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NON-MONOTONIC TRANSFORMATION KINETICS AND THE MORPHOLOGICAL STABILITY OF PHASE BOUNDARIES IN THERMOELASTIC MATERIALS

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The linear stability of an *isothermal* two-phase process involving a planar phase boundary of *normal* type is analyzed. Results are obtained for cases wherein: *i.* the temperature field is unaltered as a consequence of the disturbance which is imposed, *ii.* the process is initially static, *iii.* the phase boundary associated with the process propagates steadily with non-zero velocity. In the first case the linear stability criteria are equivalent to those obtained in a related purely mechanical investigation. In the second case the process is found to be linearly stable. In the last case it is shown that instability can arise under a variety of circumstances. Of particular interest is a case where the kinetic relation is *mechanically stable* but *thermally unstable*; in this case the transformation process is unstable with respect to disturbances which include only suitably *long waves*, implying that a state involving a highly wrinkled interface is favored over one involving a planar interface.

1. Introduction. The morphological features of multiphase equilibria in solids capable of sustaining *diffusionless* phase transformations typically involve complex arrangements of *martensitic* plates, oriented along specific crystallographic axes, within an *austenitic* matrix.[1] Much attention has been directed toward the continuum mechanical modeling of such solids. The most straightforward contexts in which to pursue such an approach are afforded by finite elasticity and finite thermoelasticity. In order to model solids which exhibit multiphase equilibria within these contexts it is necessary to consider materials characterized by constitutive response functions (*i.e.*, *elastic potentials* and *Helmholtz free energies* in the cases of elasticity and thermoelasticity, respectively) which depend upon the deformation gradient in a non-convex fashion. ERICKSEN [7,8], FONSECA [9], FOSDICK & MACSITHIGH [10,11] JAMES [15–17] and others have studied the absolute minimizers of non-convex functionals corresponding to such response functions. These minimizers display fine-scale structure akin to that observed in actual multiphase equilibria.

A question which arises at this stage concerns the manner in which such equilibria are reached. Given the apparent success of the continuum mechanical approach in the static setting, it is natural to wonder whether studies performed within a fully dynamical context will yield useful information pertaining to the evolution from single to multiphase states and, in particular, the development of plate-like morphology.

Except for the work of SILLING [24] and FRIED [12,13], most of the continuum mechanical explorations into the role of dynamics in diffusionless phase transformations have been confined to one-dimensional bar theories.[2] To address the morphological issues at hand it is clearly necessary to work in a multidimensional context. In [24] SILLING performs an asymptotic analysis which shows that a particular isotropic hyperelastic material with non-convex potential is able to sustain a steady state involving a single cusped surface of strain discontinuity, *i.e.*, phase boundary, which propagates at constant velocity. This phase boundary is reminiscent in structure to a martensitic plate of the type mentioned above. One conclusion reached in [24] is that isotropic materials can exhibit non-planar phase boundaries. This suggests that, although the important features of orientation are lost by restricting attention to isotropic materials, working within the more analytically tractable setting provided by such materials may lead to qualitative insights concerning phase boundary kinetics and the development of plate-like structures.

In [12] the linear stability of a transformation process is studied using a class of materials of which that used in [24] is an element. The process involves a steadily propagating planar phase boundary of *normal* type. The analysis reveals a condition necessary and sufficient for the process to be *linearly stable*: the *kinetic response function*—which relates the *driving traction* acting on a phase boundary to the normal velocity of the phase boundary—must be a locally increasing function of its argument at the value corresponding to the undisturbed transformation process. This result does not depend on the wave-number associated with

[1] Sample micrographs depicting such states are provided by ZACKAY *et al.* [27].

[2] See, for example, ABEYARATNE & KNOWLES [1,2,4], JAMES [18] and PENCE [23].

the initial disturbance to which the process is subjected. A necessary consequence of the results given in [12] is that the process can be unstable only if the kinetic response function depends on its argument non-monotonically. Non-monotonic kinetic response functions are admissible under the *Clausius-Duhem* version of the second law of thermodynamics (specialized appropriately to isothermal conditions consistent with [12]); furthermore, the work of OWEN *et al.* [22] suggests that a non-monotonic relation between interfacial driving traction and normal velocity may explain the emergence and growth of plate-like structures within martensitic solids. Under such kinetics, the results obtained in [12] suggest that, in a fully nonlinear context, an evolution from planar to plate-like phase boundary morphology is possible within the confines of a purely mechanical theory.

Thermal effects are absent in [12]. However, the experimental work of CLAPP & YU [5], CONG DAHN *et al.* [6], GRUJICIC *et al.* [14], and others indicates that temperature effects play a significant role in the kinetics of phase boundaries in diffusionless phase transformations. The investigation performed in [13] explores the linear stability of the isothermal extension of the process considered in [12] within a thermomechanical context. It is shown in [13] that the inclusion of thermal effects allows for a rich set of linear stability criteria where, in particular, morphological instability can occur even when the transformation kinetics are mechanically stable in the sense of [12] and, furthermore, the value of the wave-number associated with the initial disturbance to which the transformation process is subjected plays a significant role in its stability.

This paper extends the results of [13] and clarifies their relation to those of [12]. It is organized as follows. In section 2 the notation, kinematics, fundamental balance principles and imbalance principle, as well as the constitutive assumptions which will be needed thereafter are introduced. The notion of a thermoelastic *antiplane shear* is defined in Section 3. A description of the transformation process to be studied is given in Section 4. Finally, Section 5 is concerned with a *normal mode* analysis of this process.

2. Preliminaries. In the following $\mathbb{R}$ and $\mathbb{C}$ denote the sets of real and complex numbers. The intervals $(0,\infty)$ and $[0,\infty)$ are represented by $\mathbb{R}_+$ and $\bar{\mathbb{R}}_+$. The symbol $\mathcal{E}$ represents real three-dimensional Euclidean space. If U is a set, then its closure is designated by $\bar{U}$. The complement of a set V with respect to U is written as $U \setminus V$, and the Cartesian product of U and V is denoted by $U \times V$.

Vectors and linear transformations from $\mathcal{E}$ to $\mathcal{E}$ (referred to herein as *tensors*) are distinguished from scalars with the aid of boldface type—lower and upper case for vectors and tensors, respectively. Let $\mathbf{a}$ and $\mathbf{b}$ be vectors in $\mathcal{E}$, their inner product is then written as $\mathbf{a}\cdot\mathbf{b}$; the Euclidean norm of $\mathbf{a}$ is, further, written as $|\mathbf{a}| = \sqrt{\mathbf{a}\cdot\mathbf{a}}$. The set of unit vectors in $\mathcal{E}$ is designated by $\mathcal{N}$. The symbol $\mathcal{L}$ refers to the set of tensors, while $\mathcal{L}_+$ denotes the set of all tensors with positive determinant. If $\mathbf{A} \in \mathcal{L}$, then $\mathbf{A}^T$ represents its transpose; if, moreover, $\mathbf{A} \in \mathcal{L}_+$, then the inverse of $\mathbf{A}$ and its transpose are written as $\mathbf{A}^{-1}$ and $\mathbf{A}^{-T}$, respectively. If $\mathbf{A}$ and $\mathbf{B}$ are tensors then their inner product is written as $\mathbf{A}\cdot\mathbf{B} = \operatorname{tr}\mathbf{A}\mathbf{B}^T$.

When component notation is used, Greek indices range only over $\{1,2\}$; summation of repeated indices over the appropriate range is implicit. A subscript preceded by a comma denotes partial differentiation with respect to the corresponding coordinate. Also, a superposed dot signifies partial differentiation with respect to time.

Consider, now, a body which occupies a region $\mathcal{R} \subset \mathcal{E}$. A motion of the body on a time interval $\mathcal{T} \subset \mathbb{R}$ is a one-parameter family of invertible mappings $\hat{\mathbf{y}}(\cdot,t) : \mathcal{R} \to \mathcal{R}_t$, with $\hat{\mathbf{y}}(\mathbf{x},t) = \mathbf{x} + \mathbf{u}(\mathbf{x},t)\ \forall(\mathbf{x},t) \in \mathcal{M}$. Here $\mathcal{M} = \mathcal{R} \times \mathcal{T}$ is the *trajectory* of the motion. Assume that the *deformation* $\hat{\mathbf{y}}$, or equivalently the *displacement* $\mathbf{u}$, is continuous with piecewise continuous first and second partial derivatives on $\mathcal{M}$. Let $\mathcal{S}_t \subset \mathcal{R}$ be defined so that, at each $t \in \mathcal{T}$, $\hat{\mathbf{y}}(\cdot,t)$ is twice continuously differentiable on $\mathcal{R} \setminus \mathcal{S}_t$.

Adopt the global balance laws of continuum thermomechanics for mass, momentum, moment of momentum, and energy, and the Clausius-Duhem statement of the global imbalance of entropy production. The nominal forms of these principles yield the following local field equations and inequality, which hold on $\mathcal{M} \setminus \Sigma$,

$$\varrho = \rho(\hat{\mathbf{y}})J, \quad \nabla \cdot \mathbf{S} + \varrho \mathbf{b} = \varrho \ddot{\mathbf{u}}, \quad \mathbf{S}\mathbf{F}^T = \mathbf{F}\mathbf{S}^T, \quad \mathbf{S} \cdot \dot{\mathbf{F}} + \nabla \cdot \mathbf{q} + \varrho r = \varrho \dot{\varepsilon}, \quad \nabla \cdot (\frac{\mathbf{q}}{\theta}) + \frac{\varrho r}{\theta} \leq \varrho \dot{\eta}, \tag{2.1}$$

and local jump conditions, which hold on Σ,

$$[\![\varrho V_n]\!] = 0, \quad [\![\mathbf{S}\mathbf{n}]\!] + [\![\varrho V_n \dot{\mathbf{u}}]\!] = 0, \quad [\![\mathbf{S}\mathbf{n} \cdot \dot{\mathbf{u}}]\!] + [\![\mathbf{q} \cdot \mathbf{n}]\!] + [\![\varrho V_n(\varepsilon + \tfrac{1}{2}|\dot{\mathbf{u}}|^2)]\!] = 0, \quad [\![\frac{\mathbf{q} \cdot \mathbf{n}}{\theta}]\!] + [\![\varrho V_n \eta]\!] \leq 0, \tag{2.2}$$

where $[\![g(\cdot,t)]\!]$ denotes the jump in $g(\cdot,t)$ across S_t at $t \in \mathcal{T}$. In (2.1) and (2.2) $\varrho : \mathcal{R} \to \mathrm{IR}_+$ is the *nominal mass density*, $\rho : \hat{\mathbf{y}}(\mathcal{M} \setminus \Sigma) \to \mathrm{IR}_+$ is the mass density in the current configuration, $J : \mathcal{M} \setminus \Sigma \to \mathrm{IR}_+$ is the Jacobian determinant of the *deformation gradient tensor* $\mathbf{F} : \mathcal{M} \setminus \Sigma \to \mathcal{L}_+$, $\mathbf{b} : \mathcal{M} \to \mathcal{E}$ is the *nominal body force per unit mass*, $\mathbf{S} : \mathcal{M} \setminus \Sigma \to \mathcal{L}$ is the *nominal stress tensor*, $\mathbf{q} : \mathcal{M} \setminus \Sigma \to \mathcal{E}$ is the *nominal heat flux vector*, $r : \mathcal{M} \to \mathrm{IR}$ is the *nominal heat supply per unit mass*, $\varepsilon : \mathcal{M} \setminus \Sigma \to \mathrm{IR}$ is the *nominal internal energy per unit mass*, $\theta : \mathcal{M} \to \mathrm{IR}_+$ is the *nominal absolute temperature*, and $\eta : \mathcal{M} \setminus \Sigma \to \mathrm{IR}$ is the *nominal entropy per unit mass*. In (2.2), $\mathbf{n}(\cdot,t) : S_t \to \mathcal{N}$ and $V_n(\cdot,t) : S_t \to \mathrm{IR}$ orient and give the normal velocity of S_t at $t \in \mathcal{T}$, respectively. Note that in deriving (2.1) and (2.2) it has been assumed that $\mathbf{S}$, $\mathbf{q}$, ε, θ and η are continuous and piecewise continuously differentiable on their domains of definition. The stipulated smoothness of $\mathbf{u}$ and θ yields the following kinematic jump conditions, which supplement (2.2) on Σ,

$$[\![\mathbf{u}]\!] = \mathbf{0}, \quad [\![\theta]\!] = 0. \tag{2.3}$$

Observe that $(2.2)_1$ and $(2.3)_2$ together imply the following simplified versions of $(2.2)_{2,3,4}$:

$$[\![\mathbf{S}\mathbf{n}]\!] + \varrho V_n [\![\dot{\mathbf{u}}]\!] = \mathbf{0}, \quad [\![\mathbf{S}\mathbf{n} \cdot \dot{\mathbf{u}}]\!] + [\![\mathbf{q} \cdot \mathbf{n}]\!] + \varrho V_n [\![(\varepsilon + \tfrac{1}{2}|\dot{\mathbf{u}}|^2)]\!] = 0, \quad \frac{1}{\theta}[\![\mathbf{q} \cdot \mathbf{n}]\!] + \varrho V_n [\![\eta]\!] \leq 0. \tag{2.4}$$

Furthermore, the jump conditions $(2.4)_{2,3}$ can, using $(2.4)_1$ and (2.3) as in [3], be rewritten as

$$[\![\mathbf{q} \cdot \mathbf{n}]\!] + (f + \varrho\theta[\![\eta]\!])V_n = 0, \quad fV_n \geq 0, \tag{2.5}$$

where $f = \varrho[\![\psi]\!] - \langle\!\langle \mathbf{S} \rangle\!\rangle \cdot [\![\mathbf{F}]\!]$ is the *driving traction* which acts on Σ, $\psi = \varepsilon - \theta\eta$ is the *nominal Helmholtz free energy per unit mass*, and $\langle\!\langle g(\cdot,t) \rangle\!\rangle$ denotes the average of a function $g(\cdot,t)$ across S_t at $t \in \mathcal{T}$.

Suppose, from now on, that the body is composed of a *homogeneous* and *isotropic thermoelastic* material of the type used in [13,19]. Such a material can be thought of as a thermoelastic analogue of a generalized neo-Hookean material in that its thermomechanical response in all three dimensional *isochoric* deformations is fully determined by a *shear stress response function*, $\tau : \overline{\mathrm{IR}}_+ \times \mathrm{IR}_+ \to \overline{\mathrm{IR}}_+$, and a *heat flux response function*, $K : \overline{\mathrm{IR}}_+ \times \mathrm{IR}_+ \to \mathrm{IR}_+$—both of which depend only on a scalar measure of shear strain, $\gamma = \sqrt{\mathbf{F} \cdot \mathbf{F} - 3}$, and the nominal absolute temperature. For the purposes of the current investigation, it is assumed that both τ and K are twice continuously differentiable on their common domain of definition. The shear stress response function τ is allowed to lose monotonicity in the sense that

$$\partial_\gamma \tau > 0 \quad \text{on} \quad A_1 \cup A_3, \quad \partial_\gamma \tau < 0 \quad \text{on} \quad A_2, \tag{2.6}$$

where $A_1 = \{(\gamma,\theta) | 0 < \gamma < g_l(\theta), \theta \in (\theta_*, \theta^*)\}$, $A_2 = \{(\gamma,\theta) | g_l(\theta) < \gamma < g_r(\theta), \theta \in (\theta_*, \theta^*)\}$, $A_3 = \{(\gamma,\theta) | g_r(\theta) < \gamma < \infty, \theta \in (\theta_*, \theta^*)\}$, and the functions $g_l : (\theta_*, \theta^*) \to \mathrm{IR}_+$ and $g_r : (\theta_*, \theta^*) \to \mathrm{IR}_+$ satisfy the inequality $g_l(\theta) < g_r(\theta)\ \forall \theta \in (\theta_*, \theta^*)$. This renders the material non-elliptic and, hence, capable of sustaining equilibrated deformations with discontinuous gradients. The sets A_1 and A_3 are referred to as the *high* and *low* strain elliptic phases of a material of this kind. Specification of the monotonicity properties of τ on the remainder of $\bar{\mathbb{R}}_+ \times \mathbb{R}_+$ is not needed for the sequel. For future reference, the nominal stress tensor, nominal entropy per unit mass and nominal heat flux vector are given in terms of τ and K by

$$\mathbf{S} = M(\gamma,\theta)(\mathbf{F} - \mathbf{F}^{-T}), \quad \varrho\eta = -\int_0^\gamma \partial_\theta \tau(\kappa,\theta)\, d\kappa, \quad \mathbf{q} = K(\gamma,\theta)\nabla\theta, \tag{2.7}$$

for isochoric deformations of a material within the class under consideration. Here $M : \bar{\mathbb{R}}_+ \to \mathbb{R}$ is the *secant modulus in shear* for isochoric deformations of a material of the type at hand and is defined so that $M(\gamma,\theta)\gamma = \tau(\gamma,\theta)\ \forall(\gamma,\theta) \in \bar{\mathbb{R}}_+ \times \mathbb{R}_+$. For a thermoelastic material $(2.1)_3$ is automatically satisfied on $\mathcal{M}\backslash\Sigma$. Observe that the conductivity response function K is restricted so as to assume only positive values. It is easily shown that this limitation is sufficient to guarantee the satisfaction of $(2.1)_5$ on $\mathcal{M}\backslash\Sigma$. Therefore, the local balance equations $(2.1)_{3,5}$ are ignored hereafter. Note that, once $\mathbf{F}$ is known, the current mass density ρ can be calculated directly from $(2.1)_1$—which is also neglected in the following.

3. Thermoelastic antiplane shear. Suppose that $\mathcal{R}$ is a cylindrical region with cross-section $\mathcal{D}$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a fixed orthonormal basis for $\mathcal{E}$ chosen so that $\mathbf{e}_3$ is parallel to the generators of $\mathcal{R}$. If the displacement field $\mathbf{u}$ associated with the deformation $\hat{\mathbf{y}}$ takes the form $\mathbf{u}(\mathbf{x},t) = u(x_1,x_2,t)\mathbf{e}_3$ $\forall(\mathbf{x},t) \in \mathcal{M}$ and, in addition, the nominal absolute temperature field is independent of the x_3–coordinate, a process is a thermoelastic antiplane shear, normal to the plane spanned by $\mathbf{e}_1$ and $\mathbf{e}_2$. It is clear from (2.7) that the remaining field quantities, as well as any surface of discontinuity involving them, must be similarly independent of the x_3–coordinate. Refer to u as the *out-of-plane* displacement field. Note that the deformation associated with a thermoelastic antiplane shear is isochoric. It is, therefore, legitimate to subject a body composed of a material with the constitution introduced in Section 2 to such a process. For a material of this type, it can be shown, using (2.7), that $(2.1)_{2,4}$ hold on $\mathcal{M}\setminus\Sigma$ if and only if

$$\big(M(\gamma,\theta)u_{,\alpha}\big)_{,\alpha} = \varrho\ddot{u}, \quad \big(K(\gamma,\theta)\theta_{,\alpha}\big)_{,\alpha} = \varrho\theta\dot{\eta}, \tag{3.1}$$

hold on $\Omega\setminus\Gamma$, where $\Omega = \mathcal{D}\times\mathcal{T}$ and $\Gamma = \{(x_1,x_2,t)|(x_1,x_2)\in S_t\cap\mathcal{D}, t\in\mathcal{T}\}$. Similarly, $(2.4)_1$ and (2.5) hold on Σ if and only if

$$[\![M(\gamma,\theta)u_{,\alpha}n_\alpha]\!] + \varrho V_n[\![\dot{u}]\!] = 0, \quad [\![K(\gamma,\theta)\theta_{,\alpha}n_\alpha]\!] + (\varrho\theta[\![\eta]\!] + f)V_n = 0, \quad fV_n \geq 0, \tag{3.2}$$

hold on Γ. For thermoelastic antiplane shear note that $\gamma = \sqrt{u_{,\alpha}u_{,\alpha}}$ on $\Omega\setminus\Gamma$. Clearly, the jump conditions (2.3) hold on Σ if and only if the following jump conditions hold on Γ:

$$[\![u]\!] = 0, \quad [\![\theta]\!] = 0. \tag{3.3}$$

Before proceding consider the inequality $(3.2)_3$. If Γ represents the trajectory of a cross-section of a *classical shock*—a surface of discontinuity which does not separate different phases of the material at hand—$(3.2)_3$ is satisfied by requiring that the sign of $V_n(\cdot,t)$ in the *Rankine-Hugoniot* relation

$$V_n(\cdot,\cdot,t) = \pm\sqrt{\frac{[\![M(\gamma(\cdot,\cdot,t),\theta(\cdot,\cdot,t))u_{,\alpha}(\cdot,\cdot,t)n_\alpha(\cdot,\cdot,t)]\!]}{\varrho[\![u_{,\alpha}(\cdot,\cdot,t)n_\alpha(\cdot,\cdot,t)]\!]}} \tag{3.4}$$

(which is a consequence of $(3.2)_1$ and $(3.3)_1$) is chosen pointwise so that $f(\cdot,t)V_n(\cdot,t) \geq 0$ on $S_t\cap\mathcal{D}$ $\forall t\in\mathcal{T}$. On the other hand, if Γ represents the trajectory of the cross-section of a phase boundary, the foregoing procedure is insufficient to obtain uniqueness in the solution of initial boundary value problems which involve *metastable* data.[3] This difficulty can be circumvented by supplementing the constitutive

[3] For a discussion of this issue see ABEYARATNE & KNOWLES [1–4] and TRUSKINOVSKY [26].

information provided by τ and K. An approach to this, taken by ABEYARATNE & KNOWLES in [1–4], entails the provision of a ***kinetic relation*** which determines how the driving traction, the normal velocity of the phase boundary and the local absolute temperature of the phase boundary are related. Two cases motivated by the discussion in [3] can be considered. In the first case a ***kinetic response function*** $\tilde{V} : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ is specified so that, on the trajectory Γ of any phase boundary cross-section,

$$V_n = \tilde{V}(f/\theta, \theta), \tag{3.5}$$

where, sufficient to satisfy $(3.2)_3$ on Γ, $\tilde{V}(\Phi,\theta)\Phi \geq 0 \ \forall(\Phi,\theta) \in \mathbb{R} \times (\theta_*, \theta^*)$. In the second case a ***kinetic response function*** $\tilde{\varphi}$ is furnished so that, on the trajectory Γ of any phase boundary cross-section,

$$f = \theta\tilde{\varphi}(V_n, \theta), \tag{3.6}$$

where, sufficient to satisfy $(3.2)_3$ on Γ, $\tilde{\varphi}(V,\theta)V \geq 0 \ \forall(V,\theta) \in \mathbb{R} \times (\theta_*, \theta^*)$. In both cases no stipulations are imposed regarding the monotonicity of the kinetic response function. However, it is assumed that the kinetic response function is continuously differentiable on its domain of definition. For future reference, this restriction implies that whichever of

$$\tilde{V}(0,\theta) = 0, \quad \partial_\Phi \tilde{V}(\Phi,\theta)\big|_{\Phi=0} \geq 0, \quad \text{or} \quad \tilde{\varphi}(0,\theta) = 0, \quad \partial_V \tilde{\varphi}(V,\theta)\big|_{V=0} \geq 0, \tag{3.7}$$

is appropriate to the form of kinetic relation imposed is satisfied for each $\theta \in (\theta_*, \theta^*)$.

4. Description of a particular transformation process. From now on, suppose that $\mathcal{R} = \mathcal{E}$ and consider a process involving a non-elliptic thermoelastic material of the type introduced in Section 2 on the time interval $(-\infty, 0)$ with an out-of-plane displacement field $u_0(\cdot, t)$ given, for each t in $(-\infty, 0)$, by

$$u_0(x_1, t) = \begin{cases} \gamma_l x_1 + v_l t & \text{if} \quad x_1 < v_0 t, \\ \gamma_r x_1 + v_r t & \text{if} \quad x_1 > v_0 t, \end{cases} \tag{4.1}$$

and an absolute temperature field θ_0 in (θ_*, θ^*) which is constant on $\mathcal{R} \times (-\infty, 0)$, where the amount of shear-temperature pairs (γ_l, θ_0) and (γ_r, θ_0) are contained in either $A_3 \times A_1$ or $A_1 \times A_3$. Then there is no loss in generality incurred by assuming that $v_0 \geq 0$. It is clear that u_0 and θ_0 satisfy $(3.1)_{1,2}$ on $(\mathbb{R}^2 \times (-\infty, 0)) \setminus \Gamma_0$ with $\Gamma_0 = \{(x_1, x_2, t) | (x_1, x_2) \in A_t, t \in (-\infty, 0)\}$ and $A_t = \{(x_1, x_2) | x_1 = v_0 t, x_2 \in \mathbb{R}\}$. For each t in $(-\infty, 0)$ the moving line A_t is the cross-section of a planar phase boundary. Restrict v_0 so that the phase boundary propagates at a locally subsonic velocity, *viz.*

$$\varrho v_0^2 < \min\left\{\partial_\gamma \tau(\gamma, \theta_0)\big|_{\gamma=\gamma_l}, \partial_\gamma \tau(\gamma, \theta_0)\big|_{\gamma=\gamma_r}\right\}. \tag{4.2}$$

Assume, to comply with the jump conditions in $(3.2)_{1,2}$ and (3.3) on Γ_0, that the constants γ_l, γ_r, v_l, v_r, and v_0 associated with (4.1) satisfy the following equations:

$$(\gamma_r - \gamma_l)v_0 + v_r - v_l = 0, \quad \tau(\gamma_r, \theta_0) - \tau(\gamma_l, \theta_0) + \varrho v_0(v_r - v_l) = 0, \quad (f_0 + \varrho\theta_0(\eta_r - \eta_l))v_0 = 0. \tag{4.3}$$

Suppose, also, that the kinetic relation holds on Γ_0 so that the parameters γ_l, γ_r, v_l, v_r, and v_0 satisfy whichever of

$$v_0 = \tilde{V}(f_0/\theta_0, \theta_0), \quad f_0 = \theta_0\tilde{\varphi}(v_0, \theta_0), \tag{4.4}$$

is appropriate, where f_0 is given by

$$f_0 = \int_{\gamma_l}^{\gamma_r} \tau(\gamma, \theta_0)\, d\gamma - \tfrac{1}{2}(\tau(\gamma_r, \theta_0) + \tau(\gamma_l, \theta_0))(\gamma_r - \gamma_l). \tag{4.5}$$

Observe, since $v_0 \geq 0$, that f_0 must be non-negative by $(3.2)_3$.

In a coordinate frame moving with the phase boundary, the process described above involves a piecewise homogeneous strain field and a homogeneous temperature field. If $(\gamma_l, \theta_0) \in A_3$ and $(\gamma_r, \theta_0) \in A_1$ then the process is one wherein the high strain elliptic phase of the material at hand *grows*, isothermally, at the expense of the low strain elliptic phase; the interpretation when $(\gamma_l, \theta_0) \in A_1$ and $(\gamma_r, \theta_0) \in A_3$ is analogous. In both cases, for the duration of the process, the discontinuity involved is a *normal* phase boundary (*i.e.*, the angle between the limiting values of the gradient of u on either side of the phase boundary is zero at every point of the phase boundary over the time interval $(-\infty, 0)$).

The parameter $\ell_0 = \varrho\theta_0(\eta_l - \eta_r) - f_0$ is the *latent heat of transformation* associated with the process described above. From $(4.3)_3$ it is clear that ℓ_0 must be zero if $v_0 > 0$—which agrees with the intuitive notion that the heat given off during the transformation process must be zero due to the absence of *heat flux*. Recall that, since the kinetic response function is continuous, $v_0 = 0$ if and only if $f_0 = 0$. Hence, when $v_0 = 0$, the latent heat of transformation simplifies to $\ell_0 = \rho\theta_0(\eta_l - \eta_r)$; so, unlike the case when $v_0 > 0$, it is evident that $(4.3)_3$ is satisfied for any real value of ℓ_0 when $v_0 = 0$.

Suppose, in addition to all the above, that the kinetic response function is chosen so that whichever of

$$\partial_\Phi \tilde{V}(\Phi, \theta)\big|_{\Phi = \frac{f_0}{\theta_0}} \neq 0, \quad \partial_V \tilde{\varphi}(V, \theta)\big|_{V = v_0} \neq 0 \tag{4.6}$$

is appropriate holds. This precludes the necessity of a higher order expansion in the analysis which follows. Let τ be chosen so that both $\varrho c_d = -\theta_0 \int_0^{\gamma_d} \partial_\theta^2 \tau(\kappa, \theta)|_{\theta=\theta_0}\, d\kappa > 0$ and $\varrho b_d^2 = M(\gamma_d, \theta_0) > 0$ for d equal to l or r. Hereafter, the subscript d stands for either l or r; statements which involve an expression subscripted with d hold for d equal to l and r. The restrictions on c_d amount to requiring that both *nominal specific heats* associated with the transformation process are positive, while those on b_d require that the *Baker-Ericksen inequalities* are satsified throughout the transformation process. Assume also, for mathematical simplicity, that $\partial_\theta \tau(\gamma_l, \theta)|_{\theta=\theta_0} = \partial_\theta \tau(\gamma_r, \theta)|_{\theta=\theta_0} = 0$. By virtue of this stipulation, the *coefficients of thermoelastic coupling* in the tranformation process are both identically zero. The thermal and mechanical fields remain coupled, however, through the jump conditions $(3.2)_{1,2}$ and the kinetic relation (3.5) or (3.6).

5. Normal mode analysis of the transformation process. Suppose that at the instant $t = 0$ the out-of-plane displacement and velocity fields, the absolute temperature field, and the configuration of the phase boundary (associated with the transformation process specified in Section 4) are subjected to a perturbation—which is itself a thermoelastic antiplane shear. Assume that this perturbation initiates a thermoelastic antiplane shear with out-of-plane displacement field $u = u_0 + w$ and absolute temperature field $\theta = \theta_0 + T$ which are weak solutions of (3.1) on Ω and have gradients which allow $(3.2)_{1,2}$ and one of (3.5) or (3.6) to be satisfied on a single phase boundary with cross-sectional trajectory $\Gamma = \{(x_1, x_2, , t)| x_1 = v_0 t + s(x_2, t), x_2 \in \mathbb{R}, t \in \mathbb{R}_+\}$, where the amplitudes of w, T and s are all assumed to be small in some appropriate sense.

Linearization of the jump conditions $(3.2)_{1,2}$ under this assumption yields the equations

$$[\![(a^2 - v_0^2) w_{,1}]\!]_l^r = 2v_0(\gamma_r - \gamma_l)\dot{s}, \quad [\![kT_{,1}]\!]_l^r + \varrho v_0 [\![cT]\!]_l^r = \varrho v_0(\gamma_r - \gamma_l)\langle\!\langle (a^2 - v_0^2) w_{,1} \rangle\!\rangle + \ell_0 \dot{s}, \tag{5.1}$$

which hold on $I = \{(x_1, x_2, t)| x_1 = v_0 t, x_2 \in \mathbb{R}, t \in \mathbb{R}_+\}$, where $\varrho a_d^2 = \partial_\gamma \tau(\gamma, \theta_0)|_{\gamma=\gamma_d}$ and $k_d = K(\gamma_d, \theta_0)$. Linearization of the kinematic jump conditions (3.3) and kinetic relation results in the following conditions—which also hold on I:

$$[\![w]\!]_l^r = (\gamma_l - \gamma_r)s, \quad [\![T]\!] = 0, \quad \dot{s} = \frac{\gamma_l - \gamma_r}{v_*}\langle\!\langle (a^2 - v_0^2) w_{,1} \rangle\!\rangle + (v_\diamond + \frac{\ell_0}{\varrho v_* \theta_0})T. \tag{5.2}$$

The constants v_* and $v_\diamond$ in $(5.2)_3$ are defined by

$$v_* = \frac{\theta_0}{\varrho\partial_\Phi \tilde{V}(\Phi,\theta_0)\big|_{\Phi=\frac{f_0}{\theta_0}}}, \quad v_\diamond = \partial_\theta \tilde{V}(f_0/\theta_0,\theta)\big|_{\theta=\theta_0}, \tag{5.3}$$

if the kinetic relation is specified in the form (3.5), or

$$v_* = \theta_0 \partial_V \tilde{\varphi}(V,\theta_0)\big|_{V=v_0}, \quad v_\diamond = -\frac{\partial_\theta \tilde{\varphi}(v_0,\theta)\big|_{\theta=\theta_0}}{\partial_V \tilde{\varphi}(V,\theta_0)\big|_{V=v_0}}, \tag{5.4}$$

if the kinetic relation is provided in the form (3.6).

Next, taking the fact that (3.1) and (3.2) also hold on I, linearization of (3.1) gives the field equations

$$a_d^2 w_{,11} + b_d^2 w_{,22} = \ddot{w}, \quad \alpha_d T_{,\beta\beta} = \dot{T}, \tag{5.5}$$

which prevail on Ω_d—with $\Omega_l = \{(x_1,x_2,t)|\, x_1 < v_0 t, x_2 \in \mathbb{R}, t \in \mathbb{R}_+\}$ and $\Omega_r = (\mathbb{R}^2 \times \mathbb{R}_+) \setminus \bar{\Omega}_l$.

Because of the restrictions which have been placed on the values of the various constitutive response functions which comprise their coefficients, $(5.5)_1$ and $(5.5)_2$ are hyperbolic and parabolic, respectively, on their domains of definition. Observe that these equations are linear with constant coefficients and hold on rectangular domains. The equations (5.1)–(5.3) are also linear with constant coefficients and hold on the planar cross-sectional trajectory which separates Ω_l and Ω_r. This suggests the *Ansatz*:

$$\begin{aligned} w(x_1,x_2,t) &= W_d e^{\xi_d(x_1 - v_0 t)} e^{i\kappa x_2} e^{pt} \quad \forall (x_1,x_2,t) \in \Omega_d, \\ T(x_1,x_2,t) &= \Theta_d e^{\zeta_d(x_1 - v_0 t)} e^{i\kappa x_2} e^{pt} \quad \forall (x_1,x_2,t) \in \Omega_d, \\ s(x_2,t) &= S e^{i\kappa x_2} e^{pt} \quad \forall (x_2,t) \in \mathbb{R} \times \mathbb{R}_+. \end{aligned} \tag{5.6}$$

The constant *wave-numbers* ξ_d and ζ_d are required to have positive real parts for $d = l$ and negative real parts for $d = r$, so that the perturbation decays in the far-field. The constant wave-number κ is assumed to be real, but non-zero, while the constant *growth-rate* p may be complex. Observe that the initial disturbance associated with an arbitrary perturbation of the transformation process described in Section 4 can be decomposed in the form (5.6) via a two-dimensional Fourier transform. Therefore, it seems reasonable to expect that the linear stability of the transformation process under consideration can be understood by determining p as a function of ξ_l, ξ_r, ζ_l, ζ_r and κ. However, ξ_l, ξ_r, ζ_l and ζ_r are determined by (5.5) in the form

$$\xi_l = \frac{f_l(\kappa,p) - v_0 p}{a_l^2 - v_0^2}, \quad \xi_r = \frac{f_r(\kappa,p) + v_0 p}{a_r^2 - v_0^2}, \quad \zeta_l = \frac{g_l(\kappa,p) - v_0}{2\alpha_l}, \quad \zeta_r = \frac{g_r(\kappa,p) - v_0}{2\alpha_r}, \tag{5.7}$$

where $f_d(\kappa,p) = \sqrt{(a_d^2 - v_0^2) b_d^2 \kappa^2 + a_d^2 p^2}$ and $h_d(\kappa,p) = \sqrt{v_0^2 + 4\alpha_d(\alpha_d \kappa^2 + p)}$. This fixes ξ_l, ξ_r, ζ_l and ζ_r in terms of κ—which narrows the class of initial disturbances which can be decomposed in the form (5.6). From now on, restrict attention to perturbations where the initial disturbance can be decomposed in the form (5.6) with ξ_l, ξ_r, ζ_l and ζ_r given by (5.7). Such perturbations are henceforth referred to as ***admissible***. Given (5.7), (5.1)–(5.2) provide a homogeneous system of five linear equations in the five unknown *amplitudes* W_l, W_r, Θ_l, Θ_r, and S. The determinant of this system, therefore, provides a *dispersion relation* involving p and κ, *viz.*

$$\frac{\Gamma(\kappa,p)}{v_*}(1 + v_0 \Lambda(\kappa,p)) + p(1 + \frac{\ell_0}{\varrho v_*}\Lambda(\kappa,p)) = 0, \tag{5.8}$$

where $\Gamma(\kappa,p)$ and $\Lambda(\kappa,p)$ are defined as follows:

$$\Gamma(\kappa,p)=\frac{(\gamma_l-\gamma_r)^2(f_l(\kappa,p)f_r(\kappa,p)+v_0^2p^2)}{f_l(\kappa,p)+f_r(\kappa,p)},\quad \Lambda(\kappa,p)=\frac{2(v_*v_\diamond+\frac{\ell_0}{\varrho\theta_0})}{(c_l-c_r)v_0+c_lh_l(\kappa,p)+c_rh_r(\kappa,p)}. \tag{5.9}$$

Note that $v_*=0$ is precluded by (4.6), $(5.3)_1$ and $(5.4)_1$. The linear stability of the process at hand with respect to an admissible perturbation is determined by the signs of the real parts of the zeros of (5.8) as functions of κ. Zeros of (5.8) with positive real parts are called ***unstable zeros*** of the dispersion relation.

The dispersion relation can now be specialized. Three major cases will be considered. These are: *i.* $T=0$ on Ω, *ii.* $v_0=0$, and *iii.* $v_0>0$. In case *i* (5.8) simplifies to

$$p+\frac{(\gamma_l-\gamma_r)^2(f_l(\kappa,p)f_r(\kappa,p)+v_0^2p^2)}{v_*(f_l(\kappa,p)+f_r(\kappa,p))}=0. \tag{5.10}$$

Assume that $v_0\neq 0$, since this alternative is encompassed by case *ii*. Note that (5.10) is formally identical to the dispersion relation obtained in the purely mechanical analysis of [12]. Based upon the results of [12], it is clear that a necessary and sufficient condition for the existence of an unstable zero to (5.10) is that $v_*<0$. This is true independent of the value of $\kappa\in\mathbb{R}\setminus 0$. Hence, when $T=0$ on Ω, the transformation process is linearly unstable with respect to an admissible perturbation if and only if the kinetic response function allows $v_*<0$. Since $v_0\neq 0$, there are no restrictions pertaining to the monotonicity of the kinetic response function. Hence, from the alternate definitions $(5.3)_1$ and $(5.4)_1$ of v_*, the transformation process will, in the present case, be linearly unstable with respect to any admissible perturbation if and only if the kinetic response function is a locally decreasing function of its first argument. Such a kinetic response function is referred to as ***mechanically unstable.***

Next, in case *ii*, setting $v_0=0$ in (5.8) gives

$$p+\frac{1}{v_*}\left(\frac{(\gamma_l-\gamma_r)^2a_la_r\sqrt{b_l^2\kappa^2+p^2}\sqrt{b_r^2\kappa^2+p^2}}{a_l\sqrt{b_l^2\kappa^2+p^2}+a_r\sqrt{b_r^2\kappa^2+p^2}}+\frac{\theta_0(\eta_l-\eta_r)^2}{c_l\sqrt{\alpha_l(\alpha_l\kappa^2+p)}+c_r\sqrt{\alpha_r(\alpha_r\kappa^2+p)}}\right)=0. \tag{5.11}$$

Let the square roots which appear in (5.11) be defined, as is natural, so that, for $p\in\mathbb{R}$,

$$b_d^2\kappa^2+p^2>0\Rightarrow b_d^2\kappa^2+p^2>0,\quad \alpha_d\kappa^2+p>0\Rightarrow\alpha_d\kappa^2+p>0; \tag{5.12}$$

then, for $p\in\mathbb{C}$,

$$\mathrm{Re}(b_d^2\kappa^2+p^2)>0\Rightarrow\mathrm{Re}\sqrt{b_d^2\kappa^2+p^2}>0,\quad \mathrm{Re}(\alpha_d\kappa^2+p)>0\Rightarrow\mathrm{Re}\sqrt{\alpha_d\kappa^2+p}>0. \tag{5.13}$$

Then, as in case *i*, necessary and sufficient for the existence of an unstable zero to (5.11) is that $v_*<0$. From $(3.7)_{2,4}$ this cannot occur when $v_0=0$. Hence, as shown in [13], if it is static the transformation process is linearly stable with respect to all admissible perturbations.

When $v_0>0$, $\ell_0=0$—as remarked in Section 4. Hence, in case *iii*, (5.8) becomes

$$p+\frac{(\gamma_l-\gamma_r)^2(f_l(\kappa,p)f_r(\kappa,p)+v_0^2p^2)}{v_*(f_l(\kappa,p)+f_r(\kappa,p))}\left(1+\frac{2v_0v_*v_\diamond}{(c_l-c_r)v_0+c_lh_l(\kappa,p)+c_rh_r(\kappa,p)}\right)=0. \tag{5.14}$$

Assume that $v_\diamond\neq 0$ for otherwise (5.14) reduces to (5.10). In analogy to case *ii*, let the branches of the square roots which appear in (5.14) be chosen so that, for $p\in\mathbb{R}$,

$$(a_d^2-v_0^2)b_d^2\kappa^2+a_d^2p^2>0\Rightarrow f_d(\kappa,p)>0,\quad v_0^2+4\alpha_d(\alpha_d\kappa^2+p)>0\Rightarrow h_d(\kappa,p)>0; \tag{5.15}$$

then, for $p \in \mathbb{C}$,

$$\mathrm{Re}((a_d^2 - v_0^2)b_d^2\kappa^2 + a_d^2 p^2) > 0 \Rightarrow \mathrm{Re}(f_d(\kappa,p)) > 0, \quad \mathrm{Re}(v_0^2 + 4\alpha_d(\alpha_d\kappa^2 + p)) > 0 \Rightarrow \mathrm{Re}(h_d(\kappa,p)) > 0. \quad (5.16)$$

It is then easy to show that at least one of v_* and $v_\diamond$ must be negative in order for there to exist an unstable zero to (5.14). Sufficient conditions for the existence of an unstable zero to (5.14) are more involved. Three such conditions, which are presented in [13], follow. First, it can be readily shown that if both $v_* < 0$ and $v_\diamond < 0$ then there must exist an unstable zero p to (5.14), independent of the choice of κ. This condition resembles that found in case *i*. Second, it can be shown that if $v_* < 0$ but $v_\diamond > 0$ and $|v_*|v_\diamond/c_l < 1$ then there exists an unstable zero to (5.14) as long as $|\kappa|$ is sufficiently large so that

$$\frac{c_l - c_r}{c_l + c_r} + \frac{h_l(\kappa,0)}{v_0(1+\frac{c_r}{c_l})} + \frac{h_r(\kappa,0)}{v_0(1+\frac{c_l}{c_r})} > \frac{2|v_*|v_\diamond}{c_l + c_r}. \quad (5.17)$$

Third, if $v_* > 0$ but $v_\diamond < 0$ and $v_*|v_\diamond|/c_l > 1$ then it can be shown that there exists an unstable zero to (5.14) whenever $|\kappa|$ is sufficiently small so that

$$\frac{c_l - c_r}{c_l + c_r} + \frac{h_l(\kappa,0)}{v_0(1+\frac{c_r}{c_l})} + \frac{h_r(\kappa,0)}{v_0(1+\frac{c_l}{c_r})} < \frac{2v_*|v_\diamond|}{c_l + c_r}. \quad (5.18)$$

From (5.17) and (5.18) it is evident that the last two sets of conditions exhibit a dependence on the wavenumber associated with the admissible perturbation.

Of particular interest is the third set of conditions. These show that a state involving a steadily propagating planar interface can exhibit linear instability with respect to any admissible pertubation which includes sufficiently *long waves* whenever the kinetic response function is mechanically stable but allows $v_\diamond < 0$ and $v_*|v_\diamond|/c_l > 1$. A kinetic response function which permits $v_* > 0$, $v_\diamond < 0$ and $v_*|v_\diamond|/c_l > 1$ is referred to as mechanically stable but *thermally unstable*. The experimental work of Clapp & Yu [5] indicates that there is as yet no reason to rule out the applicability of mechanically stable but thermally unstable kinetic response functions. Under such kinetics, a competition is revealed between mechanically stabilizing and thermally destabilizing effects. Competition of this sort is reminiscent of that discovered in models for dendritic crystal growth (see, for example, [20], [21] and [25]). In the present circumstances, mechanically stable but thermally unstable kinetics allow for the preference of a state which involves a highly wrinkled interface over one which involves a planar interface. Similar conclusions are reached in studies of dendritic crystal growth (again, consult [20], [21] and [25]).

In closing, a few directions in which this work might be extended are put forth. The foregoing results apply only to processes which involve *normal* phase boundaries. An analysis of transformation processes which involve *oblique* phase boundaries would, therefore, be interesting. It is expected that the stability criteria corresponding to such processes will be significantly richer than those referred to above. The class of materials which has been considered is isotropic. A study of the stability of two-phase processes in anisotropic materials would be desirable, as it is anticipated that the stability criteria for such materials will exhibit a directional dependence. The kinematics of this work have been restricted to those of antiplane shear. Stability analyses within broader kinematical settings would be interesting, as it is possible that a richer set of stability criteria may emerge therein. Finally, this analysis has been confined to the linear regime. Since the kinetic response function is chosen to satisfy $(3.5)_3$, it is expected that the fully nonlinear problem is stable even when non-monotonic kinetics are present (except, perhaps, in the pathological case where the kinetics are globally decreasing). The regime of linear instability should, therefore, be confined. Limited growth may occur within this regime but then halt due to nonlinear effects—thus leading to the development of fine-scale structure. Investigatations of this possibility within a fully nonlinear setting via nonlinear stability analyses and/or numerical simulations would, hence, be desirable.

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The linear stability of an isothermal two-phase process involving a planar phase boundary of normal type is analyzed. Results are obtained for cases wherein: i. the temperature field is unaltered as a consequence of the disturbance which is imposed, ii. the process is initially static, iii. the phase boundary associated with the process propagates steadily with non-zero velocity. In the first case the linear stability criteria are equivalent to those obtained in a related purely mechanical investigation. In the second case the process is found to be linearly stable. In the last case it is shown that instability can arise under a variety of circumstances. Of particular interest is a case where the kinetic relation is mechanically stable but thermally unstable; in this case the transformation process is unstable with respect to disturbances which include only suitably long waves, implying that a state involving a highly wrinkled interface is favored over one involving a planar interface.

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