NAMS
91-12

A VARIATIONAL PROBLEM ARISING FROM A MODEL IN THERMODYNAMICS

by

M. Marcus
Department of Mathematics
Technion-Israel Inst. of Technology
32000 Haifa, Israel

Research Report No. 91-107-NAMS-12

March 1991

Report on "A variational problem arising from a model in thermodynamics" by M. Marcus

We study a model for the thermodynamics of equilibrium of materials for which the free energy density depends not only on the concentration u but also on its first and second gradients. Specifically, we assume that the free energy associated with a concentration field u is of the form,

$$\psi_u(x) = u''^2 - bu'^2 + c\tilde{\psi}_0(u)$$

where $\tilde{\psi}_0(u)$ is a two-well function such as $(u^2-1)^2$. We restrict our attention to uni-dimensional bodies so that our variational problem is

$$(*) \qquad \inf\left\{J[u]:\ u \in H_2(\alpha,\beta),\ \langle u\rangle = a\right\}$$

where

$$J[u] = \int_\alpha^\beta \psi_u(x)dx$$

$$\langle u\rangle = \frac{1}{\beta-\alpha}\int_\alpha^\beta u dx$$

and α is a specified number. From the physical point of view u should be nonnegative, but for technical reasons it is more convenient not to impose this restriction. However, our results remain valid also under this condition.

The variational problem mentioned above was studied by Coleman, Marcus and Mizel in the case of bodies of infinite extent. In this case (after an appropriate modification of the definition of J[u] and $\langle u\rangle$) it is possible to obtain rather detailed information. Let us mention just two main results.

(1) If $\phi(a)$ denotes the value of the infimum described before, then ϕ is a convex function.

(2) If a is an "exposed" point of ϕ then the variational problem has a periodic minimizer. Otherwise, there exists a minimizer which can be described as a composite of two periodic functions.

In order to apply this information to the study of bodies of finite extent one is naturally interested in the following questions. Consider the sequence of minimizers u_n of the variational problem (*) for $(\alpha,\beta) = (-n,n)$. Does the sequence $\{u_n\}$, or at least a subsequence, converge in some sense to a minimizer on the whole line? What is the nature of the limiting minimizer? In particular, is this minimizer periodic in some generalized sense?

The first step in this investigation is to establish uniform estimates for the sequence $\{u_n\}$ which will imply its compactness in some appropriate sense. This problem is currently under study by Marcus and Mizel. The results (not yet complete) indicate that the sequence $\{u_n\}$ is uniformly bounded in $C^k_{loc}(R)$ for every $k \geq 1$.

MASTER COPY KEEP THIS COPY FOR REPRODUCTION PURPOSES

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	Feb 1991	

4. TITLE AND SUBTITLE
A variational problem arising from a model in thermodynamics

5. FUNDING NUMBERS

6. AUTHOR(S)
Moshe Marcus

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Carnegie Mellon University
Department of Mathematics
Pittsburgh, PA 15213

8. PERFORMING ORGANIZATION REPORT NUMBER
NAMS-12

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

We study a model for the thermodynamics of equilibrium of materials for which the free energy density depends not only on the concentration u but also on its first and second gradients.

14. SUBJECT TERMS

15. NUMBER OF PAGES
2

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL