W_5)/4 + W_4/2,$$

Or, $V_{5'} = (W_5 + 2W_4)/3$

Similarly, $V_{4'} = (W_4 + 2W_5)/3 = V_5$ ... ... (14)

The above equations enable the offset V_t fo be known for joint 5 if the value© Vg is known ie known for joint 4.

To analyse a truss by this method, first assume V^f for all the joints ap-e zero and wiiih this understanding construct the force and equilibrium^ polygons of l/L for every panel point of the truss. Next give a displacement to the forces l/L until the above Eqs. (14) are satisfied for every joint of the truss. After the forces are definitely located the values of V are measured off and the• secondary stresses calculated from Eq.. (12) •

The following.points should be noted in the above constructions;:

(1). Negative values of <§A are to be laid off on the left side of the undisplaced forces when going over the joints in a clockwise direction.

(2). In transferring the offsets V^f from V they should be laid off in the same direction as V.

(3). The equilibrium polygon -should be redrawn for each change of offsets..

CHAPTER VII* COMPARISON OF THE DIFFERENT- METHODS.

A comparison of the different methods described above could not be made unless they are actually tested by examples. It will be found that Hitter's method is the least practical of all as any method based on trials is always inferior to those which have definite means of prodedure. The method of muller Breslau is superior in that it does not involve a large set of simultaneous equations. On the other hand it requires a greater amount of work in the computations, as V being a large multiple of f, the secondary stress, it is a large number and great accuracy must be exercised in the computing. In the example mentioned above, in Molitor's book, the computations had to be made correct to seven places which is oat to b& considered in practise. Therefore the only method that is suitable for practical engineers is the method of joints, in spite of the presence of the simultaneous equations. The merits of the methods of Manderla and Mohr are practically equal and each has its disadvantages. They will be dealt with in detail in the following articles.

<u>Article 1. Features of the Method of Manderla,</u>

(1). It is entirely analytical and gives more accurate results,

(2). No graphical constructions required with the result that there is no inteference in work.

(3). The values of M are found by taking the algebraic sum of only two terms containing T, which is more convenient than fce handling three terms as required in Mohr's method.

(4). It takes much less time to find the change of angles than to construct the displacement diagrams for the slope deflections in Mohr's method.

<u>Article 3» Features of the Method of Mohr.</u>

(1). The simultaneous equations are more easily formed.

(2). The conception of the rotation of joints is as far more superior than to that of the reference angle as the values of B etitti.d be actually measured.

(3). The values of B being nearly equal for all the joints, an approximate solution of the simultaneous equations is possible. (See Chapter II of Part IV). This feature is very important and it alone may offset all the other disadvantages of the method.

(4). ~~There are~~ Less operations are required to find f after the equations are solved, thus greatly reducing the chances of making mistakes, as the solution of the equations can be checked.

(5). The absolute terms in the equations can be found more directly than those in Manderla's method.

(6). The values of B's have the same signs at each side of a certain point which affords a good check.

PART III. NEW METHODS FOR COMPUTING SECONDARY STRESSES DUE TO RIGIDITY OF JOINTS.

CHAPTER I. FUNDAMENTAL PRINCIPLES.

Article 1. The Fundamental Conception.

Secondary stresses are fibre stresses produced in the members due to bending moments developed around the joints. There are many sources from which the moments are derived but the principal one is the rigidity of connections. If M = the bending moment, I = the moment of inertia of the member, y = the distance from neutral axis to the fibre whose stress is required and f = the secondary stresses; then

$$f = M\,y/I = M/S, \quad \text{where } S = \text{Section Modulus} = I/y.$$

Therefore, $M = S \cdot f$ ... ... (1)

In other words, "If S be assumed as a force and M be the moment it produces about a point, then the offset of the force from the point gives the value of the secondary stress."

Let AB be a member connecting the joints A and B, and S its section modulus, Fig. 10. If S be assumed as a force acting in this member, like the primary stress, it will produce moments at A and B if its line of action is displaced from the axis of AB, and the offsets f_a and f_b will then be the stresses produced at A and B, by (1). Particularly, if the moments about A and B are those due to rigidity of joints, the offsets f_a and f_b will be the secondary stresses.

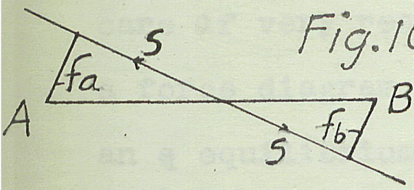


Fig. 10.

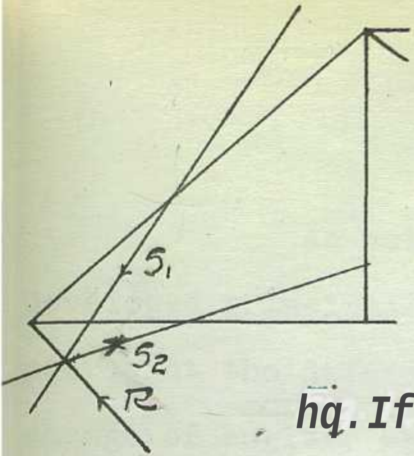
The line of action of S will be called hereafter the "Secondary Stress Line" or briefly "Stress Line", no confusion being

~~entertained~~ caused thereby as the primary stress line is not shown or understood to be the axis of the member.

It is now evident that the values ~~bf~~ of f depend only upon the location of the stress line S while the moment M will also depend on its magnitude. As the magnitude of S is constant, being the section modulus, the position and direction of S and consequently, f, will depend solely upon M.

Let the above conception be extended to every member of the truss. There will then be as many stress lines as there are members. Each stress line will be displaced to an extent ~~measurable with~~ proportionate to the bending moments produced. To utilize this fact by reversing the process, it is at once evident that if the stress lines are so located as to satisfy various imposed conditions the offsets of the stress lines will give directly the secondary stresses. This conception is fundamental.

The first condition ~~that~~ to which the various stress lines are subjected to ~~t~~is that the total resulting moments around every joint must be zero. (Eccentric moments are excluded here, but could be taken care of very readily, see p.59). Graphically this means that if a force diagram be drawn of all ~~h~~the stress lines in the truss and an ~~e~~ equilibrium polygon be likewise constructed on the truss diagram these two polygons must respectively close. This, however, is impossible for the stress lines adopted above because the values of S being constant they could ~~be~~ not be made to balance each other. Thus, in the figure, the stress lines S_1 and S_2 could not balance each other unless they are on the same straight line which is impossible. To overcome this difficulty an~~d~~ external ideal force will be



hq.If,

applied to the joint, with such magnitude add direction that the § equilibrium is maintained around the joint. Thus in tfee figaa*© a force R may be introduced which will balance the forces S-^ and Sg. In general, the magnitude value and direction of R is equal to the resultant of the stress lines acting on the joint, but with opposite sense..in dirootion.

It is important to notice that the position of R must be such that its line of action passesthrough the joint in considera- tion, so that there will be no external moment. In case of eccen- tric connections the position of R may be so adjusted that its value and the offset from the joint will give the eccentric moment.

As a convenience in terminology the term stress line will be understood to be the internal"secondary stress line S while the external force will be understood as the balancing force R.

With the conceptions of S and R thus established it is now possible to draw a force diagram around every joint £ a*wL (from \> which fo construct an*? equilibrium-polygon. If these two polygons are made to close && any given case matones the first condition that the sum of internal moments must be zero around any joint is every- where satisfied. Further, from the condition that the total ex- ternal moment is zero % for all the joints of the truss it may be inferred that both the force and equilibrium polygons of the exter- nal ideal forces R must respectively close.

Article 2, The Fundamental Equation*

As noted before the secondary stresses are due to moments developed principally by the rigidity of joints. In a frictionless pin joint the deformations of the members can be taken care of by the change.) of angles, which are not possible in a rigid joint, where the deformations must be provided for in some other way. As the members are elastic the simplest and easiest way would be to bend and twist the [illegible] that the joints may be so displaced that the deformations along the axes of the members may be just as well provided for as if the joint were frictionless pins* But the members cannot bend or twist without being subject to some outside influence, this outside ~~influence is~~ producing the internal stress. In other words some work must be performed ON the member to produce this bending and twisting and as a consequence thereof some internal work must be set up to respond. Take, for illustration, (Fig. 2), the beam 12 subject to the influence of moment M which increases from 0 at 1 to M_2 at 2* (This is equivalent to a force $P = M_2 / L$ placed at the free end 1 when the end 2 is fixed.) The effect of M_2 on the beam is to rotate the end 2 through an angle T_{21} and the end 1, an angle T_{12}. These rotations are brought about at the expense of an external work in amount equal to $1/2\, M_2\, T_{21}$. This induces an internal work in the beam equal to

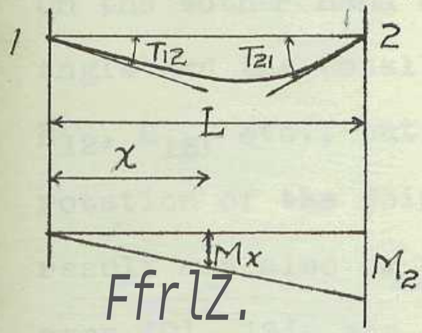


Fig. 2.

$$\frac{1}{2}\int_0 \frac{M_x^2\,dx}{EI} = \frac{1}{2EI}\int_0 (M_2 \frac{x}{L})^2\,dx = \frac{1}{2}\,\frac{M_2^2 L}{3\,EI} = \frac{1}{2} M_2 T_{21}$$

$$\therefore \quad T_{21} = \frac{M_2 L}{5\,EI} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \qquad (2)$$

That is, the end 2 of beam 12 rotates through an angle T_{21} at the expense of an external moment M_2.

Similarly, the bending and twisting of truss members due to rigidity of joints are also brought about at the expense of external moments which become internal moments when the joint is considered as a whole. The8® bending and twisting are the results of two different rotations* the first is due to the deformation of truss members and the second Is due to the rigidity of joints. Let 12, 13, 14 and lj^b© four members meeting at joint 1, which is rigid. Fig.13,

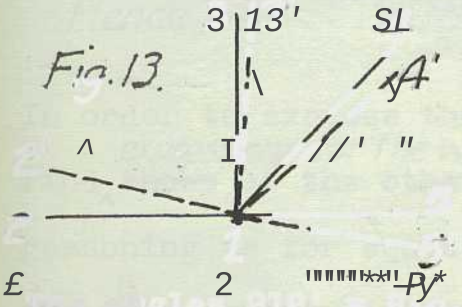


Fig.13.

After loading, on account of deformations of members the axis lines 12, 13, etc., would have been displaced eto 12*, 13*, etc. t&A ^ne joints been frintionless pins. The angles 212* = H;j2> 313* = H^g/ etc., are functions of deformations of the members,and are different for& different members. Therefore, they could not be actually realized in rigid joints where a member is prevented from rotating relativelyto the others. On the Mother hand all the members must rotate through the same angle and the total ratations of the members 12, 13, etc., are not &12> Hi^> etc., but H-jp-f- Bl, H-j ^ -f B-j, etc., for all members. where B^ is tne rotation of the joint 1, all tho mombors the same^ This s-stear result may also bfc obtained by considering, first, the sesfeep& axes 12f, 13f, etc., brought back to their respective original positions 12, 13, etc,, and then, simultaneously, all the axes turnafthDOUgh the common angle B-^.

These rotations as mentioned before, must be produced at the expense of moments,aaa& the works thus performed ON the members 12, 13, etc., would be

$$W_{12} = \frac{1}{2} M_{12} (H_{12} + B_1)$$

$$W_{13} = \frac{1}{2} M_{13} (H_{13} + B_1)$$

--- --- ---

Since θ_1 is the same for all the members at joint 1,

$$\frac{W_{12}}{M_{12}} - \frac{W_{13}}{M_{13}} = \frac{1}{2}(H_{12}-H_{13})$$

$$\frac{W_{13}}{M_{13}} - \frac{W_{14}}{M_{14}} = \frac{1}{2}(H_{13}-H_{14})$$

$$\cdots \qquad \cdots \qquad \cdots$$

Now $H_{12} - H_{13}$ is equal to the change of angle between the members 12 and 13, $= \Delta\alpha_{213}$, $H_{13}-H_{14} = \Delta\alpha_{314}$, etc.,

Hence,

$$\left.\begin{aligned} \frac{W_{12}}{M_{12}} - \frac{W_{13}}{M_{13}} &= \frac{1}{2}\Delta\alpha_{213} \\ \frac{W_{13}}{M_{13}} - \frac{W_{14}}{M_{14}} &= \frac{1}{2}\Delta\alpha_{314} \\ \cdots \quad \cdots \quad &\cdots \end{aligned}\right\} \cdots \quad (3)$$

In order to express the work W_{12}, W_{13}, etc., it is necessary to find expressions for the work for moments at the other end, i.e., W_{21}, W_{31}, etc. By similar reasoning as for equation (2), the total internal work in beam 12 due to moment M_{12} at end 1 and M_{21} at end 2 is

$$W = W_{12} + W_{21} = \frac{1}{2}\int_0^L \frac{M_x^2\,dx}{EI}.$$

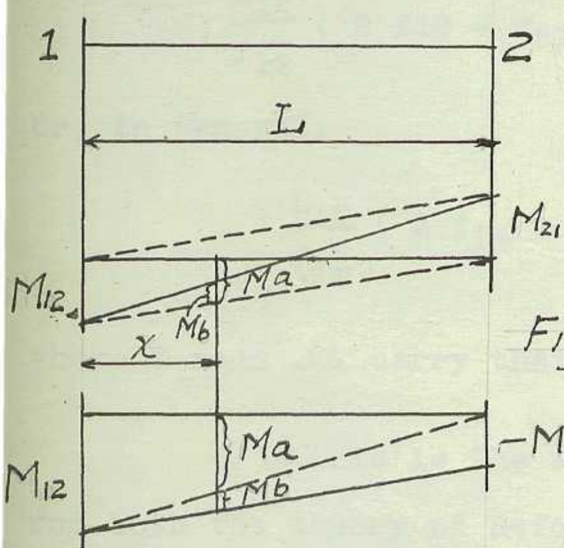


Fig. 14.

where $M_x = M_a - M_b = M_{12}\frac{L-x}{L} - M_{21}\frac{x}{L}$

$= M_{12} - \frac{x}{L}(M_{12}+M_{21})$. See Fig. 14.

$$M_x^2 = \left[M_{12} - \frac{x}{L}(M_{12}+M_{21})\right]^2$$

$$W = \frac{1}{2}\frac{1}{EI}\int_0^L\Big(M_{12}^2 - 2\frac{x}{L}M_{12}(M_{12}+M_{21}) + \frac{x^2}{L^2}(M_{12}+M_{21})^2\Big)\,dx$$

$$\therefore W_{12}+W_{21} = \frac{1}{2}\frac{1}{EI}\left[\frac{M_{12}^2L}{3} + \frac{M_{21}^2L}{3} - \frac{M_{12}M_{21}L}{3}\right]$$

It will be seen from this equation that the first two terms are the works done on ends 1 and 2 of beam 12 by moments M_{12} and M_{21} respectively, see Eq. (2), while the third term is the sum of the works done by M_{12} on end 2 and M_{21} on end 1. But by Maxwell's theorem the work done by M_{21} on end 1 and that by M_{12} on end 2 are equal and therefore the difference of W_{12} and W_{21} is simply the difference of the first two terms in the above equation.

Or, $$W_{12} - W_{21} = \frac{1}{2}\,\frac{1}{EI}\left(\frac{M_{12}^2 L}{3} - \frac{M_{21}^2 L}{3} \right)$$

Solving simultaneously,

$$W_{12} = \frac{1}{2}\,\frac{1}{2}\,\frac{1}{E\,I}\left(\frac{2\,M_{12}^2 L}{3} - \frac{M_{12}\,M_{21}\,L}{3} \right)$$

$$= \frac{1}{2}\,\frac{M_{12}\,L}{6\,E\,I}\left(2\,M_{12} - M_{21} \right),$$

Similarly,

$$W_{13} = \frac{1}{2}\,\frac{M_{13}\,L}{6\,E\,I}\left(2\,M_{13} - M_{31} \right)$$

Substituting these values in Eq. (3), there results

$$\frac{L_{12}}{6EI_{12}}\left(2\,M_{12} - M_{21} \right) - \frac{L_{13}}{6EI_{13}}\left(2\,M_{13} - M_{31} \right) = \Delta\alpha_{213}$$

Replacing M by f x S, where S = I/y,

$$\frac{L_{12}}{y_{12}}\left(2\,f_{12} - f_{21} \right) - \frac{L_{13}}{y_{13}}\left(2\,f_{13} - f_{31} \right) = 6E\,\Delta\alpha_{213}$$

Or, in general,

$$\frac{L_{1m}}{y_{1m}}\left(2\,f_{1m} - f_{m1} \right) - \frac{L_{1n}}{y_{1n}}\left(2\,f_{1n} - f_{n1} \right) = 6E\,\Delta\alpha_{n1m} \qquad \ldots\;\ldots \quad (4)$$

where f and $\Delta\alpha$ carry their own signs.

This is the Fundamental Equation which forms the basis for both the theory of Deformation Contour and the process of Successive Deduction.

CHAPTER II. ANALYTIC EQUATIONS.

Before proceeding further a few words ~~is~~ are necessary regarding the signs of the quantities in the fundamental equation:

$$\frac{I_{im}}{y_{im}}(2f_{im}-f_{mi}) - \frac{I_{in}}{y_{in}}(2f_{in}-f_{ni}) = 6E\alpha\Delta\alpha_{nim}$$

where f is the secondary stress and $\Delta\alpha$ is the change of angle. To secure a uniform system a positive f will be understood to be due to positive moment which is assumed to be counter clockwise. This would mean that the value of f, together with its sign, applies to that fibre of the member which is first met with in passing around the joint in a clockwaise direction. Thus in ~~the figure~~ Fig. 15, positive f reffers to the upper fibre, i, e., it is tension when the moment is positive.

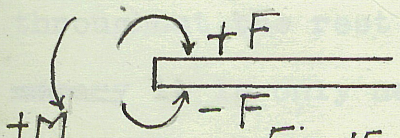


Fig. 15

Similarly, in ~~the following diagram~~ Fig. 16, if M_{12} is positive and M_{13} negative, the top fibre of 12 at 1 will be tension and that of 13 at 1 will be compression.

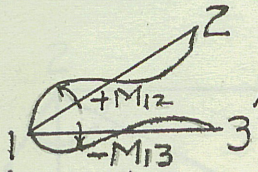


Fig. 16

Next, consider the sign of $\Delta\alpha$. As is customary it is positive if the angle α is changed to a larger angle, i.e., if the angle of rotation of 12 = H_2 is greater than that of 13 = H_3. Adopting the sign of the moment it may be inferred that if the dotted lines in ~~the figure~~ Fig. 17 are positions of the members after loading, both H_2 and H_3 are positive, being counter clockwise in direction. Therefore, if $\Delta\alpha$ carries its own sign, $H_3 - H_2 = \Delta\alpha$.

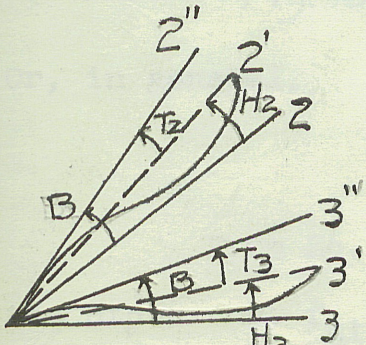


Fig. 17

Comparing this with the equation

$$\frac{W_{12}}{M_{12}} - \frac{W_{13}}{M_{13}} = \frac{1}{2}(H_2 - H_3)$$

from which the fundamental equation is derived, it is seen that the sign of $\Delta\alpha$ in the fundamental equation is negative for the present convention of signs. and that

$$\frac{I_{lm}}{y_{lm}}(2f_{lm}-f_{ml}) - \frac{I_{ln}}{y_{ln}}(2f_{ln}-f_{nl}) = -6E\Delta\alpha_{nlm}$$

if, the member lm is met with before the member ln when passing around the joint 1 in a clockwise direction.

Or,
$$\frac{I_{ln}}{y_{ln}}(2f_{ln}-f_{nl}) = \frac{I_{lm}}{y_{lm}}(2f_{lm}-f_{ml}) + 6E\Delta\alpha_{nlm}$$

Letting I/y = U and $6E\Delta\alpha_{nlm} = K_{nlm}$,

$$U_{ln}(2f_{ln}-f_{nl}) = U_{lm}(2f_{lm}-f_{ml}) + K_{nlm}$$

Or,
$$2f_{ln}-f_{nl} = \frac{U_{lm}}{U_{ln}}(2f_{lm}-f_{ml}) + \frac{K_{nlm}}{U_{ln}} \quad \cdots \quad (5)$$

This modified form of the fundamental equation will be used throughout the rest of the discussion. To ~~put~~ memorize this form ~~to memory~~ it is only necessary to remember that the quantity $(2f_{lm} - f_{ml})\,U_{lm}$ of a member lm when added to K_{nlm} in a clockwise direction ~~it~~ gives the value of $(2f_{ln} - f_{nl})\,U_{ln}$ of the next member met with in the same direction.

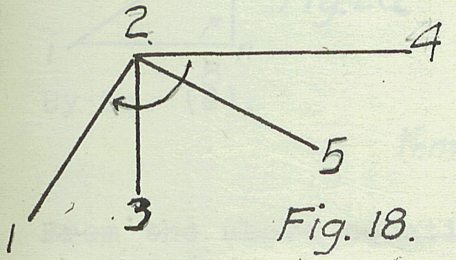


Fig. 18.

Calling $2f_{lm} - f_{ml} = r_{lm}$, and

$2f_{ml} - f_{lm} = r_{ml}$,

Then, in Fig. 18,

$$\frac{U_{24}}{U_{25}}r_{24} + \frac{K_{425}}{U_{25}} = r_{25},$$

Or, in general,

$$r_{ln} = \frac{U_{lm}}{U_{ln}}r_{lm} + \frac{K_{nlm}}{U_{ln}} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

From Eq. (5) when the stress f_{ln} is known the stress at the other end, f_{nl}, may be found if the stresses f_{lm} and f_{ml} are known. Thus, suppose the stresses in the first triangle 123 or triangle A, Fig. 19, are known. The stress f_{35} of member 35 in triangle B can be found from the equilibrium of joint 3 ~~from~~ by the "Moment Equation"

$$\sum f \cdot S = 0 \quad \ldots \quad \ldots \quad (7)$$

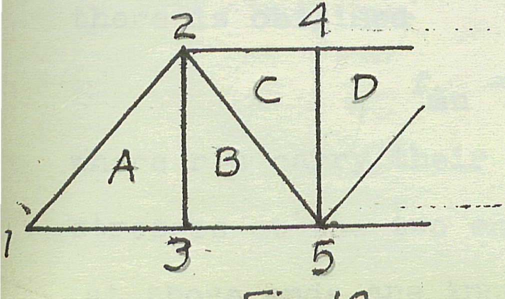


Fig. 19.

for all the members meeting at the same joint. In this particular case,

$$U_{31}f_{31} + U_{32}f_{32} + U_{35}f_{35} = 0$$

where f_{31} and f_{32} are known from the triangle A. Substituting the value of f_{35} obtained from this equation in Eq. (5),

$$f_{53} = 2f_{35} - \left[\frac{U_{23}}{U_{35}}(2f_{32}-f_{23}) + \frac{K_{B3}}{U_{35}}\right]$$

where $K_{B3} = K_{235} = 6E\Delta\alpha_{235}$

Or in general, from (5),

$$f_{ni} = 2f_{in} - \frac{U_{im}}{U_{in}}(2f_{im}-f_{mi}) - \frac{K_{nim}}{U_{in}} \quad \cdots \quad (8)$$

The stresses in member 25 are found from the following two simultaneous equations:

$$2f_{52}-f_{25} = \frac{U_{35}}{U_{25}}(2f_{53}-f_{35}) + \frac{K_{B5}}{U_{25}}$$

$$2f_{25}-f_{52} = \frac{U_{23}}{U_{25}}(2f_{23}-f_{32}) - \frac{K_{B2}}{U_{25}}$$

Solving, $f_{25} = \frac{1}{3}\left(\right.$ [illegible] $\left.\right)$

Or, in general, using the notation r, Fig. 10

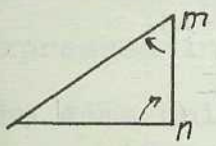


Fig. 10.

$$f_{nm} = \frac{1}{3} r_{mn} + \frac{1}{3}\left[\frac{U_{im}}{U_{nm}} r_{mi} - \frac{K_{imn}}{U_{nm}}\right] \quad \cdots \quad (9)$$

By Eq. (6),

$$r_{nm} = 2f_{nm} - f_{mn} = 3f_{mn} - 2\left[\frac{U_{im}}{U_{nm}}(r_{mi}) + \frac{K_{imn}}{U_{nm}}\right]$$

From the above equations,

$$r_{nm} = 3f_{mn} - 2r_{mn} \quad \cdots \quad (10)$$

This equation shows that the value of $r_{nm} = 2f_{nm} - f_{mn}$ may be obtained from the values of f and r at the other end of the member without knowing the value of f_{nm}.

Applying Eq. (10) to any member *mn*, chord or web members, there is obtained

$$f_{mn} = \frac{1}{3}(2r_{mn} + r_{nm}) \quad \cdots \quad (11)$$

where r's carry their own signs. This equation shows that the stresses at the two ends of a member are known if the values of r at those ends are known.

CHAPTER IIL., GRAPHICAL CONSTRUCTIONS.

This Chapter intends t-e describe some of tlB graphical constructions that are common to both of the methods of Deformation Contour and "fete* mothod of fiUaarcnotori!Lt±c Inoromonts> Successive Dec/ucJrbn* It is advisable in reading this chapter to <sonf«or refer to frequently the examples workdd out in Chapters V and VII.

Article 1. Preliminary Considerations.

(a). Scales.

For graphical constructions the scales? of f is evidently dependent on that of *EA0(* . From the fundamental equation (5) it is seen that the scale of f is *&£* the same dimensional degree as that of $\frac{K}{\text{nnim}} \atop U_1$ or of (6E^^|rJj / (L/y)$_{in}$. Since L is always expressed in Peet dlxi y^in inches^ L and y may be represented by ' the same unit if 6E«^W(/ is changed into l/2 *EA0(* • Hence, hereafter K is to be understood as *EAOO /2* with the understanding that y. is expressed in inches but represented as if it were in feet. As the values of *EAQh* are found from the primary stresses, by graphic or analytic methods, it is convenient to consider only one half of the primary stresses. In graphic methods the scale of primary stresses may be made only one half as large as that used for measuring the values of *EAoL* . This will give the value of K directly.

The scale of S, the section mudulus, used, in constructing the force diagrams may be anytting independent of f as it is only the direction that is ^required.

The scale used for r must be the same as for K or f.

(b). Signs.

In the present paper there are two instances where a conception of signs is necessary. One is rotational and the other is linear. The first applies to moments, angles, circular sections and directions of contours while the second applies to the stress lines, their offsets, representations of f and r, and the co-ordinates of variables. The rotational direction will always be considered positive if it is COUNTER CLOCKWISE. For linear directions there are many conventions, but the principle is, with the exception of the stress lines, that if ab is the base of a straight line through a and b with a as origin, the distance from a to x is positive if x is laid off from a toward b and negative if away from b. That is, it is positive if it is between a and b and negative if it is on the opposite side of a from b.

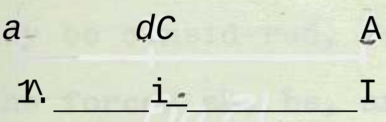


Fig. 27.

For stress lines the following convention is necessary. It has been shown that a positive moment is that which will rotate a member in a counter clockwise direction. As the moment is represented by the product of f and S the direction of is should be such that a counter clockwise moment would give a positive stress f.

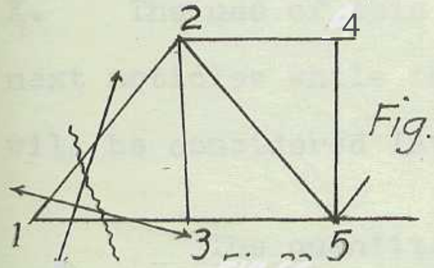


Fig.22

For the sake of uniformity all the stress lines S will be considered as compressive stresses in the members. Under this convention, if the stress lines in 12 and 13 are as shown, f_{12} and f_{21} will be negative, (moment is clockwise about 1 and also about 2), and f_{13} and f_{31} will be positive (moments counter clockwise about 1 and 3.) Hence, when the position of the stress lines are known both the sign and the magnitude of f are at once obtained. Further, by this convention the force diagram

of stress lines S assumes a definite form which twnds to facilitate
Fi'g.£3.
its construction. Consider the simple truss as snown,^ The

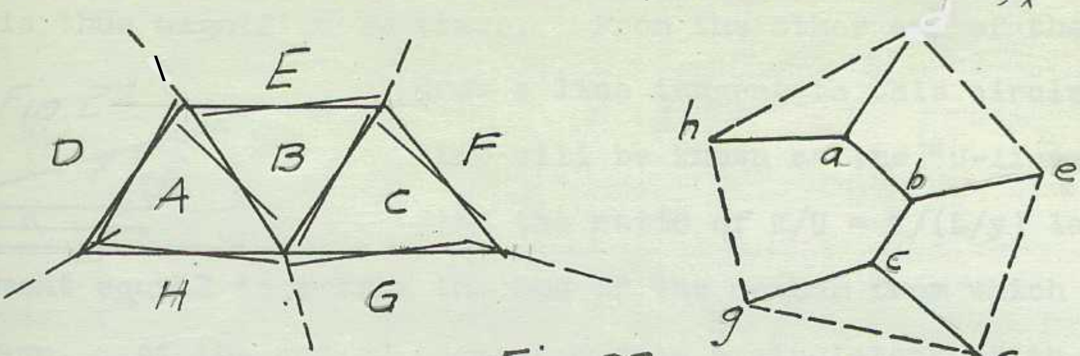


Fig. 23.

letters A, B, and C corresponds to a, b, and c of the force diagram. It will be seen that the forces da, ab, be, and cf may be considered, for illustration, as the trunk of a tree while the forces ah, be, and eg are to its branches. This makes it easy to remember that ail the stress lines in web members and end posts formthe trunk of a tree while those of the chord members form the spreading branches.

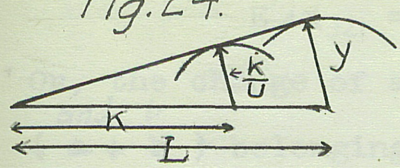
(c) Truss Diagrams.

Two or three truss diagrams are necessary # in the graphic methods presented in this paper. One of the diagrams will be used to record given information^ and to construct the values of K. The use of this diagram will be described here and also in the next articles ?/hile the construction© on the remaining diagrams will be considered later on.

The quantities S and primary stresses in lbs. per sq. in. are first marked on ail the members of the truss. There are then next constructed the "y-clrcles". These are the circles with radii egual to y of the member drawn at one of its ends. Let L be the l̫gth of the member in feet and y in inches. At one end e£

of the member draw a circle with radius equal to y, expressed in inches. The scale used is the same as that for L which is in feet, the value of y is thus magnified 12 times. From the other end of the member

Fig. 24.



draw a line tangent to this circle. This line will be known as the "U-line". To find the ratio of K/U = K/(L/y) lay off a segment equal to K from the end of the member from which the U line is drawn. At the end of segment K draw a circle tangent to the U line. The radius of this circle then gives the ~~th~~ ratio required.

Article 2. Change of Angles.

In Part II there are discussed the various methods used to find the change of angles in a triangle when the sides of the latter are deformed. ~~To adopt to~~ For the use of graphic methods the following construction is the most convenient. Except~~a~~-ing the systematic arrangement the principles of this method ~~is~~ are due to Ritter. It has been found that in a triangle 123,

$$E\Delta\alpha_{213} = (P_{23} - P_{13}) \text{ Cot } \underline{132} + (P_{23} - P_{12}) \text{ Cot } \underline{123}$$

$$E\Delta\alpha_{132} = (P_{12} - P_{23}) \text{ Cot } \underline{123} + (P_{12} - P_{13}) \text{ Cot } \underline{213}$$

$$E\Delta\alpha_{321} = (P_{13} - P_{12}) \text{ Cot } \underline{213} + (P_{13} - P_{23}) \text{ Cot } \underline{132}$$

Or,

$$E\Delta\alpha_{213} = P_{23} (\text{Cot } \underline{132} + \text{Cot } \underline{123}) - P_{13} \text{ Cot } \underline{132} - P_{12} \text{ Cot } \underline{123}$$

$$E\Delta\alpha_{132} = P_{12} (\text{Cot } \underline{123} + \text{Cot } \underline{213}) - P_{23} \text{ Cot } \underline{123} - P_{13} \text{ Cot } \underline{213}$$

$$E\Delta\alpha_{321} = P_{13} (\text{Cot } \underline{213} + \text{Cot } \underline{132}) - P_{12} \text{ Cot } \underline{213} - P_{23} \text{ Cot } \underline{132}$$

Suppose In triangle 123, Fig. 25, through 1 draw a line perpendicular to 23 with ~~an amount~~ of length equal to P_{23} and through its end draw a line parallel to 23, giving segments a and b. Similarly find the segments c, d, e, and f. Then,

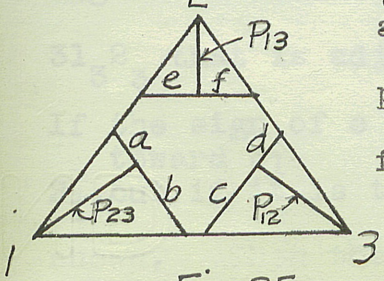
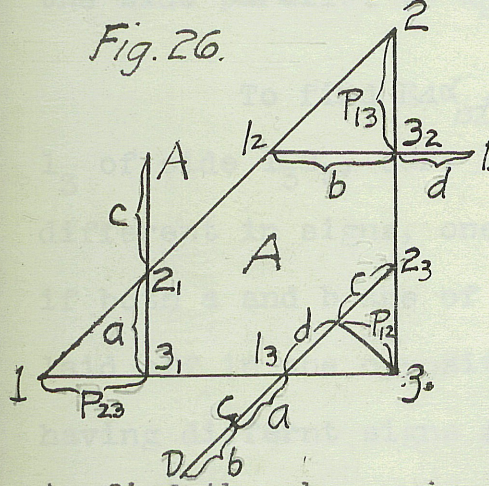


Fig. 25.

$$E\Delta\alpha_{213} = (a + b) - (f + d)$$
$$E\Delta\alpha_{132} = (c + d) - (a + e)$$
$$E\Delta\alpha_{321} = (e + f) - (c + b)$$

Or, the change of angle of any angle 213 is equal to the sum of segments a and b ~~(a + b)~~ belonging to this angle minus ~~substract~~ the sum of the segments of the other two angles f and d which are not adjacent to the segments of the angle under consideration, a and b.

To secure a systematic arrangement both in sign and magnitude the following scheme is advisable. The example is for a triangle with one right angle but the method could be easily extended to any kind of triangles. Fig.26. Lay off the primary stresses per sq. in. as shown. Draw the lines parallel to the sides and obtain the segments a, b, c and d. These segments will be of the same signas the corresponding stresses. Further, there is obtained the small triangles, similar to the large triangle 123, $1 2_1 3_1$, $2 1_2 3_2$, and $3 1_3 2_3$. Next adopt a sign of contour in following the consecutive sides of the different triangles. Let this be counter clockwise, to secure uniformity. Then



to find the change in angle 213, proceed as follows. At the end 2_1 of segment a, met with in a counter clockwise direction after 3_1 when following the contour $1 3_1 2_1$, measure off 2_1A ~~=~~ along $2_1 3_1$ extended equal to the segment c which is that segment of the side $1_3 2_3$ of the triangle $3 1_3 2_3$ that is adjacent to the side 23, ~~the one parallel to $2_1 3_1$~~. If the sign of c is the same as that of a it is laid off ~~away~~ from 2_1 toward 3_1 but if it is the negative of a it is laid off away from 3_1, as shown, Then the distance of 3_1A, shown. Then the distance ~~e~~ 3_1A, the segment from the initial point 3_1A to the end point A, gives the value of $E\Delta\alpha_{213}$. The sign of 3_1A is the same as that of a because A is reached from 3_1A in a counter clockwise direction. In case c and a ~~are~~ have ~~different~~ the same signs and ~~that c~~ c is greater thn a, then the point A is found as follows. Fig. 27. Lay off segment c from 2_1 toward 3_1, the point A will then be found below the point 3_1. The distance 3_1A, giving $E\Delta\alpha_{213}$, is now opposite in sign ~~from~~ to that of a because the point A is reached from 3_1 in a negative direction, clockwise, with reference to the contour 13_12_1. Similarly the change in angle 123 may be found ~~by~~ from the segments b and d, the latter being that part of ~~of~~ the side 1_32_3 of triangle 31_32_3 which is adjacent to the side parallel to 1_23_2.

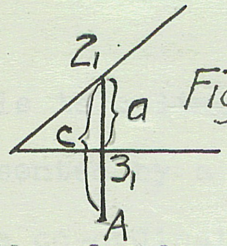


Fig.27.

To find $E\Delta\alpha_{132}$, lay off the segments a and b from the end 1_3 of side 1_32_3, care being taken of the signs. If a and b are different in sign, one should be laid off toward 2_3. Similarly if both a and b are of the same sign as c and d they should be laid off in the opposite direction as shown, which is for a and b having differnt signs from c and d. As a check the sum of the $E\Delta\alpha$'s should be zero.

It should be noted that since y is ~~taken~~ magnified 12 times, ~~as big~~ the primary stresses should be laid off with a scale one half as large as used in measuring the segments 3_1A, etc.

Article 3. Deformation and Property Lines.

Consider Eq. (6) which is the abbreviated form of the fundamental equation,

$$r_{ln} = \frac{U_{lm}}{U_{ln}} r_{lm} + \frac{K_{nlm}}{U_{ln}}$$

As this is a linear equation connecting r_{ln} and r_{lm} it may be represented by a straight line ab if the side lm and ln be considered as the co-ordinate axes of r_{lm} and r_{ln}. Positive r_{lm} will be laid off toward m from l while negative r_{lm} will be laid off away from m. Similarly for r_{ln}. To locate this straight line ab the simplest method would be to find the intercepts on the two axes.

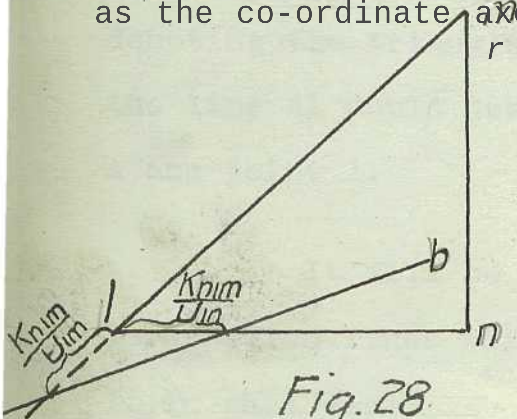


Fig. 28

If $r_{lm} = 0$, $r_{ln} = K_{nlm}/U_{ln}$, and

if $r_{ln} = 0$, $r_{lm} = -K_{nlm}/U_{lm}$,

that is, the intercepts on the two adjacent sides are respectively equal to the values of K_{nlm} divided by the values of U for the corresponding sides. The scale for K/U should be the same as that used for r's and f. Regarding the signs of the intercepts it will be seen that positive value of K/U belongs to the side which is first met with by passing round the joint l in a counter-clockwise direction (if K is positive), and the negative value of K/U belongs to the side next met with in the same direction. Accordingly, the value of K_{nlm}/U_{ln} is laid off toward n while that of K_{nlm}/U_{lm} is laid off away from m, as shown. This is for positive K, if it is negative the sign is reversed. Joining the end points of the intercepts the straight line ab is obtained.

For any value of r_{lm} to find the corresponding value of r_{ln} it is

only necessary to draw a line from r_{lm} parallel to the si_ne ln, the
n
legth of this line intercepted "between the side lm and ab will
give the value of r_{ln}.

In each triangle of the truss there are three of these lines ab and they completely express the fundamental equation (5). These lines will "be known as the "Deformation Lines" meaning that they are derived from the fundamental equation involving the deformation of the members. They will be designated by a letter denoting the triangle and a numeral denoting tne joint. Thus, . the line Al would designate thak line which & belongs to the triangle A and joint 1.

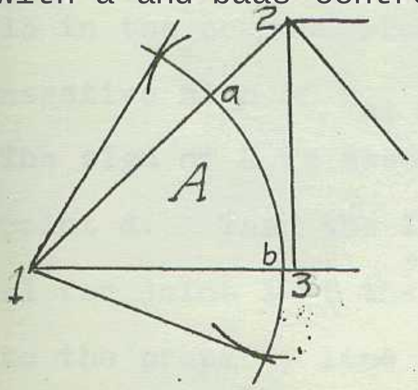
It will be seen from Eq. (6) that if K^m = 0, these deformation lines will pass through the origin, that is, the joint 1 in this case. These lines will then be known as the "Property Lines" of the truss, since they represent the ratios of U which is the ppcoperty of the truss. They are the same for any kind of loading.

There are two features of the property lines which de^serve notice. (1) In any triangle the three property lines must meet in one point. (2) The values of r may be laid off in any scale. (For deformation^ fck lines trie scale for r must be the same as for K/U). The first feature furnishes a good check of the deformation lines since they are parallel to the property lines.

For details of construction of the deformation and property lines the following methods are recommended. First let the property lines be constructed. Use will be made of the U-lines discussed in (c), Art. 1 of this chapter. To draw the

Fig. 28.

property line A1 in triangle A draw any arc with joint 1 as centre on the truss diagram used for constructing the values of K. It is preferable to make the radius as large as possible to secure accuracy. Let this arc cut the sides 12 and 13 at a and b. With a and b as centres draw the arcs tangent to the U-lines. Let



the radius having a as centre be called u_{12} and that having b as cent be called u_{13}. On the Second diagram, in triangle A, lay off the segmet u_{12} on the side 13, not side 12, letting the end point be c. With c as centre draw an arc with radius = u_{13}. Through c draw a line parallel to the side 13 cutting the arc at d. Join 1d. This is the property line A1.

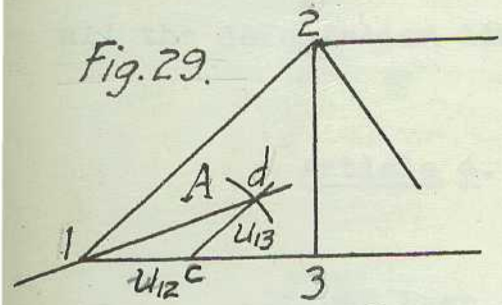


Fig. 29.

Similarly all the other property lines in triangle A may be obtained. As a check, the three property lines in the triangle should meet in one point. The same process may be extended to every triangle of the truss.

For deformation lines since the values of K have already been found the ratio of K/U should next be constructed. Take the

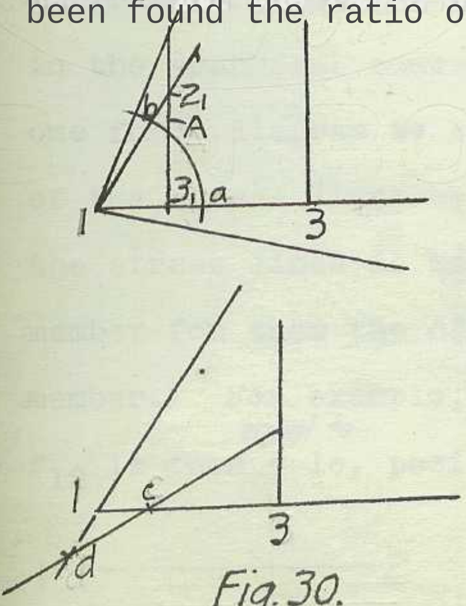


Fig. 30.

joint 1 for illustration. From 1 lay off 1b and 1a equal 3.A = 1/2 E40(2/3. With a and b as centres draw arcs tangent to the U lines, the radii of which will give the values of K_{A1} / U_{12} and K_{A1} / U_{13} respectively. On the truss diagram where the property lines are drawn lay off the values of K_{A1} / U_{13} on side 13. The sign of the segment is determined by passing around the joint 1 in a counter clockwise direction. Since 13 is the side first met, the segment on it has the same sign as that of K_{A1} / U_{13} itself. If it is positive it is laid off from 1 toward 3. This gives the point c. Since the side 12 is met after the side 13 in the counter clockwise direction, the segement on it has the negative Hign of K_{A1} / U_{lg} and is therefore laid £off away foom 2. The sign of K is assumed to be posititre here. This gives the point d. Then the line joining c and d gives the deformation line Al for joint 1 of the triangle A. This line should be parallel to the property line Al found in the previous paragraph. Similarly all the deformation lines of the truss could be constructed.

Article 4• The Equilibrium Contour.

It has bsen shown in Chap. I, Part III, that the stress lines S could be made to balance each other around any joint &H by the introduction tit an ideal external force R. In the process of construction, however, the conception of stress lines as there adopted is not convenient as it would involve a different force diagram for each change of the stress lines, which is unavoidable in the graphical construction*• A speedy method demands that one force diagram ~~to~~ serve the whole process; that is, the direction of the stress lines be kept constant. This fixed direction of the stress lines is best chosen perpendicular to the axis of the member for then the offset (= f) would be found along the axis of the member. For example, in fig-31 let S_{12} cut the member 12 at c. The stress f_{12} is then equal to lc, positive if c is between 1 and 2 and negative if on the opposite side of 2 from 1. For the end 2 the same force S_{12} should be shifted

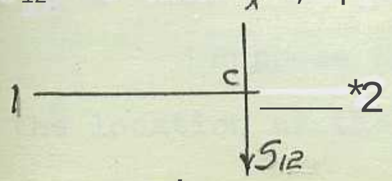
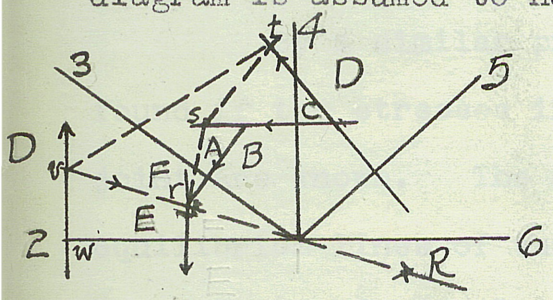


Fig-31.

to a point whose distance from the end 2 gives the stress f_{21}. By this modification the direction ~~and~~ of stress lines S ~~are~~ is made constant for the members and one force diagram is suffieient for different values of f, at the same end of the same member. Here, these S lines are no longer the "Stress Lines" in the correct use of the term, but they will be known as the "Equilibrium Lines"

The direction of the equilibrium lines should be such that they give a counter clockwise moment around the joint when the values of f are positive; i.e., when the lines cut the members somewhere between the joints. Thus, the direction of S_{12} considered above should be upward whether it is on the right or left hand side of joint 1. If on the left side of a the moment is clockwise and f is then negative.

Consider a joint of the truss as shown in Fig.32. The force diagram is assumed to have been constructed here. If the stresses in members 12, 13, 14, and 15 are known, the position of the equilibrium lines ~~are~~ is known. Then the moment of 12 equals the force af multiplied by the perpendicular distance f_{12}. If f_{12} is negative it should be laid off on the opposite side of 12, i.e., on side 16. The closing line fe furnishes the external force R at the joint. The direction of the force is constant and R may be drawn on the diagram as a part of the truss.



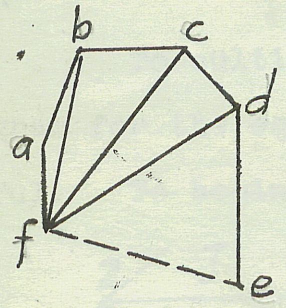


Fig.32.

Suppose that the stress in 16, or f_{16}, is required, i.e., the location of the force de whose direction is upward, found by passing around the joint in a counter clockwise direction. In the force diagram take f as the pole and draw the rqys fb, fc, and fd. At the intersection of r of the forces FA and AB, draw a line parallel to fb meeting the force BC at s. From s draw line parallel to fc meeting the force CD at t. From t draw a line parallel to fd meeting the force FE at v. Then the force DE must pass through v. Therefore drop a perpendicular from v on the side 12 and the distance lw would give the value of f_{16}. It is negative here, as it is on the opposite side of 6 from 1. Thus it will be seen that only three lines are necessary for a joint of 5 members in order to determine the stress in 16. For joints with a less number of members only one or two lines are necessary.

If there are eccentric connections at the joints the eccentric moment M can be taken care of by displacing the force R a distance = M/R.

By a similar process the stress in any member may be found if the stresses in all the other members meeting at the joint are known. The contour formed by the force R and the equilibrium lines of the different members will be known as the "Equilibrium Contour".

To cultivate speed and accuracy in drawing the force diagram for the equilibrium lines the following scheme will be found useful. ~~To begin with the diagram in~~ For a truss as shown, first

Fig. 33.

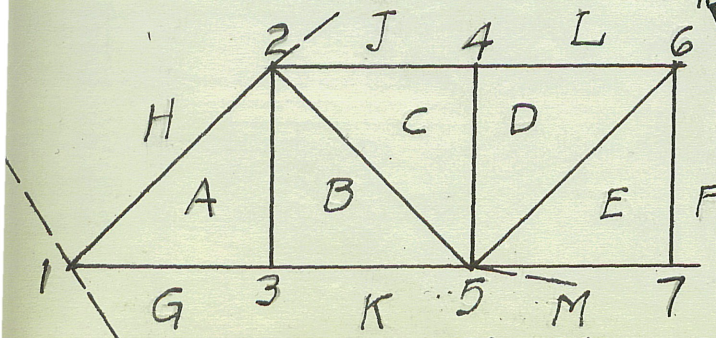
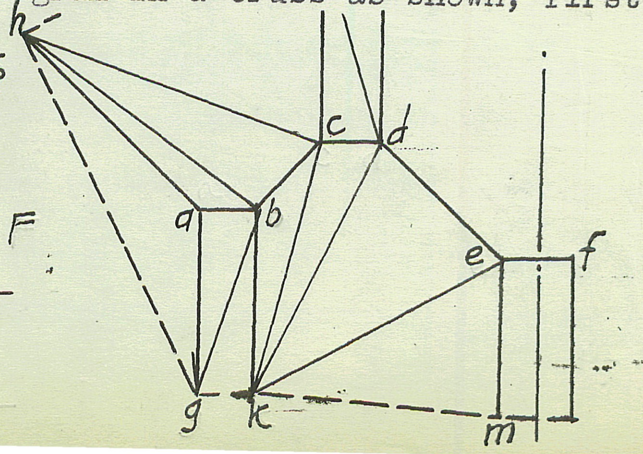


Fig.33.



draw a line perpendicular to 12 and measure ha = S_{12}. Then from a draw a line pendicular to 25 and measure off bc = S_{25}. From b draw a line perpendicular to 25 and measure off bc = S_{25}. ~~Keep on going~~ Continue for all the web members of the truss until the points e and f are obtained. Only one half of the truss will be considered as the other half will be symmetrical. From a draw a line perpendicular to 13 and measure off ag = S_{13}. The point g should be ~~on the under side of~~ below a so that the force ag will be downward. Similarly from all the points b, c, d, and e draw lines perpendicular to the respective chord members and measure off the segments bk, cj, etc., equal to S_{35}, S_{24}, etc. All the values of the equilibrium lines are then constructed. Next join the end points l, j, h, etc., with dotted lines as shown. These will give the directions of the external forces R, which are to be transferred to the truss diagram, as shown. Lastly the rays hb, hc, etc., are complete~~ly~~d. They are the lines which join the left end of the external force R to the inner points of the diagram.

CHAPTER IV. THE THEORY OF DEFORMATION CONTOUR.

Consider a joint of the truss as shown in Fig. 34, say 5, and apply the Eq. (6) successively to the triangles B, C, D and E.

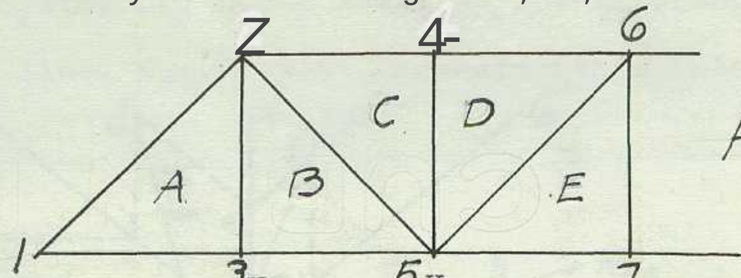


Fig. 34.

Then,

$$r_{52} = \frac{U_{35}}{U_{25}} r_{53} + \frac{K_{B5}}{U_{25}}$$

$$r_{54} = \frac{U_{25}}{U_{45}} r_{52} + \frac{K_{C5}}{U_{45}}$$

$$r_{56} = \frac{U_{45}}{U_{56}} r_{54} + \frac{K_{D5}}{U_{56}}$$

$$r_{57} = \frac{U_{56}}{U_{57}} r_{56} + \frac{K_{E5}}{U_{57}}$$

By successive substitution,

$$r_{57} = \frac{U_{35}}{U_{57}} r_{53} + \frac{K_{B5} + K_{C5} + K_{D5} + K_{E5}}{U_{57}}$$

Or,

$$r_{57} = a\, r_{53} + b. \qquad (12)$$

where a and b are constants. From this equation it is seen that when the value of r for one member is known, it is known for all the other members meeting at the same joint. This fact is analogous to the relation existing between the deflection angles of members around a joint, as found in the method of Manderla. As it stands, this equation does not offer much advantage in the discussion but when applied to the graphical construction of the deformation lines there is obtained a remarkably simple and useful figure which is known as the Deformation Contour.

By the method of Article 3 in previous chapter suppose the deformation lines are constructed for all the triangles of the truss, as shown in Fig. 35. For any joint 5, if the value of r_{54} is known that of r_{52} may be obtained by drawing a line parallel to side 52, being given by the intercept on the deformation line, K_{B5}. Next, transfer this intercept and obtaining

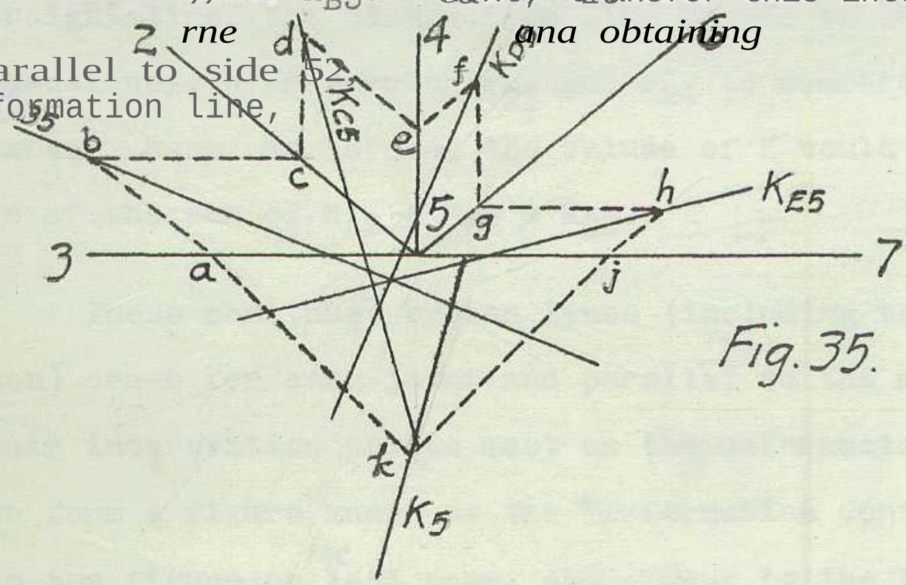


Fig. 35.

to member 25 by a line parallel to member 35. From r_{52} r_{54} may be similarly obtained by parallel lines. Repeating the process the value of r_{57} is finally obtained by successive parallelograms. In order to check the results and also to close the polygon a deformation line K_5 is drawn for joint 5 which gives the relation between r_{53} and r_{57} considering the change in angle that is outside of the truss, or the negative of the total changes in angles 352, 254, 456, and 657. Since both r_{53} and r_{57} are on the same straight line this deformation line K_5 could not be obtained by the usual Cartesian co-ordinates but must be found as follows, (See Article 1, Chapter V). Find a pair of corresponding values of r_{53} and r^r? and at the end of each segment draw a 45° line directed each toward the other. These two lines will determine a point. Similarly^ find another point;,these two points will then give the deformation $ine,designated as K_5, for joint 5. For a joint like 2, where the outside members do not lie on the same straight line, the deformation line Kg may be obtained in the usual way, by referring r_{21} and r_{g4} to members 21 and 24 as axes. Here, as before, the valuefee of K would be the negative of the sum of K_Ag •+• K_{BS} + ^{K}c2*

ou

These continues broken lines (including the two 45° lines) drawn for each joint and parallel to the members, with their intersention points meet on the deformations lines, together form a figure known as the "Deformation Contour". Thus, in the figure on^last page, abedefghjk is the Deformation Contour for joint 5. There is evidently one deformation contour for each value of r_5^, but they are all parallel to s each other. The Deformation Contour that is to give the actual secondary stresses is known as the "Correct Deformation Contour", its location being dependent on the relations existing between the different deformation contours for the different joints. Obviously these relations are derived from the. con-

fhe

siderations of equilibrium of^joints. As there are as many contours as there are joints the probelem is always solvable-

To fix the idea of the deformation contour it is necessary to give it a value designated by the value of r of one of the members meeting at the joint. For reasons stated in Afct. 2

the

in next chapter this member will be chosen to be the chord member which is at the left of the joint in consideration. These members will be known as "Reference Members for the Deformation Contour".[1] Thus the value of contour fpor joint 5 just considered will be designated by r_{53}, and that of joint 2 by r_{21}. The members 53 and 21 are reference members for the joints 5 and 2.

Suppose the correct deformation contours for the joints 1 and 2 are known. The contour for joint 3 could then be found by the relation between r_{13} and r_{31} from Eq. (10); that is,

where r_{13} is known from the given contour. Since $f_{13} = -S_{13}$ ...

$$f_{13} = - \frac{S_{13}}{S_{13}} \frac{r_{12} - r_{21}}{3}$$ by Eq. (11);

and $r_{12} = a\, r_{13} + b$ where a and b are constant.

Therefore, $$r_{31} = - \frac{S_{13}}{S_{13}} (2\, a\, r_{13} + 2b + r_{21}) - 2\, r_{15} \cdot$$

that is, the value of r_{31} may be found from r_{13} and r_{21} by a linear equation, or

$$r_{31} = m\, r_{13} + n\, r_{21} + p,$$

where m, n, and p are constants. Since r_{13} and r_{21} are known from the given deformation contours around the joints 1 and 2 r_{31} may be found by a very simple process.

Next, consider the relation between r_{53} and r_{35}. By Eq. (10), $$r_{53} = 3\, f_{35} - 2\, r_{35}$$

Since $$f_{35} = - \left(\frac{S_{23}}{S_{35}} f_{32} + \frac{S_{13}}{S_{35}} f_{31} \right)$$

$$= - \left(\frac{S_{23}}{S_{35}} \frac{2\, r_{32} + r_{23}}{3} + \frac{S_{13}}{S_{35}} \frac{2\, r_{31} + r_{13}}{3} \right)$$

it is seen that r_{53} is a linear function of r_{31} and r_{35} because the relations between r_{31} and r_{13}, r_{32} and r_{31}, r_{23} and r_{21} are

r_{31} and r_{35},

all linear. Therefore,

$$r_{53} = m' r_{35} + n' r_{21} + p'$$

Since r_{35} can be found from r_{31} and r_{31} is known, r_{53} may be computed. Similarly consider the relation between r_{42} and r_{24}.

$$r_{42} = 3 f_{24} - 2 r_{24}$$

$$f_{24} = -\frac{1}{S_{24}} \left(S_{25} f_{25} + S_{23} f_{23} + S_{21} f_{21} \right)$$

$$= -\frac{1}{S_{24}} \left(S_{25} \frac{2r_{25}+r_{52}}{3} + S_{23} \frac{2r_{23}+r_{32}}{3} + S_{21} \frac{2r_{21}+r_{12}}{3} \right)$$

Now the following relations are linear: r_{25} and r_{24}; r_{52} and r_{53}; r_{23} and r_{24}; r_{32} and r_{35} (and hence r_{32} and r_{53}); r_{21} and r_{24}; r_{12} and r_{13} (and hence r_{12} and r_{31}, r_{12} and r_{35}, and finally r_{12} and r_{53}). Hence the relations between r_{42} and r_{53} and r_{42} and r_{24} are also linear, and it follows that

$$r_{42} = m'' r_{24} + n'' r_{53} + p''$$

Since r_{53} and r_{24} are known, r_{42} may be obtained.

By a similar process it ~~d-en~~ can be shown that for every triangle of the truss a linear relation can always be expressed between the values of the r's at the three vertices. In general, (Fig. 36) for every triangle lmn of a truss, where lp is the reference member,

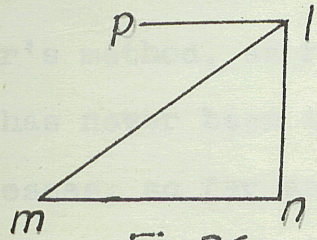


Fig.36.

$$r_{nm} = g\, r_{mn} + h\, r_{lp} + k \quad \ldots \quad \ldots \quad (13)$$

where g, h and k are constants. By successive application of this equation the value of any contour of the truss may be found from the given contours around the joints 1 and 2.

The Equations (12) and (13) together form the basis for the Theory of Deformation Contour, which may be stated as follows:

"In any framed structure composed of triangular elements the values of the deformation contours for the joints of the structure are related to each other by a linear equation." If the least number of members entering into a joint of the structure is "n" the value of each of the deformation contours may be expressed linearly in terms of n others. In a bridge truss n is generally equal to 2 so that the deformation contours of any 3 joints of the truss are connected by a linear equation and, by the process of successive substitution, all the deformation contours of the truss can be made to depend, by linear expressions, on only two other deformation contours. These two contours may be chosen at random but for practical purposes they are best taken to be the deformation contours for the two joints at the end of the truss.

By the use of the above theory there is overcome the necessity of solving a large set of simultaneous equations which is always considered as the most laborious part in the solution of secondary stresses. While the truth of this theory is simple and almost evident (in fact it could be derived from any set of the simultaneous equations involving T in Manderla's method or B in Mohr's method, as r in this paper is linearly related to T and B) it has never been taken cognizance of in the solution of secondary stresses, so far as the writer is aware. The advantage of considering the existence of such a "theory" will be apparent in the next chapter, although in the strict sense of the word "theory" it may be questionable whether its use is correct as the present method, like all others, is not a mathematically exact solution of secondary stresses.

CHAPTER V. GRAPHIC METHOD BY THE THEORY OF DEFORMATION CONTOUR.

Article 1. Graphic Representation of a Linear Equation involving Three Variables.

In the previous chapter the values of deformation contours for any three joints in a truss are shown to have been connected by a linear equation. If r_{53}, r_{35} and r_{21} are the contours at the vertices of a triangle 235, as shown in Fig. 37, then by Eq. (13),

$$r_{53} = a\, r_{35} + b\, r_{21} + c.$$

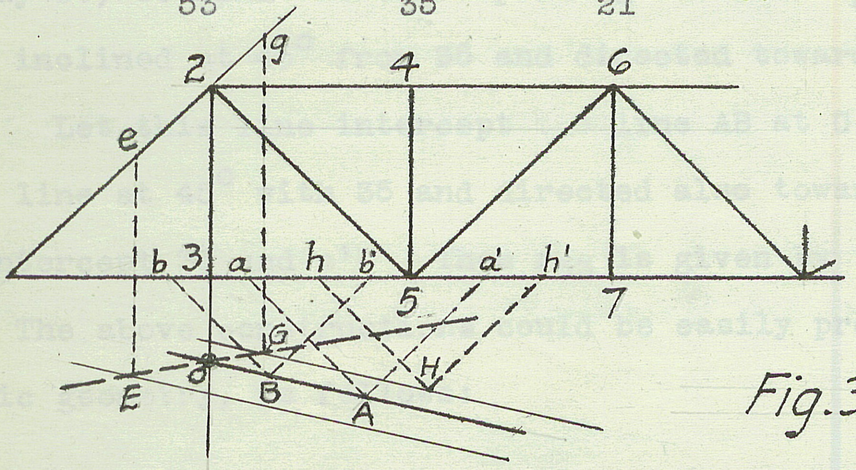


Fig. 37.

If one of the variables, r_{21}, be assumed fixed or arbitrarily assigned, the other two variables r_{35} and r_{53} can be graphically represented by a straight line. This, however, implies that the two variables are referred to two axes which are not in the same straight line, as they are in the present case, both r_{35} and r_{53} being referred to line 35. To overcome this difficulty and facilitate the construction the following device has been evolved.

Find a pair of values of r_{35} and r_{53} connected by the above equation, with r_{21} arbitrarily assigned, and locate the points a and a' by making 3a = r_{35} and 5a' = r_{53}. From a draw a line at 45° with 55 and directed toward the right of joint 5 and below the truss. Similarly, from a' draw a line at 45° with 53 and directed toward the left of joint 3, and also below the truss. These two lines will intersect at a point A. Next find another pair of values of r_{35} and r_{53} with the same r_{21} and locate the points b and b'. From b and b' draw lines at 45° with 35 and directed toward each other, intersecting at B. Draw a straight line through A and B. This line will then give the relations between r_{35} and r_{53} for the assigned value of r_{21}. For any value of r_{35}, say 3c, to find the corresponding value of r_{53} draw a line from c inclined at 45° from 35 and directed toward the opposite end 5. Let this line intercept the line AB at O. From O draw a line at 45° with 35 and directed also toward 5. Let this intercept 35 at c'. Then r_{53} is given by the segment 5c'. The above construction could be easily proved by analytic geometry, as follows;

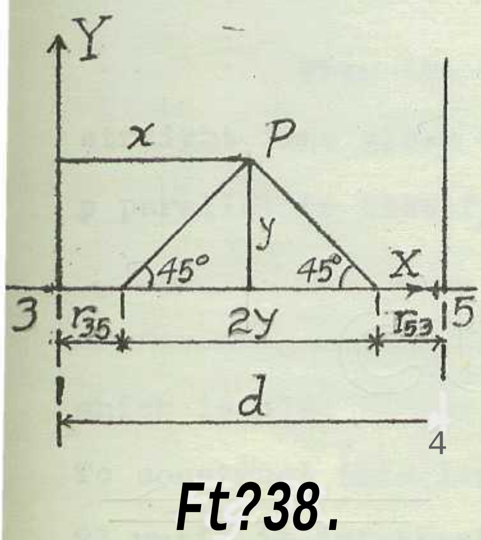


Fig. 38.

Let the equation between r_{35} and r_{53} with an arbitrarily assigned value of r_{21} be

$$r_{53} = a\, r_{35} + f \quad \ldots \quad (a)$$

where f = b r_{21} + c = constant. Let P, the point obtained by the above construction with any two corresponding values of r_{35} and r_{53}, be referred to 35 and 3Y as X and Y axes, so that its coordinates are x and y. From the figure,

$$y = \frac{1}{2}(d - r_{35} - r_{53}) \quad \ldots \quad \ldots \quad (b)$$

where r_{35} and r_{53} carry their own signs. Also,

$$x = r_{35} + y \qquad \ldots \qquad \ldots \qquad \ldots \qquad (c)$$

Substituting the value of r_{53} from (a) in (b) and eliminating r_{35} between the resulting equation and (c) there is obtained a relation between x and y:

$$y = \frac{a + 1}{a - 1}x - \frac{d}{a - 1} + \frac{f}{a - 1}$$

$$y = \frac{a + 1}{a - 1}x - \frac{d}{a - 1} + \frac{b\,r_{21} + c}{a - 1}$$

$$y = \frac{a + 1}{a - 1}x + \frac{b\,r_{21} - (d - c)}{a - 1} \qquad (d)$$

As this is a linear equation the locus of P is a straight line.

It is evident from equation (d) that the slope of the line is a function of a alone and independent of r_{21}. That is, the slope of the line is constant for all the assigned values of r_{21} and may be most readily obtained by making $r_{21} = 0$. This line will be known as the "Base Line" for triangle 235, for the reason that it expresses the relation between the r's of base 35 with r at the vertex of triangle = 0.

When the value of r_{21} is different than zero the straight line given by equation (d) will occupy a new position parallel to itself with the y-intercept p given by the equation

$$p = \frac{b}{a - 1} r_{21} - \frac{d - c}{a - 1} \qquad \ldots \qquad (e)$$

which is also linear and may be represented by a straight line. To construct this line proceed as follows: Prolong the member 23 until it intersects the base line at 0. (See p. 66). Assume a value for $r_{21} = 2e$, preferably as large as possible, and by the above method find the line EF representing the relations of r_{35} and r_{21} for r_{21} = [illegible]. This line, as proved above, must be parallel to the base line AB. From e draw a line parallel to member 23 and intersect EF at E. Join OE. This line OE then gives the y-intercept p for any value of r_{21}. If 2g is the value of r_{21}, the intercept is found by drawing a line through g parallel to member 23 until it meets OE at G and through G drawing a line parallel to the base line AB. This line GH then gives the relation between r_{35} and r_{23} with r_{21} = 2g.

The line OE, giving the relations between p and r_{21} at the vertex of the triangle, will be known as the "Vertex Line".

To utilize the base and vertex lines in finding r_{23} from r_{35} and r_{21} proceed as follows. Let r_{21} = 2g (negative) and r_{35} = 3h. Through g draw a vertical line cutting the vertex line at G. Through G draw a line GH parallel to the base line AB. Through h draw a downward 45° line cutting GH at H. From H draw an upward 45° line directed toward joint 5 cutting 35 at h'. Then r_{23} for the above values of r_{35} and r_{21} is given by 5h' and (5 negative). Similarly the values of r_{23} for any other combination of r_{35} and r_{21} may be found.

By a simple reverting process the value of r_{21} may be found from any given values of r_{35} and r_{23}. For these two values, say 3h and 5h' respectively, determine a point H by 45° lines and through H draw a line parallel to the base line, cutting the vertex line at G. Through G draw a vertical line cutting member 21 at g. Then 2g is the value of r_{21} required.

By the same method the base and vertex lines may be constructed for any other satriangle of the truss if two of the r's are known. In case the triangle has a "base on the top chord of the truss, it is advisable to draw the 45° lines directed upward instead of downward as for e member 35.

As a means of standardization all the base lines will be shown by full heavy lines and the vertex lines by dashed heavy lines.

Wfc&iA advantages of representing relations of. r by the above method are essentially as follows:

(1). It gives tl3 values of r both in magnitude and in sign,

(2). The value of r is found right on the mBber, no attention, therefore, need be paid to the signs,

(3). It is much easiper and more accurate to draw 45° lines with the aid of a triangle and T square than parallel lines required in Cartesian coordinates.

(4), This method takes much less time than to represent the relation by Cartesian coordinates where the values of r have to be transferred to the members from a set of axes.

Article 2« Construction of Base and Vertex Lines.

Since the base line is a straight line it can be completely determined by two points. A point for tfcB vertex line is given by the intersection of the base line and the vertical member produced, so that only one more point is necessary for the vertex line. Therefore, theoretically three points are sufficient for constructing both the base and the vertex lines. For practical purposes, however, it is advisable to find one more point for each of the two lines, both as a check and a means of improving the accuracy of the results. These points may be so chosen as to require the least amount of work.

Let the base and vertex lines be constructed for a truss as shown in Fig. 39. This is a Warren truss with verticals but the method is applicable to all kinds of trusses, the only difference being found in the order of procedure. This is governed by the number of members entering into the joints. It is well to remark here that the base of a triangle is always chosen as the member which forms the outline of the truss.

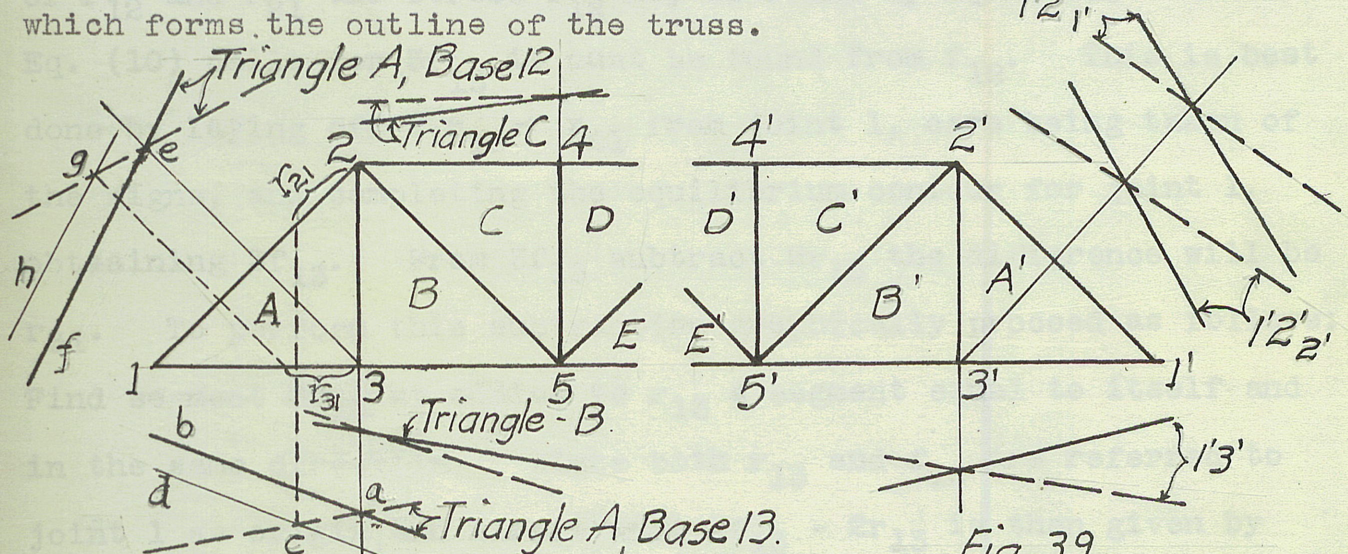


Fig. 39.

(1). Base and Vertex Lines for Triangle A.

Here, both members 12 and 13 may be chosen as the base. While only one base line is necessary in the solution of the truss, it is advisable to have one more in order to check the final results.

(a). Member 13 as Base.

Since the vertex for triangle A is joint 2 the base line is obtained by taking $r_{21} = 0$, as 21 is the reference member for joint 2. To construct this base line three points are necessary which are determined by corresponding values of r_{13} and r_{31} connected by Eq. (10). The detailed process is as follows. Assume a value for r_{13} and complete the deformation contour for joint 1, giving r_{12}. Also, for $r_{21} = 0$ complete the deformation contour for joint 2, giving r_{21}, r_{23}, r_{25} and r_{24}. From the values of r_{12} and r_{21} the stress f_{12} may be found by Eq. (11). Since Eq. (10) calls for $3f_{13}$ it must be found from f_{12}. This is best done by laying off $2\,r_{12} + r_{21}$ from joint 1, care being taken of the signs, and completing the equilibrium contour for joint 1, obtaining $3f_{13}$. From $3f_{13}$ subtract $2r_{13}$ the difference will be r_{31}. To perform this subtraction graphically proceed as follows: Find segment $2r_{13}$ by adding to r_{13} a segment equal to itself and in the same direction. Since both r_{13} and f_{13} are referred to joint 1 as origin, the difference $3f_{13} - 2r_{13}$ is then given by the segment from the end of segment $2r_{13}$ to the end of segment $3f_{13}$ (this is given by the intersection of equilibrium line 13 and member 13), both in magnitude and in sign. The difference is positive if the segment is in the direction from 1 to 3, negative if from 3 to 1.

The value of r_{31} thus obtained together with the assumed value of r_{13} determine a point for the base line, use being made of the 45° lines as described in the previous article. By a similar process, still keeping $r_{21} = 0$, two more points may be obtained which, together with the one previously found, must lie on the same straight line. This gives the base line ab required.

To find the vertex line corresponding to the base line just found, keep one of the assumed values of r_{13} as constant and find a pair of corresponding values of r_{21} and r_{31}. ~~which are corresponding~~. To do this, first give a value to r_{21} which is fairly large and complete the deformation contour for joint 2 with this value of r_{21}. The contour for r_{13} has already been drawn for constructing the base line so that r_{12} and r_{13} are known. From these contours obtain $2r_{12} + r_{21} = 3f_{12}$ and from $3f_{12}$ obtain $3f_{13}$. Subtract $2r_{13}$ from $3f_{13}$, r_{31} is obtained. This r_{31} together with the assumed r_{13} determine a point. Keeping the value of r_{21} constant find another pair of values of r_{13} and r_{31}, determining another point. These two points must lie on a straight line cd that is parallel to the base line. Through the end of segment r_{21} thus assumed draw a vertical line cutting cd at c. Let the base line ab cut member 23 produced at a. Join ac. This is then the vertex line required.

These base and vertex lines are indicated in the figure as "Triangle A, Base 13".

(b). Member 12 as Base.

Here joint 3 is the vertex of the triangle and the base line is obtained by taking $r_{31} = 0$, as 31 is the reference member for joint 3, To find a pair of values of r__ and r_{01} first assume a value for r^g. For this r^ complete the deformation contour for joint 1, obtaining r^. Find $3f_{13} = 2r_{13}$-h r_{31} = 2r$_{13}$. =2«$_3$+o

Construct an equilibrium contour for joint 1, 3f^g is found from Sf-^g. Ffcome3fYg anduSr-jo, T9.\ *-S obtained* This r^^ and the assumed r_{12} determine a point on the base line, use being made of the lines inclined at 45° with member 12. Similarly two more points may be obtained and the base line ef is completely known. Find the intersection of this line with a line drawn through joint 3 and perpendicular to member 12 at e.

To locate the vertex line, assign a fairly large value to r_{31} and keeping it constant, obtain two pairs of corresponding values of r?_1 and r_1g. These two points must leie on a line gh parallel to ef. Find the intersection g of this line gh and a perpendicular to member 12 through the end of segsment i»$_{31}$ assumed. The vertex line is then given by a line drar/n through points e and g.

These base and vertex lines are indicated in the figure as "Triangle A, Base 12".

To find the corresponding values of r_{12} ^d- r21 for any assumed P value of r^ drop a perpendicular to 12 from the end of segment r_{31} and find the intersection g of this perpendicular with -&h& vertex line eg. Draw a line through g parallel to the base line ef. This line then gives the relation© between r^ and r21 for t:he assume(3- rgi> ttie corresponding values being given by 45° lines.

(2)« Base and Vertex Lines for Triangle B.

The base for triangle B is member 35 and the vertex, joint 2, so that the base line is obtained by making $r_{21} = 0$, the deformation contour for which has already been drawn. As before, assume a value for r_{53} and find the corresponding value for r5g. These two values will determine a point on the base line. On account of the greater number of members entering into joint 3 the process is a little more complicated than for triangle A but the principle will be the same.

For the assumed value of v^r, construct a deformation contour for joint 3, giving r_{32} and r_{51}. Since the value of r53 ^ePen(3-s on that of F fgg and whiofe, in turn, depends on frzo aiind fg]* or indirectly, on r^2> r23> r3i an(^- rl3> ^ ^- s necessary to find the values of r_{n} a for the corresponding values of r^-, and ^g]_> as r23* r32 an(i r21 are a1^ known* To obtain this r_{ln} use is to be made of the base and vertex lines for triangle A, l?Ese 13, precisely as explained a in the previous article. For this reason it is advisable to choose r = 0 instead of r = 0 for this triangle, for then the base line for triangle A could be used. Now that the values of r for members 13 and 23 are known, the sums $Sr_{32} + r = 3f_{32}$ and $2r_{31}$-H $r_{13} = 3f_{31}$ may be obtained by graphic addition and the value of $3f_{5g}$ obtained from anequilikforium contour for joint 3. Then, by graphic subtraction, v = 3f - 2i* is obtained' as explained in (1) for triangle A. These corresponding values of r and r_{53} determine a point on the base line for triangle B. Similarly two more points may be obtained and the base line is completely known. For the vertex line give r_{si} a value which is fairly large, preferably the same one as used in constructing the vertex line for triangle A, base 13. This changed value of r_{21} affects r_{23} directly and r_{13} indirectly, for then the base line for triangle A could not be used in finding r_{13} from $r_{g,1}$ and z^l' Instead, the vertex line for triangle A should be used for the assumed value of r_{21}. With this exception aii- the remainder of the method is the same as described before.

(5). Base and Vertex Lines for -Triangle C.

The. base & for triangle C is member 24 and the vertex, joint 5. The base line is therefore obtained by makinggr-- = 0. Since the value of r^g depends on 3fg4 and 2rg^, the latter being arbitrarily assigned, it H is necessary to find fg^.* or indirectly, f£c> ^03' an(3- ^91* Tllese v-alueaof f are found from T%Q and r_{5Q}, $r_{g(7}$ and r^g, and r_{gl} and r^g* The deformation contour for joint 2 for the assumed r gives r__, r and r, whieMle that for joint 5 for r§g = 0 gives r. The only r's unknwown are therefore* r^ and r_{12}. To find r_{52}, first obtain r from the base and vertex lines for triangle B from rg]_ and r§£ = 0, then complete the deformation contour for joint 3 for this value of r^. The contour will then give zVp* Lastly, the va-lue of r_{12} is found from r-^g, which in turn is found from the fease ad a4nd vertex lines fpor triangle A, where rg-j_ and r^ are known. With these values of r known, 3f0g, 3fgg' and 3fg-i may be obtained "-6 by addition and 3fg4 by an equilibrium contour for joint 2.

The resultant 3f$_{Q}$> then furnishes r_{40} for the assumed ^{2r}pA> $^{E(}$i* _(_10)» These two values of r determine a point on the "base line for triangle © 0. By a similar method, two more points may be obtained, still keeping r^g = 0, and the base line is then completely located* As a means fcoffacilitatiijythe construction, it is well to have one value of r_{24} w&ife which will give r_{21} = 0.

The vertex line for triangle 0 is obtained in much the same way as for triangles A and B»

By a similar process the base and vertex lines could be located for all the triangles of the truss. For a triangle like E where five members enter into Joint 5 the process is necessarily complicated but that is about the extreme case that is likely to occur in ordinary trusses.

It will be found in actual cases that with, the exception of the triangles at the two ends of the truss, the base and vertex lines are always very close together and in many cases the vertex lines could be omitted, entirely. In this case the base line is to be used for all values of r at the vertex.

For a symmetrical truss with symmetrical loading the base and vertex lines need be constructed for only one half of the truss, as for the other half, the$r are identical in form: If the truss or the loading or both are not,symmetrical the base and .ver- tex lines must be obtained for- every triangle of the truss.

A few words are now necessary re&garding the selection of

;-rig t>

reference members. They are seen to be the chord members, end posts included, that are at the left side of the joints in consideration. This is due to the fact that the above process of constructing base and vertex lines advances from left to right and only a part of the deformation contour that is on the left side of the joint need be constructed.

Article 3. Solution of the Problem.

Let the base and vertex lines for all the triangles of the truss be constructed by the methods of the previous article. At the right end of the truss there are then drawn the base and vertex lines for triangles B', C', etc. For triangle A' three sets of base and vertex lines are drawn; first, for side 1'3' as base; second, for side 1'2' as base considering equilibrium about joint 1'; and third, also for side 1'2' as base but considering equilibrium about joint 2'. These three sets of lines are indicated as 1'3', $1'2'_{1'}$ and $1'2'_{2'}$ in the figure. Since they all represent the same relation among $r_{3'1'}$, $r_{1'2'}$ and $r_{2'1'}$ they may be used for solving the three unknowns as analytically they represent three equations. To do this, first assume a value for $r_{3'1'}$ and apply it to the sets $1'2'_{1'}$ and $1'2'_{2'}$, thus giving two lines parallel to their respective base lines. These two lines intersect at a point which will give a pair of values of $r_{1'2'}$ and $r_{2'1'}$ for the assumed $r_{3'1'}$. Next apply the values of $r_{1'2'}$ and $r_{2'1'}$ thus found to the set 1'3' and find the corresponding value of $r_{3'1'}$. This value should check with that assumed before, if not, another trial may be made. Repeat the process until the values of $r_{3'1'}$,

$r_{1'2'}$ and $r_{2'1'}$ all satisfy the three sets of base and vertex lines. They are then the values that will give the correct secondary stresses. From the value of $r_{2'1'}$, that of $r_{2'4'}$ may be found from a deformation contour for joint 2'. Similarly the value of $r_{3'5'}$ may be found from $r_{3'1'}$ by a deformation contour for joint 3'. These two contours are the correct deformation contours for the problem since from them the correct secondary stresses may be obtained. From the values of $r_{2'4'}$ and $r_{3'5'}$, that of $r_{5'3'}$ may be obtained from the base and vertex lines for triangle B', and consequently, the correct deformation contour for joint 5'. Repeating the process the correct contours are obtained for all the joints of the truss. To check the accuracy of the results the values of r_{31}, r_{12} and r_{21} must satisfy the base and vertex lines for triangle A with member 12 as base, as this set of lines has not been used in constructing the correct deformation contours. This ensures the correctness of every step of the procedure and is one of the important features of the method of deformation contour.

From the correct deformation contours the actual secondary stresses may be found by Eq. (11). Graphically this may be accomplished by adding to the segment r_{1n} of member 1n a distance equal to r_{1n} plus r_{n1}, with proper signs, and measuring off the resulting segment with a scale three times as large as that used for r. This gives f_{1n} directly. Same for f_{n1}. It will be seen that on account of the large scale used for measuring f, the result could be read to three significant figures which is accurate enough for all practical purposes.

When the loading and truss are symmetrical the stresses at the two ends of the member of symmetry will be zero, hence the values of r at these two ends are also zero. This locates two correct deformation contours for the two joints of symmetry, from which the problem may be solved as before.

To obtain a clearer view of the distribution of secondary stresses in the truss and also the way they affect each other, it is advisable to draw in the stress lines for the different members and their force diagrams, as mentioned in the first chapter. As a final check on the values of f, the equilibrium polygons drawn around each joint for the stress lines ~~A~~ S and ideal external force R, applied at the joint, must ~~respectively~~ close.

Article 4. Checks.

There are the following checks found in this method of Deformation Contour.

(1). In constructing the values of K, the sum of the K's in each triangle must be zero.

(2). All the property lines in a triangle must meet in one point.

(3). All the deformation lines must be parallel to the property lines in the same triangle.

(4). All deformation contours must close for any assigned value of r.

(5), All equilibrium contours must close.

(6). The base and vertex lines for any triangle are checked by constructing one extra point for each line. These extra points must lie on the respective lines.

(7). The correct deformation contours are checked by the base and vertex lines for the end triangle of the truss, with the end post as base.

(8), For any joint of thefcrtussthe equilibrium polygon for the stress lines S and external forces R must close.

Article 5» Exmample. Details of Procedure.

To illustrate the graphic method as presented in this chapter an «ЗЕ»a ©a example will be work&d out and compared with that analysed by the ordinary methods. The truss and loading taken are the same, as those used in illustrating the methods of landerla and *Motor,* in Engineering Hews. Refer to Drawing Plate III•

On the drawing two truss diagrams are shown. The one at the right is for the change of angles and for the construction of Uslines.

Below

It is also used for recording useful informations. *M* tfee *&4&e &£* the two diagrams a force diagram of the equilibrium lirs s *íB~* drawn, data being taken from the section moduli of the members. On the second diagram are drawn the property lines, the deformation lines, the external forces R and the co-ordinate axes for the base and vertex lines. From the method given in the previous article the base* and vertex lines area located for all triangles of the tnuss* Since the leading is symmetrical with the centre of the truss the stress in member *67* la zero and therefore r_{g7} and $r_{?6}$ are each equal to zero. From thesefc -two values all the other r[f]s are found sua£ from which the stress lines may be located. To avoid confusion all the construction lines for the x base and vertex lines will be omitted and &k only the correct deformation contours are shown, A force diagram for the correct position of stressH lines atfe also shown*

It will be found that the values of f obtained found by this graphic method check very closely with those obtained by the methods of Mohr and Manderla. *The* total time consumed in the solution, from beginning to end is about 6 l/2 hours, distributed as follows: 2 hours for the construction of the change of angles and the deformation lines, 5 l/2 hours for the construction *t* of all the bass and vertex lines, and 1 hour for the location of the stress lines and the force diagram.

Article *6.* Oonstruction of Influence Lines.

It will be seen from the derivation of Eq. (12),

$$r_{lm} - a\, r_{ln} \sim D$$

that the v±alue of a depends on the values of U while fchat of 'b* depends also upon the values of *&* which are different fKor different loading. Also, from the derivation of El Eq. (15),

$$r_{nm} = h\, r_{mn} + g\, r_{lp} + k,$$

the values of h and g are constant for the truss while that of k. varies with the loading. Hence, the frase and vertex lines discussed in the previous articles will be parallel to ea&h other for different kinds of loading. If they are drawn for one kind of loading the slopes of the lines sass will be fixed and for each extra loading only one point will be suffidient for *the* corresponding lines. The method of Muller-Breslau for influence lines is especially addpted to this graphic method for then the base andac vertex lines will be the same for one half of the truss for the different loadings.

CHAPTER VI. THE PROCESS OF SUCCESSIVE DEDUCTION.

Analogous i bo the relations between deformation contours there is also a linear law existing between secondary stresses P f. Consider a structure e composed of triangular elements. In each triangle there are then six values of f, a one at each end of the member. But the trianglesa are connected together and for every pair of adjacenttriangles there is ones side in common. One of tiriangles may be considered as added to th© other by the introduction of two sides, and therefore has only £our values of f. If the structure isfe built up of triangles by successive addfeition of to a "base triangle" all the added triangles will then £ have only four unknowns while the tease triangle will have six. To find these unknowns there is requires, an equal number of equations which must be found from the nature of thep problem. Let the number of triangles in the structure be n. If one of the triangles be considered as the base triangle and the other (n - {j| triangles be added to this triangle by successive introduction of two sides,, at a time, there will be 6+ 4(n - 1) = 4 n f 2 unknowns. For each triangle there are three angles giving three fundamental equations. This furnishes 3 n equations and only n + 2 more are required. But this is equal to the number of joints in n triangles and for each such jioint there is also a moments equation (7), A solution is therefore always possible. Since both the fundamental and memnent equations are linear in form, it follows that all the secondary stresses are connected by equations linear in form.

Since the base triangle has six values of f and there are only four equations available, three from the fundamental equations and one from the moment equation for the joint that is free from other triangles, two of the values of f ~~them~~ could not be found until two more equations are introduced. This base triangle therefore has two unknowns at the outset of the solution. As the stresses in the other triangles depend on those in the base triangle all of them will be expressed in terms of two unknown quantities. Hence if f be the stress in any member and f_x and f_y are the two unknowns in the base ~~tri~~ triangle,

$$f = a\,f_x + b\,f_y + c, \quad \dots \quad \dots \quad \dots \qquad (14)$$

where a, b, and c are constants. To determine f_x and f_y it is simply necessary to notice that the very last triangle added to the structure has three joints but only one is used, for the moment equation. There are therefore two more equations expressing the moments at these two joints. It is now evident that the value of f_x and f_y cannot be obtained until the above equation (14) has been applied to every member of the structure. Conversely if f_x and f_y are known all the other stresses are known.

To illustrate the process consider a truss as shown in Fig. 40. Let $f_{12} = f_x$ and $f_{21} = f_y$ be the unknowns. From these values find

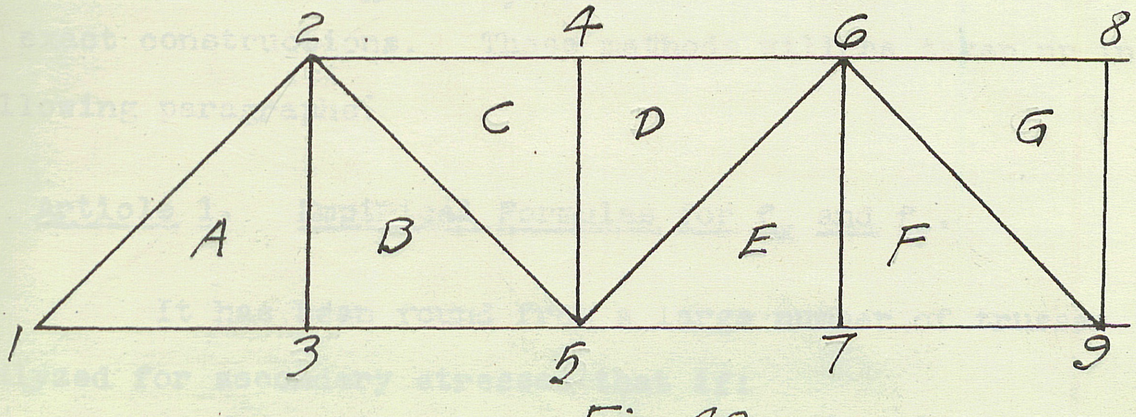


Fig. 40.

r_{12} and r_{21} and construct the deformation contours around the joints 1 and 2. Applying an equilibrium contour to joint 1 stress f_{13} is obtained from f_{12}. By Eq. (10) r_{31} is found from f_{13} and r_{13} and hence f_{31} by Eq. (11). From r_{31} construct a deformation contour around joint 3. From this contour and the one around ~~the~~ joint 2 stresses f_{23} and f_{32} are known. Now at joint 3 f_{31} and f_{32} are known and f_{35} may therefore be found by an equilibrium contour. From f_{35} and r_{35}, r_{53} is obtained from Eq. (10) and f_{53} from Eq. (11). Construct the deformation contour around joint 5. This gives r_{52} which together with r_{25} furnishes f_{25} and f_{52}. At the joint 2 since stresses f_{21}, f_{23}, and f_{25} are known f_{24} may then be found from an equilibrium contour. From r_{24} and f_{24}, r_{42} and f_{42} may be obtained from Eq. (10) and (11). By repeating the above process the stresses f in all the members may be obtained for the known values of f_{12} and f_{21}.

It is now evident from the above that when f_x and f_y are known the solution of the problem may be effected in a very short time. But in an actual cases they could not be found until at the end of the process when two moment equations furnish the unknowns required. For this reason four methods have been suggested by which f_x and f_y may be obtained either by trial or by exact constructions. These methods will be taken up in the following paragraphs.

Article 1. Empirical Formulas for f_x and f_y.

It has been found from a large number of trusses ~~analu-~~ analyzed for secondary stresses that if:

(1). The type of the truss is Pratt or Warren with verticals,

(2) The loading is uniform,

(3) The connections are concentric, and

(4) The proportions of the truss ~~is~~ conform ~~with~~ to the principles of economical design, i.e., when the inclination of the diagonal members ~~are~~ is about 45°,

then, the secondary stresses at the ends of the hip verticals are approximately equal, both in magnitude and in sign. Further, the stresses at the two ends of the end post are proportional to each other as the algebraic sums of ~~sum-e~~ the change of angles at the corresponding joints connecting the member. These two stresses are also equal in sign. In other words if the truss be as shown, then

$$f_{32} = f_{23}, \qquad \frac{f_{12}}{f_{21}} = \frac{K_{A1}}{K_{A2} + K_{B2} + K_{C2}},$$

where f_{32} and f_{23}, f_{12} f_{21} have the same signs.

Applying the fundamental equation (5) to the above triangle A,

$$(2f_{12} - f_{21})U_{12} + K_{A1} = (2f_{13} - f_{31})U_{13}$$

$$(2f_{31} - f_{13})U_{13} + K_{A3} = (2f_{32} - f_{23})U_{23}$$

$$(2f_{21} - f_{12})U_{12} - K_{A2} = (2f_{23} - f_{32})U_{23}$$

and the moment equation to joint 1,

$$f_{13} = -\frac{S_{12}}{S_{13}} f_{12},$$

there is obtained,

$$f_{12} = \frac{-K_{A1}}{U_{12} - \frac{S_{12}}{S_{13}} U_{13}}, \quad \ldots \quad \ldots \quad (15),$$

$$f_{21} = f_{12}(K_{A2} + K_{B2} + K_{C2}) / K_{A1} \quad "$$

where the ratio of the K's is ~~in~~ a absolute value. Applying the above equations to the example worked in the previous chapter it was found that $f_{12} = -.14$ and $f_{21} = -.06$. From the above values it was found that $f_{67} = -.16$ and $f_{76} = +1.00$, while they should be zero. By a careful consideration of the effect of one stress upon the other the correct values of the f's may be obtained by several trials.

Article 2^. Mathematical Formulas for f_x and f_y.

Thftsemay be obtained from the fundamental and moment equations instead of the -from deformation and equilibrium contours in the graphic method. Undoubtedly the formulas will be extremely long but there are the following simplifications which must be considered: (1) All bottom chords have the same length and sometimes the same y, (2) for parallel chords the values of 0 for ana vertical* top chords^ are equal, aieo -£ea* verticals, (3) the values of S in some web mSbers may b© neglected, (4) the truss elements are symmetrical about the centre line,, (5) thesum of.change of angles in eabh triangleequate -fe zero,

The distinct advantage of©a? a mathematical formula lies in the fact that once it is derived it is good for ail others and hec© has a permanent value. It is hoped that this will prove useful in bridage offices where th© designs for structures are largely standardized and only a few sets of formulas arefneeded.

Article 3. The Process of CharacteristjgEic Increments.

It has been shown in Eq. (14) that the stress f in any member is connected with that in member 12 by a linear equation,

$$f = a \; f^{\wedge} \; \text{-h} \; b \; f_{-} \; \text{-b} \; c$$

where a, b, and c are constants. An examination of the methods by which this equation is derived will show that the constants a Had and b are functions of U only while c is trk&4> fundtion of both U and K • Hence a and b depend only on the truss dimensions and th© sections of members, (U = L/y) while c depends injiaddition on the loading, as K is a function of the change of angles which depends on the loading. This fact shows that a and b are constant for the^truss and may be found without any knowledge of the actual loading. Further, by differentiating Eq. (14) the rate of change of f with respect to f_x or f SEX will be found to be a constant being equal to a and b respectively. When fj^ receives an increment equal to c-^g and fg-j_, anJr increment 02I9 the stress f receives an increment equal to d c^ -f g §21 jv>4/c/i 15 independent of the loading. In other words, if c is the increment of the stress f,

c =: d Cis"^" & ^{c}21 •*• •** •••'• **. (16)

whatever the value of K may be. These increments c will be known as the "Characteristic Increments" meaning that they depend on the truss dimensions only which are characteristic. These characteristic increments may now be applied to the correction & of stresses f_{12} and f_{21} which weres assumed and found to b© incorrect. Suppose for the assumed values of f_{12} and f_{21} there are external mements around the two joints 1* and 2' in the last triangle A' instead of being zero moment which would be follow a correct assumption. fre**&ew3 thoy ohould 2r# fee aaoumption is oorroot* Let these moments be Mi« = Ri» x hi' and M2' = Es' x h2', Since these moments are* linearly connected with the stresses f^ an<^ ^21 as they are linear functions of the stresses of the members meeting at the joints, they may be expressed in the form:

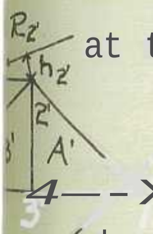


Fig. 41.

Ml» = ul» f12 + vl» f2l"t* wl»

4—-X* 2 2 1 2 t 2 ^ 21 " t 2»

where u, v, and w are constants, By similar reasoning as for Eq, (16) the increments of M are also dependent on &, u and v only and are independent of w. Thatis,

$$M_{c1'} = u_{1'} c_{12} + v_{1'} c_{21}$$

$$M_{c2'} = u_{2'} c_{12} + v_{2'} c_{21}$$

Now the consideration of equilibrium of joints 1' and 2' requires that $\Sigma M_{1'} = 0$ and $\Sigma M_{2'} = 0$. That is, an increment must be given to M so that it will be zero, or

$$M_{c1'} = -M_{1'} \qquad M_{c2'} = -M_{2'}$$

Hence,

$$-M_{1'} = u_{1'} c_{12} + v_{1'} c_{21}$$

$$-M_{2'} = u_{2'} c_{12} + v_{2'} c_{21}$$

From these two equations c_{12} and c_{21} may be found and the correct stresses f_{12} and f_{21} are obtained by adding these increments to the assumed values. After the correct values of f_{12} and f_{21} are known all the other stresses may be solved.

For a truss under symmetrical loading the moments are symmetrical about the plane of symmetry, say member 67, and

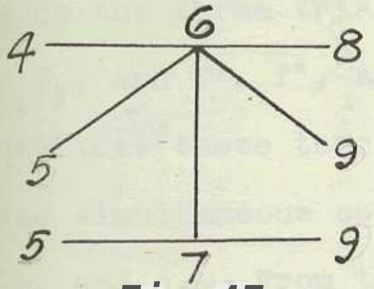


Fig. 42.

$M_{64} = -M_{68}$, $M_{65} = -M_{69}$ and $M_{75} = -M_{79}$. This shows that $M_{67} = M_{76} = 0$ and hence f_{67} and $f_{76} = 0$. If from the assumed values of f_{12} and f_{21} they are not equal to zero but equal to f'_{67} and f'_{76} respectively, then the increments for f_{12} and f_{21} must be such that

$$c_{67} = -f'_{67} \quad \text{and} \quad c_{76} = -f'_{76}$$

By Eq. (16),

$$-f'_{67} = d_{67} c_{12} + g_{67} c_{21}$$

$$-f'_{76} = d_{76} c_{12} + g_{76} c_{21}$$

From these two equations c_{12} and c_{21} may be found and the assumed f_{12} and f_{21} corrected as before.

Article 4. The Method of Substitution.

Consider the same equation (14),

$$f = a f_x + b f_y + c$$

If the values of a, b, and c are known the values of f_x and f_y may be found when the value of f is subject to certain conditions. To find a and b the previous articles made use of c = 0, but in some cases it might be well to include the value of c. Then, if f_x and f_y are assumed or found by empirical formulas of Art. 1 in this chapter there is a chance of having the values correct and thus saving much time in finding the increments although the process of characteristic increments is mathematically correct. If the assumed values are not correct a second trial may be made or even a third one. But there is no need for a fourth trial even if the third trial is not correct, which is not apt to be hardly the case, for the correct values of f_x and f_y can then be found from the three trial values, as follows; Let f_{x1}, f_{x2}, and f_{x3} be the three trial values for f_x and let f_{y1}, f_{y2}, and f_{y3} be the same for f_y, and f', f'', and f''' be the corresponding values for f. Then substituting these three sets of values in Eq. (14) there are obtained three simultaneous equations for the solution of the three coefficients, a, b, and c. From these coefficients and the correct values of f which are known from imposed conditions, the correct values of f_{12} and f_{21} may be found. It should be noted that it is not the intention of this article to be of service in any actual case as it applies only when the three trial values of f_{12} and f_{21} have failed to give the correct result, which is not believed possible.

CHAPTER VII. GRAPHIC METHOD BY THE PRODESS OF SUCCESSIVE DEDUCTION.

The graphic method by the prodess of successive substitution differs from that of deformation contour in the order of procedure. Instead of expressing the relations between r's by graphs and finding the correct stresses from the correct r's, this method handles the stresses directly. The general prodedure has been given in the previous chapter while the various details have been taken up in connection with the method of deformation contours. Therefore, only the particulars involved in the finding of f_{12} and f_{21} are necessary here.

For the empirical formulas of f_{12} and f_{21} there is no further discussion except to mention the methods of correction to be applied after each trial. It has been found from analytic expressions that in a Pratt or Warren truss with verticals the effects of f_{12} and f_{21} on the member of symmetry are different in signs and widely different in magnitudes. ~~That~~ The effect of f_{12} is generally much greater than that of f_{21}, ~~and~~ ranging from 2 to 4 times for the upper end of the member ~~to~~ and from 4 to 8 times for the lower end of the member. Further, it will be found that both ~~thes~~ the effects due to f_{12} and f_{21}, individually, are very large compared with the stresses f themselves, amounting to 50 - 400 times in a 3 panel truss. Based on these facts ~~the~~ a judicious correction may be applied to f_{12} and f_{21} after one or two trials. It is believed, however, that under the conditions the empirical formula is derived the values of f_{12} and f_{21} should be more than a rough approximation.

In the method of characteristic increments the coefficients of c_{12} and c_{21} for any c are derived from two processes; first, for $c_{12} = 1$, and $c_{21} = 0$ and then $c_{12} = 0$, and $c_{21} = 1$. In each of these processes the method is the same as for the general solution described in the previous chapter, except that the property lines should be used instead of the deformation lines in drawing the deformation contours. These contours then become the "Characteristic Contours". It will be found in this method that the values of the c's increase very rapidly toward the other end of the truss and it is necessary to change the scales for the c's ~~along~~ throughout the whole process. This is immaterial for the property lines, see Art. 3, Chap. 3. For the assumed values of f_{12} and f_{21} in this method it is better to so choose them that they are nearly correct, in order to get rid of unduly large scales for f toward the other end of the truss. This, however, is not very serious as the scale for f may also be changed by changing the scale for the deformation lines. To obtain the correcting increments for f_{12} and f_{21} after the whole truss is analyzed, the simplest method would be to draw two lines representing the linear equations referred to the same axes and find their intersecting points.

With regard to the method of substitution no further statement is necessary except to note that the third trial should give a fairly correct solution and the necessity for solving the three simultaneous equations does not exist in fact, if the assumptions are properly made. The empirical formula will be of value here.

Article 1. Checks

The principal source of mistakes that may occur in this graphic method comes from the wrong use of the deformation lines. To check this, let Eq. (5) be applied to any triangle lmn, Fig. 43, then,

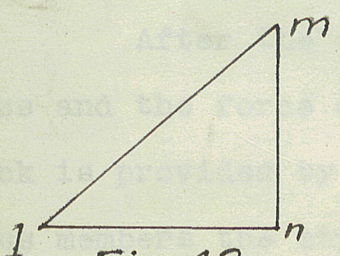


Fig. 43.

$$(2\,f_{ln} - f_{nl})\,U_{ln} = (2\,f_{lm} - f_{ml})\,U_{lm} + K_{mln}$$

$$(2\,f_{nm} - f_{mn})\,U_{nm} = (2\,f_{nl} - f_{ln})\,U_{ln} + K_{lnm}$$

$$(2\,f_{ml} - f_{lm})\,U_{lm} = (2\,f_{mn} - f_{nm})\,U_{nm} + K_{nml}$$

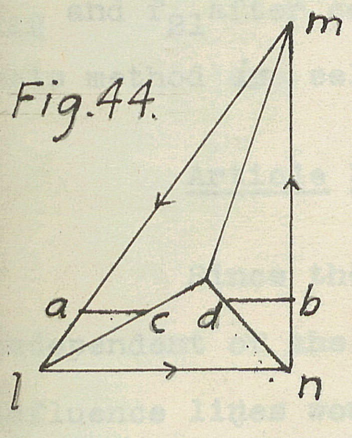
Adding together and noting that the sum of the K's should be zero in a triangle,

$$U_{ln}\,(f_{ln} - f_{nl}) + U_{nm}\,(f_{nm} - f_{mn}) + U_{lm}\,(f_{ml} - f_{lm}) = 0,$$

or,
$$(f_{ln} - f_{nl}) + \frac{U_{nm}}{U_{ln}}\,(f_{nm} - f_{mn}) + \frac{U_{lm}}{U_{ln}}\,(f_{ml} - f_{lm}) = 0.$$

This test may be applied as follows: Passing around the sides of the triangle in a counter clockwise direction, find the difference of the stresses in each side by subtracting the one at the end next met with from the one at the end first met with, multiply each by its corresponding U and the total sum of the differences should be zero. Making used of the property lines the test may be greatly simplified. Fig. 44. Choose any convenient side as a base, say ln, and find the difference, $f_{ln} - f_{nl}$. On the side nm find find $f_{nm} - f_{mn}$ and on the side ml, $f_{ml} - f_{lm}$. Lay off these two values with proper signs on the corresponding sides of the triangle and at the ends of the segments, a and b, draw lines parallel to the side ln meeting the property lines at c and d. Then the algebraic sum of ac and bd should be the negative of the difference $f_{ln} - f_{nl}$.

Fig.44.



The only other source of mistakes comes from the equilibrium of joints. But in the method of equilibrium contour th© construction is mad© so simple that this is hardly possible iS a little caret is exercised. This is far more free immuno from mistakes than the analytic expression *2* M =0.

After the stress lines ar© completely located for the truss and the fore© diagram drawn for th©s© lines a very unique check is provided by the fact that in any section cutting the truss members th© stress lines in these members and all th© external forces R in one side of the said section must balance each other. That is, th© force and equilibrium polygons for these forces must respectively clos©. This furnishes a check which insures th© equilibrium at all the joints.

Article *2j*> Example. Details of Procedure.

Th© same drawing mad© in connection with th© method of deformation contours may b© used to illustrate this method if (1) the bas© and vertex lines'and th©ir axes be omitted, and (2) if th© stress lines as shown b© regarded as the final solution for f_{lg} and fg_1 after correction. The other constructions needed in this method *d&* self evident and doemot need a separate drawing.

Article 3^ Construction of Influence Lines.

Sine© th© coefficients of f_x and f_y in Eq.(14) ar© independent of the loading they only construction required *vsiTor the* influence li^es would *bw* the values of c for each case of loading. The characteristic increments therefore remain the same as before. Th© same assumed values of fi2 and fgl may be used for all the cases requiring different sets of correcting increments. Each position of load has a set of stress lines of its own.

CHAPTER VIII. THE SCIENTIFIC ARRANGEMENT OF COMPUTATIONS.

It is evident that both the methods of deformation contour and successive deduction admit of analytic solutions. As is well known an analytic method, while it gains accuracy in result, is generally accomplished at great expense of time. It therefore behooves us to devise some such methods which will reduce the amount of work to a minimum. The computations for secondary stresses by the ordinary methods have been systematized to a great extent in recent years but still there is room for improvement, mainly in the arrangement of computations. Among the many items which are inefficient and are found in all of the ordinary methods may be mentioned the following:

(1) All these analytic methods need some graphic construction to show the proper location of certain computed quantities.

(2) In all ordinary methods the computations are arranged in tabular form in which at least two or three columns of figures are copied from the previous tables. This not only wastes time and energy but also invites error.

(3) Only the final results are shown in the tables, the rough computations like addition and subtraction being made on separate pieces of paper which are not filed. Such computations are most liable to be performed in an unsystematic way.

(4) Time is wasted in transferring the results of the rough computations to the tables and the practice also is apt to introduce mistakes.

(5). There are no means of checking the computations except by repeating the numerical work which not only wastes time but also requires a different computer.

(6) The figures arranged in a table do not convey as much idea as when marked on the truss diagram in a semi-graphical way. To transfer the results from the tables to the truss diagram again wastes time.

To get rid of the above objections, which are very gommon, a simple and self-evident scheme would be to dispense with the idea of the tables entirely and to record all the results directly on the truss diagram. Second thought makes it plain that to save time it is also advisable to include all the rough computations on the diagram. By this arrangement it is possible to so record the figures that each one is written only once and so that the computation could be most easily performed. Further, by arranging the computations so that each' figure has a definite space on the diagram, the chances of using wrong figures are entirely eliminated.

The writer acquired this idea of arrangement when
the
he attempted the solution of/eleven sets of ^ simultaneous equations each in connection with the secondary stresses in a 16-panel two-hinged arch. It was his good fortune to spend some time latter on the subject of scientific management which not only convinced him of the necessity for such an arrangement but also added much to his knowledge in achieving the end.

CHAPTER IX. ANALYTIC SOLUTION BY THE SCIENTIFIC ARRANGEMENT OF COMPUTATIONS

Article 1. General Considerations.

A careful comparison of the two graphic methods discussed in the previous chapters will disclose that while they are ~~in~~ different in form and construction they are nevertheless deducible one from the other. In the method of deformation contour the unknown quantity is r, $(= 2 f_{1n} - f_{n1})$ while in the method of successive ~~of~~ deduction the unknowns are the stresses, f, themselves. Since the deformation contours are the same in both methods the only difference will be found in the relations between the r's and those between the f's. But these relations are equal, differing only by a constant factor. For,

$$r_{1n} = 2 f_{1n} - f_{n1},$$

By Eq. (10) $$r_{n1} = 3 f_{1n} - 2 r_{1n},$$

Subtracting, $$r_{1n} + r_{n1} = f_{1n} + f_{n1},$$

Also by Eq. (11), $$3 f_{1n} = 2 r_{1n} + r_{n1}$$

$$3 f_{n1} = 2 r_{n1} + r_{1n}$$

Subtracting, $$3 (f_{1n} - f_{n1}) = r_{1n} - r_{n1}.$$

Hence, analytically, the quantities used in the two methods are either the same or differ by a constant factor. The two methods could therefore be reduced to the same basis by suitable combinations.

It should be noted here that by the same reasoning as above, the method of Muller Breslau and those of Manderla and Mohr are equally deducible one from the other and, broadly speaking, ~~they~~ are practically the same. The method of Muller Breslau, however, has the advantage of expressing the unknowns in simple linear equations which are not connected together simultaneously. The existence of the simultaneous equations in the other two methods is indeed unnatural and unnecessary.

Article 2. General Procedures.

In the analytic method the features of the two graphic methods will be combined to the best advantage. The unknown quantities will be made the stresses f themselves as this is more direct than the use of r in the method of deformation contour. The deformation contours will be applied to the different members around a joint by Eq. (12) which states that "the value of r of any member 1n at joint 1 is equal to that of r for any member 1p, multiplied by the ratio of u_{1p}/u_{1n} and increased by the summation of K's from 1p to 1n divided by u_{1n}". That is, Fig. 45,

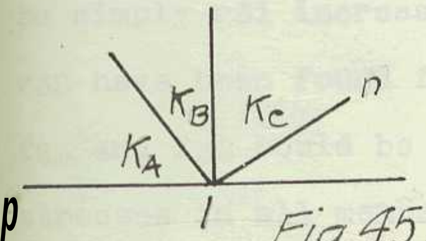


Fig. 45.

$$r_{1n} = \frac{u_{1p}}{u_{1n}} r_{1p} + \frac{\Sigma K}{u_{1n}} \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

The relation between the two f's at the ends of each member is given by Eq. (8), i.e.,

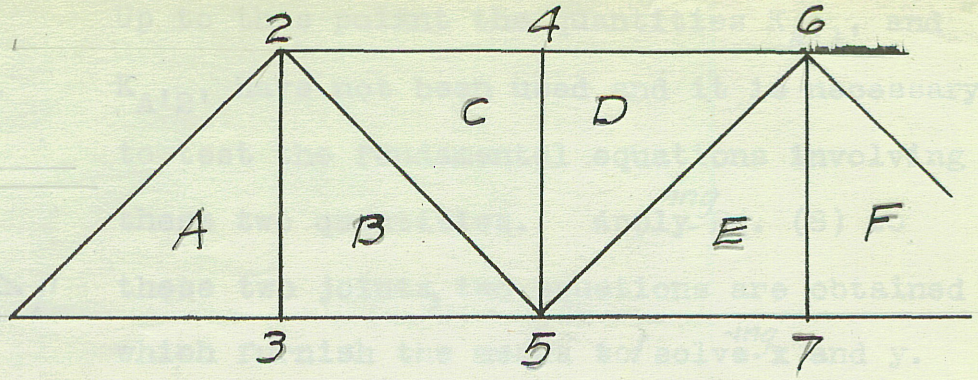
$$r_{1n} = 2 f_{1n} - f_{n1}$$

From the values of r at the two ends of a member the stresses at those two ends may be expressed by Eq. (11),

$$f_{1n} = \frac{1}{3} (2 r_{1n} + r_{n1})$$

Let the truss as shown in Fig. 46 be analyzed by the analytic method. Let stress f_{12} in member 12 be x, and ~~that~~ f_{21} be y. Then, $r_{12} = 2x - y$, and $r_{21} = 2y - x$. From these two values of r find

Fig. 46.



those of all the members meeting at the joints 1 and 2 by Eq. (17) Applying the moment equation (7) to joint 1, f_{13} is obtained from x. To find f_{31}, first subtract f_{13} from r_{13}, obtaining $f_{13} - f_{31}$ ($r_{13} = 2 f_{13} - f_{31}$), and then subtract $f_{13} - f_{31}$ from f_{13} giving f_{31}. Subtracting again $f_{13} - f_{31}$ from f_{31}, $r_{31} = 2 f_{31} - f_{13}$ is obtained. From r_{31} find r_{32} and r_{35} by Eq. (17). If ~~x~~ $U_{13} = U_{35}$, r_{35} will be simply r_{31} increased by $\Sigma K/U_{35}$. Now for member 23, r_{23} and r_{32} have been found from r_{21} and r_{31} and therefore the stresses f_{23} and f_{32} could be found from Eq. (11). Around joint 3 the stresses in all members are known except that of 35. By the moment equation around joint 3,

$$S_{31} f_{31} + S_{32} f_{32} + S_{35} f_{35} = 0$$

f_{35} is therefore known. By a similar process as ~~as~~ for member 13, r_{53} is obtained, ~~and~~ from which the values of the r's for all the members meeting at joint 5 are determined. The stresses in member 25 may then be found from r_{25} and r_{53} and this makes all the stresses around joint 2 known, except that of member 24 which may be found from the moment equation. From the value of r_{24} that of r_{42} may be found by the same method as for member 13 and the remainder ~~rest~~ of the process repeated. Eventually the stresses in the members around joints 2' and 1' are found ~~and~~ from ~~which~~ the moments around the joints 2' and 1', Fig. 47. ~~may be found~~

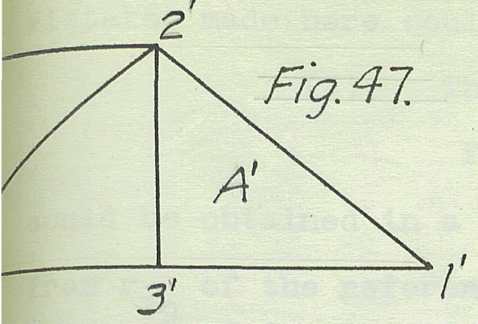


Fig. 47.

Up to this point the quantities $K_{A'1'}$ and $K_{A'2'}$ have not been used and it is necessary to test the fundamental equations involving these two quantities. Applying Eq. (8) to these two joints, two equations are obtained which furnish the means ~~to~~ of solving x and y. After they are obtained all the other stresses can be found by substitution.

If the loading is symmetrical about a member at the centre of the truss, the stresses at the two ends of that member will be zero. Since these stresses are expressed by linear equations in terms of x and y, the latter can be found from the two equations equated to zero.

In the above method the following points should be noted.

(1). In applying Eq. (17) to members it should be remembered that the signs of the K's in ΣK should be changed for all those members ln which are reached from the reference member lp in a counter clockwise direction around the joint in consideration. See Chap. II.

(2). The moment equation could not be applied to any joint until the stresses in all members around the joint except one are known.

(3). The check in Art. 1, Chap.VII, i.e.,

$$U_{ln}(f_{ln} - f_{nl}) + U_{nm}(f_{nm} - f_{mn}) + U_{lm}(f_{ml} - f_{lm}) = 0 \qquad \ldots \ldots \ldots (18)$$

must be satisfied as soon as the stresses in one triangle are completely determined.

(4). Particular attention should be paid ~~in~~ to the solution of the moment equation and also ~~in~~ to the correct substitution of the U's.

Mistakes made here could not be detected by Eq. (18).

(5). The values of f from Eq. (11),

$$f_{ln} = \frac{1}{3}(2r_{ln} + r_{nl})$$

could be obtained in a simpler way as follows: First find $r_{ln}/3$ from r_{lp} of the reference member by using $3U_{ln}$ instead of U_{ln}. This gives

$$\frac{r_{ln}}{3} = \frac{r_{lp}U_{lp}}{3U_{ln}} + \frac{\Sigma K}{3U_{ln}}$$

Similarly find $r_{nl}/3$. Then add the sum of $r_{ln}/3$ and $r_{nl}/3$ which, when added to $r_{ln}/3$ gives f_{ln} and to $r_{nl}/3$ gives f_{nl}. Since only the stresses in web members are found from the r's at the end of the members, only the web members need be computed for 3 U's.

Article 5. Change of Angles.

The equations for the change of angles as previously discussed could be arranged in a more useful form as shown in the figure, which is constructed as follows: First write down the values of the P's on the sides of the triangle and then fill the spaces inside the triangle with the P's as shown, each primary stress being entered twice. Next form the differences $P_{12} - P_{23} = a$ and $P_{12} - P_{13} = b$

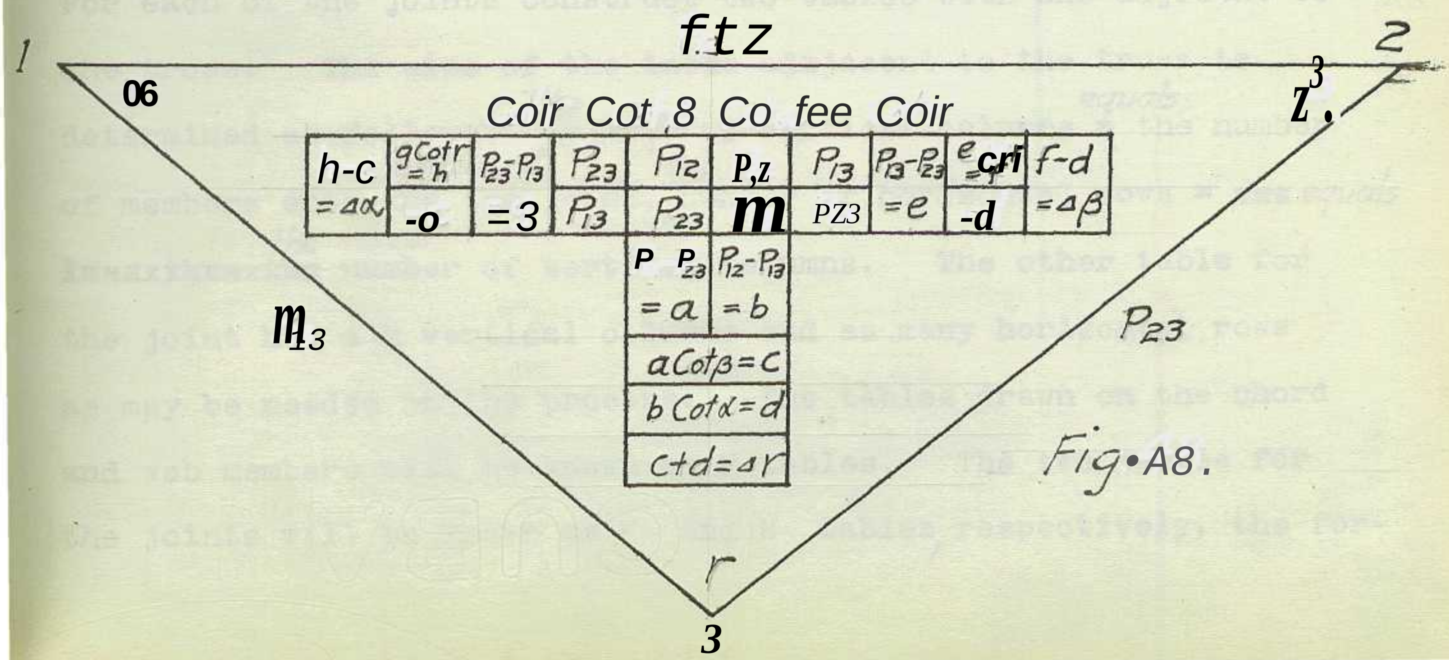


Fig. A8.

and Multiply each by the cotangents of the corresponding angles, which are written above the columns for the stresses P. The sum of these products, c and d, gives Af. Next find $P_{23} - P_{13}$ = e, and e Cot f = f. Add f to the negative of d is found. Similarly for . If f is a right angle, f and h = 0 and = -d and = -c. This is the case shown in the example below.

Article 4. Example. Details of Procedure.

Let the truss analysed in the previous chapters be used to illustrate this analytic method. See the Drawing, Plate IV.

On the upper portion of the drawing construct a small truss diagram to record the given data and to solve for the values of K. Construct another diagram with a scale as large as the drawing will permit. This diagram will be used for the analytic method. On each of the web members construct two small tables, one with 7 rows and the other with 3 rows, all having 3 columns and placed one above the other. In the first table extend the third and fifth rows one space to the right, as shown. For each of the chord members construct a small table of 3 columns and 6 rows with the second row extended one space to the right. For each of the joints construct two tables with one adjacent to the truss. The size of the table adjacent to the truss is determined as follows: The Number of vertical columns equals the number of members entering the joint, number of horizontal rows equals the number of vertical columns. The other table for the joint has 4 vertical columns and as many horizontal rows as may be needed in the process. The tables drawn on the chord and web members will be known as P tables. The two tables for the joints will be known as K- and M tables, respectively, the former applying to the one adjacent to the truss. The small tables below the F-tables for the web members will be known as the C-tables. Mark all the vertical columns with letters as shown in the first horizontal rows. For the F-tables these will be x, y and k and for the M-tables, S, x, y and k. For the K-tables the identification marks for all the members entering the joint should be noted except the reference member.

Next fill the K-tables with the values of K found in the small truss diagram. For example, at joint 5, the value of K_{B5} is put in the 2nd row and 2nd column, that of K_{C5} in the 3rd row and 2nd column, that of K_{D5} in the 4th row and 3rd column, and finally that of K_{E5} in the 5th row and 4th column. There will then be one column vacant at the right of K_{E5}. At joint 2, the value of K_{A2} with the sign changed is put in the 2nd row and 2nd column, that of K_{B2} with the sign changed in the 3rd ~~an~~ row and 2nd column, and finally that of K_{C2} with the sign changed in the 4th row and the 3rd column. One column is vacant at the right of K_{C2}. Similarly, all the K-tables may be filled with the values of K transferred from the truss diagram at the top of the drawing. Next there are performed the summations of the various K's as follows: Find the algebraic sum of the amounts in the second column and write the results in the 3rd column and 3rd row. This sum is next added algebraically to the amount below, the result being put in the 4th column and the 4th row. Similarly, this result may again be added to the amount below if there is any, and finally the space at the lower right hand corner of the table is filled. Thus, at joint 5, the sum of -.55 and -2.10 is -2.65, that of -2.65 and -.75 is -3.40 and that of -3.40 and .94 is -2.46. Similarly, for the joints at the top chords. It will be seen that the first figure in the 2nd column is the value of *ZK* for the first member met with in a clockwise direction around the joint from the reference member if the joint is at the bottom chord and in a counter clockwise direction if the joint is at the top chord. Similarly, the first figure in the ^rd column is the value of £ K for the 2nd member met with in the proper directions around the jointsj the first figure in the 4th column is that for the 3rd member, etc. The proper member's to which these sums apply are written down in the first row* Thus, at joint 5, the' figure -.55 is for member 5-2, -2.65 is for member 5-4, etc.

Above and below the tables constructed on the members, marked as F and G, there are recorded the values of U^s L/y where L is in feet and y in inches,' and S = l/y. For the web members tke values of 5 U should be put down at the side of U.

If the various sums in the K-tables are divided hy U, the constant term in Eq. (17) will be obtained for the respective members. As mentioned, before the values of r/5 are to be used for the web members and the summations of the K's should then be divided by 5 U of the web members. These values of £ K/U and *Z* K/ 2U are next recorded in the F-tables at the extended portions provided at the right. Thus for memberb2-5 the value of *Z* K/^U from the reference member 5-2 is $-.55/^{1}5*^{2l}-5$ = -.0?<5, and is put in the extended portion of the 5th row in the F-table for 2-5. fh: This is the increment for r^2. Similarly, for r_2<j the value of *TJ* K/ 3U from the reference member 1-2 is -2.04/15.43 • -.125 and is put in the extended portion at

the right of the 3rd row of the F-table for member 2-5. As another example the value of $\Sigma K/U$ for chord member 5-7 from the reference member 3-5 is -2,56/2.93=-.874. By the same process all the extended portions of the F-tables will be filled up with $\Sigma K/U$'s.

Now the solution of the problem begins. Let $f_{12} = x$, and $f_{21} = y$ and put the proper figures in the F-table for member 1-2. Subtracting the coefficients of x and y of f_{21} from those of f_{12}, $f_{12} - f_{21}$ is obtained. Reverse the sign of the difference and put it above the original sign. By using this reversed sign the difference $f_{21} - f_{12}$ is obtained. When f_{12} is added to $f_{12} - f_{21}$, r_{12} is obtained; if f_{21} is added to $f_{21} - f_{12}$, r_{21} is obtained. In other words r_{12} is obtained by adding f_{12} and $f_{12} - f_{21}$ with the lower signs, while r_{21} is obtained by adding f_{21} and $f_{12} - f_{21}$ using the upper signs. From the value of r_{21} complete the deformation contour around joint 2 as follows: Multiply the coefficient of x, -1, for r_{21}, by U = 4.38, giving -4.38. Divide this by 3 U = 13.55 and put the quotient -.324 in the column for x and row for $1/3\ r_{23}$ in the F-table for member 2-3. Divide -4.38 by 3 U = 15.43 and put the quotient -,284 in the proper space in the F-table for 2-5. Similarly, divide -4.38 by U = 3.03 giving -1.45 and put the result in the proper space for member 2-4. Since the rows for r's have an extended portion at the right there is no chance of recording the figures in the wrong places. It is only necessary to notice that for chord members these r's are put in the 2nd row and for the web members they put in either the 3rd or 5th row depending on whether the i joint is at the top chord or the bottom chord. By similar processes fill the columns for y in the F-tables for the members around joint 2. For values of the constant k the same process is to be pursued except that the figures are put directly on the top of the extended portions of the respective horizontal rows. Thus, k for 2-5 = k for 2-1 x 4.38/15.43 = 0, because k for member 1-2 is equal to zero. This figure 0 is then put on the top of the extended portion of the 3rd row.. Now add the figures in the extended portion to the one above and write the algebraic sum in the 3rd row in column k. Thus 0 - .133 = -.133 for member 2-5. Similarly, the constant term k is obtained for all the members around the joint and the contour for the joint is then completely determined by analytic expressions. By the same process complete the contour ~~the~~ around joint 1.

To find f_{13} from f_{12} apply the moment equation to joint 1. Multiply the coefficients of x and y and k of f_{12} by S_{12} and enter the product which equals the moment due to f_{12}, in the M-table at joint 1. This part is believed to be self-evident. Reverse the sign of the moment~~s~~ and enter the results in the row for member 1-3 whose S = 155. Dividing the moment~~s~~ of 1-3 by S_{13}, f_{13} is obtained. Enter the result~~s~~ in the 3rd row of the F-table for member 1-3. There are already in this table figures in the 2nd row representing r_{13}. Subtract f_{13} from r_{13}, giving $2f_{13} - f_{31} - f_{13} = f_{13} - f_{31}$. Thus, for coefficient of x, 2.995 - (-2.405) = 5.400. Next reverse the sign of $f_{13} - f_{31}$ and put the new sign below the old one. Thus for the coefficient of x, $\pm$5.400.is obtained. Next subtract $f_{13} - f_{31}$ from f_{13} or, using the lower signs, add $f_{13} - f_{31}$ to f_{13}. Thus for the coefficient of x, -2.405 - 5.400 = -7.805. This gives the value of f_{31}. Again subtract $f_{13} - f_{31}$ from f_{31}, giving $2 f_{31} - f_{13} = r_{31}$, or, using the lower signs of $f_{13} - f_{31}$, add f_{31} to it. Thus for the coefficient of x, r_{31} = -5.400-7.805 = -13.205.

From the value of r^i thus obtained a deformation contour around- joint 5 may be obtained by analytic expressions as explained for joint 2, obtaining thereby 1/5 $r^{\wedge}_2$ and r ^. , Now for member 25 there are already filled the values of 1/5 $r_P{}^*$ and 1/5 r^*_2 in the F-table, the difference of these two quantities may bext be obtained. This, as explained in Art, 1 of this chapter, is equal to fgz - f*2* These figures are to be filled in the 6th row, Next find the sum of 1/5 r_{25} and 1/5 r_{32} and put the result ia the 4th row. This quantity when added to that in the 5rd row gives fg- in the 2nd row and whan added to that in the 5th row gives f*2 in the 7th row. All the stresses in the first triangle are now completely determined it is necessary to apply the test of Eq. (18), Noting the direction of the contour by which the differences of the f's are taken, it will be seen that the figures in the 6th row of the F-table for 2-5> plus the figures in the 4th row of the F-table for 1-5> using the lower signs, plus those in the 4th row of the F-table for 1-2, using the lower signs, each multiplied by the corresponding value of U, should be zero. These-results are recorded in the C-table under the F-table for member 2-5*

Now around joint 5 the stresses in all members are known except that of 5-5 which can be found from the moment equation. This summation is performed in the' M-table for Jjoint 5. From f*= thus obtained and r•« already found, the F-table for member 5-5 may be filled as for member 1~5» From r^jj as obtained, complete the deformation contour around joint 5»

'In the F-table for 25, 1/5 $r_2{}^{\wedge}$ and 1/5$r^{\wedge}_2$ are known and it can then be filled as for member %-*>.ya As before the figures in the 6th row of the F-table for 2-5, and those in the 4th row of the F-table for 3-5, using the lower signs, each multiplied by the proper U's are next put in the first two rows of the 0-table. Note that U for member 2-5 should be used instead of 5 U. The last row in the table is copied from the 1st row of the 0-table for member 5-2 with the signs reversed. The sum of the figures in each column of the 0-table should then be zero.

By the same process the stresses in all members are obtained in terms of x and y in a linear equation. The two simultaneous equations derived from the fact that f^-j and fy^ should be zero then furnish the values of x and y required. From these values of x and y the stress f in any member may be found in the following manner. Fig. 49. On the side of the member where the fibre stress is obtained, that is, on the side first met with in passing around the end in a clockwise direction, the values

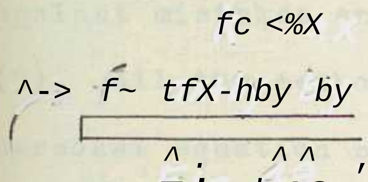


Fig. 43.

of the terms containing x and y are written one above the other after substitution. The constant k is next written on the side of these two figures toward the end of the member. Find the algebraic sum of the terms of x and y and write the result below k. Adding this to k algebraically the value of f is obtained, which is to be written at the end of the member.

Article 5. Construction of Influence Lines.

As mentioned before the coefficients of x and y are characteristic of the truss and do not undergo changes with the loading. Therefore for influence lines, the only change in the above method is to extend the various tables so that there will be room for each change of loading which means a change of k.

2M215S £• MATURES OF THE NEW METHODS AND COMPARISON.

Article V. Features Common .to all. Methods*

(1) No simultaneous equations involving *more* than two unknowns are required.

(2) All the methods deal with a quantity which is of the same dimensional degree as the secondary .stress itself. This gives more accurate results than when a quantity involving large multiples of f is used,

(2) AJ.1 the methods take less time than the ordinary methods* For a..six-panel truss with uniform loading they require only six to seven hours for the complete solution.

(4) There is at least one chieck for every method which insures against mistakes and errors.

(5) All the methods are reduced to a mechanical basis and no personal equation can affect the results.

(6) All the methods can be easily applied to influence lines of secondary stresses.

(7) All the methods are accomplished in a continuous process from one end of the truss to the other.

(8) In all the methods the complete information is contained on only one sheet of paper, including the rough computations, figuring, formulas and data.

(9) All the results can be easily duplicated by taking blue prints, which is not possible or at least inpracticable in ordinary computations made in tabular forms.

Article 2. Features of the Graphic Methods.

(1) They give a complete representation of secondary stresses in every member of the truss and convey the complete information in the least amount of time.

(2) The methods not only show stresses at the ends of the members, but also the variation of the stresses along their whole length.

(3) They give the points of inflexion and show the general form of bending of the different members.

(4) With the aid of the force diagram for the stress lines they show how the secondary stresses are affected by changing the section moduli of the members.

(5) The secondary stresses obtained by the graphic methods are never seriously in error although they may not be exact.

Article 3. Features of the Graphic Method of Deformation Contour.

(1) It is strictly graphical. From beginning to end no computations are required. No slide rules are necessary.

(2) It is the most accurate of the three methods if in the analytic methods slide rules are used with which only three significant figures can be obtained. This is due to the fact that in analytic expressions the secondary stress is found from the difference of two large quantities. These quantities must be very accurate since the difference is small. See equation 7, p.23.

(3) The errors made in locating the base lines and vertex lines are not cumulative as the effect of one line upon the other is comparatively small.

(4) The method is perfectly general and applies to any kind of

truss without any knowledge of the distribution of the stresses.

(5) Only a few construction lines need be retained on the drawing.

(6) It has the most unique checks.

Article 1. Features of the Graphic Method of Successive Deduction.

(1) It is the quickest method of the three if the assumed values of f_2 and f are nearly correct.

(2) It can be used to check the stresses computed by other methods with the least amount of time.

(3) It is good for experienced computers who can make close estimates at the start.

(4) It is the most direct method in that it gives the stresses in various members from the very beginning of the solution.

(5) The method is easy to remember.

Article 2. Features of the Analytic Method with the Scientific Arrangement of Computations.

(1) In bridge offices where a large number of standardized structures is to be analyzed for secondary stresses, the truss diagram together with the small tables can be blue printed, thus saving fully one-sixth of the time.

(2) This method is semi-graphical in that it shows the stresses in the proper places and the effects of one upon the other.

(3) The process is very mechanical. Every figure has a definite space in the table; every procedure has a definite order.

(4) No figures are recorded twice and no necessary figures are omitted.

(5) All the rough computations are shown on the diagram thus rendering possible a check at any future time.

(6) The figures on the diagram are so arranged that all the additions and subtractions do not involve more ~~tan~~ than two lines of figures and such computations can be made without the use of extra sheets of paper. All other computations can be made with the slide rule.

(7) The method is capable of affording any degree of acuuracy that may be desired.

(8) Practically all back references are eliminated.

(9) It gives a check of the secondary stresses as soon as they are found.

Article 6. Comparison of the Three Methods.

Briefly speaking, the graphic method of deformation contour and the analytic method are about equal in merit as far as the amount of work and accuracy of reault are concerned. For ordinary routine work in bridge offices the analytic method may appear best to the draftsmen while the graphic method will be preferred by experienced designers.b The graphic method of successive deduction is very satisfactory for ordinary trusses where an empirical formula can be ~~su~~ used; if this is possible this method takes the least amount of work.

PART IV. OTHER CONSIDERATIONS OF SECONDARY STRESSES DUE TO RIGIDITY OF JOINTS

CHAPTER I. EXACT METHODS.

Article 1. The Exact Method of Manderla, including the Effects of Primary Stresses.

In the methods of analysis discussed in the previous chapters the influence of the primary stresses has been neglected in deriving the deflection equations which appear as Eq. (3) of Part I and Eq. (4) of Part III. As a matter of fact these stresses produce bending moments along the axis of the member proportional to the deflection of the elastic curve, and, strictly speaking, should be considered as a part of the moments producing the flexure. Analyses made on this basis form the so called exact methods which are more accurate than the seven methods mentioned before. It is surprising to note that while Manderla's method, published in 1880, was the first adequate treatment of secondary stresses it was also the exact method in the above sense.

To begin with the exact method it is necessary to modify the deflection equations so that they will include the effects of the direct primary stresses.

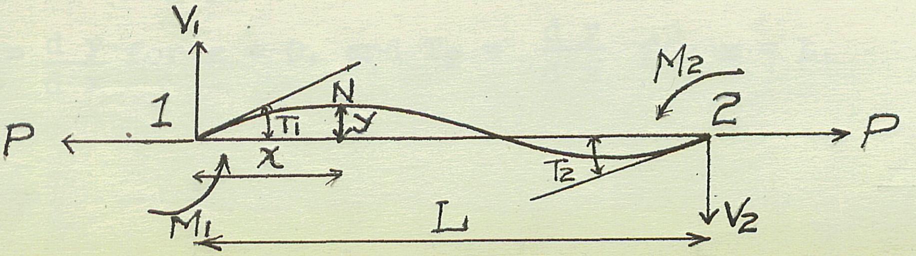


Fig. 50.

Fig. 50.

From the above figure the moments at any point N of the beam distant x from 1 is

$$M_x = M_1 - P y - V_1 x$$

where P is considered as tension but may also be considered as compression with reversed sign.

The differential equation of the elastic line is

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI} = -\frac{M_1}{EI} + \frac{Py}{EI} + \frac{V_1 x}{EI}.$$

Let $\frac{P}{EI} = Q^2$, a constant,

$$\frac{d^2y}{dx^2} - Q^2 y = -\frac{M_1}{EI} + \frac{V_1 x}{EI},$$

This is a linear differential equation with constant coefficients and may be solved readily by finding (1) the complementary function with $y = e^{mx}$, and (2) the particular integral by the method of undetermined coefficients. This gives

$$y = C_1 e^{Qx} + C_2 e^{-Qx} + \frac{M_1}{P} - \frac{V_1 x}{P}$$

where C_1 and C_2 are constants of integration and are to be determined by the conditions that for both x = 0 and x = L, y = 0. This gives

$$C_1 = -\frac{M_1}{P}\left(\frac{1 - e^{QL}}{e^{QL} - e^{-QL}}\right) + \frac{V_1 L}{P}\left(\frac{1}{e^{QL} - e^{-QL}}\right)$$

$$C_2 = \frac{M_1}{P}\left(\frac{1 - e^{-QL}}{e^{QL} - e^{-QL}}\right) - \frac{V_1 L}{P}\left(\frac{1}{e^{QL} - e^{-QL}}\right)$$

The slope of the ealstic line at any point I is

$$\frac{dy}{dx} = Q C_1 e^{Qx} - Q C_2 e^{-Qx} - \frac{V_1}{P}.$$

But $T_1 = \frac{dy}{dx}$ for x = 0, and $T_2 = \frac{dy}{dx}$ for x = L.

Hence, $T_x = Q\,C_1 - Q\,C_2 - \frac{M_1}{P}$

$T_2 = \ldots\, e^{-QL} - \frac{M_1}{P}$

Substituting the values of C_1 and C_2 in the above equations,

$$T_1 = -\frac{M_1}{P} Q \left(\frac{e^{QL} - 1}{e^{QL} + 1}\right) + \frac{V_1}{P} \left(\frac{2}{e^{QL} - e^{-QL}} QL - 1\right)$$

$$T_2 = -\frac{M_1}{P} Q \left(\frac{e^{QL} - 1}{e^{QL} + 1}\right) + \frac{V_1}{P} \left(\frac{e^{QL} + e^{-QL}}{e^{QL} - e^{-QL}} QL - 1\right)$$

The above may be transformed in terms of hyperbolic functions, thas,

$$T_1 = -\frac{M_1}{P} Q \tan h \frac{QL}{2} + \frac{V_1}{P} \left(\frac{1}{\sin h\, QL} QL - 1\right)$$

$$T_2 = -\frac{M_1}{P} Q \tan h \frac{QL}{2} + \frac{V_1}{P} \left(\frac{\cos h\, QL}{\sin h\, QL} QL - 1\right)$$

Eliminating V_1 from the above two equations, a relation is found between M_1 and the deflection angle T. This gives

$$M_1 = \frac{2EI}{L} (a T_1 + b T_2),$$

where,

$$a = \frac{1}{8}\left(\frac{QL}{QL \cot h \frac{QL}{2} - 2} + QL \cot h \frac{QL}{2}\right).$$

$$b = \frac{1}{4}\left(\frac{QL}{QL \cot h \frac{QL}{2} - 2} - QL \cot \frac{QL}{2}\right),$$

Replacing M_1 by M_2 and interchanging T_1 and T_2,

$$M_2 = \frac{2EI}{L} (a T_2 + b T_1)$$

The above values of a and b may be expressed in the forma of series by develop.ing

$$\cot h\, x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} - \ldots\ldots\ldots$$

Thus,

$$a = 1 + \frac{(QL)^2}{30} + \frac{11\,(QL)^4}{25\,000} + \cdots\cdots$$

$$b = 1 - \frac{(QL)^2}{60} - \frac{15\,(QL)^4}{25\,000} - \cdots\cdots$$

If P is compression instead of tension for which the above series are derived, Q^2 will be negative and the signs should be reversed for all the even terms in the above series:

$$a = 1 - \frac{(QL)^2}{30} + \frac{11\,(QL)^4}{25\,000} - \cdots\cdots$$

$$b = 1 + \frac{(QL)^2}{60} + \frac{13\,(QL)^4}{25\,000} + \cdots\cdots$$

Summarising, the fundamental deflection formulas in the exact methods of solution are as follows:

$$M_{12} = \frac{2EI}{L}\left(3\,a\,T_{12} + b\,T_{21} \right)$$

$$M_{21} = \frac{2EI}{L}\left(3\,a\,T_{21} + b\,T_{12} \right) \qquad \cdots\cdots \quad (1)$$

where

$$a = 1 \pm \frac{(QL)^2}{30} + \frac{11\,(QL)^4}{25\,000} \pm \cdots\cdots$$

$$b = 1 \mp \frac{(QL)^2}{60} \mp \frac{15\,(QL)^4}{25\,000} \mp \cdots\cdots$$

the upper signs are used for P = tension and the lower signs for P = compression.

By a similar method it can be found that

$$T_{12} = \frac{L}{EI}\left(2c\,M_{12} - d\,M_{21} \right)$$

$$T_{21} = \frac{L}{EI}\left(2c\,M_{21} - d\,M_{12} \right), \qquad \cdots \quad \cdots \quad (2)$$

where

$$c = 1 \mp \frac{(QL)^2}{15} + \frac{2\,(QL)^4}{315} \mp \frac{(QL)^6}{1575} + \cdots\cdots$$

$$d = 1 \mp \frac{7\,(QL)^2}{60} + \frac{9\,(QL)^4}{2520} \mp \frac{12[illegible]\,(QL)^6}{100\,800} + \cdots\cdots$$

the upper signs are used for P = tension and the lower signs for P = compression.

Comparing Eqs. (1) and (2) with Eq. (3) of Part 1 and Eq. (4) of Part III, it is seen that the effect of the primary stresses on the deflection equation is to introduce two infinite series as the coefficients of M, in which the the primary stresses enter in higher powers. If the primary stresses are assumed as zero, i.e., if the effect of the primary stresses ~~is~~ neglected, all the terms in the series after the first one will be omitted and ~~the~~ Eqs. (1) and (2) reduce to those derived by the previous methods. This assumption, it should be noted, introduces more errors in tension members than in compression members. But the bending moment along the elastic line is not necessarily a maximum at the two ends of a compression member due to its form of bending. The resulting moments found by the ordinary methods are therefore of the same degree of accuracy for all the members, compression or tension.

The method of computing secondary stresses by the exact analysis does not differ much from those described before, except that all the graphical methods could not be used to advantage. The additional labor required is of course very great but the increased accuracy secured by the process does not give sufficient justification for its general use. As far as practical engineers are concerned the exact method will never come into vogue, although its nature of analysis should be fully grasped for a better understanding of the generally accepted correct methods.

Article 2. The Exact Method including the Weight of the Member*

preceeding :
In «*i the abovo analyses of secondary stresses the members are not supposed to bear transverse loads distributed between their two ends. This, in fact, is not true because the weight of the& member itself and also the applied loads are a not concentrated at the joints as assumed. These transverse loads produce additional moments along the axis of the member and may be considred in the differential equations just as are the primary stresses. Let this moment at any point 1M of the aemfeer be M' and let its effects be included in the flexure equations (3)rfjfPart I. For the sake of simplicity the effect of primary stresses will not be considered here.

By a similar mathematical process it can be shown that

$$M_{12} = \frac{2EI}{L} (2 T_{12} + T_{21}) + \frac{2}{L^2}\int_0^L M' (L - 3x)\, dx$$

$$M_{21} = \frac{2EI}{L} (2 T_{21} + T_{12}) + \frac{2}{L^2}\int_0^L M' (3x - 2L)\, dx,$$

where M' is computed on the assumption that the members are simply supported. If the integrals containing M' are performed tha last terms of the above equations will be reduced to the moments which the transverse loads would pcoduce if the members are fixed at both ends. Let these moments for fixed ends be M_f'. Then

$$M_{12} = \frac{2EI}{L} (2 T_{12} + T_{21}) + M_f'{}_{12}$$

$$M_{21} = \frac{2EI}{L} (2 T_{21} + T_{12}) + M_f'{}_{21} \qquad \ldots \quad \ldots \quad (3)$$

If M_f' is due to the weight of the member it would be 1/12 wL^2, where w = the weight of the member per unit length. To include the effect of the primary stresses in (5) a oimilar process as similar to that in the previous article may also be used but it is so cumbersome and impractical that its use is not at all advisable.

CHAPTER II. APPROXIMATE METHODS.

Strictly speaking, all the methods described in Parts II and III are approximate ~~methods~~ in that the effects of the primary stresses are not considered. As this influence ~~are~~ is generally small the said methods are not far from correct and are generally accepted as exact methods. It is ~~upon~~ with this understanding that the methods to be described in this chapter are classified, that is, they are approximate only ~~in~~ with reference ~~with~~ to the methods described in Parts II and III.

There are four groups into which the approximate methods may be divided:

(1). Those in which the effects of the web members are neglected.

(2). Those in which the effects of the joints beyond a certain range are neglected.

(3). Those in which the solution of the simultaneous equations is approximated, and finally

(4). Those in which empirical formulas are used.

To the first class belong the graphic methods of Muller Breslau, ~~and~~ of Engresser and Landsberg and ~~also~~ their analytic solutions. The second class is described by Turneaure in his book on Modern Framed Structures. The third class is proposed by Mohr while the last one is suggested by the writer. On account of its bulk of material and also of its importance the first class will be treated in a separate chapter while the remaining classes will now be taken up. ~~presently~~.

Article 1. The Approximate Methods of Turneaure.

This method is useful in case the secondary stresses are required in only certain parts of the structure or in certain members only. It is here assumed that the effect of the joints that are remote from the joint or joints in question is negligible in magnitude and ~~could~~ can be disregarded entirely.

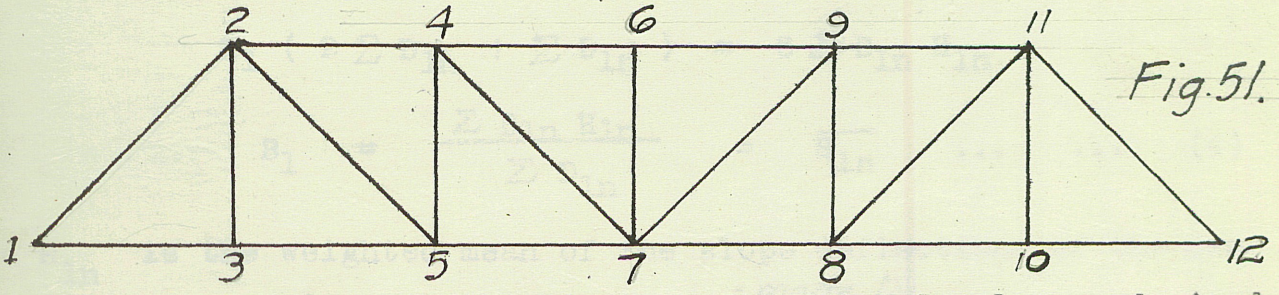


Fig.51.

For instance if the stresses at joint 5 only are desired, Fig.51. the joints beyond 2, 3, 4 and 7 may be neglected. In other words, only the members which meet at joint 5 are considered to have rigid connections at their ends, all the joints beyond the reach of these members being considered to have frictionless pins. This makes the moments around the last named joints ~~to~~ equal zero and greatly simplifies the solution of the simultaneous equations. This method gives results very nearly correct but its advantage is not obvious in small structures.

Article 2. The Approximate Method of Mohr.

It will be noticed from the simultaneous equations in Mohr's method that in each equation the coefficient of B for the joint for which the equation is formulated is very large compared with the other coefficients. It is further found from experience that the valuese of B for different joints of a structure are not widely different from each other and may be assumed equal for approximate solutions. These two facts furnish a solution which is remarkably simply and correct, as follows. Prom Eq. (6), Part II, the general form of the simultaneous equations is as follows:

$$2 B_1 \Sigma D_{1n} + \Sigma D_{1n} B_n - 3 \Sigma D_{1n} H_{1n} = 0$$

If in each equation B_n is assumed to equal to B_1, as the effect of variation of B_n is small compared with that of B_1, due to the magnitudep of the coefficients,

$$B_1 (2 \Sigma D_{1n} + \Sigma D_{1n}) = 3 \Sigma D_{1n} H_{1n}$$

$$B_1 = \frac{\Sigma D_{1n} H_{1n}}{\Sigma D_{1n}} = \overline{H_{1n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $\overline{H_{1n}}$ is the weighted mean of the slope deflections of the members meeting at joint 1, reference being made to the value of $D = I/L$.

By Eq. (4) the values, of B for any joint may be found as soon as the values of H for the joints are known. This reduces the analysis of secondary stresses to a simple arithmetical process and greatly enhances the practical value of Molarts Method. The writer has tested this equation in various occasions and has found it to be entirely dependable. See also Bulletin #1, Studies in Engineering, University of Minnesota, by G, A. Maney.

Article 5. The Approximate Method involving Empirical Formulas.

This method if perfected would be the quickest of all.

Unfortunately these is not much information available upon which to form a definite process. The one suggested by the writer in Chapter VI, Part III, is very useful for Pratt and Warren trusses but is not applicable to other structures. It is hoped that more light may be thrown on this subject by further investigations.

It is interesting to mention here the so called Patton's curve. This curve, deduced by Patton, shows the relations of the ratio of h/j and that of $(P + f)/P$ where P and f are the primary and secondary stresses. From a large number of trusses Eatton found by both calculations and measurements that the equation connecting these two ratios is a hyperbola of the form

$$n = \frac{a}{m - b} + c$$

where $m = L/y$, $n = (P + f)/P$, and a, b, c are constants.

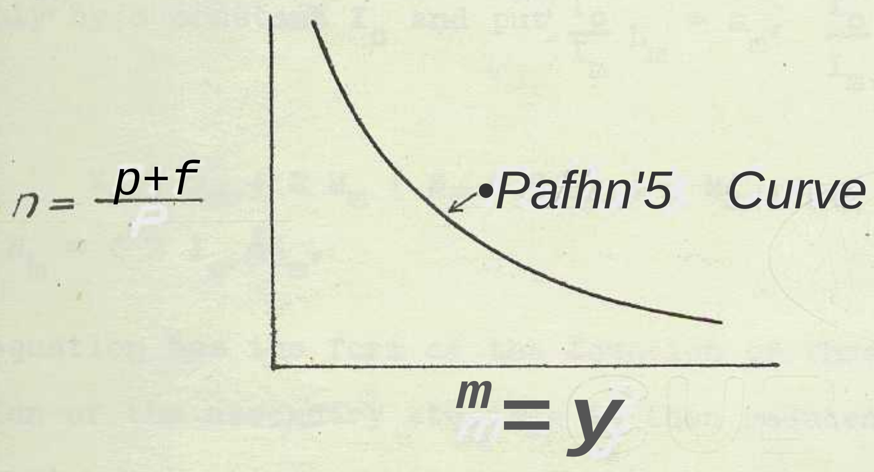


Fig.52.

CHAPTER III. THE APPROXIMATE METHODS BY THE THEORY OP CONTINUOUSBEAMS.

It has been found that for any member the secondary stresses are directly proportional to the moments of inertia. If the moments of inertia in the web members of a truss are considerably smaller than those in the chords, as they are in fact, the bending moments developed in web members may be neglected while considering those in the chords. This is the basis upon which the approximate methods in the present chapter are derived and it has been found justifiable in many practical cases. Consider a truss as shown in Fig. 53. Let T_{m-1} and T_{m+1} be the deflection angles of the members having lengths L_m and L_{m+1} respectively. From Eq. (2) in Part II,

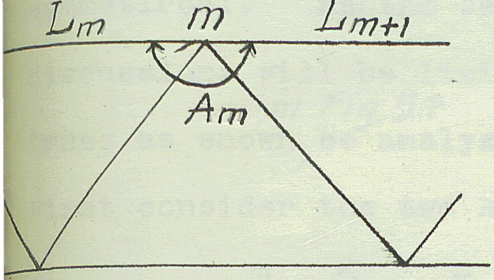


Fig. 53.

$$T_{m-1} + T_{m+1} = \delta A_m,$$

where δA_m is the change of angle of A_m or the sum of the angular changes of the vertices of the triangles meeting at m. Substituting the values of T in terms of M as determined by Eq. (3), Part I, there is obtained

$$M_{m-1}\frac{L_m}{I_m} + 2\,M_m\left(\frac{L_m}{I_m} + \frac{L_{m+1}}{I_{m+1}}\right) + M_{m+1}\frac{L_{m+1}}{I_{m+1}} = 6\,E\,\delta A_m$$

Multiplying by a constant I_c and putting $\frac{I_c}{I_m} L_m = S_m$, $\frac{I_c}{I_{m+1}} \cdot L_{m+1} = S_{m+1}$;

then,

$$M_{m-1}\,S_m + 2\,M_m\,(S_m + S_{m+1}) + M_{m+1}S_{m+1} = N_m \quad \ldots \quad (5)$$

where $N_m = 6\,E\,I_c\,\delta A_m$,

This equation has the form of the Equation of Three Moments. The solution of the secondary stresses is then reduced to that for the moments in a continuous beam. There are three methods based on this equation:

(1) The graphical method of Muller Breslau,

(2) The graphical method of Engresser and Landsberg,

(3) The analytic solution.

Article 1. The Graphical Method of Muller Breslau.

Muller Breslau considered three cases for the solution: (1) where there is at least one joint where M = 0, (2) where there is not joint where M = 0 but the truss and loading are symmetrical, and (3) where there is no joint with M = 0 and the truss is not symmetircal. As the second case is the most general the following discussion will be limited to that case of treatment only. Let the truss as shown in Fig. 54 be analysed for secondary stresses by this method. First consider the two joints of symmetry, 7 and 6.

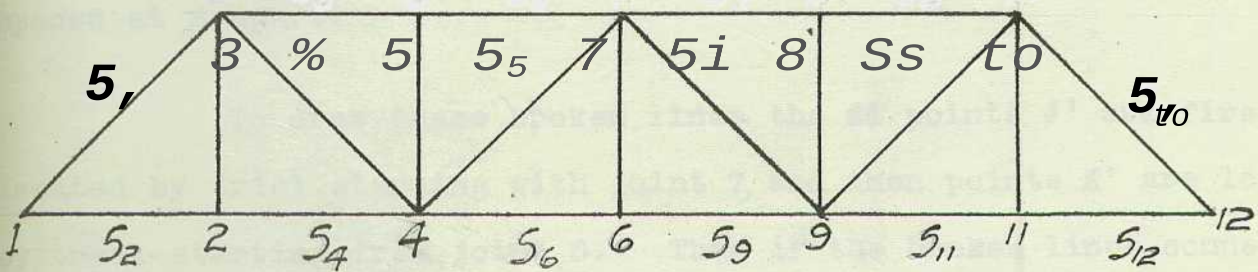


Fig. 54.

By Eq. (5), $M_5 S_5 + 2 M_7 (S_5 + S_7) + M_9 S_7 = N_7$,

Since $M_5 = M_9$, $S_5 = S_7$ by symmetry,

$$\frac{1}{3} M_5 + \frac{2}{3} M_7 = \frac{N_7}{3(S_5 + S_5)} = X_7,$$

where $X_7 = \frac{2 E I_c \, dA_7}{(S_5 + S_5)}$

Similarly for joint 6,

$$\frac{1}{3} M_4 + \frac{2}{3} M_6 = \frac{2 E I_c \, dA_6}{S_6 + S_6} = X_6.$$

Since X can be computed for all the joints of the truss, the process is then to so determine M that the above equations and Eq. (5) are satisfied by all the joints. To accomplish this graphically Muller Breslau made use of a moment diagram as shown in Fig. 55

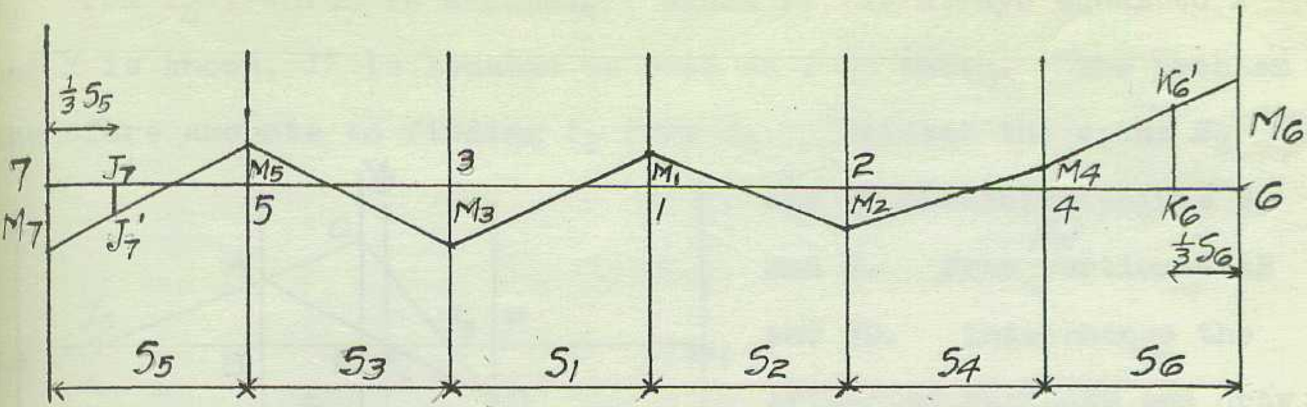


Fig.55.

in which the a ordinates to the broken lines give the moments at the various points. The joints from 7 to 6, following the outline of the chords, are here considered are to lie on a straight line and spaced at S apart.

To draw these broken lines the 34 points J' are first located by trial starting with joint 7, and then points K' are located by trial starting from joint 6. Then if the broken lines connecting the points K' coincide with those connecting the points J', these broken lines will then give the moment diagram required. The method of constructing the J' and K' polygons are the same; the following descriptions for the J' polygon serves at the same time for the K' polygon.

The point J_7 is first located by taking its distance from 7 equal to 1/3 S_5, for then J_7J_7' would equal $\frac{1}{3}M_5 + \frac{2}{3}M_7$. Since that sum is equal to X_7 and is known, the point J_7' is then fixed. If the moment line for 53 is known, M_7 could be determined from J_7J_7'. But since the moment line 53 is not known unless J_5' and the moment line 31 are known, it is therefore necessary to find J_5' and J_3'. The latter determines the moment line 31 in conjunction with the moment line 212. Since both J_5' and J_3' are unknown one must be assumed and corrected later by the K' polygon. The following example shows how to find J known' ng.dv.

J_3 ¹ from Jgf, which is assumed^ Since JJ^f is always equal to X and X is known, J* is located as soon as J is known. The problem therefore amounts to finding J_3 from J_R. Trisect the spans S* and Sn obtaining points B and F.< Draw the verticals AB and FD. Interchange the trisected segments and draw the verticals VgE. From the assumed position of J_5 draw any line JgC cutting the vertical AB at A and V^&E at C. Through A and 3 draw a line cutting FD at D. Join CD. This line then gives the point J_3.

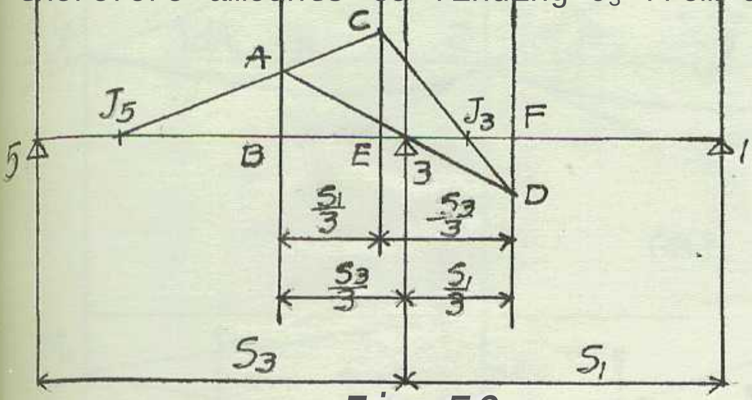


Fig.56%

By a similar process the points J^f on each side of the joints are found and a J' polygon could be drawn through these points, raB-e ali-fcfee *ay& -hRke^&eekiiag-ak with the adjacent sides intersecting on the verticals through the supports. Similarly a K^f polygon could be drawn beginning with joint 6. These two polygons are next made to coincide by correcting the assumed positions of J and K, the resulting polygon then gives the moment diagram from which the moments at various points may be determined.

<u>Article 2,</u> <u>The Graphical Method of Engrosser and Landsberg</u>.

This method is based on Mohr's construction of elastic curves by considering the moment diagram as load. Consider the chord members' as being cut at the joints and laid out with their end points touching a straight line. By EJJ. (2) of Part II the end tangents of -tike two consecutive lines at any joint would then intersect at an angle equal to δA for the joint. In ~~the figures~~ Fig. 57

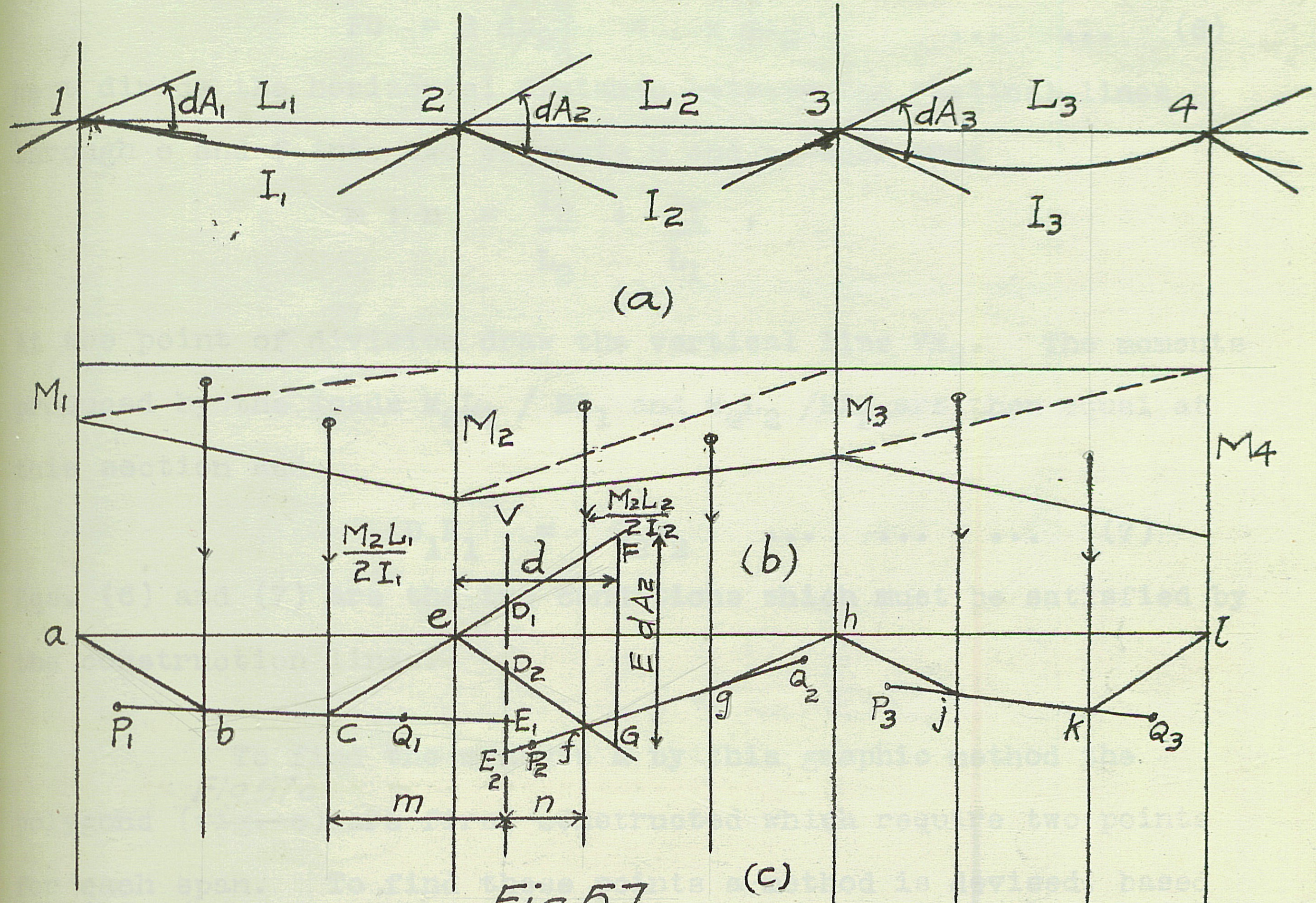


Fig. 57.

~~above~~ the connecutive chord members 12, 23, and 34 are shown with their ends cut. The moment diagram for each of the spans is shown in ~~Fig. (b)~~ Fig. 57b. These diagrams are of course assumed as the moments are yet unknown. Divide each diagram into two parts and through the center of gravity of eahh draw the vertical lines representing the load lines of ML/2I. Let a force diagram be drawn ~~of~~ for these loads and construct the equilibrium polygon a b c e, etc. with a pole distance equal to d. The lines e c and e f then represent the directions of the end tangents for the elastic curves 12 and 21 at joint 2. The angle included between these two lines is therefore a measure of the change of angle δA_2, in fact it equals $\delta A \frac{E}{d}$. Draw a line pG at distance d from joint 2 cutting the lines e c and e f at F and G. Then,

$$PG = d \,<£A_2| = E <£A_2 \quad . \quad . \quad . \quad . \quad (6)$$

Next divide the horizontal distance between the vertical lines through e and f into two segments m and n, such that

$$m s n = \frac{£§}{L_2} : \frac{I_i}{L_l},$$

At the point of division draw the vertical line VE_g. The moments produced by the loads $M_pL_n / 2E_1$ and $M_QL_0 /2I_0$ are then equal at this section and

$$D_1E_1 = D_2E_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Eqs. (6) and (7) are the two conditions which must be satisfied by the construction lines.

To find the moments M by this graphic method the *Fi$.57c* polygons (figi—e) are first constructed which require two points for each span. To find these points a method is devised, based on Eqs. (6) and (7), by which if one point is known for one polygon the corresponding points are known for all the polygons. Thus a point may be assumed for^the first span from which the corresponding point is found for the last span which, in the truss, is adjacent to the *the* first span. From the point in the last span a corresponding point could be found for the first span a«*i which should coincide with the point assumed if the assumed point actually lies on the polygon. If not several' trials may be made until they coincide. By a similar process two sets of points may be found which completely determine the polygon of £4rg*—f^).

The following method shows how to locate Pg in one span when P_1 in an adjacent span is known. It is here assumed that the line VE_2 and FG have already been drawn.

Through P_1, the given point, draw any arbitrary line

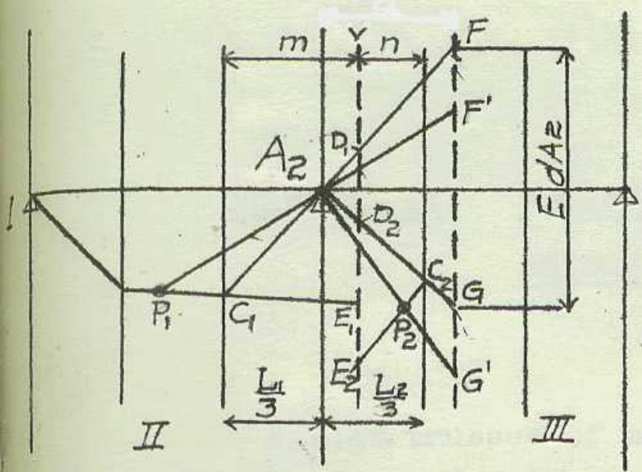


Fig.58

$P_1C_1E_1$. Join C_1A_2 and prolong it to cut VE_2 at D_1 and FG at F. Measure off FG = $E\,dA_2$, thus determining point G. Join A_2G cutting VE_2 at D_2. Find point E_2 on VE_2 by making $D_1E_1 = D_2E_2$. Join E_2C_2.

Through P_1 draw a line through A_2 cutting FG at F'. Lay off F'G' = FG. Join A_2G' cutting the line C_2E_2 at P_2. The point P_2 is then the point required corresponding to P_1 in span II.

It will be seen that if C_1E_1 revolves about P_1, C_2E_2 will revolve about P_2. P_2 is therefore fixed if P_1 is fixed.

Fig. 59.

To find any moment M, say M_2, it is simply necessary to draw a vertical line at a distance d from C_1 and consider KL as the static moment of the force $\frac{M_2L_1}{2I_1}$ acting at C_1. Since the pole distance is d,

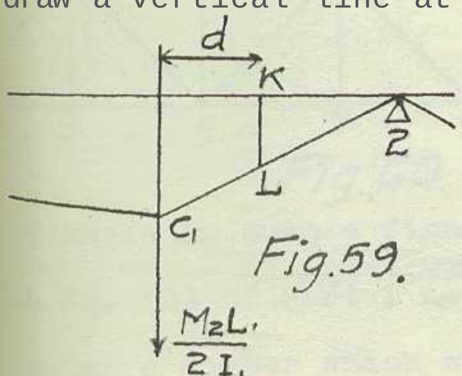


Fig.59.

$$\frac{M_2L_1}{2I_1}\,d = \overline{KL} \times d,$$

Therefore, $$M_2 = \frac{2I_1}{L_1} \times \overline{KL},$$

and $$f_2 = \frac{2I_1}{L_1} \times \overline{KL}.$$

Article 5. The Analytic Solution.

This is ~~nothing~~ only but ~~to~~ aHHum*- ^ = 0 for all the web members in the usual methods described in Parts II and III. Since the simultaneous equations take the form of 4k Equation of Three Moments Olapeyeron's method of solution may be applied to advantage in preference to the method of Gauss,

CHAPTER IV, SECONDARY STRESSES IN SPECIAL FORMS OF BRIDGE TRUSSES.

Bridge trusses of SHsnal- unusual types require special consideration* in the analysis of secondary stresses. They offer difficulties both in the methods and details of computations, which must be removed by judici-al changes in the procedure* Take, for example, a bridge truss having sub-panels as shown* in Fig. 60. The figure 4698 is a triangle when the bridge is unloaded but it reduces to a 4-sided figure as soon as the members are under stress, as then the members 46 and 69 are no longer on the same straight line. To find the change of angle in such a figure, therefore, requires a special procedures, as Eq. (2) of Part I can be used Ui g»©d. only for a triangle. In this case assume a member which would divide this figure into trian-gles, as shown by dotted line, 68. The figure is then reduced to two triangles each of which, may then be submitted to the usual formulas. To find the change in length in this imaginary member 68 it is only necessary to compute the deflection of one point as 6, relative to the

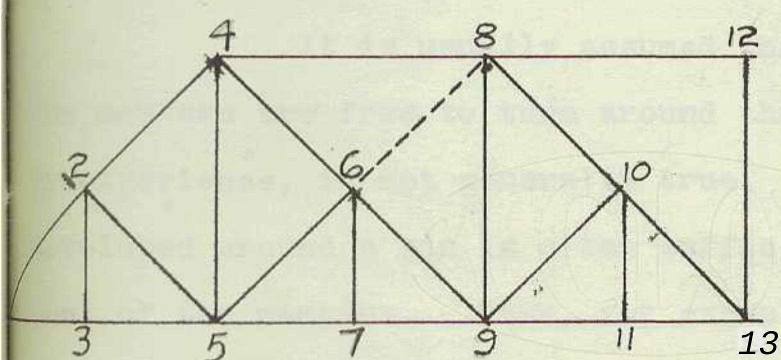


Fig. 60.

other as 8, the sectional area of the member being assumed zero.

Further, in analyzing this kind of truss by the method of Muller-Breslau or that of Chapter IX, Part III, difficulties will be encountered in finding the moments in members 46 and 56 from that in 45. In this case let one of the moments be represented by an unknown x and proceed by the above methods until the moments in all the members in panel 59 are computed. The moment equation around joint 6 would then furnish the unknown x required.

There may be other instances of the same or similar nature but in every case there is always a solution. So long as the truss is composed of triangular elements there is no indeterminateness in the field within which the methods considered before are confined.

CHAPTER V. SECONDARY STRESSES IN PIN CONNECTED TRUSSES.

It is usually assumed that in a pin connected structure the members are free to turn around their joints. This fact, as shown by experience, is not generally true. On the other hand the friction developed around a pin is often sufficient to prevent the free movement of the members. Take, for example, the pin and eye bar as shown in Fig. 61. When the stress P is of such a magnitude as to cause turning around the pin its displacement would be r and moment P x r. This moment is also equal to that developed by the friction F, which equals the normal pressure on the pin multiplied by the coefficient of friction. If A is the angle of friction, F = P sin A. Therefore,

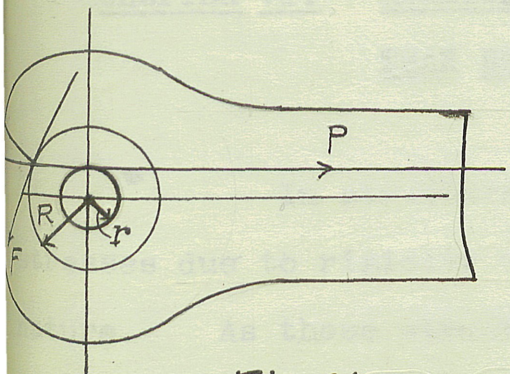


Fig.61.

$$P \times r \quad = \quad P \sin A \times R,$$

Or, $\qquad r = R \sin A.$

This equation shows that if the ~~diameter~~ radius of the pin is greater than the quantity $r/\sin A$, the friction moment would be greater than the applied moment and the member connected to the pin could not turn at all. In such a case the pin joint would be about as rigid as a riveted joint and the secondary stresses developed in such members could not be much less than those calculated by the assumption of rigid joints. In fact if the diameter of the pin is made 3/4 the width of the bar, as is usually required, the secondary stress cannot be less than 45% of the primary if the coefficient of friction is taken as 0.2, which is a fair value.

In case the members are built up of sections instead of eye-bars, the ratio of diameter of pin to the width of member is much smaller than for the eye bars and the friction moment developed around the pin ~~are~~ is generally sufficient to make the members turn. In this case the friction moment could be treated as an eccentric moment in the analysis of secondary stresses. If any member is supposed to turn freely around a pin, its effect on other members is then null and it may be considered ~~as~~ to have a zero moment of inertia.

CHAPTER VI. ANALYSIS OF SECONDARY STRESSES IN STRUCTURES OTHER THAN BRIDGE TRUSSES.

In structures other than bridge trusses the secondary stresses due to rigidity of joints are generally of a different nature. As these structures are mostly of rectangular forms, the secondary stresses are essential for the stability of the structure.

Take a rectangular portal frame, Fig. 62, for instance. If loads are applied on the top AB the whole frame will deform until the joints A and B come into some such positions that equilibrium is maintained throughout the frame without, at the same time, altering the angles at the corners. The stresses thus developed in the posts and struts are necessary to support the loads because if they are absent,

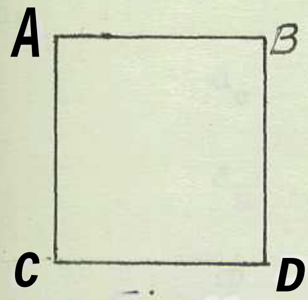


Fig. 62.

i.e. if the joints are pin connected, the frame would collapse with the application of the load. This kind of stresses is therefore more important than those in structures composed of triangular forms as an exact knowledge of the stresses is necessary to design the frame.

As it is not the intent of this paper to give a more than passing note of structures other than the bridge trusses, only the general method for the solution of Rectangular frames will be given.

Let this frame, Fig. 63, be cut at a convenient point as E and replace the insternal forces by the thrust H_o, shear V_o and moment M_o. Assuming one end of the broken frame as fixed the deflection of the other end due to the load with respect to this end may be found as follows:

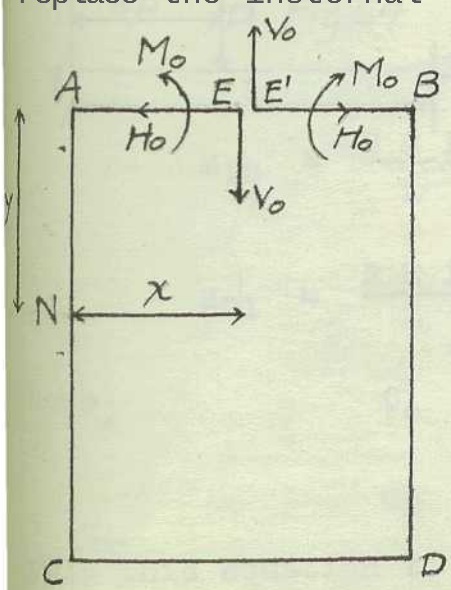


Fig. 63.

Let M' = moment at any point E of the frame due to external load,

M = same due to all the forces,

$= M_o - V_o x + H_o y - M'$,

d_v, d_h = vertical and horizontal deflections of E with respect to E'

ϕ = a Angular rotation of tangent at E with respect to tangent at E',

Then,

$$d_v = \int_E^{E'} \frac{M\,x\,dx}{EI} = 0,$$

$$d_h = \int_E^{E'} \frac{M\,y\,dx}{EI} = 0,$$

$$\phi = \int_E^{E'} \frac{M\,dx}{EI} = 0,$$

The solution of the above simultaneous equations furnishes the three unknowns required, from which the moment M at any point N may be determined.

For structures with a multiple number of rectangles the above method becomes very cumbersome and the need of some simpler method is very evident. The following is a solution based on the method of Mohr and developed by G. A. Maney in Bulletin #1, University of Minnesota.

It has been shown that if a beam 12, Fig. 64, besides being subjected to the moments at the two ends, has also concentrated loads P at kL from 1 and uniform loads ~~at~~ of w per unit length, then

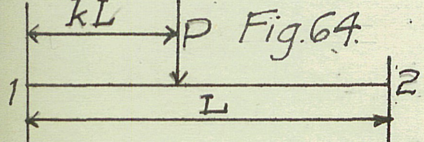


Fig.64

$$M_{12} = \frac{2\,E\,I}{L}\,(\,2B_1 + B_2 - 3\,H\,) + \frac{2\,C_2 - C_1}{3}$$

$$M_{21} = \frac{2\,E\,I}{L}\,(\,2B_2 + B_1 - 3\,H\,) + \frac{C_2 - 2\,C_1}{3}$$

Where,

$$C1 = P\,(\,2k - 3k^2 + k^3\,)\,L + \frac{w\,L^2}{4}$$

$$C2 = P\,(\,k - k^3\,)\,L + \frac{w\,L^2}{4}.$$

Apply-ing this equation to all the members meeting at a joint, there will be obtained as many p equations as the number of joints and the unknowns M may be found by solving ihe equations. In buildings the axial deformations of the members are generally small and may be neglected compared with the deflections of the joints due to applied forces as wind. The slope deflections of the horizontal members are then equal to zero and for all the vertical members in one story are equal to each other, This requires one equation for each story in order to find the common value of H, which is furnished by equating the moments in each story due to external and internal forces.

PART V, SECONDARY STRESSES DUE TO OTHER INFLUENCES THAN THE RIGIDITY OF JOINTS.

All the discussions in the previous chapters are limited to only one kind of secondary stresses which, of course, is the most important. There are, however, other sources of secondary stresses

prove of if not more

which may oxhibit^ ju&% as much importance. i# «ei -mes^e. Of a the many kinds of stresses mentioned in the following chapters only few are susceptible of mathematical analysis and even then they are not as

analyzed

completely as are the stresses due to rigidity of joints. Further investigations in this field, both theoretically and experimentally, are therefore very welcome.! The stresses to be discussed in this

andare

paper are divided into four classes^ as follows:

(1), Bending stresses in a transverse frame due to primary stresses in the Posts,

(2)# Stresses in a horizontal plane due to unequal deformations in chords and stringers,

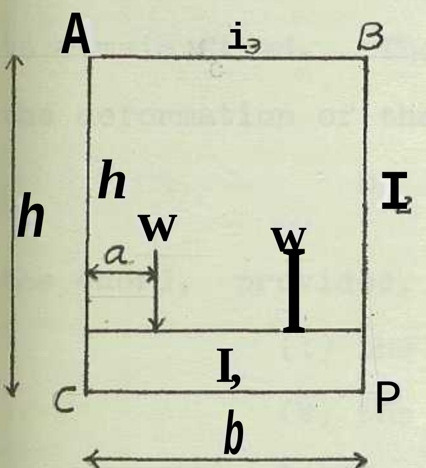
(5). Bending stresses due to variation of axial stresses in different elements of # members, and

(4), Torsional stresses due to various combinations of secondary stresses both from known and unknown sources.

CHAPTER I, BENDING STRESSES IN A TRANSVERSE FRAME DUE TO PRIMARY STRESSES IN THE POSTS.

Properly speaking^ these stresses being due to rigidity iof joints may also be classified as such under some headings in Parts I-IV. It is found advisable, however, to adopt the pEesent form of arrangement as the method of procedure in this case differs entirely from ^those considered before*

The exact solution of the problem requires the use of the three equations considered in the last chapter, but as the ILabor involved tfeor-ofor is generally unwarranted an approximate solution may be made by assuming the posts to have been connected by hinges to the transverse overhead bracing. The following formulas are then obtained:



M_o =s Moment at a, Fig.65,

$$- W \frac{5a\,(b - a)\,\%2}{2\,h\,I_x + 3\,b\,Ig}$$

If f_1 and fg are the secondary stresses in the beam CD and post CA at C and o-, and op are their respective widths,

$$f_o \;-\;.\;¥\;g\;\frac{»<\;>\;-\;*.)\;08}{3\,h\,Ii + 3\,b\,Ig},$$

Considering I_2 as negligible if the beam

Fi.g.65. -)ng

is very deep and tak«-a = ,3 b,

$$f_2 = W\,\frac{3 \times .3b^2 \times .7\,c_2}{2\,h\,I_1} = W\,\frac{.3\,b^2\,c_2}{h\,I_1}$$

If f_w is the working stress in the floor beam, $f_w = W\frac{a\,c_1}{I_1} = W\,\frac{.5\,b\,c_1}{I_1}$

Therefore,

$$\frac{f_2}{f_w} = \frac{b}{h} \times \frac{c_2}{c_1},$$

that is, the stresses in the ~~baemams~~ beam and posts of a transverse frame in a bridge truss are directly proportional to their widths and indirectly proportional to their lengths.

CHAPTER II. STRESSES IN A HORIZONTAL PLANE DUE TO UNEQUAL DEFORMATIONS OF CHORDS AND STRINGERS.

The floor system of a truss bridge generally consists of stringers riveted to the webs of the floorbeams. When such a spam is loaded the chord members of the truss deform longitudianally but the axial length of the stringers undergoes little or no change. If the stringers are riveted continuously from end to end of the span the floor beam must bend horizontally thereby producing secondary stresses. Consider a floor beam situated at a distance d from another beam assumed to remain fixed. Then the horizontal deflection of this beam due to the deformation of the chords is

$$D = \frac{P\,d}{E},$$ where P = average unit stress in

the chord, provided,

(1) the axis of the stringer does not elongate,

(2) the stringer connections are unyielding, and

(3) the ends of the floor beams remain vertically over the joint centers of the chords.

Now the deflection in the beam, assuming the ends hinged, is also equal to

$$D = \frac{M\,a}{6EI}\,(3\,b - 4\,a),$$

where a is the distance from the centre of the truss to the nearer stringer and b the length of the beam;

hence, $\therefore \quad \frac{M\,a}{6EI}\,(3b - 4a) = \frac{P\,d}{E}$,

Replacing M by fy/I, where y= the half width of the flange,

$$f = \frac{6\,c\,d}{a\,(3b - 4a)}\,P,$$

The values of a and b are generally constant so that the value of f is proportional to the width of the flange of the beam and also to its location from the centre of the span. On account of the various assumptions made in the formula the actual ~~a~~ stress is probably not as great as computed but the importance of this source of secondary stress cannot be overlooked, especially in bridges of unusual spans.

CHAPTER III. BENDING STRESSES DUE TO VARIATION OF AXIAL STRESSES IN DIFFERENT ELEMENTS OF A MEMBER.

If the primary stresses in different elements of a member are unevenly distributed, the difference of the stresses will cause bending moments in either a horizontal or a vertical plane. These are largely due to the improper design of connections and errors in shop length in manufacture. To analyze these stresses, theoretical considerations are of little avail as the data required in these computations would never be complete. Turneaure in his book on Modern Framed Structures noted that a variation of stress of 5 to 10 % in riveted member and 10 to 20 % in eye-bars ~~are~~ is not uncommon. As these variations generally occur in a horizontal plane the bending stresses resulting therefrom will give torsional stresses ~~together with~~ in addition to the bending stresses in the vertical plane of the truss.

CHAPTER IV, TORSIONAL STRESSES DUE TO VARIOUS COMBINATIONS OF SECONDARY STRESSES.

If any of the previously mentioned secondary stresses occur at the same time and in different planses torsional stresses will be produced. These stresses will be very difficult to compute if the effects of the several bending moments are to be considered simultaneously, to say nothing of the complications involved in ascertaining the nature of the bending moments. Some of the bending stresses may come from unknown sources while there are others which act in planes of unknown directions. The mathematical solution of thid problem is therefore almost impossible.

PART VI. SECONDARY STRESS: A FACTOR IN DESIGN.

CHAPTER I. INTRODUCTION.

While the importance of secondary stresses is still a moot question at the present time, it cannot be denied that great improvements have been made in the design of structures as a result of the studies and observations made on the subject during recent years. The achievements so far attained are very beneficial and convincing although it is not uncommon to hear it said that secondary stresses accompany good design, and it makes the structure rigid and stiff which is highly desirable. To be sure, a good design does aim at a rigid structure but certainly not at the expense of its strength. Structures may be of different types; some are rigid, others are strong, and still others are both right and strong. A bridge may be designed and detailed in various ways; some make the structure rigid, others make it strong, and still others make it both rigid and strong. Would it be logical to consider only rigidity in a structure--in the selection of types, propertioning of memebers, and designing of details--and neglect its effects on the most vital factor in the design--the strength?

The consideration of secondary stresses in a design tends to make the structure both rigid and strong, not only in type and design but also in details. Eventually the design is also economical as the uncertain "factors of safety" may be greatly reduced.

It is for this reason that the secondary stress is so important in design and has been so widely considered. It must be admitted, however, that the subject is a difficult one-not in the thgoreti ical analysis of course, but in its practical application. Oftentimes a design made to satisfy secondary stresses violates very decidedly other good principles which must be respected. In other cases, the gain in reduced secondary stresses does not ba&lance the iosa of impaired rigidity. Here good judgement must be exercised to determine which course to pursue.

Before going into the details of design some general conclusions about the secondary stresses will be given.

(1) The secondary stresses are, in general, propor- tional to the primary stresses and,ttherefore, are conveniently expressed in percentages of primary stresses.

(2) Other things being equal, the percentages of secondary stresses are proportional to the distance from the gravity axis to the outer fibre in the plane of bending, and in- versely proportional to the lengths of the members.

(5) The secondary stresses in any member depend on the distortion of all the members of the truss, but primarily upon the distortion of members of the triangles of which this member is a part•

(4) A design in any individual member cannot be Jlianged without affecting the secondary stresses in the other mem- bers.

In the following eteapters are given some principles of desicm which must be observed in reducing secondary stresses.

These are largely derived from theoretical studies and practical observations and are very valuable on that account* During the past few years as increasing attention was paid to secondary stresses considerable space in books and periodicals has been devoted to the design of structures from this point of view. This valuable information, however, is not within easy reach, as the literature is scattered and the subject has never been systematically treated. As one of the purposes of the present paper is to present the facts about secondary stresses which must be understood by the designer, these principles of design will be listed under proper headings and discussed and disgested in a logical order. For each principle stated the source of information is accredited by using a letter (explained in the following key) &o indicate the title of the work. This is followed by a number referring to the page. These principles are next followed by a chapter on the correction of secondary stresses—a matter which is receiving increased attention.

Key

a 4 m proceedings of the American Railway Eng. Ass'n, 19^4, p. 438

a 5 B i' . » " " " n u 1916, p. 129 ,

e = Engineering News Eecord.

g = Grimm'a Secondary Stresses in Bridge Trusses

j = Merriman and Jacoby's Roofs and Bridges, Part III,

k - Kunz's Design of Steel Bridges

M • Thesis by T. E.Mao presented*to Cornell University

m « Molitor'* Kinetic Theory of Engineering Structures

p * Secondary Stresses in Framed Structures by Pitman in Proceedings of Engineers' Society of Western Penna., Vol. 25•

R = Notes on Design by Reichmann in Journal of Western Society of Engineers, Vol. 17,

TII = Modern Framed Structures, Part II,

TIII = " " " Part III,

T = Thayer's Structural Design,

W = Waddell's Bridge Engineering,

W = Effects of Secondary Stresses on design by Wilson in the Journal of Western Society of Engineers, Vol. 21,

S = Stress Measurement on the Hell Gate Arch Bridge by Steinman in Transactions of the American Society of Civil Engineers, Vol. 82.

CHAPTER II. SELECTION OF TRUSSES.

(1). In choosing between different styles of trusses, those of the statically determinate class should always receive preference, other things being equal. The primary stresses will usually be less than in similar indeterminate systems, especially when the temperature stresses are included. Also the deformation and secondary stresses may be less and frequently the connections will be simpler than for the indeterminate ones. (T 268)

(2). Among the statically indeterminate structures the two hinged arch is especially immune from secondary stresses. (S 1840, W 497)

(3). The secondary stresses in continuous trusses are very large at the centre and in the neighborhood of the end supports; for this reason pins at these points appear to be advisable in order to reduce the stresses. (S 129)

(4). The best modern practice in bridge engineering does not countenance the building of trusses having more than a single system of cancellation. (W 271). The secondary stresses in such a system are generally very high, often reaching 100% of the primary (TII 490), as the distortion in truss members due to loads ~~at~~ on one system only is very great. (t 235, g 129).

(5). In double triangular trusses the secondary stresses could be greatly reduced by the insertion of verticals connecting every pair of upper and lower joints. These verticals effectively connect the two single systems. (TII 491, g 123).

(6). Trusses consisting of approximately equilateral triangles, and without hangers or vertical struts, present the most uniform condition and will have, in general, the lowest secondary stresses. (TIII 93, a4).

(7). A truss composed of right-angled triangles will show somewhat higher secondary stresses, and such stresses will be large if the ratio of height to panel length is large. (TIII 94, a4).

(8). The truss systems should be as simple as possible and all members which make the stress distribution uncertain should be avoided. (k 169).

(9). As far as the elimination of secondary stress is concerned, the Pratt truss, ~~and~~ the Warren truss with verticals, and the K-truss are very desirable. (W 200).

(10). The amount of secondary stress in ordinary Pratt

(17). The use of collision struts cannot be recommended, because they divide the end posts into two fragments, decreasing the ratio between length and width of these members, pulling the post out of line and increasing the secondary stresses. The collision struts are as a rule rather weak members and it is a ~~w~~ question whether it would not be better to use the material on the end posts instead of on these struts, thereby increasing their strength in the direction of the plane of the truss as well as ~~also~~ at right angles to their plane. (g 130, TII 458).

(18). The double triangular truss with sub-panels is satisfactorily so far as the action of the secondary stress is concerned. (TII 495).

(19). A truss with polygonal chord as the parabolic truss or the Schwedler truss, is a good selection. (g 129).

(20). In a truss aim to make the curve of deflection an approximate circle with vertical members radial. This means uniform stress in chords and low stresses elsewhere. (t 235).

(21). Curved members whose neutral axis is not straight before the application of the load, should never be tolerated under any circumstances. (W 272, g 129, k 170).

CHAPTER III. ARRANGEMENT OF SPAN.

(22). Skew bridges and bridges on curves should never be built except in very rare cases where no other disposition is possible. These types should be regarded as measures of last resort. (m 267, W 371).

(23). Wherever possible the loads should be applied at the panel points only. (k 170).

(24). The plane of the lateral system should coincide with that of the chords, otherwise the stresses in web and chords will produce bending moments in the posts. (r 102, m 208, k 185).

(25). The plane of the floor should be as close as possible to that of the lateral system. (m 208).

(26). The gravity axes of all the main members of trusses and lateral systems coming together at any apex of a truss should intersect at a common point wherever such an arrangement is practicable. In other words the panel points of the bracing should coincide with the panel points of the main trusses. (W 275, k 185, g 130).

(27). If horizontal trusses are used in the floor system to resist traction stresses, the members should be placed near the centre of a span or midway between expansion joints. (Till 105, 86).

(28). The double intersection Warren truss with verticals is very suitable for lateral bracing. (Til 48[illegible])

(29). Bracings on posts should be connected as high as possible. (g 130).

(30). The pedestal pins in riveted trusses should not be placed below the bottom chords but on the centre lines. This eliminates the bending moments that may be developed in the end post and the end panels of the bottom chords by the train thrust. (W 284).

CHAPTER IV. DESIGN OF FLOOR SYSTEM.

(31). The floor system should be so designed that it is rigid in itself with respect to its main duty, but independent so far as practicable of the action of the chord system of the main trusses. (TII 505).

(32). Floor-beams should be made deep to reduce secondary stresses in the cross frames. (m 268, r 101)

(33). The flanges of the floor beams should be made relatively narrow. (TII 505).

(34). The floorbeams should be centrally connected to the post and not too far below the plane of chords. (r 101).

(35). Flexible connections between floorbeams and truss members are advisable. These should be so arranged that the end connections are rigid in a vertical plane but flexible in a horizontal plane. (m 268, g 130).

(36). Floor beams should be provided with brackets or any other suitable constructions to prevent the bending of the horizontal flanges. (g 132).

(37). The stringers should be made heavy and continuous and should be designed to transmit the tractive forces to the panel points of the loaded chord instead of to the floor beams by inserting proper tie members between the stringers. (W 400, m 268).

(38). When the floorbeams extend considerably below the bottom of the stringers they should have stiffeners at or near the stringer connection to transmit the bending caused by the deflection of the stringers to the flanges of the floorbeam.

(39).- In long span bridges stringers should be provided with expansion joints at intervals of a few panels. (TII 505, r 101).

(40). The connection of lower laterals to stringers is of doubtful value. Owing to the relative movement between the chords and the floor system, such a connection would cause considerable lateral bending in the stringers. The laterals are therefore better merely supported on the stringers. (TII 104, a6).

CHAPTER V. PROPORTIONING OF MEMBERS.

(41). The more uniform the proportions of a truss, the less, in general, will be the secondary stresses. Sudden changes in length, width or in moment of inertia are likely to result in relatively large secondary stresses. (TII [illegible]5).

(42). A reduction of moment of Inertia of any member tends to increase the deflection of that member and reduce that of others. To maintain a balance in the fibre stresses themselves, it is necessary to make the width of a member correspond in a measure with its moments of inertia. Wide members of small moment of inertia are likely to have high secondary stresses; narrow and compact members will have low secondary stresses. This statement does not take account of the long column action in compression members, the effect of which tend in the opposite direction. (TII 487).

(43). It is advisable to keep the moments of inertia of web members as small as possible (W 265).

(44). The axes of all members of a truss should be in the same plane and should intersect at a common point at all connections. (W 267, W 273).

(45). In any truss no torsion on any member should be allowed if it can possibly be avoided; otherwise the greatest care must be taken to provide ample strength and rigidity for every portion of the structure affected by such torsion. (W 272).

(46). If the axes of members do not intersect at one point the secondary stresses developed due to eccentric connections may be as high as 1 to 1 1/2 times the primary stresses. (W [illegible]).

(47). Excessively deep members should be avoided (W 495), especially in compression chords and in end posts of trusses with sub-divided panels (W 96). However, the members should not be so slender as to impair economy of design (W 177) or to reduce the effectiveness of the riveted connections. (W 595).

(48). To secure a comparatively narrow width and at the same time preserve the stiffness as against buckling, a compression member may be tapered from the centre toward both ends. (W 267).

(49). To avoid excessive bending stresses in posts to which floorbeams are riveted, the posts should be made only of moderate width in a transverse direction. In the best design these bending stresses are likely to be as much as 25% of the primary stresses. (T II 105, W 129).

(50). If the floorbeams connected to the posts are shallow the latter should be given increased sectional areas to provide for the secondary stresses due to the deflection of the floorbeams (¶ 401).

(51). Truss members and portions of truss members should always be arranged in pairs symmetrically about the plane of the truss. (¶ 173). This will tend to equalize the stresses carried by the component parts of the member.

(52). The cross sections of members should be so chosen that the material is concentrated as far from the neutral axis as possible, thus securing the largest moments of inertia for the smallest over all dimensions (¶ 129). The cross form is thus the least advantageous while a square box form is the most desirable. However consideration must be given to the fact that when the secondary stresses occur simultaneously in the plane of the truss and in a cross frame the stresses are additive in members of the box type but not in those with cross form (¶ 267).

(53). Since the effect of the hip verticals and suspenders in a truss is to pull the chord members out of line, they should be provided with liberal cross sections (¶¶ 487, a6).

(54). Where the suspenders are of considerable length and attached to the lower chord it is desirable to make them slightly shorter than the calculated length. (a6).

(55). Where a top chord in a sub-divided truss is supported by means of secondary members these should be made slightly longer than the calculated length. (a6).

(56). The secondary stresses in chord members may be reduced by the proper selection of cross sections for the diagonals and their attachments to the chords. (r 101)

(57). In case a member is made up of a single part, like an angle, the gauge lines for rivets should be as close to the gravity axis as possible. (p).

(58). For a truss with built-up, continuous chords and built-up diagonals, pin connected throughout, the secondary stresses will be very low if the chords are made sufficiently shallow and the diagonals sufficiently wide in the plane of the truss, so as to overcome the pin friction. (r 96).

(59). In eye-bar trusses, the eye bars should be made as wide as permissible in the plane of the truss. (r 97).

CHAPTER VI. DESIGNING OF DETAILS.

(60). Riveted connections must be made concentric by so grouping the rivets that they will balance about centre lines and centre planes to as great an extent as possible. (W 274, g 129).

(61). The end lateral connections should be as concentric as practicable. This is very important in the case of the end panel of the lower chord, as eccentric connections at this point result in heavy secondary stresses in end posts and lower chords. (TIII 105).

(62). The use of thick gusset plates and large diameter rivets is advisable as it would materially reduce the number of rivets and the size of the plates. (m 267, W 491).

(63). In case two gusset plates are used at each joint these should be firmly connected together by the use of diaphragm plates.

(64). In a pin-connected truss, the diameters of the pins should not be so large that the friction developed around the joints will virtually make the connections rigid. They should therefore be made as small as is consistent with the design. (m 268, g 132).

(65). If the diameter of the pin is made three quarters the width of the eye bar, as is usually ~~employed~~ done, the secondary ~~stresses~~ stresses developed will be about 45% of the primary stresses with the coefficient of friction equal to 0.2. (g 77)

(66). A double pin arrangement has been evolved for bettering the pin bearings in cantilever spans whereby a second pin is placed side by side with the bearing in the bottom chord. The object is to relieve the bending stresses in the bottom chord which would result from the simultaneous deflections of the two adjacent spans. (W 1067).

(67). For lacing of compression members the arrangement of double lacing with transverse cross bars is advisable. (TII 498).

(68). Long diagonals which are subject to the bending due to deflection of floorbeams should preferably be provided with lacings. (j 401).

(69). For heavy members composed of two parts, one on each side of the center line, the use of cross diaphragms spaced at 8 to 10 feet is necessary. (Fig. 86). This not only holds the members true to shape and line but also tends to equalize the stresses in the two component parts.

(70). The use of lug angles for connections is advisable for members having single angle sections.

CHAPTER VII. CORRECTIONS OF SECONDARY STRESSES.

The subject of correction of secondary stresses is of very recent origin. Formerly it was often assumed that the use of reduced unit stresses would cover the effects of bending but a little study will disclose the unsoundness of this theory as the distribution of secondary stresses in various members in a truss is by no means uniform. The secondary stresses must, therefore, be either properly cared for as the primary stresses or judiciously corrected by appropriate methods. The latest developments in bridge design seem to indicate the tendency of the second resort, that is, to correct the secondary stresses by manufacturing and erection methods.

In almost all bridge specifications there are clauses dealing with the camber of the truss. This is probably the simplest and easiest method for correcting secondary stresses, although its degree of accuracy largely depends on the amounts of camber. As usually specified, the correction is never complete, as the assumption of uniform lengthening and shortening of various members could not be true for any instance. A correct method like the following is much more desirable, both from theoretical and practical points of view:[1] "The secondary stresses in riveted trusses are to be modified by lengthening and shortening the various members the amounts of their respective shortening and lengthening under dead load plus one half of the live plus impact load, drilling or reaming the chord splices while the chords are assembled in straight lines, and then forcing the truss members into their proper positions for connection to each other before drilling or reaming the holes in the joints."

Perhaps the most important work along this line was done on the Sciotoville Bridge[2] in which the secondary stresses were corrected by erecting the members with a bend exactly the opposite of the bends which they would normally experience at a given stage of loading--dead plus one half live load. Thus the bridge ~~thus~~ has high secondary stresses under dead load alone, but the secondary stresses decrease as the live load comes on and are fully neutralized under half live load. The bending was done as follows: Every member was originally built to such a length that when erected and under the influence of one half full live load the bridge would be of its diagram size. The members, when laid together unstressed, would therefore not have the diagram angles at the joints. The joint connections were reamed to the diagram angles by fitting two corners a time of each triangle. After these were reamed one corner was disconnected and the member shifted to make possible the reaming of the third position. To connect these members in the field, they had to

*** *** *** *** ***

1. Waddell's Bridge Engineering, p. 203.
2. Engineering News-Record, Jan. 10, 1918, p. 62.

be forced together by means of jacks.

To provide for the secondary stresses in posts, due to deflections of the floorbeams, the following method may be used.[1] The upper lateral and transverse bracing are made of lengths slightly shorter than normal. In erection, the trusses are sprung inward one half inch at the top to connect to the bracing, thus causing some bending moments in the verticals, which are relieved when the floor beams deflect under the live load traffic.

The secondary stresses in the posts may also be corrected by giving the floor-beam a downward camber.[2]

*** *** *** *** *** ***

1. Merriman and Jacoby's Roofs and Bridges, Part III, p. 401.
2. Grimm's Secondary Stresses in Bridge Trusses, p. 130.