Shear Bands as Discontinuities

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1. Introduction

This report provides the essential aspects of an invited presentation made by the first author during a "Workshop on Shear Bands" held at Carnegie Mellon University, March 23-25, 1992. The workshop was under the auspices of the Center for Nonlinear Analysis, a Center of Excellence funded by the Army Research Office. The organizers were Professor Morton E. Gurtin of Carnegie Mellon University and Dr. John Walter of the Ballistics Research Laboratory.

The purpose of this investigation is to demonstrate a mathematical description of the formation of a shear band which exploits the extreme thinness of the band. The assumption of a very narrow scale for shear bands is well justified by experimental observations (e.g. [1],[3]). The analysis will be developed within the context of a one-dimensional theory analogous to that considered in [2],[4],[5],[6].

In the one-dimensional theory, the shear band is represented as a spatial line of discontinuity, across which there are jumps in the value of certain physical quantities. In particullar, expressions for the jumps in velocity, temperature gradient and stress gradient are derived in terms of a single function. The continuity of temperature and stress across the shear band, as well as the inherent symmetries of the problem are maintained.

In dimensional form, we consider the governing equations of momentum, elasticity and energy, respectively as

(1.1) $\hat{\rho} V_{\hat{t}} = S_{\hat{y}}$,

(1.2) $S_{\hat{t}} = \mu(V_{\hat{y}} - \dot{\Gamma})$,

(1.3) $\hat{\rho} c_v T_{\hat{t}} = k T_{\hat{y}\hat{y}} + AS\dot{\Gamma}$, $-L < \hat{y} < L$, $\hat{t} > 0$,

where $V(\hat{y},\hat{t})$ is the velocity, $S(\hat{y},\hat{t})$ is the shear stress and $T(\hat{y},\hat{t})$ is the temperature in a slab of thickness 2L centered at $\hat{y} = 0$. The material constants $\hat{\rho}$, μ, c_v and k are the density, elastic shear modulus, specific heat, and thermal conductivity, respectively. The constant A scales the conversion of plastic work into thermal energy. The plastic strain rate function $\dot{\Gamma}$ is defined by a flow law,

(1.4) $\dot{\Gamma} = \dot{\Gamma}(S,T,\Gamma)$,

in which certain types of work hardening effects have been incorporated through the explicit dependence on Γ.

The boundary conditions at the ends of the slab are

(1.5) $T(\pm L,\hat{t}) = T_0$, $V(\pm L,\hat{t}) = \pm V^*$,

with appropriate compliance of the stress. The initial conditions are

(1.6) $T(\hat{y},0) = T_0$, $S(\hat{y},0) = S_0$, $V(\hat{y},0) = V_0(\hat{y})$.

Here T_0, S_0 and V^* are positive constants, while the initial velocity $V_0(\hat{y})$ is a continuously differentiable function with the properties

(1.7) $V_0(-\hat{y}) = - V_0(\hat{y})$, $V_0(\pm L) = \pm V^*$, $V_0'(\hat{y}) > 0$.

The continuity of $V_0(\hat{y})$ together with (1.7) implies that $V_0(0) = 0$, so that there is no initial velocity slip.

It is consistent with (1.1)-(1.7) to impose the symmetry conditions

(1.8) $$T(\hat{y}, \hat{t}) = T(-\hat{y},\hat{t}), \quad S(\hat{y},\hat{t}) = S(-\hat{y},\hat{t}), \quad V(\hat{y},\hat{t}) = - V(-\hat{y},\hat{t}).$$

In our analysis to follow we will assume that the possible presence of a shear band can be described by the strong localization of $\dot{\Gamma}$ along the line of symmetry $\hat{y} = 0$. Moreover, our nondimensionalization of the slab thickness will be chosen so as to locate the ends of the slab at an (essentially) infinite distance from the centerline. These concepts are physically reasonable and allow for some mathematical simplifications of the problem.

To nondimensionalize (1.1)-(1.7) we introduce the scalings

(1.9) $$\theta = T/T_0, \quad s = S/S_0, \quad v = V/V^*,$$
$$t = \hat{t}/\bar{t}, \quad y = \hat{y}/\ell, \quad \gamma = \mu\bar{t}\Gamma/S_0, \quad \dot{\gamma} = \mu\bar{t}\dot{\Gamma}/S_0$$

and parameters,

(1.10) $$\ell = kS_0/\hat{\rho}c_v\mu V^*, \quad \bar{t} = kS_0^2/\hat{\rho}c_v(\mu V^*)^2,$$
$$\rho = \hat{\rho}\mu(V^*/S_0)^2, \quad \lambda = AS_0^2/\hat{\rho}c_vT_0\mu .$$

Under the assumption that $L/\ell >> 1$, we express (1.1) to (1.3) as

(1.11) $$\rho v_t = s_y,$$

(1.12) $$s_t = v_y - \dot{\gamma},$$

(1.13) $$\theta_t = \theta_{yy} + \lambda s\dot{\gamma}, \quad -\infty < y < \infty, \quad t > 0.$$

The flow law for the plastic strain rate takes the form

(1.14) $\dot{\gamma} = \dot{\gamma}(s,\theta,\gamma)$.

The boundary conditions become

(1.15) $\theta(\pm\infty,t) = 1, \quad v(\pm\infty,t) = \pm 1,$

with appropriate compliance of the stress. The initial conditions are

(1.16) $\theta(y,0) = 1, \quad v(y,0) = v_0(y),$

where $v_0\ (y)$ is continuously differentiable with

(1.17) $v_0(-y) = -\ v_0(y), \quad v_0(\pm\infty) = \pm 1, \quad v_0'(y) > 0, \quad v_0(0) = 0.$

The symmetry conditions (1.8) become

(1.18) $\theta(y,t) = \theta(-y,t), \quad s(y,t) = s(-y,t), \quad v(y,t) = -v(-y,t).$

2. Derivation of the Jump Conditions.

We treat the (possible) presence of a shear band by requiring that it be confined to a small neighborhood of the origin ($-\epsilon \leq y \leq \epsilon$). This confinement then allows for (possible) jump discontinuities in velocity, temperature gradient and stress gradient across the infinitesimally thin shear band located along the axis of symmetry ($y = 0$). It will be seen that such discontinuities must be associated with a strong localization of the plastic strain rate at $y = 0$.

By assuming that $\dot{\gamma}$ can localize into a spatially singular behavior at the origin, it follows that the limit defined by

$$(2.1) \qquad F(t) = \lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} \dot{\gamma}\, dy$$

is not necessarily zero. The jump conditions for velocity, temperature gradient and stress gradient can be derived in terms of $F(t)$. It follows from (1.12), (1.13) and (1.11), respectively, that

$$(2.2) \qquad \langle v \rangle = v(0^+,t) - v(0^-,t) = \lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} v_y dy$$

$$= \lim_{\epsilon \to 0} \left[\frac{d}{dt} \int_{-\epsilon}^{\epsilon} s dy + \int_{-\epsilon}^{\epsilon} \dot{\gamma}\, dy \right] = F(t),$$

$$(2.3) \qquad \langle \theta_y \rangle = \theta_y(0^+,t) - \theta_y(0^-,t) = \lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} \theta_{yy} dy$$

$$= \lim_{\epsilon \to 0} \left[\frac{d}{dt} \int_{-\epsilon}^{\epsilon} \theta dy - \lambda \int_{-\epsilon}^{\epsilon} s\dot{\gamma}\, dy \right]$$

$$= - \lambda\, s(0,t)\, F(t),$$

$$(2.4) \qquad \langle s_y \rangle = s_y(0^+,t) - s_y(0^-,t) = \rho\, F'(t).$$

These results assume that s and 8 remain continuous at y — 0.

To illustrate how F(t) might be determined from (2.1), we consider the rather typical circumstance in which the functional form of $\dot{7}$ contains a very large multiplicative parameter. That is,

(2.5) $\dot{7}(s,e,_7)$ - $B\dot{7}_0(s,e,7_Q)$, B » 1 .

Many of the power law and Arrhenius law models of $\dot{7}$ yield expressions in the general form of (2.5). Typically for s and 8 below certain critical values, $\dot{7}_9$ is so small with respect to B that $B\dot{7}_Q$ is essentially zero (no plastic straining). Then, when s and 8 reach their critical values, $\dot{7}_Q$ becomes order unity with respect to B, and hence $B\dot{7}_{\tilde{U}}$ is quite large (plastic straining has begun) . If we take

(2.6) B - B(c) - $\overset{B}{2?}^{+}$ ••• '

as a scaling relationship between the large parameter B and the width 2e of the shear band, it follows that

(2.7) $F(t)$ - $\lim_{\epsilon\to0} \int^{\epsilon}$ ^ $\dot{\gamma}\, dy$ = ^ $\lim_{\epsilon\to0} \int_{-1}^{1} \dot{\gamma}_0\Big|_{y=\epsilon y'} dy' = B_0\dot{\gamma}_0\Big|_{y=0}$.

Thus,

(2.8) $F(t)$ - $B_0\dot{7}_0[s(0,t),\ 0(0,t),\ 7_0(0,t)]$.

3. Formulation of the Half-Slab Problem

In view of the symmetry conditions (1.18), we can now pose an appropriate problem for the half-slab ($0 < y < \infty$) with boundary conditions at $y = 0$ deduced from the jump relations (2.2)-(2.4). An important implication of (2.1) is that the localization of $\dot{\gamma}$ to the symmetry axis $y = 0$ allows us to ignore this term in (1.12) and (1.13) when considering only a half-slab. The contribution from $\dot{\gamma}$ will be reflected through the boundary conditions at $y = 0$. It is also convenient to eliminate v from the statement of the mathematical problem by integration of (1.11) with respect to time and utilizing the initial conditon of (1.16).

The half-slab problem ican then be expressed in the form

$$s_t(y,t) = \frac{1}{\rho}\int_0^t s_{yy}(y,t')dt' + v_0'(y), \tag{3.1}$$

$$\theta_t(y,t) = \theta_{yy}(y,t), \quad 0 < y < \infty, \quad t > 0 \tag{3.2}$$

$$\theta_y(0,t) = -\frac{\lambda}{2}s(0,t)\,F(t), \quad s_y(0,t) = \frac{\rho}{2}F'(t), \tag{3.3}$$

$$\theta(\infty,t) = 1, \quad s(\infty,t) = 1, \tag{3.4}$$

$$\theta(y,0) = 1, \quad s(y,0) = 1. \tag{3.5}$$

The nonlinear coupling of $s(y,t)$ and $\theta(y,t)$ arises through the dependence of $F(t)$ on $s(0,t)$ and $\theta(0,t)$ as determined by a specific flow law.

It is straightforward to obtain expressions for $\theta(y,t)$ and $s(y,t)$, in terms of $F(t)$, which satisfy (3.1)-(3.5). We have

$$(3.6)\qquad s(y,t) = 1 + \frac{\sqrt{\rho}}{2}\left[v_0\left(y + \frac{t}{\sqrt{\rho}}\right) - v_0\left(y - \frac{t}{\sqrt{\rho}}\right)\right]$$

$$- \frac{\sqrt{\rho}}{2} H\left(t - \sqrt{\rho}\, y\right)\left[F\left(t-\sqrt{\rho}\, y\right) - F(0)\right],$$

$$(3.7)\qquad \theta(y,t) = 1 + \frac{\lambda}{2}\int_0^t [\pi(t - t')]^{-\frac{1}{2}} \exp[-y^2/4(t - t')]s(0,t')F(t')dt'.$$

It should be noted that if the slip velocity $\langle v\rangle = F(t)$ were given <u>a priori</u>, then (3.6)-(3.7) provides the solution of the half-slab problem. However, that represents a very elementary view of the problem, which ignores any dependence of the slip on stress and temperature, as implied by the flow law (1.14).

In the more significant physical situations the slip velocity will depend upon the stress and temperature in the shear band. In the case of (2.8), if we suppress strain hardening effects for simplicity, and set $B_0 = 1$, then

$$(3.8)\qquad F(t) = \dot{\gamma}_0[s(0,t),\ \theta(0,t)] \quad .$$

Equations for the determination of $s(0,t)$ and $\theta(0,t)$ then follow by setting $y = 0$ in (3.6)-(3.7) so that

$$(3.9)\qquad s(0,t) = 1 + \sqrt{\rho}\ v_0\left(\frac{t}{\sqrt{\rho}}\right) - \frac{\sqrt{\rho}}{2}\left\{\dot{\gamma}_0[s(0,t),\ \theta(0,t)] - \dot{\gamma}_0(1,1)\right\} \quad ,$$

$$(3.10)\qquad \theta(0,t) = 1 + \frac{\lambda}{2}\int_0^\tau [\pi(t-t')]^{-1/2} s(0,t')\dot{\gamma}_0[s(0,t'),\ \theta(0,t')]dt' \quad .$$

Typically, (3.9) will represent an algebraic equation, from which $s(0,t)$ can be determined in terms of $\theta(0,t)$. This determination would allow (3.10) to be reduced to an integral equation for $\theta(0,t)$. Once $\theta(0,t)$ has been determined, then s(0,t) is known from (3.9). Thus F(t) becomes known from (3.8), which implies that s(y,t) and Θ(y,t) are then given by (3.6)-(3.7).

As an illustrative example, consider the plastic strain rate flow law to be of the form

$$\dot{\gamma} = \frac{1}{2\epsilon} s^N e^{\beta\Theta} \quad , \quad N \geq 1 \quad , \quad \beta > 0 \quad . \tag{3.11}$$

This combination power and Arrhenius law ignores strain hardening effects. With an initial velocity profile $v_0(y) = \alpha y$, then (3.9)-(3.10) take the form

$$s(0,t) = 1 + \alpha t - \frac{\sqrt{\rho}}{2} \left\{ [s(0,t)^N e^{\beta\Theta(0,t)} - e^{\beta} \right\} \quad ,$$

$$\Theta(0,t) = 1 + \frac{\lambda}{2} \int_0^t [\pi(t-t')]^{-1/2} [s(0,t')]^{N+1} e^{\beta\Theta(0,t')} dt' \quad .$$

A numerical solution of this problem is currently under investigation.

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