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Design Of Digital Filters
By Pole-Zero Decomposition

B. Yegnanarayana and T. V. Ananthapadmanabha
Department of Computer Science
Carnegie-Mellon University
Pittsburgh, PA15213

This research was sponsored by the Defence Advanced Research Projects Agency (DOD), ARPA Order No. 3597, and monitored by Air Force Avionics Laboratory under Contract F33615-78-C1151.

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ABSTRACT

A new technique for design of digital filters based on a pole-zero model is proposed. The technique is capable of realizing any desired magnitude response specified in discrete frequency domain to the required degree of accuracy without significantly increasing the complexity for the computation of filter coefficients. It is shown that excellent characteristics for all types of the standard filters can be realized. The distinct features of the technique are: (a) Both the passband and the stopband have nearly flat characteristics; (b) A highpass (bandstop) filter can be realized as the reciprocal of a lowpass (bandpass) filter; (c) A high degree of flexibility exists in the choice of design parameters permitting trade-off between the realized response and the number of filter coefficients. The coefficients for the pole part and the zero part of the model are obtained in an identical manner using FFT algorithms and recursive relations, without the need for solving complex nonlinear equations. These features in the design are a result of application of a pole-zero decomposition technique for digital filter design. The technique, which was developed for modelling speech spectra, is based on the properties of the derivative of phase spectrum of a minimum phase signal.

I. INTRODUCTION

Synthesis of linear systems to satisfy a desired input-output relation is a general problem of interest to electrical engineers. Filter design is one such problem where the specification of the system is made either in terms of the impulse response or frequency response or both. In this paper we present an effective method for designing a linear discrete-time system to realize any desired magnitude response specified in discrete frequency domain. The method is based on the properties of the derivative of phase spectrum (DPS) of a minimum phase signal and a new technique for pole-zero decomposition of speech spectra [1]-[3].

A general linear discrete system $H(z)$, shown in Fig.1, is given by

$$H(z) = G N(z)/D(z) \quad (1)$$

where

$$N(z) = \sum_{n=0}^{M_z} a^+(n) z^{-n}, \text{ with } a^+(0)=1, \quad (2)$$

and

$$D(z) = \sum_{n=0}^{M_p} a^-(n) z^{-n}, \text{ with } a^-(0)=1, \quad (3)$$

and G is a gain term. The output signal $y(n)$ is obtained from the input signal $x(n)$ by the following recursive relation [4]:

$$y(n) = G \sum_{j=0}^{M_z} a^+(j) x(n-j) - \sum_{j=1}^{M_p} a^-(j) y(n-j). \quad (4)$$

The frequency response of the system $H(z)$ is defined as

$$H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}}. \quad (5)$$

The response $\bar{H}(e^{j\omega})$ is periodic in ω with period 2π . In discrete frequency domain the

normalized frequency range $(0-2\pi)$ is divided into N equal parts. The discrete frequency response of the system is then given by

$$H(e^{j2\pi k/N}) = H(e^{j\omega}) \Big|_{\omega=2\pi k/N}, \quad k=0,1,2,\dots,N-1. \quad (6)$$

For convenience we shall adopt the following simplified notation throughout this paper:

$$H(\omega) = H(e^{j\omega}) \quad (7)$$

and

$$H(k) = H(e^{j2\pi k/N}). \quad (8)$$

For the system $H(z)$ to be stable, all the roots of the denominator polynomial $D(z)$, called poles, must lie within the unit circle in the z -plane. If the roots of the numerator polynomial $N(z)$, called zeros, also lie within the unit circle in the z -plane, then the impulse response of $H(z)$ is called a minimum phase signal [5], [6]. An important property of the minimum phase signal is that the magnitude and phase characteristics of its Fourier transform are related through Hilbert transformation [6].

The problem in digital filter design is to realize the desired frequency domain specifications through a filter of the type $H(z)$, such that the output $y(n)$ of the filter is obtained from the input $x(n)$ using a finite precision arithmetic. Digital filter design techniques are usually classified as infinite impulse response (IIR) and finite impulse response (FIR) filters. The filter $H(z)$ in (1) is in general an IIR filter because of the denominator polynomial. If $D(z)=1$ then the filter is referred to as an FIR filter. The choice between the two types depends upon the relative weight one attaches to the advantages and disadvantages of each type of filter. IIR filters have the advantage that a variety of frequency selective filters can be designed using closed form design formulas. Such a simplicity is available only for a limited class of filters. Complex IIR filter design uses optimization techniques leading to solution of nonlinear equations to determine the coefficients of the filter. The phase response of IIR filters is generally nonlinear, whereas it is desirable to realize a filter having linear phase characteristics. FIR filters, on the other hand, can be designed with linear phase

characteristics. They utilize the advantages of flexibility and computational speed of FFT algorithms. But many FIR design methods involve iterative procedures even for modest specifications such as equiripple in the passband and stopband. Thus the flexibility of an FIR filter design is achieved at the expense of simplicity whereas the simplicity of an IIR filter design is achieved at the expense of flexibility [7].

More recently, design techniques in quefrequency domain are proposed in which the desired log magnitude and phase responses are realized by approximating the log frequency response with a finite length quefrequency response [8]. The quefrequency response, also called the cepstrum of the impulse response, is the inverse Fourier transform of the given log frequency response. Each of the quefrequency components is realized by a set of elemental filters, the order of the elemental filter being dependent on the value of the quefrequency component. By designing digital filters in the quefrequency domain the best approximation (in the mean-square sense) for the desired log frequency response can be realized. The resulting error in this procedure is caused by truncation in the quefrequency domain and the approximation of the quefrequency components by the elemental filters.

In this paper we present a new technique for determining the coefficients in $H(z)$ to meet a desired log magnitude specification in discrete frequency domain. The design procedure is simple and highly flexible. The filter can be designed for any desired accuracy by appropriately choosing the orders of numerator and denominator polynomials of $H(z)$, without increasing the computational complexity. This has been possible due to application of FFT algorithm and recursive relations for determining the coefficients of the IIR filter. The passband and stopband characteristics of the realized filter are nearly flat in most cases. This enables transformation of a lowpass filter to a highpass filter or vice versa by merely considering the reciprocal of $H(z)$. Also by cascading a lowpass filter and a highpass filter it is possible to realize a bandpass or a bandstop filter.

All these advantages of this new method are achieved due to application of the pole-zero decomposition technique recently proposed for modelling speech spectra [2]. In Sec.II the technique of pole-zero decomposition is discussed. The design procedure for digital filters is

described in Sec.III. The procedure is illustrated with examples in Sec.IV and also several design choices available for a designer are discussed.

II. PRINCIPLE OF POLE-ZERO DECOMPOSITION

In this section some important properties of the derivative of phase spectrum (DPS) of a minimum phase signal and the principle of pole-zero decomposition are briefly described. Details can be found in [1] and [2].

A. Properties of DPS:

A minimum phase signal has, by definition, all its poles and zeros within the unit circle in the z -plane [6]. The pole part of the signal spectrum can be represented as a cascade of several first order sections with real poles and second order sections with complex conjugate poles. The DPS of a typical first order pole filter usually has all its significant values confined to frequencies close to the origin. The DPS of a second order filter (resonator) is approximately proportional to the squared magnitude response of the filter around the resonance frequency. The DPS of the overall filter is a superposition of the DPS of each filter in the cascade. These properties were shown to be very useful for unambiguous identification of resonance peaks from the DPS of an all-pole filter obtained in linear prediction analysis [1]. The zero part of the signal spectrum can similarly be considered as a cascade of several first order sections with real zeros and second order sections with complex conjugate zeros. The DPS of the zero part will have similar behaviour as that of the pole part except for a sign change. In particular, the DPS due to poles is negative and that due to zeros is positive [2]. These simple properties were shown to accomplish the pole-zero decomposition of speech spectra [2].

B. Pole-Zero Decomposition

Let $V(\omega)$ be the Fourier transform of a minimum phase signal. Since we consider only sampled signals, $V(\omega)$ is periodic in ω with period 2π . The properties of minimum phase signals permit the expansion of $\ln V(\omega)$ in Fourier series as follows [9]:

$$\ln V(\omega) = \hat{c}(0) + \sum_{n=1}^{\infty} \hat{c}(n) e^{-j\omega n} \quad (9)$$

where $\{\hat{c}(n)\}$ are called *cepstral coefficients*. Writing

$$V(\omega) = |V(\omega)| e^{j\theta_V(\omega)}, \quad (10)$$

we get the real and imaginary parts of $\ln V(\omega)$ as

$$\ln |V(\omega)| = \sum_{n=0}^{\infty} \hat{c}(n) \cos n\omega \quad (\text{real part}) \quad (11)$$

and

$$\theta_V(\omega) + 2\lambda\pi = - \sum_{n=1}^{\infty} \hat{c}(n) \sin n\omega \quad (\text{imaginary part}) \quad (12)$$

where λ is an integer. Notice that $\theta_V(\omega)$ represents the phase spectrum of the minimum phase signal. Taking the derivative of $\theta_V(\omega)$ with respect to ω , we get

$$\theta'_V(\omega) = - \sum_{n=1}^{\infty} n \hat{c}(n) \cos n\omega. \quad (13)$$

As discussed before, the contributions of poles and zeros can be separated from $\theta'_V(\omega)$ because they have opposite signs. Let

$$\theta'_V(\omega) = [\theta'_V(\omega)]^- + [\theta'_V(\omega)]^+ \quad (14)$$

where

$$\begin{aligned} [\theta'_V(\omega)]^- &= \theta'_V(\omega) & \text{for } \theta'_V(\omega) < 0 \\ &= 0 & \text{for } \theta'_V(\omega) \geq 0 \end{aligned} \quad (15)$$

and

$$\begin{aligned} [\theta'_V(\omega)]^+ &= \theta'_V(\omega) & \text{for } \theta'_V(\omega) \geq 0 \\ &= 0 & \text{for } \theta'_V(\omega) < 0. \end{aligned} \quad (16)$$

We can express $[\theta'_V(\omega)]^-$ and $[\theta'_V(\omega)]^+$ separately in terms of the cepstral coefficients for the pole part and the zero part respectively, since the relation given in (13) is valid for poles or

zeros or for both, except for a constant term (C) on the right hand side. That is

$$[\theta'_V(\omega)]^- = -C - \sum_{n=1}^{\infty} n \hat{c}^-(n) \cos n\omega \quad (17)$$

and

$$[\theta'_V(\omega)]^+ = C - \sum_{n=1}^{\infty} n \hat{c}^+(n) \cos n\omega, \quad (18)$$

where $\{\hat{c}^-(n)\}$ and $\{\hat{c}^+(n)\}$ represent the cepstral coefficients for pole and zero spectra of $V(\omega)$ respectively. Notice that $\hat{c}(n) = \hat{c}^-(n) + \hat{c}^+(n)$ for $n=1, 2, \dots, \infty$. In other words, the cepstral coefficients are split into two parts, one corresponding to poles and the other to zeros. Here $[\theta'_V(\omega)]^-$ represents the significant portion of DPS for the poles of $V(\omega)$ and $[\theta'_V(\omega)]^+$ represents the significant portions of DPS for the zeros of $V(\omega)$. By significant portion we mean that the shape of the curve in the negative portion of $\theta'_V(\omega)$ is mainly due to poles and that in the positive portion of $\theta'_V(\omega)$ is mainly due to zeros.

We now describe a method for deriving the coefficients in $D(z)$ and $N(z)$ from $\{\hat{c}^-(n)\}$ and $\{\hat{c}^+(n)\}$ respectively. Let us consider the expansion of $\ln D(z)$ in Taylor series as follows [10]:

$$\ln D(z) = - \sum_{n=1}^{\infty} c_1(n) z^{-n} \quad (19)$$

where $\{c_1(n)\}$ are the cepstral coefficients corresponding to $1/D(z)$. Using the expression for $D(z)$ from (3) and differentiating both sides of (19) with respect to z , and rearranging the terms, we get

$$\sum_{n=1}^{M_p} n a^-(n) z^{-n} = - \left[\sum_{n=1}^{\infty} n c_1(n) z^{-n} \right] \left[\sum_{n=0}^{M_p} a^-(n) z^{-n} \right]. \quad (20)$$

Multiplying out the series and using the fact that $a^-(0)=1$, we get

$$a^-(1) = -c_1(1),$$

$$j a^-(j) = -j c_1(j) - \sum_{n=1}^{j-1} n c_1(n) a^-(j-n), \quad j=2,3,\dots,M_p \quad (21)$$

It is to be noted from (21) that we can obtain $\{a^-(n)\}$ from $\{c_1(n)\}$ or vice versa. Only M_p values of $\{c_1(n)\}$ are required to determine $\{a^-(n)\}$.

It is worth noting that the coefficients $\{a^-(n)\}$ obtained from $\{c_1(n)\}$ using the recursion (21) is meaningful only if it is known a priori that $\{c_1(n)\}$ correspond to the cepstral coefficients of a minimum phase polynomial like $D(z)$. This is the reason for splitting the cepstral coefficients in (9) into $\{\hat{c}^-(n)\}$ and $\{\hat{c}^+(n)\}$ through DPS.

III. DESIGN OF DIGITAL FILTERS

In the previous section the theoretical relations among various parameters have been derived using Fourier series in which the upper limit in the summation is infinity. However, in practice the design has to be made using sequences of finite length. The design of digital filters when the specification is in discrete frequency domain will now be discussed. For a discrete sequence $x(n)$ the discrete Fourier transform (DFT) is defined as [4]:

$$X(k) = \sum_{n=0}^{N-1} x(n) [\cos(2\pi kn/N) - j \sin(2\pi kn/N)], \quad k=0,1,\dots,N-1 \quad (22a)$$

and the inverse DFT (IDFT) is defined as

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) [\cos(2\pi kn/N) + j \sin(2\pi kn/N)], \quad n=0,1,\dots,N-1 \quad (22b)$$

The DFT and IDFT are computed using FFT algorithms. Let the desired log magnitude response $\ln|V(\omega)|^2$ be specified for $\omega=2\pi k/N$, $k=0,1,\dots,N-1$, with $\ln|V(\omega)|^2$ being symmetric around $\omega=\pi$. The IDFT $x(n)$ of $X(k)=\ln|V(k)|^2$ is called in literature as power cepstrum [4].

The cepstral coefficients $\{c(n)\}$ for the given data are given by

$$\begin{aligned}
c(0) &= 0.5 x(0), \\
c(n) &= x(n), & n=1,2,\dots,N/2-1 \\
c(n) &= 0, & n=N/2,N/2+1,\dots,N-1.
\end{aligned} \tag{23}$$

For the discrete frequency domain specification there are only $N/2$ cepstral coefficients $\{c(n)\}$. The discrete DPS $\theta'_V(k)$ is obtained as the real part of the DFT of the sequence $\{nc(n)\}$.

$$\theta'_V(k) = \sum_{n=0}^{N-1} n c(n) \cos(2\pi kn/N), \quad k=0,1,\dots,N-1. \tag{24}$$

The discrete DPS is separated into positive and negative part as given by

$$\begin{aligned}
[\theta'_V(k)]^- &= \theta'_V(k) & \text{for } \theta'_V(k) < 0 \\
&= 0 & \text{for } \theta'_V(k) \geq 0
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
[\theta'_V(k)]^+ &= \theta'_V(k) & \text{for } \theta'_V(k) \geq 0 \\
&= 0 & \text{for } \theta'_V(k) < 0.
\end{aligned} \tag{26}$$

The IDFT of $[\theta'_V(k)]^-$ is given by

$$\begin{aligned}
n c^-(n) &= \frac{1}{N} \sum_{k=0}^{N-1} [\theta'_V(k)]^- \cos(2\pi kn/N), & n=1,2,\dots,N/2-1, \\
-c &= \frac{1}{N} \sum_{k=0}^{N-1} [\theta'_V(k)]^-.
\end{aligned} \tag{27}$$

Similarly $\{nc^+(n)\}$ can be obtained through the IDFT of $[\theta'_V(k)]^+$ as

$$n c^+(n) = \frac{1}{N} \sum_{k=0}^{N-1} [\theta_V(k)]^+ \cos(2\pi kn/N), \quad n=1,2,\dots,N/2-1 \quad (28)$$

$$C = \frac{1}{N} \sum_{k=0}^{N-1} [\theta_V(k)]^+.$$

The coefficients $\{a^-(n)\}$ and $\{a^+(n)\}$ of the denominator and numerator polynomials of the realized filter $H(z)$ are computed using the following recursive relations:

Pole Coefficients:

$$a^-(1) = c^-(1)$$

$$j a^-(j) = j c^-(j) + \sum_{n=1}^{j-1} n c^-(n) a^-(j-n), \quad \text{for } j=2,3,\dots,M_p \quad (29)$$

Zero Coefficients:

$$a^+(1) = -c^+(1)$$

$$j a^+(j) = -j c^+(j) - \sum_{n=1}^{j-1} n c^+(n) a^+(j-n), \quad \text{for } j=2,3,\dots,M_z \quad (30)$$

The above recursive relations are valid for M_p and M_z not exceeding $N/2$. The log spectra in dB of the pole part and that of the zero part are given by the following relations:

Pole Spectrum:

$$-20 \log |D(\omega)| = -20 \log \left| 1 + \sum a^-(n) e^{jn\omega} \right| \quad (31)$$

Zero Spectrum:

$$20 \log |N(\omega)| = 20 \log \left| 1 + \sum a^+(n) e^{jn\omega} \right| \quad (32)$$

The log spectrum in dB of the overall filter is given by

$$20 \log |H(\omega)| = 20 \log |N(\omega)| - 20 \log |D(\omega)| \quad (33)$$

The response $H(\omega)$ will have smooth passband and stopband characteristics. At the edges of

the transition band, however, there will be an overshoot and an undershoot, the values of which depend upon the width and the nature of the transition band.

In order to apply this method, the cepstral coefficients $\{c(n)\}$ in (23) should be free from aliasing errors as far as possible. Aliasing is caused by the finite N -point DFT used to compute $\{c(n)\}$. This is because, for a given frequency response $V(\omega)$, the cepstral coefficients are infinite as shown in (9). Discretizing ω and computing $\{c(n)\}$ in (23) through DFT results in a finite number of cepstral coefficients $\{c(n)\}$ which are related to $\{\hat{c}(n)\}$ in (9) by the relation

$$c(n) = \sum_{r=0}^{\infty} \hat{c}(n+rN), \quad n=0,1,\dots,N/2-1, \quad (34)$$

where r is an integer. What is desirable is a rapid decay of the values of $c(n)$ with n or a large value of N or both in order to reduce the effects of aliasing errors in $\{c(n)\}$. The values of $c(n)$ for large n depend on the nature of the transition band and the dynamic range of the magnitude response specification. Once a sufficiently large value of N is chosen to reduce the effects of aliasing, it may be possible to use filters with orders considerably lower than $N/2$. It may also be possible to realize the desired characteristics through a lower order filter by multiplying the cepstral coefficients with a suitable weighting function. But multiplying the cepstral coefficients with a weighting function alters the desired specification itself. Some of these issues are discussed in detail through illustrative examples in the following section .

IV. RESULTS AND DISCUSSION

A lowpass filter for the following specifications is designed using the proposed method.

$$\begin{aligned} \ln|V(k)| &= \ln 100 & k=0,1,\dots,62 \\ &= \ln t(k) & k=63,64,\dots,74 \\ &= \ln 1 & k=75,76,\dots,256 \end{aligned}$$

and

$$\ln|V(k+255)| = \ln|V(257-k)| \quad k=1,2,\dots,256, \quad (35)$$

where $t(k)$ is the specification over the transition band. Initially a linear variation for $\ln t(k)$ was chosen and the filter was designed for $M_p=M_z=63$ in (29) and (30). The specified magnitude response in dB and the negative derivative of the phase spectrum i.e., $-\theta'_V(k)$ are shown in Fig.2. The distribution of poles and zeros can be seen from the negative DPS curve. The positive part of the curve, which occurs only in the passband, corresponds entirely to poles. Similarly the negative part of the curve, which occurs only in the stopband, is entirely due to zeros. The log spectra of the pole part and the zero part are shown separately in Fig.3. The individual responses have poor lowpass characteristics, with the accompanying minimum phase characteristics. This shows that the pole part and the zero part are optimized simultaneously to yield the desired overall magnitude response. The realized magnitude and phase responses are shown in Fig.4.

The deviation from the desired passband and stopband characteristics is shown in Fig.5. The error over the passband and stop band is negligible except near the edges of the transition band where there is an overshoot in the passband and an undershoot in the stopband. The maximum deviation due to overshoot is 0.2dB in this case. The amplitudes of the overshoot and the ripples in the passband and stopband depend upon the width of the transition band and the dynamic range of the filter, which in the present case are 12 samples and 40dB respectively.

The impulse response of the realized filter is shown in Fig.6. The response decays with time,

indicating that the system is stable.

The same filter was designed with $\ln t(k)$ having a raised cosine characteristic. The overshoot was reduced to 0.17dB. In this case the reduction in the overshoot appears to be insignificant. But when the transition width was made 8 samples, the overshoot was 1.1dB for linear transition compared to 0.37dB for a raised cosine transition, which is a significant reduction. Obviously, the width and the nature of transition band specification is a design choice.

In order to obtain a good overall performance of the filter for a given order $M_p (=M_z)$, the filter specifications may have to be relaxed. The most convenient way of designing such a filter is to start with an ideal filter specification for the transition band in the discrete frequency domain. The width of the transition band then becomes one sample. The filter is designed by preprocessing the cepstrum before computing the DPS. The preprocessing consists of multiplying the cepstral coefficients $c(n)$ with a weighting function $w(n)$, so that the values of $w(n)c(n)$ for large n are significantly reduced before computing the DPS. An exponential window function of the form

$$w(n) = e^{-\alpha n/M_p} \quad (36)$$

has been found to be very convenient for this purpose. The decay constant α can be varied according to the order of the filter desired. The effect of this windowing is to broaden the transition band for the desired response. The factor α is a design parameter. The realized magnitude response and phase response for the lowpass filter specification in (35) for $\alpha=1$ are shown in Fig.7. It may be noted that there is no overshoot or undershoot and also that the transition characteristics are smooth.

The effect of α on the realized magnitude response is shown in Fig.8 for $M_p=15$. It may be noted that as α is increased the ripples in the filter response disappear. However the transition width becomes broader. For a given order of the filter a trade-off exists between the transition width and the magnitude of ripples in the passband and stopband. A narrower transition band with a smooth filter response (i.e., with reduced ripples) can be obtained by

using a higher order filter. The filter responses for different orders of the filter for $\alpha=1$ are shown in Fig.9. The curves on the left side are the desired responses after the exponential weighting. The curves on the right side are the corresponding realized responses.

Since the lowpass filter designed by this method has nearly flat passband and stopband characteristics, it is possible to realize a highpass filter using the reciprocal of a lowpass filter i.e., $D(z)/N(z)$. For the same reason, a bandpass or a bandstop filter can be designed by cascading a lowpass filter and a highpass filter. Fig.10 shows the response of a bandpass filter obtained by cascading a lowpass filter and a highpass filter, each of order 127, with $\alpha=0.5$.

V. CONCLUSIONS

A highly flexible and effective design technique for digital filters has been proposed. The technique can be used to realize a desired magnitude response characteristics to the required degree of accuracy. The effectiveness of the technique is due to a pole-zero decomposition method based on the derivative of phase spectrum. More stringent requirements on the filter characteristics can be met either by using a higher order filter or by cascading several low order filters. The complexity of the filter design remains almost independent of the order of the filter being designed. This is because of the use of FFT techniques for the design.

The design technique proposed here differs from the conventional FIR or quefrequency domain designs, where the impulse response or the quefrequency response is truncated using suitable windows. In the present method the cepstral coefficients of the desired response are matched by the realized filter upto a specified order and the cepstral coefficients are then *extrapolated* upto infinity by the realized filter. This provides the desired smooth behaviour in the magnitude response which is responsible for achieving nearly flat response characteristics in the passband and stopband. Presently, we are exploring the possibility of extending the method to realize a given phase response specification. The technique has already been found to be very effective in modelling speech spectra and we expect it to be useful for applications in general linear system modelling as well.

ACKNOWLEDGMENTS

The authors wish to thank Professor D. Raj Reddy for many useful discussions and encouragement.

REFERENCES

- [1]. B. Yegnanarayana, "Formant extraction from linear prediction phase spectra," *J. Acoust. Soc. Amer.*, vol. 63, pp. 1638-1640, 1978.
- [2]. B. Yegnanarayana, "Pole-Zero decomposition of speech spectra," Computer Science Department, Carnegie-Mellon University, Pittsburgh, PA 15213, Report CMU-CS-79-101.
- [3]. B. Yegnanarayana and D. Raj Reddy, "A distance measure based on the derivative of linear prediction phase spectrum," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, Washington D.C., April 1979.
- [4]. A.V. Oppenheim and R.W. Schaffer, *Digital Signal Processing*, Prentice-Hall, 1975.
- [5]. A.J. Berkhout, "Related properties of minimum-phase and zero-phase time functions," *Geophysical Prospecting*, vol. XXII, No. 4, pp. 683-709, 1974.
- [6]. Ref. 4, ch. 7.
- [7]. L.R. Rabiner, J.F. Kaiser, O. Herrmann, and M.T. Dolan, "Some comparisons between FIR and IIR digital filters," *Bell Syst. Tech. J.*, vol. 53, No. 2, Feb. 1974, pp. 305-331.
- [8]. S. Imai, "Digital filter design in quefrequency domain," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-26, pp. 226-236, June 1978.
- [9]. Ref. 4, p. 346.
- [10]. A.H. Gray and J.D. Markel, "Distance measures for speech processing," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-24, pp. 380-391, Oct. 1976.

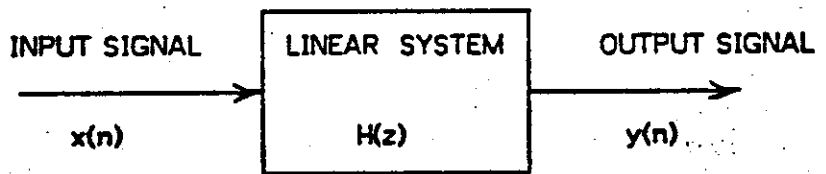


Fig.1 Linear discrete-time system

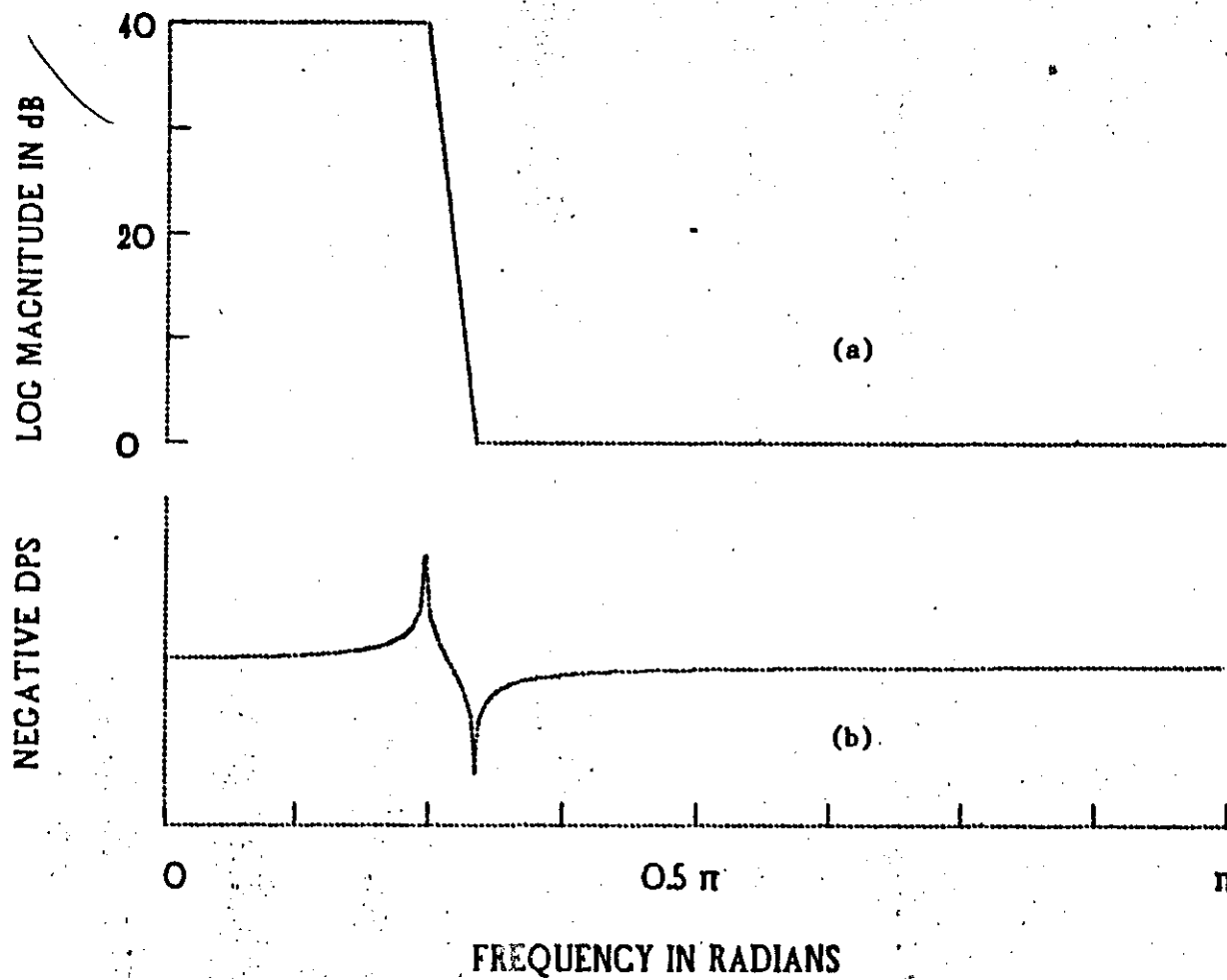


Fig.2 a) Specified magnitude response characteristics of a lowpass filter
 b) Negative derivative of phase response

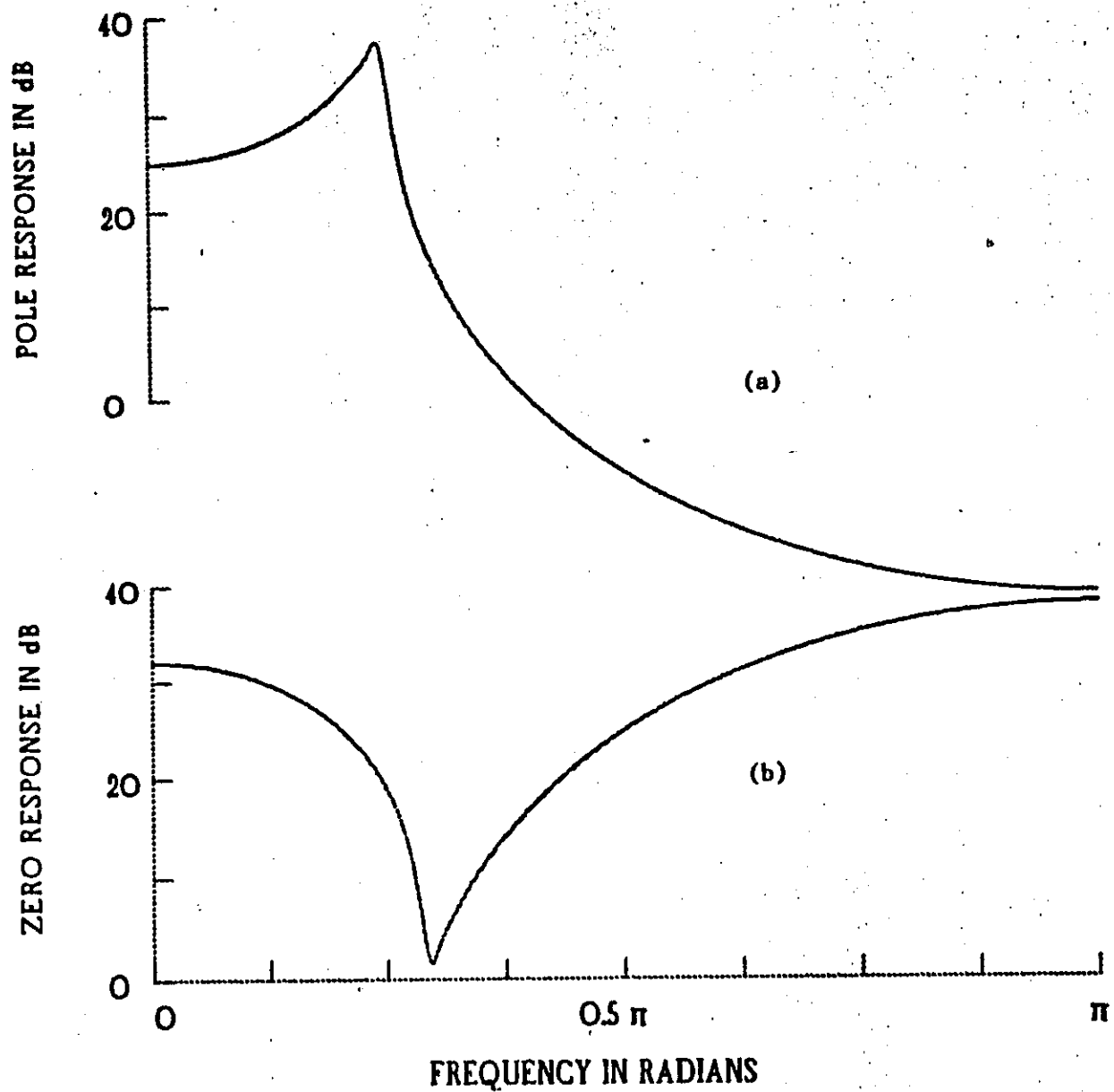
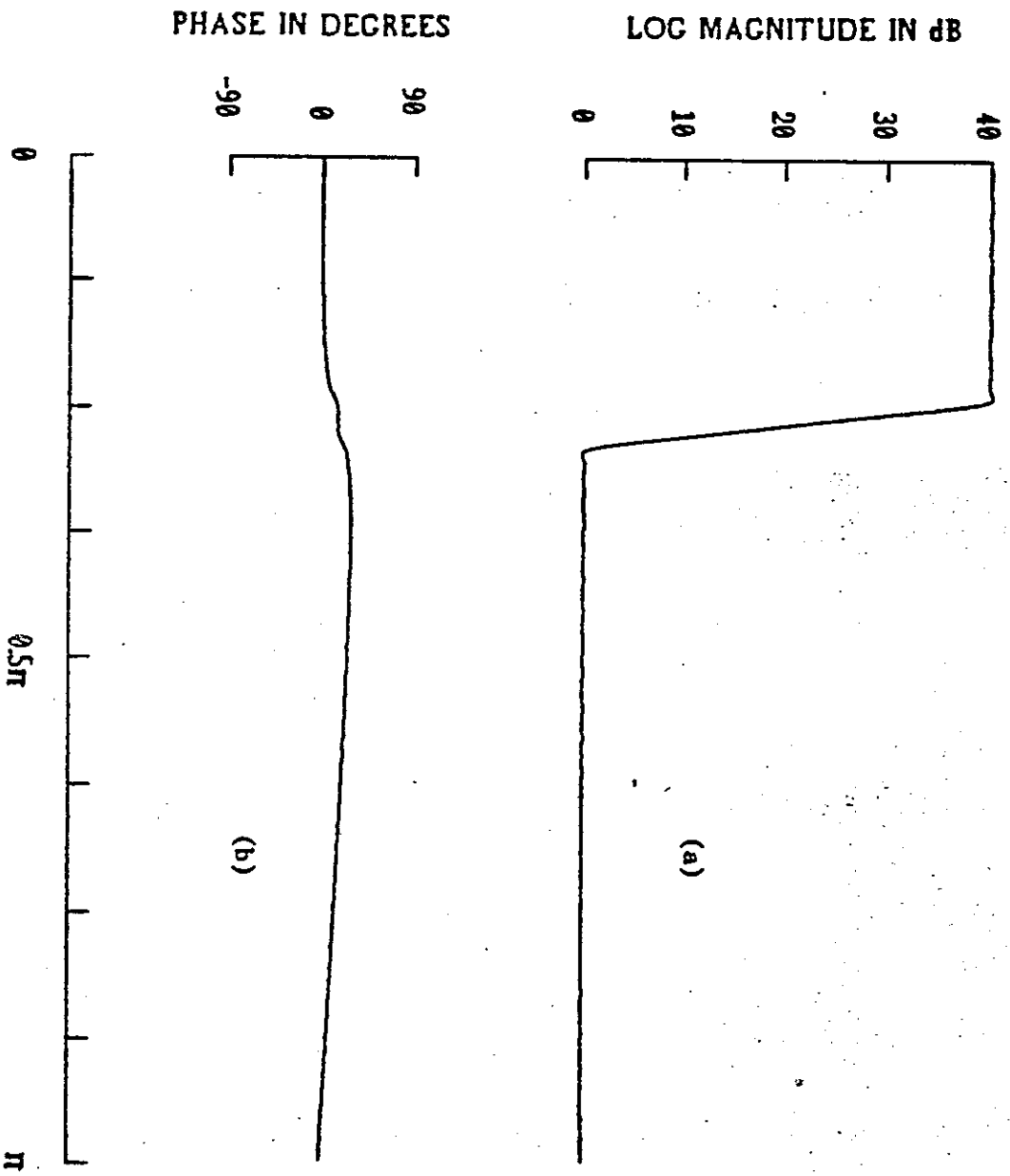


Fig.3 a) Log spectrum of the pole part of the realized lowpass filter
 b) Log spectrum of the zero part of the realized lowpass filter



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Fig. 4 a) Realized magnitude response characteristics of the lowpass filter
 b) Realized minimum phase response

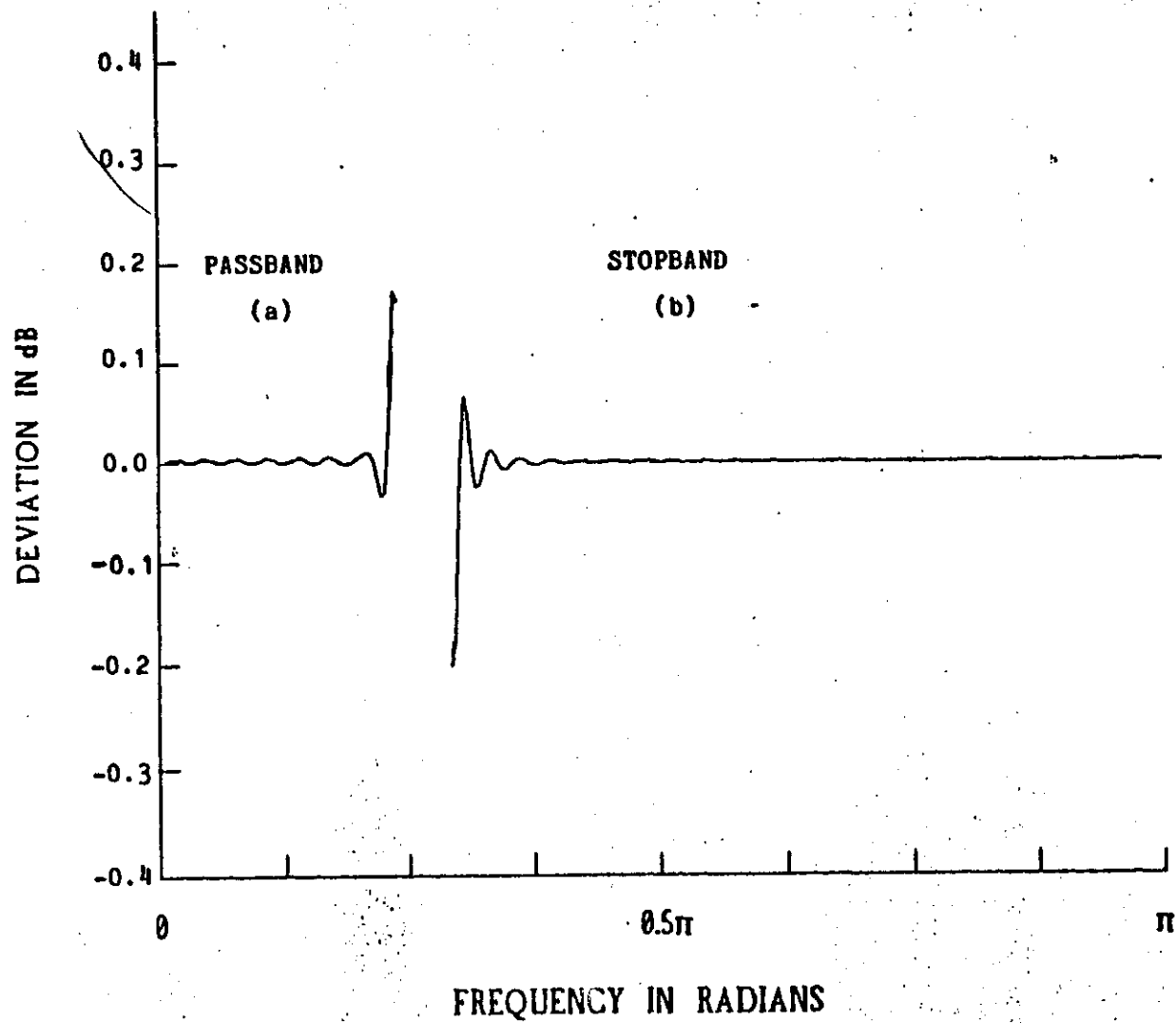


Fig.5 a) Deviation from the desired passband characteristics
 b) Deviation from the desired stopband characteristics

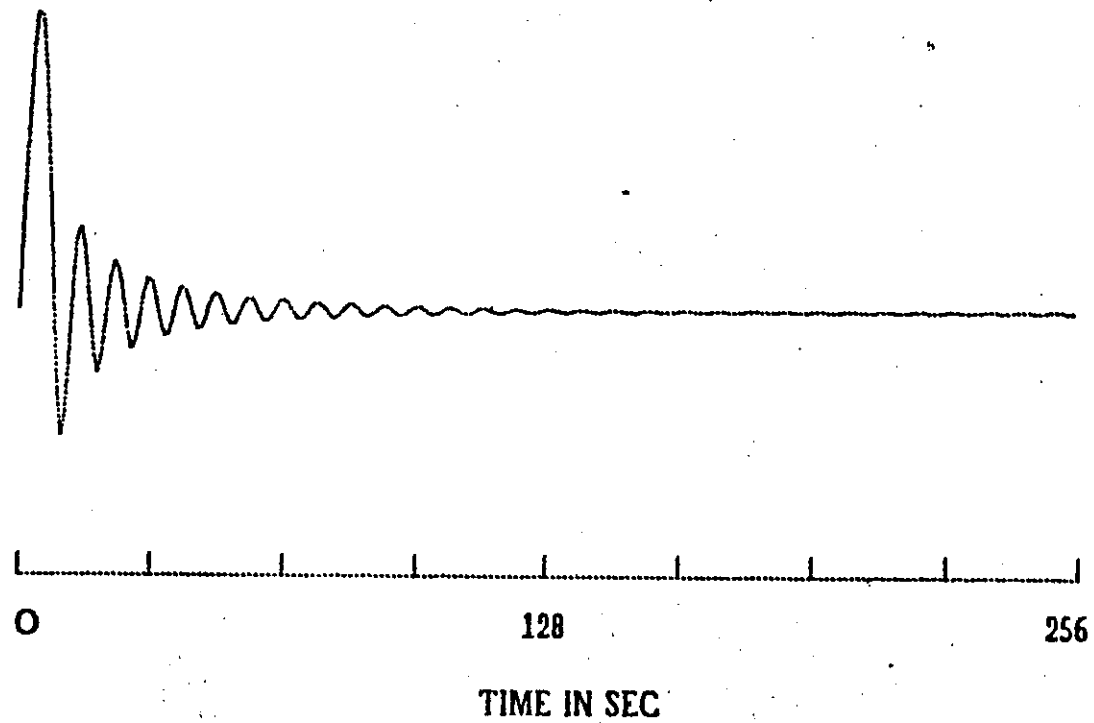


Fig.6 Impulse response of the realized filter

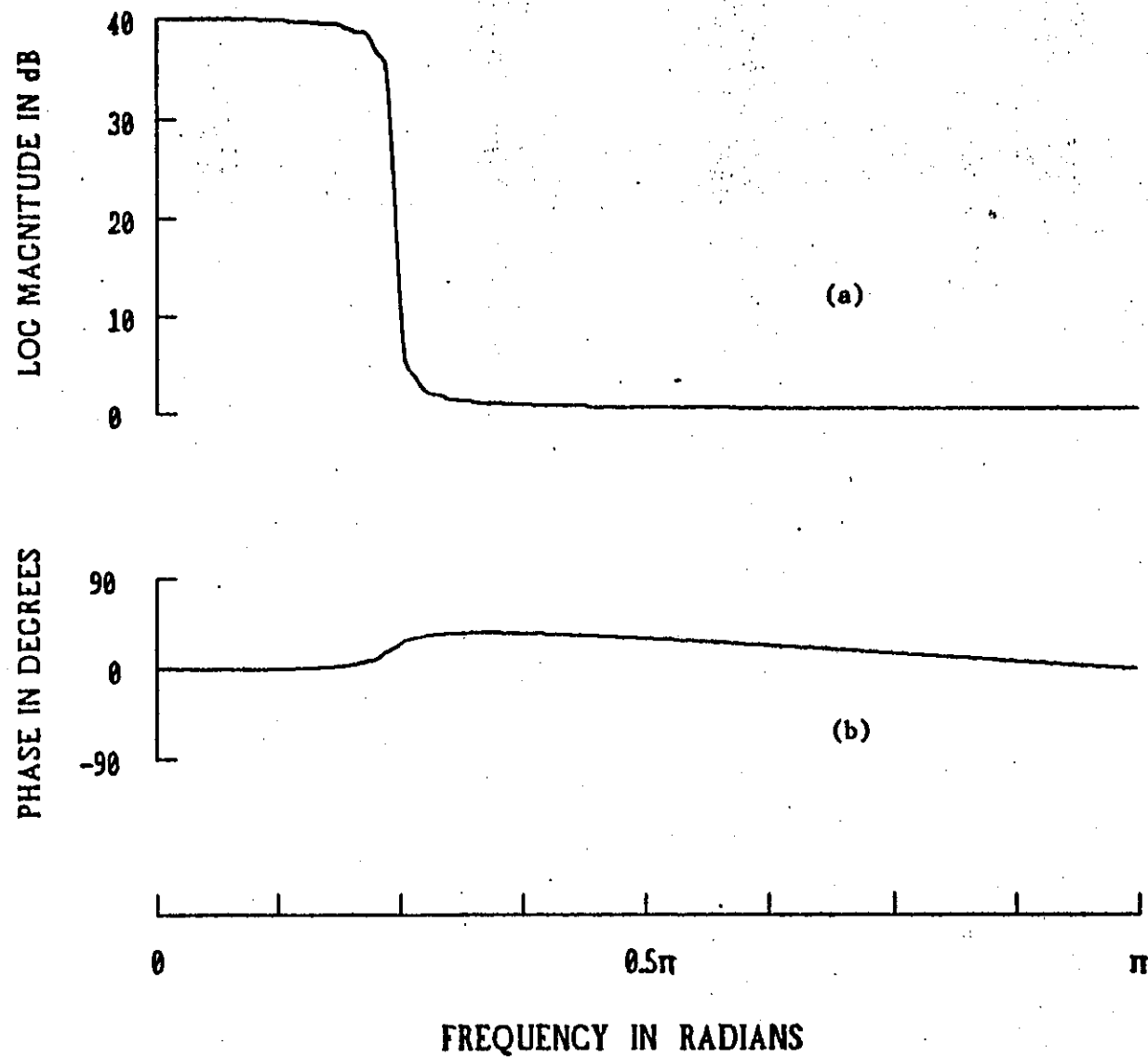


Fig.7 Realized response for the filter in Fig.2 after exponential weighting of cepstral coefficients.
 (a) Magnitude response
 (b) Phase response

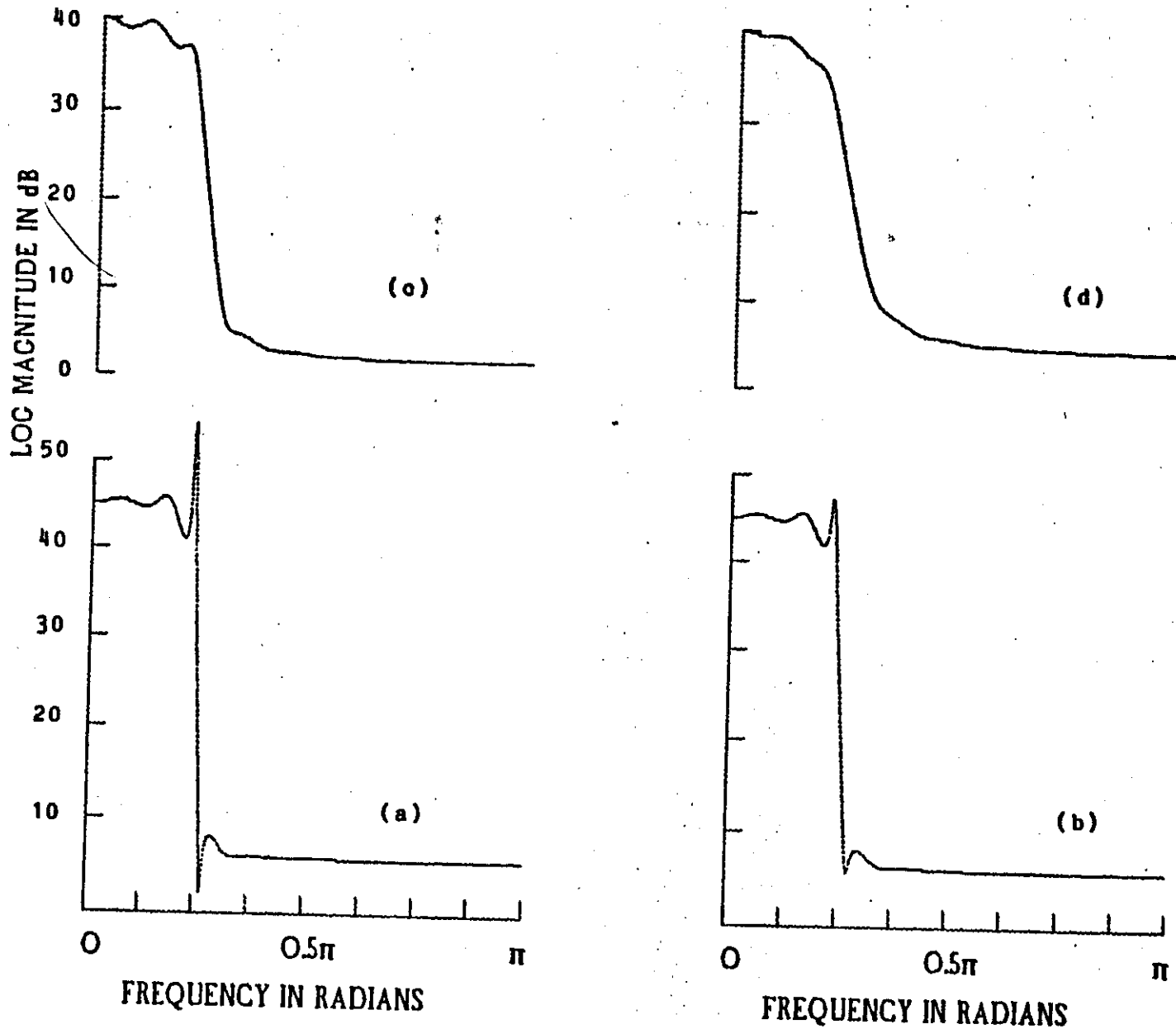


Fig.8 Realized magnitude response for a low order ($M_p=M_z=15$) lowpass filter for different decay rates of the exponential weighting function. (a) $\alpha=0.2$, (b) $\alpha=0.3$, (c) $\alpha=1.0$, (d) $\alpha=2.0$.

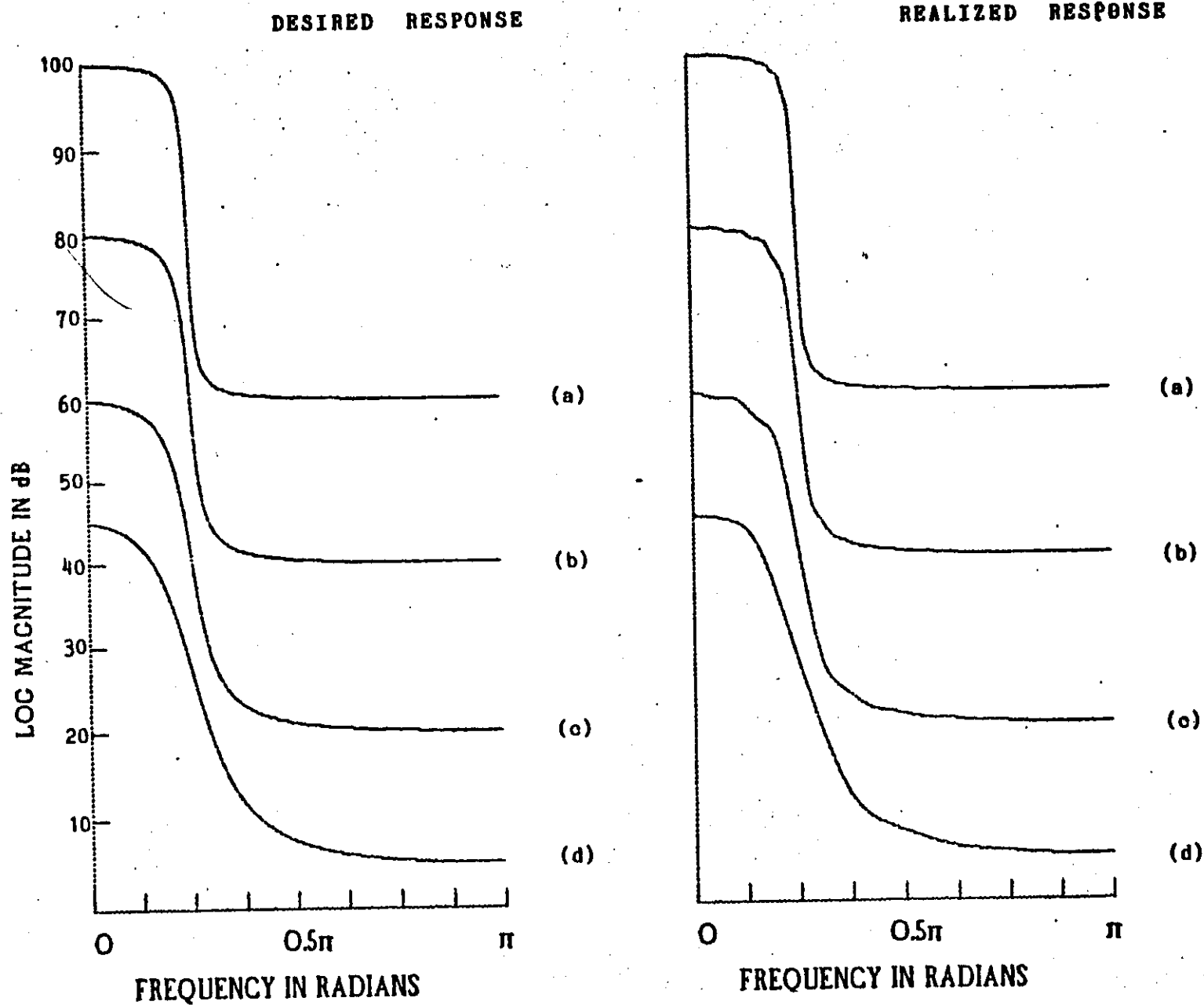
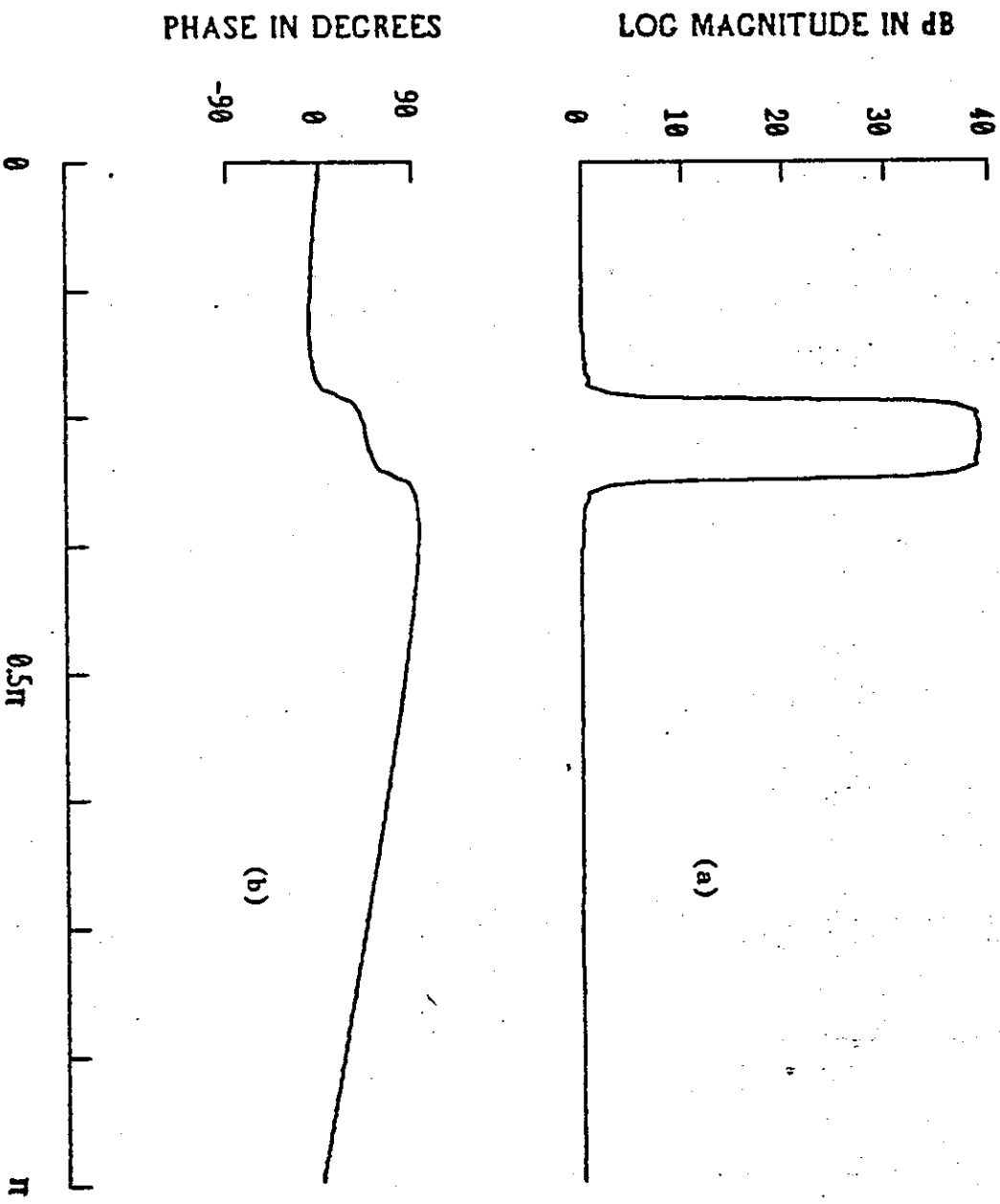


Fig.9 The desired and realized magnitude responses for different orders of the filter

- a) $M_p=M_z=63, \alpha=1$
- b) $M_p=M_z=31, \alpha=1$
- c) $M_p=M_z=15, \alpha=1$
- d) $M_p=M_z=7, \alpha=1$



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Fig.10 Response of a bandpass filter realized by cascading a lowpass filter and a highpass filter ($M_p=M_z=127, \alpha=0.5$).
 (a) Magnitude response
 (b) Phase response