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# The Studnt Production System 

A Study of Encoding Knowledge in Production Systems
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Abstract. This paper describes a production system implementation of Bobrow's STUUENT program. The main features of the new program, Studnt, are described. Contrasts between the two versions are pointed out. A discussion of the implementation brings out several properties of production systems, especially with regard to control.

Studnt is then used as an example of the embedding of knowledge in a production system. The knowlesige in Studnt is expressed as 218 natural language statements of three types: task-oriented knowledge, implementation and programming techniques, and knowledge about production system control. Task-oriented knowledge is characterized by an abstract model with 16 statements, which can be organized as a problem space. A detailed example illustrates how the knowledge is mapped to the production rule form. The knowledge is largely at the problem space lovel, with about a fourth of the statements dealing with programming techniques, and a much smaller fraction dealing with production system control. The knowledge analysis brings out the importance of the explicitness of unordered production systems with respect to determining the knowledge encoded in each production. The model of knowledge acquisition suggested by the analysis indicates unique properties for production systems with respect to programming, debugging, and augmentation. The analysis gives rise to some measures along eight understanding-system dimensions. Comparisons with other research and consideration of the processes involved in the analysis point up the need for further work on this approach.

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Studnt

## A. Introduction

This paper is concerned with Studnt, a production system implementation of the STUDENT program of O. Bobrow (1964a, 1964b). The analysis of STUDENT grows out of a more general research program whose aim is to rationalize the field of artificial inteligence (Al). The purpose is to clarify the scientific issues involved in Al, to characterize and justify the methods, and to firm up the theoretical and conceptual basis of AI. It is hoped that this would give better direction to research, bring about better teaching and learning of Al, improve the quality of reporting of research, and in general make AI more productive. The approach is to try to extend some sound preliminary work (Newell, 1969) by looking at specific Al prograns. Given any system, questions were to be asked along the lines of: "Where is the intelligence in it?", "How does its behavior come about?", "What are the methods it uses?", "Is there some measure of its effectiveness?", and "Can we measure the relative contribution of its parts?". These questions arise naturally in the context of Al prograns whose basis is heuristic search, where analysis and experimentation can lead, in a straight-forward way, to satisfactory answers. For instance, in evaluating a chess heuristic like the sorting of capture moves according to the value of the captured piece, it is possible to test various versions of a chess program and contrast their behavior. That kind of evaluation is in consonance with the scientific tradition of gathering knowledge by controlled experiments. It is not possible to carry over that approach to an analysis of STUDENT because apparently minor variations in STUDENT's structure can give rise to major deficiencies in its behavior, so major that comparisons lose their significance. Therefora, we take the approach of making explicit and analyzing the knowledge embodied in STUDENT, and in measuring the degree to which that knowledge is understood by STUDENT. Then we can go on to determine what parts of the knowledge represent methods, what parts contribute intelligence, and so on. This paper presents some initial progress, including some tentative measures, and puts forth a conceptual structure that may shape future work.

The goal of exploring the properties of production systems (PSs) as an AI language provides a second motivation. A PS program specifies everything in its behavior in terms of condition-action rules. The conditions all refer to a conmon Working Memory which is the complete dynamic knowledge state of the program, and actions are simply changes to that knowledge state. In practice, the numbers of conditions and actions are both in the range of half a dozen to a dozen. There are no control primitives as such, but rather control is achieved through explicit elements of the Working Memory. From this small collection of rather abstract properties, there are some features of PSs that we might look for in a PS program: uniformity and explicitness of expression of the knowledge content; flexibility and intelligence in the sense of doing a significant amount of conditiontesting for each small sequence of actions; flexibility also in the sense of being able to respond to unexpected items in the knowledge state; and modularity of knowledge organization, following from the way knowledge is encoded in small, independent units. In addition to these attractive properties, there is evidence that a PS-like organization is prominent in human cognition (Newell and Simon, 1972). The task area of Studnt is hardly

[^0]one that places demands on the language that will exercise all of those properties, but nevertheless we will get some preliminary data from examining the extent that STUDENT's structures and concepts have changed in order to be functional in a different programming environment.

The choice of STUDENT was based on personal preference, on the availability of a good description of the program, including a listing of the program in a rule-based language, and on simplicity and expected ease of implementation.

Input to STUUENT (the original) was a story problem expressed in a highly restricted subset of natural language. STUDENT converted that to a set of equations plus a set of unknowns to be solved for, and then solved the problem. It was able to apply optional transformations, consult a global store of "knowledge", and ask the user for more information, in case the set of equations derived from the input was insufficient for a solution. A typical problem is:
"The price of a radio is 69.70 dollars. If this price is 15 per cent less than the marked price, find the marked price."

STUDENT's version of the equations and variables to be found can be expressed as:

```
(price of radio) =( (69.70) X (dollars))
(price of radio) = ((.85) X (marked price))
(solve-for (marked price))
```

STUIDENT's answer is: the marked price is 82 dollars.
Studnt is designed to do only part of the above, namely, the translation from English-subset expressions into algebraic equations. Studnt thus includes the most interesting segments of STUDENT from the point of view of problem solving and natural language processing. In addition that portion of STUDENT was written in a readable PSlike language (Meteor), and the relevant parts of STUDENT were included in Bobrow's report (1964a), so that the present implementation follows the content of original rather closely. The omitted portions, except for the equation-solving process, seem to be straight-forward extensions of Studnt, while the equation-solver is a distinct piece of program and rather peripheral to the interesting natural language and problem-solving issues.

So, given a problem similar in form to those given to STUDENT, Studnt outputs: a set of equations; the set of variables in those equations as represented by the natural language text of the input; and a set of variables to be solved for. In addition, Studnt outputs the equivalences that it is assuming between certain phrases (which became variables) in the natural language text.

Section B contains; a description of Studnt, with progressively more detail towards the end of the section. The material starting with Section B.4 is optional for the first reading. Section C discusses the knowledge content of Studnt, and investigates knowledge interactions in forming the Ps. Some of the appendices deal with details of the Studnt processing, while the others are relevant to the knowledge section, as will be explained below.

Studnt is implemented in Psnist (PS analyst), a PS language specifically designed for Al applications. A PS is an unordered set of rules, Ps, specifying changes to a symbolic model of a situation, to be applied according to satisfaction of explicit conditions on that model. In Psnlst, condition- or left-hand-sides (LHSS) of Ps match an associative, unstructured Working Memory of data instances (items), each of which is a list headed by a predirate, followed by arguments. On matching, changes as specified by the action- or right-hand-sides (RHSs) are made to the Working Momory, either adding or deleting instances. The match distinguishes between new and old data, and Ps are selected for matching according to a stack regine whereby those relevant to the newest data are tried first, with older ones pushed down for later consideration. The stack is called :SMPX, stack memory for production examinations. The set of $P s$ is thus ordered dynamically, not statically, if indeed it can be considered to be ordered at all. The following is a typical $P$ :

> T1; "HOW OLD->WHAT" :: TFSCAN(X) \& EQHOW(X) \& LEFTOF(X,Y) \& EQOLO(Y) \& LEFTOF(Y, $Z$ )
> $\Rightarrow$ MODLEN(-1) \& EQWHAT(X) \& WORDEQ(X,'WHAT) \& NOT WORDEQ(X,'HOW)
> \& LEFTOF $(X, 7) \&$ N: $\operatorname{GATE}(A L L) ;$
"T1" is the label, "HOW OLD->WHAT" is a comment string, and the condition (LHS) and action (RHS) are conjunctions separated by " $m>$ ". T1 is intended to recognize the sequence "HOW OLD" and change it to "WHAT", deleting and updating "LEFTOF" links. This brief description should be sufficient for the reader to follow the examples scattered throughout the text. Appendix A gives a more systematic explanation of Psnlst features and explains in detail the various characters that are output by the running interpreter.

Studnt

## B. The Studnt Production System

## B.1. General overview

The main processing of Studnt is driven by a single left-to-right scan of the input, dividing it into smaller units called chunks, which are then parsed before continuing the scan. During this initial scan three things are done to provide information for the parsing process. First, simple string transformations are made, mapping the input to a form more acceptable to later processes, for instance, "twice" is converted to "two times". Second, dictionary tags are attached to key words, for instance, "times" is tagged as an operator of class "OP1". Third, the initial scan detects the operator, in the portion scanned, which has the highest "precedence", according to the parsing scheme to be described below. After the occurrence of a question word or phrase, the initial scan goes into FV mode (FV for find-variable). Each type of FV, as determined by the first word, has its own chunking cues, and each chunk becomes a variable, which requires no parsing.

The parsing of a chunk is based on a system of precedences, in such a way that the chunk is split at the leftmost operator of the set of those operators having the highest precedence in the chunk. The chunk is split into two chunks, and each of these is processed in the same way. The precedence system, for instance, assigns a high value to "is", the main equation operator, and lower values to "plus", "times", and "the sum of", respectively. That is, the higher-precedence operators are assumed to apply to higher levels of the resulting expression tree, for instance, "a times b plus c times $d$ " is taken to mean "(a times b) plus (c times d)".

When a chunk can be split no further, it is taken to represent variable. Thus, noun phrases are determined by their boundaries (operators and delimiters), and the only knowledge about internal structure consists of the features used in determining equivalence with previous noun phrases. Each variable is compared to each previouslydetermined variable. Two variables are the same if they have the same words in the same positions, with the following exceptions: a phrase which is the "head" of a previous phrase is taken to refer to the same object, for instance, "the number of fish" will match to a previous phrase "the number of fish in the pond"; "the" corresponding to "a" is taken as a match; and so on. The features used are independent of the meaning of the nouns used, and dependent on properties of structure and function words (pronouns, determiners). A variable containing "this" might be taken as referring back to some previous variable, in particular the "subject" of the previous sentence (for sentences of the form "xxx is equal to ...", where $x x x$ contains no operators). Alternatively, "this" refers to a whole expression, as in "this product", provided the previous sentence had an operator as its main connective different from EQUAL.

After each variable has been examined, the pieces of the original sentence are put back together into a tree-structured expression according to labels that were formed as the chunks were split. That is, as each chunk is split, a marker is formed for each half of the chunk, with a pointer back to its parent; the halves become operands, the parent becomes the operator at the node of the tree. The label of the parent chunk in turn points to its parent, and so on. The tree is built from the bottom up until labels run out, and if the operator at the top of the tree is "EQUAL", it is noted as an equation.

The subdivision of FV (find-variable) chunks is quite distinct from the preceding. An FV chunk is simply a list of one or more FVs, delimited in special ways according to the initial words of the FV chunk. For instance "What are" is followed by two or more FVs separated by "and". As another example, "How many ... do ... have ?" is taken to mean "what is the number of ... ... have ?", that is, the FV starts out, "the number of". Each portion of an FV chunk delimited in these special ways is taken to reter to a variable of the problem, and a comparison is made to previous ones until a match is found.

When the end of the input is reached, unreadable internal representations are transformed into lists suitable for output. The natural-language text corresponding to each variable is collected into a list, and variables determined to be FVs are gathered into a single list.

## B.2. An example problem in detail

This subsection summarizes Studnt's processing on the example TEST2. This should give a good idea of how Studnt works in a general way; fine details of the actual Ps and data representations are given in later subsections.

The run begins by inserting the full representation of the text of the problem into the Working Memory (Figure B.1). The last insertion gives the external representation of the text.

```
INSERTING (ASCAN PB-1) (PROBLEM PB-1) (TGSCANFIN SB-1)
    (LEFTOF SB-1 A1-1) (EQA A.1-1) (WORDEQ A1-1 A)
    (LEFTOF A1-1 F2-1) (EQFIRST F2-1) (WORDEQ F2-1 FJRST)
    (LEFTOF F2-1 N3-1) (EQNUM, ABER N3-1) (WORDEQ N3-1 NUMBER)
    (LEFTOF N(3-1 P4-1) (EQPLUS P4-1) (WORDEQ P4-1 PLUS)
    (LEFTOF PA-1 \#5-1) (EQ6 \#5-1) (WORDEQ \#5-1 6)
    (LEFTOF \#5-.1 16-1) (EQIS 16-1) (WORDEQ 16-1 IS)
    (LEFTOF 16-1 E7-1) (EQEQUAL E7-1) (WORDEQ E7-1 EQUAL)
    (LEFTOF S3A-1 N35-1) (EQNUM, MBER N35-1) (WORDEQ N35-1 NUMBER)
    (LEFTOF N(35-1. ?36-1) (EQ? ?36-1) (WORDEQ ?36-1 ?)
    (LEFTOF ? 36-1 SE-1)
    (STIRIENGTH 36) (ENIDMARK SB-1) (ENDMMARK SE-1)
    (TEXT
        (A FIRST NUMBER PLUS 6 IS EQUAL TO A SECOND NUMBER.
        TWICE THE FIRST NJMMBER IS THREE TINES ONE HALF OF THE SECOND NUMBER .
        WHIT ARE THE FIRST NUMBER AND THE SECOND NUMBER ?))
```

Figure B. 1 Initial Working Memory contents for TEST2

The portion starling with the first LEFTOF and ending with the last LEFTOF is the internal representation of the text, which is the argument of TEXT. Each word of the text has associated with it a token, A1-1, F2-1, N3-1, etc. A token consists of the first letter of the word concatenated with the position of the word in the text, then "-" and a number which gives the number of tokens that have been generated from the identifier which precedes the "-" (the final number insures uniqueness for all such generated tokens). Relations are then attached to these tokens (the structure of token names is never used internally). LEFTOF gives relative positions of tokens in the string, while EQwww (for some word www) and WORDEQ relate the tokens back to the external representation. (Why two predicates are necessary for this is explained in Section B.4.) The two tokens SB-1 and SE-1 are ENDHAARK's marking the left (beginning) and right (ending) ends of the string, respectively. The first insertion, (ASCAN PB-1), is a signal that the problem is to be checked for clues as to whether it is an age problem. This age-problem check must be done before everylhing else, because transformations and other processing depend on the result. PROBLEAK gives the problem an internal name, which is very rarely used. STRI.ENGTH means "string lengih", and its value is used in making estimates of certain quantities having to do with the monitoring processes (1 $\mathrm{Ps}_{\mathrm{s}}$, which will be explained in Section B.A. TGSCANFIN is the single most important predicate in the above list, since it initiates the scanning process, at token SB-1.

The first major piece of processing has to do with the text up to the first period. The following describes the essence of this processing, ignoring many of the finer details. The first segment is the chunk C-1: (A FIRST NUMBER PLUS 6 IS EQUAL TO A SECOND NUh 1 (3ERR). After the initial scan, PLUS is marked as an operator of class OP2, with precedence 7. The EQUAL TO is deleted by a transformation, and IS is assigned precedence 8. The highest presedence in $\mathrm{C}-1$ is thus 8 , and the chunk is split at the IS , to form $\mathrm{CL}-1$ : (A FIRST NUhMBER PLUS 6) and CR-1: (A SECOND NUMBER). CL-1 and CR-1 are labelled so that when fully parsed the tree for the arithmetic expression can be re-built from the fragments. For instance, we have (LABELU C-1 1 TOP) and (LABELU CL-1 $2 \mathrm{C}-1$ ); thus, CL-1 has a level- 2 label, with parent node C-1. The $U$ in LABELU stands for "untinished".

A precedence scan is now done on CL-1 (picked by virtue of its being leftmost of the "unfinished" chunks, computed by a numerical priority; the effect of the numerical ordering is similar to that of a slack) and a split occurs at PLUS, which is the onty thing in CL. -1 which has a precedence value. In general, the precedence scan picks the element with highest precedence for the next split, and in case of ties picks the leftmost such. CL-1 becomes CL-2: (A FIRST NUMMBER) and CR-2: (6). CL-2 undergoes the precedence scan, and the absence of any precedences indicates that it is a variable chunk. The variable identification process is done, and since no other variables have the same form, it is given a new token, VAR-1, as its expression (a chunk has associated with it an expression, which may be trivially a single VAR token). CR-2 similarly becomes VAR-2. In the process of giving the two chunks expressions, LABELU is changed to LABELF, F for "finished", and the presence of two "finished" chunks with the same "unfinished" parent node ( $\mathrm{Cl},-1$ ) results in assigning $\mathrm{CL}-1$ the expression formed from its operator, which was noted when it was split, and its two descendant nodes, namely (PLUS VAR-1 VAR-2). Having done this, control passes again to the precedence scan, which now examines CR-1; CR-1 was formed in the first split, but was "forgotten" while the left half of the split was being parsed. CR-1 has no precedences, and becomes VAR-3, after checking that it is not identical to any of the other VAR's. This prompts the construction of (EQUAL (PLUS VAR-1

VAR-2) VAR-3), since the two descendants of C-1 are now "finished". This expression is marked as an equation (ISEQN) by noting that it has EQUAL as its operator, and that its expression-tree level is 1. The first chunk is now complete, and the scan resumes, starting at TWICE.

The second main chunk is processed in a way similar to the first. Three new transformations are applied before it is parsed: TWICE becomes 2 TIMES, ONE HALF becomes 0.5 , and the OF after the 0.5 becomes TIMES.

The third main chunk, starting at WHAT, is an FV chunk, since WHAT is recognized as a QVORD (question-word). The action on the third chunk involves splitting it at the ANID, and processing the two halves as variables. The variables (A FIRST NUMBER) and (THE FIRST NUJMBER) are recognized to be the same, differing only in A as opposed to THE, so that (THF: FIRST NUMMBER) is known to be VAR-1. Similarly, (THE SECOND NUMBER) is VAR-3.

The portion of the Working Memory that gives the final solution is in Figure 8.2.

```
ISEQN {C..1 (EQUAL (PLUS VAR-1 VAR-2) VAR-3))
    (C-2 (EQUAL (TIMES VAR-4 VAR-1) (TIMES VAR-5 (TIMES VAR-6 VAR-3))))
HASREPR (VAR-1 (A FIRST NULMBER)) (VAR-2 (6)) (VAR-3 (A SECOND NUMBER))
    (VAR-4 (2)) (VAR-5 (THREE)) (VAR-6 (0.5))
FVLIST (PB-1 ((VAR-1 VAR-3)))
EQVARCHUNK (C-3 CL-2) (C-4 CR-1) (CR-4 CL-2) (CR-6 CR-1)
```

Figure B. 2 Final output for TEST2

ISEQN denotes the two equations found; HASREPR gives external representations for each of the VAR's; and FVI.IST gives the list of FV.. Instances of each predicate are ordered lexicographically by their first element. The EQVARCHUNK instances give whith chunks are assumed to be equivalent. We see that two occurrences of VAR-1 (CL-2) are noted in addition to the first, and also two other occurrences of VAR-3 (CR-1). (The chunk names, C-1, etc., refer to actual text segments, whereas the VAR's are more abstract, and can be represented by several different C's.)

## B.3. Comparison with the original

One of the primary differences in the overall processing between Studnt and STIJI:NT is due to Studnt's being driven by the left-to-right scan. The Meteor language had built-in facilities for efficient scanning over arbitrary string segments to pick out patterns; Psnlst is more general, and must do the scan more deliberately. The original repeatedly applicd its templates to the entire input string until no more valid applications could be made, thus imposing an order on template application as opposed to Studnt's order of examining text. This means, for instance, that sentence-boundary templates in STUDENT were all applied before, say, the breaking of sentences into equations was starled. Studnt proceeds in contrary fashion, making full use of all information seen in the
scan up to a boundary, before continuing beyond that boundary. This contrast is quite visible in the actual programs. A significant portion of STUDENT consisted of sets of rules, with individual rules in those sets consisting of processing plus a branch to the initial rule in the set. Exhaustion of one set of templates led to a branch to another set. The corresponding left-to-right sequencing is evident in Studnt's "S" group of Ps, which control applications of the various rule sets at each scan point.

A second major difference arises from the internal representation. STUDENT was written in a language specifically oriented towards processing data organized as onedimensional lists. The underlying language for Studnt, Psnlst, is designed to require all such structure to be explicit rather that built-in, partially for the purpose of allowing examination of just how much use is made of the string structure of the input, and partially for the purpose of retaining generality.

This can be illustrated by comparing a specific rule from STUDENT:
(* (HOW OLD) (WHAT) IDIOMS)
to the corresponding rule from Studnt:
T1; "HOW OLD->WHAT" :: TFSCAN(X) \& EQHOW(X) \& LEFTOF(X,Y) \& EQOLD(Y) \& LEFTOF(Y,Z)
$\Rightarrow$ MODLEN -1 ) \& EQWHAT(X) \& WORDEQ(X, WHAT) \& NOT WORDEQ(X,'HOW) \& LEFTOF(X,T) \& N:GATE(ALL);

In the former rule, there are four elements: the label of the rule (actually * is just a placeholder, with control passing implicitly from the previous ruie); the left-hand-side; the right-hand-side; the "GOTO" field of the rule. Some rules have an optional action sequence between the third and last positions. Note that the Studnt $P$ makes explicit the LEFTOF links and the updating necessary for the transformation, while this is implicit in the STULDENT rule. Also, the Studnt rule has a data signal TFSCAN instead of the combination of a label, which might be the target of a GOTO, and a GOTO field. Overall, STUDENT had about 290 rules, which included high-level control and output printing, whereas Studnt has about 260 Ps , so that the advantages of the specialized notation seems to result in compression in size of rules rather than changing the number of rules in the entire system.

Minor differences can be noted in some of the details of the processing. Not everything done by STUDENT was in the program as published; thus certain assumptions were made along the way that resulted in some differences in the final results. For instance, STUIDENT used a plural convention, converting occurrences of singular forms to their plurals (" 1 span" becomes " 1 times spans") whereas Studnt converts plurals to singulars ("6 feet" is "6 times foot"). STUDENT deleted occurrences of "the" and "a", so that noun phrase comparisons have some automatic equivalences, while Studnt retains those words, and uses explicit Ps to encode the knowledge that the difference between "the" and " $a$ " is non-essential. In this case, and perhaps others, Studnt is less general, since it doesn't have Ps to handle all of the cases implied by STUDENT's mechanism; this specificity seems desirable from the standpoint of analysis of just what knowledge is required for the task. Studnt doesn't check for error conditions; STUDENT recognized a : fow limited types of "errors" in the input problenis. Overall, Studnt performs as well as

STUDENT on the test problems published in the original report (given the more modest definition of "solution"), so that there is good reason to assert close similarity in knowledge content of the two versions (see Appendix E for results on that set of tests).

The ways in which the control of the two programs differs can be illustrated by displaying the actual code for processing that results in parsing the input according to the operator precedence tags. First, the rules from STUDENT, with commentary enclosed in Y's:
(* (\$ $(\$ 1 / \mathrm{OP} 1) \$) \quad((\mathrm{FNCAR}(* \mathrm{~K} 2)))$
(/ (*S LEFT (*K 1)) (*S RIGHT (*K 3))) OPTST)
7. this stacks the left operand onto LEFT, the right onto RIGHT 7
(OPTST (\$) \$)
(1)
8)

7 the operator itself is used to determine branch target \%
7. the recursive step: these rules are all part of OPFORM \%

Studnt does the same thing by a loop for the precedence scan (P20-P29, P50), followed by the split into operator and operands (C25, C60), followed by the assembly (C70):

P2O; "N:W HIGH PRREC" :: PRECSCAN(C,X) \& HIGHPREC(C,N,Y) \& HASPREC(X,M)
\& SATISFIES2(M,N, (GREATERP M N)) \& LEFTOF ( $X, W$ )
\& NOT CHUNGEENDR (X,C)
$\Rightarrow$ PRECSCAN(C,W) \& HIGHPREC(C,M,X) \& NEGATE(1,2);
P23; "PREC SCAN ON" :: PRECSCAN(C,X) \& HIGHPREC(C,N,Y) \& HASPREC(X,M)
\& NOT SATISFIES2(M,N,'(GREATERP M N)) \& LEFTOF(X,W)
\& NOT CHUJMKENDR(X,C)
$\Rightarrow$ PRECSCAN(C,W) \& NEGATE(1);
P26; "PREC SCAN ON" :: PRECSCANKC,X) \& NOT( EXISTS(N) \& HASPREC(X,N) )
\& LEFTOF $(X, W)$ \& NDT CHUNKENDR $(X, C)$
$\Rightarrow$ PRFESCAN(C,W) \& N:GATE.(1);
P27; "PREC SCAN DONE" :: PRECSCAN(C,X) \& HIGHPREC(C,N,Y) \& HASPREC( $X, M)$ \&SATISFIES2(M,N,'(GREATERP M N)) \& CHUNKENDR(X,C)
$\Rightarrow$ HIGHPREC(C,M,X) \& PRECSCANIJ(C) \& NEGATE(1,2);
P28; "PREC SCAN DONE" :: PRECSCAN(C,X) \& HIGHPREC(C,N,Y) \& HASPREC( $(X, M)$
\& NOT SATISFIES2(M,N,'(GREATERP M N $)$ ) \& CHUNKENDR $(X, C)$
$\Rightarrow$ PRECSCANIP(C) \& NEGATE(1);
P29; "PREC SCAN DONE" :: PRECSCANKC,X) \& NOT( EXISTS(N) \& HASPREC(X,N) )
\& CHUNKENDR ( $X, C$ )
$\Rightarrow$ PRECSCANDIC. \& NEGATE(1);
P50; "HASOPI" :: PRECSCANDXC) \& HICHPREC(C,M,X) \& SATISFIES(M,'(EQ M 5)) $\Rightarrow$ HASOPI(C,X) \& NEGATE(2);

C25; "OP1 BRK" :: HASOP1(C,X) \& WORDEQ $(X, X W)$ $\Rightarrow$ CSPI.IT $(C, X, X) \&$ HASOP(C,XW) \& NEGATE (1);

C60; "SPLIT CHUNUK" :: CSPLIT(C,LOCL,I.OCR) \& L.EFTOF(X1,LOCL) \& LEFTOF(LOCR,X2) \& LABELU(C,N,P) \& MXCPRIOR(M)
$\Rightarrow$ EXISTS(CL,CR) \& NFWPLOP(C) \& RRENAME(X2,C,CR) \& LRENAME (X1,C,CL)
\& LABELU(CL,N+1,C) \& LABELU(CR,N+1,C) \& HASCPRIOR(CL,M+2) \& HASCPRIOR(CR,M+1) \& MXCPRIOR(M+2) \& CHUNKENDL(X2,CR) \& CHUNNKENDR(X $1, C L) \& \operatorname{NEGATE}(1,2,3,5)$;

C70; "FINISH SEG" :: $\operatorname{L.ABELU(C,N,P)~\& ~L.ABELF(C1,M,C)~\& ~LABELF(C2,M,C)~}$ \& HASOP(C,X) \& SATISFIES(P,P NEQ 'TOP) \& HASCPRIOR(C1,PR1) \& HASCPRIOR(C2,PR2) \& SATISFIES2(PR1,PR2,PR1 ?*GREAT PR2) \& SATISFJES2(M,N'(EQUAAI. (?*DIF M N) 1)) \& HASEXPR(C1,Y) \& HASEXPR(C2,Z)
$\Longrightarrow \operatorname{HASEXPR}(C,\langle X, Y, Z>) \& \operatorname{LABELF}(C, N, P) \& \operatorname{NEGATE}(1) ;$
(For help in understanding those Ps, the reader might refer to Section B.5.) How Studnt encodes the choice of which chunk to do the precedence scan on ( P 10 ) is not shown here, but it suffices to note that the choice is based simply on a numerical priority (HASCPRIOR). assigned to the chunks. How STUDENT makes the same selection is implicit in the recursive calling of OPFORM illustrated above.

One further examiple illustrates the differences in the languages used to express the two versions. STUDENT uses the following:

RFMEMBER ( ... (PEOPLE IS THE: PLURAI. OF PERSON) ... )
where there are many similar phrases as arguments to REMEMBER, to set up internal properties which are then used by the rule:
(WORDS (\$1)
0 (/ ( $\pm$ Q SHFLF (FN GETDICT 1 DICT $)$ )
WORDS)
which cycles repeatedly over the entire problem string. Studnt's corresponding rule is:
DG1; "PEOPLE PL." :: TGSCAN(X) \& EQPEOPLE(X) m ISPLURAL(X,'PERSON) \& NEGATE(ALL);
Thus STUDENT could be augmiented by adding rules of a natural form, but the class of such forms was rather small, and the larger issue of significant augmentation could certainly not be encompassed by this mechanism. One of the aspects of the Studnt knowledge analysis below is an approach to the more general problem of augmentation.

## B.4. Description of the productions

Now we describe the Ps of Studnt in some detail, in groups according to their function, pointing out features of interest with respect to the use of PSs. Some of the descriptions include a typical $P$ and a trace segment (starting at "!") showing its operation. In order to understand everything in full detail, the reader will need to refer to the
meanings of the predicates, Section B.5, the program listing, Appendix 8, and perhaps the cross-reference, Appendix $C$. The groups of $P s$ in this subsection are ordered by importance, which corresponds to their order in the program (though such order has no effect on program behavior). There are twelve groups: S (scan), $T$ (transformations), $D$ (dictionary tags), $P$ (precedence tags), $M$ (main verbs), $C$ (chunking), $R$ (renaming), $V$ (variable identification), $F$ (FV chunking), $A$ (age problem), $B$ (building output), and I (information monitoring). $P$ names in Studnt are a single letter (the letter of the containing group) followed by one or two digits, e.g. S13, perhaps in rare cases followed by another tetter, e.g. V33R.
$S P_{s}$ Scanning the problem string ( $14 P_{s}$ )
The $S$ Ps make the primary scan of the input, resulting in the application of transformations, the addition of dictionary tags, the segmentation into sentences, and the determination of the highest operator precedence seen in each segment scanned. The important predicates are: LEFTOF, TFSCAN, TFSCANFIN, ISSCANCHUNK, TGSCAN, TGSCANIIN, TGSCANFIN?, TFASCAN, TFASCANFIN, HIGHPREC, HASPREC, FVSCAN, ISSCAN:V. These $P_{s}$ have the effect of sequencing the firing of other sets of $P_{s}$ to accomplish the things mentioned. This sequencing is explicit, using two signals for each evoked process. For instance, TFSCAN evokes the transformation processing, and TFSCANFIN signals that the TFSCAN signal has been examined. These two signals are both asserted by 513 (and others), bul TFSCANFIN follows TFSCAN in being asserted, and is therefore starked in :SMPX until all the consequences of the TFSCAN have been examined. The signals for major processing are asserted as follows: TFSCAN (transformations, see T Ps), TFASCAN (age problem transformations, called optionally, see A Ps), TGSCAN (dictionary tags, D Ps), and TGSCANFIN2 (leads either to precedence checks of S20-S30, or to FVGCAN, see F Ps). S20-S30 determine the leftmost position that has the highest precedence.

S40 is the key to segmentation of the input at the period delimiter. The PRECSCAND assertion in the RHS of 540 evokes the extensive parsing process on the chunk just scanned, passing control to the P Ps. S40 also contains the start of the scan of the next segment (TFSCAN and TFSCANFIN); these signals are stacked in :SMPX throughout the parsing. 570 notes that the end of the input is reached, and signals the answerbuilding process ( $\mathrm{B} P \mathrm{~s}$ ).

A typical S P:
S13; "TF SCAN" :: TGSCANFIN?(X) \& LEFTOF ( $X, Y$ ) \& NOT ISDELIMKX) \& ISSCANCHUNK(C) \& CHUNHK.EN(L.)
$\Rightarrow \operatorname{TFSCAN(Y)}$ \& TFSCANFIIU(Y') \& INCHUNK (X,C) \& CHUNKLEN(L+1)
\& NI: GATE $(1,5)$ \& NOT TGSCAN(X);
! 7. S13-1 "TF SCAN"
USING (TGSCANFIN2 A1-1) (LEFTOF A1-1 F2-1) (ISSCANCHUNK C-1) (CHUNKLEN 1)
INSERTING (TFSCAN F2-1) (TFSCANIJN F2-1) (INCHUNK A1-1 C-1) (CHUNKLEN 2) (NOT (TGSCANFIN? A1-1)) (NOT (CHUNKLEN 1)) (NOT (TGSCAN A1-1))

This $P$ firing moves the initial scan pointer from $A 1-1$ to $F 2-1$, i.e., from " $A$ " to "FIRST", in
problem TEST?. C-1 is the current chunk. Transformations are invoked on F2-1, A1-1 is added to $\mathrm{C}-1$, and the length of the chunk goes from 1 to 2 . This is the seventh P firing in the process of solving TEST2.

TPs: Transformations on the input string ( 38 Ps )
These Ps specify that certain sequences of tokens in the input are to be replaced by equivalent sequences, so that the parsing process can work with a standard form of input. Examples of transformations were mentioned in Section B.2. Some Ps achieve this by changing external names associated with tokens, while others assert new tokens and remove the old ones. In doing this, the LEFTOF links are maintained, sometimes requiring changes to the scan pointers that were set up originally by the $S$ Ps. There are many uses of the macros STRINGEQ and STRINGINS; for an explanation of what these expand into, see the comment at the very beginning of the Studnt program listing, Appendix $B$.

External names of tokens are encoded in two ways, by EQwww and WORDEQ, as we saw in Section B.2. WORDEQ's could be used everywhere, without a need for the EQwww's, except that since WORDEQ has an instance for every input token, there would be much more searching during the matching process. On the other hand, WORDEQ is required to give a direct link from a token to its external name, for instance in comparing arbilrary phrases for identity.

The $T P_{s}$ form a non-deterministic if-statement (COND). All of their conditions are keyed to the TFSCAN signal, and the checking of the conditions is done in nondeterministic order. When a $P$ succeeds in matching, the result is to delete the TFSCAN signal, thus disabling any further firings of other transformations. Another view would call these Ps a subroutine, control being passed by a data condition instead of in the conventional way. Other sets of $\mathrm{Ps}_{s}$ in Studnt also maintain control of processing in a coherent way, but use a larger set of signals to achieve communication.

T50-T52 are used (as a sort of subroutine) by several other Ps to properly rearrange the global scan pointers in case old tokens become inoperative as a result of replacement. The S Ps function as if nothing had happened.

Example:

$\Rightarrow$ MODLEN $(-2) \& \operatorname{LEFTOF}(X, Y) \& \operatorname{NEGATE}(A L L,-2)$;

```
!26. T2-1 "IS EQUAL TO->IS"
USING (TFSCAN I6-1) (EQIS 16-1) (LEFTOF 16-1 E7-1) (EQEQUAL E7-1)
    (LEFTOF E7-1 T8-1) (EQTO T8-1) (LEFTOF T8-1 A9-1)
INGERTING (MODLEN -2) (LEFTOF I6-1 A9-1) (NOT (TFSCAN 16-1))
    (NOT (LEFTOF 16-1 E7-1)) (NOT (EQEQUAL E7-1)) (NOT (LEFTOF E7-1 T8-1))
    (NOT (EQTO T8-1)) (NOT (LEFTOF T8-1 A9-1))
```

"IS EQUAL. TO" is transformed to "IS" by removing the two extra words, E7-1 and T8-1, and by fixing LEFTOF pointers to make 16-1 left of A9-1. The first insertion is a signal to the I Ps that a change in problem length has taken place.

D Ps: Dictionary tags (43 Ps)
The tags applied to word tokens are: ISOP2, ISOP1, ISOPO, ISVERB, ISPERSON, ISPRON (optionally, only in age problems), ISPOSSPRON (another optional one), ISPLURAL, ISSINGUIARR, ISQWORD, and ISDELIM. These tags are applied in a control environment similar to the that for the T Ps.

P Ps: Precedence scanning and lagging (23 Ps)
P1-P9 are sensitive to the tags applied by the $D \mathrm{Ps}_{\mathrm{s}}$, adding precedence values for operators. P10..P29 form a precedence-scanning process that is called after chunks scanned by the S Ps are split. P10 and P15 determine which chunk to scan next, according to the explicit sequencing tag, HASCPRIOR. The unscanned chunk with highest value is chosen.

Actually P10 also notes the next-highest chunk, and re-inserts the ISCHUNK predicate for that chunk. This is necessary to be sure that P10 or P15 will be tried again after a precedence scan is completed, because ISCHUNK, as used in P1O and P15, actually means a new ISCHUNK, at least for the CO one. Each time the match is done, though (even if it fails to succeed using a particular ISCHUNK as the new one), all new ISCHUNK's become old, and without the re-assertion, P10 or P15 would not be examined again, resulting in neglecting some ISCHINJKs. So, in P10, the next-highest chunk is re-asserted, making it new again, and stacking it in :SMPX behind other data which cause other processing to be done before coming back for more precedence scanning. P15 checks that no other unprocessed ISCHUNM's exist, so that no re-assertion is necessary.

P20-P29 make up a precedence-scanning loop, going from left to right in the chunk, with the result that the leftmost instance of the highest precedence is selected. PRECSCAN is the scanning signal, CHUNHENDI. is used to start the scan at the left end, and HIGHIPREC records the progress. The set of $P s$ is a loop because each new assertion of PRECSCAN results in examination of the elements of the set to determine the next action.

P30 P75 emit signals that are picked up by $C, M$, or $V$ Ps, depending on the particular signal; so, after the precedence is determined, the chunk is split at an operator, transformed according to its verb structure, or taken as a variable chunk with no further splits possible.

## Example:

## P10; "GTART PREC SCAN" :: ISCHLNK(CO) \& CHUNKENDL(X,CO) \& HASCPRIOR(CO,MO) <br> \& NOT PRECSCANI)(CO) \& ISCHUNK (C1) \& HASCPRIOR(C1,M1) <br> \& SATISFIES2(M0,k1,M0 ?*GREAT M1) \& NOT PRECSCAND(C1) <br> \& NOT( EXISTS(C2,M2) \& HASCPRIOR(C2,M2) <br> \& SATISFJES2(M0,M2,M2 ?*GREAT MO) \& NOT PRECSCANDXC2) ) <br> \& NOT( EXISTS (C3, 人 3 ) \& HASCPRIOR(C3,M 1 ) <br> \& SATISFIES3(MO,M1,M3,'(GREATERP M0 M3 M1)) <br> \& NOT PRECSCAND(C3) ) <br> $\Rightarrow \operatorname{PRECSCAN}(C O, X) \& \operatorname{HIGHPREC}(C O, 0, X) \& \operatorname{ISCHUNK}(\mathrm{Cl})$;

!68. P10-1 "START PREC SCAN"

USING (ISCHHNKK CL-1) (CHUNNKEINDL. A1-1 CL-1) (HASCPRIOR CL-1 3) (ISCHUNK CR-1) (HASCPRIOR CR-1 2)
WARNING (CR-1) ALREADY UNIDER ISCHUNH -*+
INGE:RTING (PRECSCAN CL-1 A1-1) (HIGHPREC CL-1 0 A1-1) (ISCHUNK CR-1)
A precedence scan is initiated on $\mathrm{Cl}-1$ at position A1-1, its left end. (ISCHUNK CR-1) is re-asserted so that PlO will be examined again, after C1-1 is processed, to look at CR-1. P10 insures that CO, assigned to C1-1, is the chunk with highest priority, and that no chunk has priority between CO and Cl , assigned here to $\mathrm{CR}-1$.

M Ps: Main verbs, Miscellaneous post-tag transformations (10 Ps)
M10-M55 split or re-arrange chunks according to the main verb. M10 handles the simple "is" case. The others are much more complex. For instance, M2O applies in situations such as "Tom has twice as many fish as Mary has guppies", transforming it to "The number of fish Tom has is twise the number of guppies Mary has".

M60 M 75 are sensilive to outputs of D Ps, either un-doing their effects, or carrying them somewhat further, according to context not taken into account in the tagging. These actions could be incorporated into D's; their form is a carry-over from the original STUIDENT, which did the tagging and transforming in such a way that assumptions about the contexts used in M60-M75 could not be made until after all of the transformations had been done. The left to right scan in Studnt removes that difficulty.

C $P_{s: ~}$ Chunk splitting and re-combining (19 Ps)
C2.C55 act on the signals sent by P1-P9, by setting up to split chunks at the marked operators. The actual splitling and attendant bookkeeping is done by C60. C70C78 put the chunks back together after they are parsed fully, with a separate $P$ for each of three cases. C75 and C78 are concerned with saving referents of future "this" (this is only done for the highest level in the sentence, so that C70 handles other cases). C80C85 handle bookkeeping for the "this" referents. C90 notes that a completed expression is an equation. The important predicates for this segment are: CSPLIT, URENAME, HASUOFCHUINK, ISUOPDUHM, N:WREFEXPR, ISREFEXPR, ISEQN.

C15-C52 (except C25) are somewhat more complex than the other Ps. Their purpose is to control the parsing of unary operators (square, squared) in such a way that the single operands of the operators are parsed before further action is taken. This is as if parentheses were put around the operands. It is necessary to do this because the other operators in Studnt are binary, and expect a variable as argument. But in the case of, say, "two times the square of the number", the second operand of the "times" is the unary-operator expression. Thus the unary operators insert a dummy where the unary expression used to be, rename the unary expression as another chunk (using URENAM佂 and Ps C20-C22), parse the unary expression, and signal that the dummy stands for the unary expression, so that it won't be treated as text when the ordinary processing gets to it (see V10).

C70; "FINISH SEG" :: LABELU(C,N,P) \& LABELF(C1,M,C) \& LABELF(C2,M,C)
\& HASOP(C,X) \& SATISFJES(P,P NE: 'TOP) \& HASCPRIOR(C1,PR1)
\& HASCPRIOR(C2,PR2) \& SATISFIES2(PR1,PR2,PR1 ?*GREAT PR2)

# \& SATISFIES2(M,N,'(EQQUAI. (?*DIF M N) 1)) <br> \& HASEXPRR(C1,Y) \& HASEXPR(C2,Z) <br> $\Rightarrow$ HASEXPR (C, $\langle X, Y, 7>) \&$ I.ABELF(C,N,P) \& NEGATE(1); 

```
1112. C70.1 "FINISH SEG"
UGING (LABELU CL-1 2 C-1) (LABELF CL-2 3 CL-1) (LABELF CR-2 3 CL-1)
    (H/IASOP CL-1 PL.US) (HASCPRIOR CL-2 5) (HASCPRIOR CR-2 4) (HASEXPR CL-2 VAR-1)
    (HASEXPR CR-2 VAR-2)
INSERTING <HASEXPR CL-1 (PLUS VAR-1 VAR-2)) (LABELF CL-1 2 C-1)
    (INOT (LAFSELU CL-1 2 C-1))
```

Two finished chunks, $\mathrm{CL}-2$ and CR-2, which are variables VAR-1 and VAR-2, are formed into an expression using the operator PLUS of the parent chunk $\mathrm{CL}-1 . \mathrm{CL}-1$ is marked finished (LAFBELF) and is ready to be formed into the expression of its parent C-1 (that won't occur, though, until the second operand, CR-1, is finished).

R Ps: Renaming chunks after splitting of a chunk ( 6 Ps)
R2-R4 rename a chunk going from right to left. R6-R9 rename a chunk going from left to right. R6-R9 additionally are able to name pieces of a sequence of text that were not previously in any chunk ( $R 2$ and R4 assume a previous chunk). New pieces of chunks as checked for by R6-R9 are added by $P_{s}$ like M20. The important predicates are: INCHUNK, I.EFTOF, CHUNKENDL, CHUNKENDR, LRENAME, RRENAME. Each group of R Ps is a loop, maintaining control structure through LRENAME and RRENAME instances. After completion of the renaming, the ISCHUNK signal is emitted, to be picked up by P Ps.
$\checkmark$ Ps: Variable comparison, for equivalences (26 Ps)
V5-V37 perform a number of tests on new variable chunks (chunks with no operators), in order to determine if the chunk, or something very close to it, has been seen before. These tests are performed in a particular sequence, as controlied by instances of the predicates UNTESTED, THISTESTED, EQVARREMD, and EQCHUNKTEST. V5 emits the UNTF:STED, after a check for a unary operator dummy; V10 handles the dummy case. V15-V21 check for "this" in the chunk, and resolve references accordingly. V23V24 remove comparisons to variables that have already been proven equivalent to others (such comparisons would just be duplication of effort). V25 initiates comparison of the new variable to all previous variable chunks, except as just mentioned. The comparison is done by stepping through the variables to be compared, on the LEFTOF links, with either check for equality or check for correspondence according to several special equivalence conditions. These special conditions are checked by V31-V37, as follows: "the" = a previous "a"; "they" matches "the $x x x$ ", where $x x x$ is an unspecified word, e.g. "the Russians"; "the" may be skipped; a singular form matches "the number of xxx", where xxx is the plural-form of the singular word (only for words that have been tagged by D's); "first number" = "one number" (the latter is in a new variable); "first number" = "one of the numbers" (latter is new); "second number" = "other number" (latter is new).

V40.V V 50 note that two variables are equivalent, when the comparison goes through the entire chunks being compared. V 55 counts the variable chunks as they are compared to the new one, in a particular sequence to prevent the $P$ match from finding multiple
assignments; if it were allowed to find multiple ones, incrementing the count as kept by CHTESTEO would be done only once, effectively, since each increment would use the value of CHTESTED before any of the multiple firings. Allowing multiple firings is a feature of Psinst; it was used to advantage in V25, to find all comparisons to be made with a single match, but in V25, the order didn't matter, and no values depended on non-multiple firings.

The presence of $V 55$ is actually not necessary, by analogy with a similar comparison process elsewhere in Studnt, A63-A69. The latter test makes better use of the implicit stacking mechanism of Psimst; it was coded somewhat later in time than the V tests. V55 was left in because it seemed desirable to use it as an illustration of atternative methods of expression in Psnlst, and because it illustrates an epproach applicable in more general situations, where stricter control is essential.

V60 notes that all tests are finished, and creates a new VAR token. V65-V90 are used to remove all testing signals from the Working Memory; this is useful in case one test succeeds before all the others are done, so that they need not be continued.

```
V3O; "VAR m" :: EQCHUNKTEST(C1,C2,X,Y) & WORDEQ(X,XW) & WORDEQ(Y,XW)
        & L.EFTOF(X,X2) & L.EFTOF(Y,Y2) & NOT CHUNKENDR(X,C1)
        & NOT CHUHNENDD(Y,C2)
    # EQCHUNUTEST(C1,C2,K2,Y2) & NEGATE(1);
```

!123. V30.1 "VAR ="
USING (EQCHUNKTEST CR-1 CL-2 A9-1 A1-1) (WORDEQ A9-1 A) (WORDEQ A1-1 A)
(LEFTOF A9-1 S10-1) (LEFTOF A1-1 F2-1)
INGERTING (EQCHUNKTEST CR-1 CL-2 S10-1 F2-1)
(NOT (EQCHUNKTEST CR-1 CL-2 A9-1 A1-1))

This is an example of the variable comparison process. In this case the next positions to be tested will not be the same, since CR-1, "A SECOND NUMBER", is being matchad to CL-2, "A FIRST NUJMBER".

F Ps: FV scanning and segmentation (15 Ps)
The type of scanning and segmentation for FV chunks depends only upon the initial question-words. For instance, if a sentence starts with "What are", Studnt expects more than one variable, separated by "and". These expectations are set up by asserting instances of: RTANIDQMGOING, RTQMGOING, RTDOGOING, RTOOESGOING, RTHAVEGOING, RTANDPERGOING. The scan is actually sequenced by the $S$ Ps, using FVSCAN. In a couple of cases, more complicated transformations are done, for instance, F45 will change phrases like "How many fish does Mary have?" to "the number of fish Mary has". Example:

F5; "WIHAT ARE FV" :: FVSCAN(X) \& EQWHAT(X) \& ISSCANFV(C) \& CHUNKENDL(X,C) \& LEFTOF( $X, Y) \&$ EQARE(Y) \& LEFTOF(Y, Z .
$\Rightarrow$ CHUNKENDL. $(Z, C) \&$ RTAND.QMGOING(C) \& NEGATE $(1,4)$;
! 439. F5-1 "WHAT ARE FV"
USING (FVSCAN W27-1) (EQWIHAT W27-1) (ISSCANFV C-3) (CHUNKENDL W27-1 C-3)
(LEFTOF W27-1 A28-1) (EQARE: A28-1) (LEFTOF A28-1 T29-1)
INSERTING (CHUNKENIDL T29-1 C-3) (RTANIDQMGOING C-3) (NOT (FVSCAN W27-1))

## (NOT (CHUNKENDI. W27-1 C-3))

Here the beginning of an FV chunk is noted, T29-1, starting "THE FIRST NUMBER", keyed to "WHIAT ARE.". A signal is set up so that "AND" and "QMARK" are treated appropriately when encountered.

A Ps: Age-problem transformations (44 $\mathrm{P}_{\mathrm{S}}$ )
The age heuristics in Studnt closely parallel those in STUDENT, so that the following descriplion is somewhat cryptic; scanning the Ps should help to fill in the details. Most of the relevant predicates start with "AGE". A1-A3 detect clues to whether a problem is an age problem; the occurrence of any of the special words is conclusive evidence. A11-A12 delete superfluous phrases. A15-A20 translate the occurrences of verbs like "will be" into more suitable forms. A2A-A28 note the occurrence of phrases that may be used later on to modify age variables that are not otherwise modified. A31-A35 translate age operators into arithmetic operators, for instance "age 5 years from now" becomes "age pluss 5" (pluss has a different precedence from plus). A38-A43 detect the need for an age operator, as first noted by A24-A2B, collect that operator, and place it in the string after the current age variable. A50-A59 replace an occurrence of "their ages" by a list of all age variables seen so far, separated by "and". These AGEREF's are collected in the order seen, by using a numeric argument. Pointers to all age variables are coliected as scanned, by A61-A69, which also do a comparison, so that several occurrences of the same age variable do not appear in the replacement for "their". A71-A75 replace the occurrence of a personal pronoun by the first age variable seen. A81-A85 do a similar thing for a possessive pronoun.

B Ps: Build up answers (6 Ps)
Several functions are performed in building answers: chunks that are FVs are collected into a list, replacing the chunk name with the variable it stands for ( $\mathrm{B} 1-\mathrm{B} 2$ ); a cheok is made for an answer unit (as in, "How many spans ..."), by B3; and the external representation of problem variables is collected for output, by $\mathrm{B5}-\mathrm{B8}$. Note that the FVs are collected in a particular order, by using HASCPRIOR. B2 constitutes a singleproduction loop, continually firing until all the ISFV's have been collected onto the FVLIST.

B5 is also a single-production loop of sorts: the RHS specifies that BUILDREPR is to be done, followed by a re-assertion of an ANSWERBUILD2 instance, which causes B5 to be examined again for more possibilities, and so on until the variables to be represented are exhausted. In the variable-representation collection process started by the B5 BUILDREPR assertion, since several variables may be equivalent, and since those that are equivalent have the same expression but not necessarily the same string representation, HASCPRIOR sequencing is used, so that the first representation seen in the scan is used as the collected list (the second HASREPR argument).

I Ps: Information gathering (13 Ps)
These Ps are not part of Studnt proper. Rather they monitor Studnt's progress by counting operators, variables, equations, and FVs, and by estimating how many more of those are likely to be found, assuming the worst case. These counts and estimates are
recorded in SPACESITES instances. The information as recorded was at one time used to attempl to measure the contribution of each $P$-firing towards reducing the combinatorial possibilities of the final output of the process. Thus, as each piece of new information is added, more is known about the form of the output, in terms of a reduction in the number of a priori possibilities. On the basis of that reduction, the ultimate "value" of each $P$ might be measured, with due account being taken of the fact that it depends on outputs of previous Ps, and so on.

## X Ps: Examples for testing ( 27 Ps )

Each $\times P$ contains the initial data for an example, including signals to start the Studnt processing. These tests are in sets of three, so that during testing, only a small amount of storage is taken up by problen statenents. The modules represented by the EXPR's were loaded separately, and after testing, deleted, before loading the next set. Each test uses the macro INITPROB to translate from a string representation into a sequence of predicates with arguments, for the internal representation. INITPROB is explained in a comment at the very beginning of Appendix B.

## B.5. Description of the predicates

In the following alphabetical listing of predicate descriptions, conventions on the types of arguments have been adopted to shorten the descriptions and to ease comprehension. Unfortunately, this typing is not done in exactly the same way in the body of the program (its value was not realized soon enough). Six argument types are distinguished, based on the first letter of the argument:
c: chunk; a chunk is a sequence of tokens linked by LEFTOF which forms a unit.
l: list structure.
$n$ : number.
p: position in string; each position is represented by a token, for which varibus properties can apply.
$w$ : word; the external name for a chunk element, e.g. "TIMES".
$x$ : other, to be explained with specific uses.
Arguments that are multiply used within a predicate description are numbered. If numbers for different types correspond, then the arguments also correspand, for instance, ( $\mathrm{c} 1, \mathrm{c} 2, \mathrm{p} 1, \mathrm{p} 2$ ) refers to two chunks, and two positions in those chunks, with p1 in cl, and p2 in c2.

The reader can refer to Appendix $C$ to find names of $P_{s}$ (Appendix B) that use these predicates.

AGECOMP ( $p 1, p$ ? ) loop status for comparing age variables in an nge problem to see if n now one is the same as one already seen; the tokens int pl and p2 are to be compared naxt.
AGECOMPFIN(p) signal that an age varibble comparison has been initintad, for a now variable starting int $p$; crestes a new AGEREF if not removed by the AGECOMP loop.
AGECOMPREM(p) doboto all AGRECOMP signale, since the tost heo failod.

AGEOP ( $p, c$ ) patats an age oporator for $c$; thn operator may be used later in the chunk to modily an ase veriable that is otherwise uncualified.
AGEOPNEED $(p 1, p 2, p 3,1)$ coll:et the woids of an AGEOP, as list 1 , with current callactine pobition $p 3$; the resull will rill in bntween $p$ l and $p$ 2.
AGEPOSSCOL( $1,1, p$ ?) collect worde starling at $p 1$ into $l_{i}$ result is to replace the possessive pronaun of p2.
AGEPROB $(x) x$ is an age problem; this onables special heuriatic trenaformationn and processing. AGEPRONCOL $\left(p 1,1, s ?\right.$ ) colfict words starting at $p 1$ into $l_{i}$ result is to replace the pronoun at p2.
$\operatorname{AGEREF}(p, n) \quad p$ in the starting position of $n n$ age variable with priority $n$ (lawor moans seen bofore); an nes variable in any age problem variable which starta with a person.
AGEREFCHK $(p)$ phrnan starting ot $p$ is to be checked to seo if it is new distinct age variable (AGEREF).
ACEREFCNT(n) count AGEREF'r, for assigning priaritios to now onas.
ANSUNITCHK $(x)$ check for creation of an ANSUNIT, in the procese of answor-buibling for problem $x$.
ANSWERBUJLD $(x)$ signal that the annwor--buisifing precess shouk begin for problem $x$.
ANSWERBUILD2 $(x)$ signal the chack for initintion of the colbaction of the exfernal representation of variabkes, in nnewor-bubtling, problem $x$.
ASCAN( $x$ ) do proliminery chock for keywords signifying en ase problem; $x$ is the current problem
BUILDREPR $(x)$ buikl up the axternal string representation for variable $x$.
CHTCOUNTED ( $c 1, c 2$ ) in the variable-test counting process, marke $c 1$ as having been counted wilh respect to tests on C ?
CHTESTED $(c, n) c$ has been insted with reapoct to $n$ other chunks; initialized to $i t o$ incluxle $c$ ifself.
CHIJNKENDL $(p, r)$ element at $p$ is at the inft end of $c$.
CHLINKENDR $(p, c)$ element at $p$ is at the right end of $c$.
CHUNKLEN(n) current length of current scan chunt; in $n$; ueed in I Ph.
CSPLIT (c,pi,p2) chunt: $r$ in to bo eplit into two chunks, with pi directly to the left of tho oporator phrabn at tho aplit, and p? diroctly to the right.
DEFOPLIST $(n, w)$ the $n^{\prime}$ th dofinifo operator found it w.
DELAYEXPND(x) Panisit primilive for delayed oxptinrion of PSMACRO; ueed here beceune of insertion of now, variable text during the problem runs.
ENDMARK ( $p$ ) an ond of the preblem toxt elving io at $p$ (laft or right end).
EQCHUNKTEST ( $c 1, s 2, p 1, p$ ? ) fobt for Esfuinisence botweon $c 1$ and $c 2$, which are assumed to be variableg. -
EOVARCHUNK $(c 1, c 2)$ cl and $c 2$ reprosent the same variable.
EQVARRFMI(c) signal thit all EQVARCHUNK's have been removed from consicieration in the varinble compariaonn
EQwww(p) the woid at p in the string is squal to "www".
FVLIST $(x, 1)$ is a list of $F V_{s}$ for problem $x$.
FVSCAN(p) signal to initinta chock for special FV tranaformations at $p$.
FVSCANEND $(p, c) \quad p$ in $c$ marka the ans of an $F V_{i}$ resulta in the at-up for another FV fo follow, or in detection of the end of the input etring.
HASCPRIOR( $c, n$ ) $c$ has priority $n$; lower meana seen firgt, if the chunk wat crated in the initial scan; otherwise higher value is given to the laft chunk than to the right, whon a chunt: in aplit in two; valuos from lator aplite are higher than for earlior ones.
$\operatorname{HASEXPR}(c, y)$ chas expression $x_{;} x$ ia aither a taken referring to e varibble, or a liet structure for the experssion.
HASIS ( $c, p$ ) $c$ has IS as highest precodence oloment, ot p.
HASOP(c,w) $c$ has oporator with name $w$; this will be used in conetructing tho output oxproseinn.
HASOPin( $c, p)$ chas OPin, for mm 0 1 2, as highast precedence olement, at $p$.
HASPREC $(p, n) \quad n$ har procodenoe $n$.
HASREPR $(x, 1) \times$ has external repmesentation $l$; Usually the fial of worde for the token $x$ of variabler chunt:
HASSQUARE (c,p) c han highest--precedonce oporetor SQUARE, at p.
HASSQUARED $(c, p) c$ has highest..precedonce operator SQUARED, it p.

HASUOPCHUNK ( $p, \mathrm{c}$ ) p in a unary oporator dummy, get up to hokl a pooition in $c$ white tho unary oposotor oxpmession it represents is parsed; result will replace the dummy as an oporand in $c$.
HASVERKS $(c, p)$ chas a vorb as highest precodenoe element, at $p$.
HIGHPREC( $c, n, p)$ the highest procodenoe for $c$ is $n$, at $p$.
IFDELETED(x) aignal thal on IF has been dowiod in the scan; $x$ is a dummy areument.
INCHUNK $(p, c)$ plempent of $p$ is in $c$.
ISANSUNIT(w) $w$ in the unil in which the answer is to be expesesed; before the anawar-buikling procese, it is just a position in the atring.
ISCHUNK(c) $c$ is a (nnw) complate chunk; innarted after the ontire chunt, hee boen inifinily scannect, or ofter it has been renamed as in result of the splitting procecs.
ISDEI.IM(p) $p$ is a delimitar.
$\operatorname{ISE}(Q N(c, x) \quad c$ is an aquation, with expresgion $x$
$\operatorname{ISFV}(c) c$ is an FV.
ISIS( $n$ ) $p$ is "in"; used to establich precodence value.
ISOP $m(n)$ $p$ is oporator of class $m, m=0,1,2$, usod to establish precedence value.
ISPERSON(p) $p$ is a person.
ISPI.IJRAL $(p, w) p$ is the pluml form of $w$.
JSPOSSPRON(p) pis a pobbfrive pronaun (only age probleme).
ISPRON(p) $p$ is a pronoun (only ago pretbiems).
JSOW (SRD(p) $p$ ia a quontion word.
ISREFEXXPR(c) $c$ in a reference oxprissisin, io., a canclixlete for a future "thig"; $c$ is either a sentonce that isn't an equation or the subject of e centence.
ISSCANCHUNK(c) c is curconlly beilig scanned; it is not an FV.
ISSCANFV $(c) c$ is an $F V$, and in curponlly boing ecanned.
ISSINCULAR(p) $p$ is the ningulay form of bome ward.
ISUOPDUMip) p is a unary aporator dummy, seo HASUOPCHUNK.
ISVARCHUNK(e) $e$ is a variable chunt, io., no oporators, noun phrese; this is eisnal for inilitition of variable comparisan procespas.
ISVERB( $p$ ) $p$ is verb.
LABELF(cl,n,r2) c1 is labeted finisihed, expressinn-tree lovel $n$, parent c2.
LABEIUU(cl,n,cद) cl is labeled unfininhed, explassion-iree lovel n, parent c2.
LEFTOF ( $p 1, p 2$ ) pl is dienctly to the left of $p 2$.
LRENAMF $(p, c l, c 2$ ? $) \mathrm{cl}$ is renamed to c 2 , curcont position $p$, proceeding to tho loft from $p$.
MODLEN(n) modify the iength of the string of the problem by $n$; used for eatimating epace sizen in I $\mathrm{P}_{8}$.
MOOLENC( $x$ ) $x$ is a dummy argument; chunk boumpary has been raached; the string length uned to computo worst-case opsee-sizen (I Pe) can be adjusted based on the length of thn chunk; just ecanned.
MXCPRIOR(n) maximum chunt, priseity number is $n$; used to assign to each chunt: a uniqua order nufrber.
NEWOVAR(c) $c$ is a now diptinct variable; gignal to I Pe.
NEWE:ON(x) signole a now equation to I Pe.
NEWFV(c) $c$ is a now $F V$; signol to $I \mathrm{~Pa}$.
NEWOP $(x)$ signel that $x$ is a new operatori for I Pe.
NEWPLOP $(x)$ signal a newly.-piaced operator to thn I Ps.
NEWPLVAR(c) $c$ is a nowly..placed variable; signal to I Pa.
NEWREFEXPR(c) signal a now reference expression, to become the ISREFEXPR.
NEWREFDP $(w)$ signal that $w$ in the operstor of ereference expreasion, to I Pa.
NEWSIZE $(x)$ eignal that a new space-sire vector meode to be computed; $x$ is dummy argument.
NUMVARCHUNKS(n) n dintinct variabie chunlis are known.
PLACOPLIST(n,w) the n'th plenced operator in $w$.
PRECSCAN $(c, p)$ pracetente scan is boirg dane on $c$, curcent point $p$.
PRECSCAND(c) pracedonoe acan has beon dotro on chunk c; ignol to note result and proceed occordingly, oither to eplil chunk or test as varioble.
PROBLEM $(x) x$ is tho name of the curvent problem

PROBxxx(n..) whero $x \times x$ in VARS, EQNS, OPS, or FVS; argumenta are values contributing to spnre-size af noted in thn commente accompanying II (see Apponxlix; B); INDEF is an estimain based on string length of what is conniklered the worst cnse for the givon quantily; in, aspumptionn are made on leng the of entities giving rise to the largeot expoctod count; DEF refiecte actual count so far found; PLACED reflacts that an oporator or variable may be determined but ite position in the output expringsion tree remains undetarmined.
RRE:NAISF:( $p, c i, c$ ?.) $c 1$ is renamed to $c 2$, current ponition $p_{1}$ praceeding to the right from $p$.
RTANDPERGOING(c) signal to npply FV trannformationn whon "PERIOD" or "AND" is scanmed, soms where to the right of the current scan position; $c$ is the current scan FV, limiting the acspo of the aignal.
RTANIDOMCDDING(c) similar to RTANDPERGOING, for "AND" or "QMARK".
RTDOESGOING(e) aimilar to RTANDPERGOING, for "DOES".
RTDOGOING(c) similar to RTANDPERGOING, for "DO".
RTHAVEGOING(c) similar to RTANDPERGOING, for "HAVE".
RTOMGOING(c) similar to RTANDPERGOING, for "QMARK".
SPACESIZEN(n) the number of apace-size vectors.
SPACESIZES (n, 1 ) $l$ is the $n$ 'th space-size vector; components correspond to argumente for all of the PROBxxx's.
STRINGEQ macro for generating gtrings of EQwww's, LEFTOF's, ote. - cee comment in program listine.
STRINGINS macro for genorating stringe of EQwww'b, LEFTOF's, ote. - ese comment in program lintine.
STRLI:NGTH(n) the lencth of the input string remeining to be acanned.
TANDDIFF(c) trannform "AND" in $c$ to "MINUSS", since the difference oporator has precoded it.
TANDSUM(c) trannform "AND" in c to "PL.USS", sinoe the SUM oporetor has been seon.
TEYYS(c) trennform "BY" to "IS", as requirod by "EXCEEDS".
TFASCAN(p) Bignal to check for apocial ago problom tranaformationa
TFASCANFIN $(p)$ birnal comptrion of TFASCAN at $p$.
TFOUT(p1,p?) rearrang the TFSCAN poinlers that ured to be at p1, to be at p2; necessary in some trannformationn that actually re-order the string.
TFOUTDELAY( $p 1, p$ ?) do a TFOUT on pl and what bocomea to the left of p2, after inmertion of gentrated, variable fext.
TFOUTLEN( $\mathrm{p} 1, \mathrm{p} 2, \mathrm{n}$ ) TFOUT with a string longth adjuatment of $n$.
TFSCAN(p) aignal to initintn chock for string trannformationn at $p$.
TFSCANFIN(p) signal completion of TFSCAN al p, ready for next step in the scan process.
TGSCAN(p) signal to initiate shock for diclionary tage at $p$.
TGSCANFIN $\langle p\rangle$ dane with TGSCAN at $p$, record precedences or do FVSCAN; also a special signal to initiato the scan to begin the probiem.
TGSCANFIN2(p) completion of initial scan processing at $p$, ready to move scan pointor.
THEIRCOLL( $p 1, p$ ? , pi, phill colbet an age variable starting at $p 3$, current collaction position $p 4$, list of text $t$, to bo innerind along with othor variablea between $p l$ and $p$ ? when collected.
THEIRCOLLD( $p 1, p$ ? ) age reference starting at pl has bean colimeted, for "THEIR" which in to be repliced at p 2 .
THETRREF ( $p$ ), p? ) signal to colifct a liat of all nge日 seen bo far, which are referred to by "THE:IR", and put thom botwoon pl and p ? whon collecied.
THETRREFFI.(i) a lint of all toxt collectod a far for "THEIR" replacement; each variable is colhectad anprivately and than adsled to thin lise.
THISTESTEO(c) the warimble loat for "THIS" has boon done for $c$; gignals the initiation of the match of c against other variable chunl: k
UNTESTED(c) $c$ is not tosiod with respoct to equivalonose with other variables; aignale for the first of a beries of testa to bo started.
URENAME: (c1, e?, n3,pA,p5) e1, which is the operand for a unary oporator, is to be renamed to be c2; renaming is currently at $p 3$, to be ferminatod at $p 4$ i on termination, the chunk is to be spliil at p5.
$\operatorname{VARCHCOUNT}(\mathrm{c} 1, \mathrm{c} 2$ ) signats failura of equivalence tests of cl with respect to c 2 ; chunks are countad after being testad.
VARCLEANUP(c) clean up absertiona hnving to da with the teating of $c$, since the result is known.
WCOLLECT ( $c, x, p$ ) colhet wards for $c$, with axpressian $x$, it $p$.

WORDEQ( $p, w$ ) the word at $p$ is $w$<br>WORDINS macro for gonnrating EQwww and WORDEQ for : string position - see comment in prescram listing

## B.6. Conclusions on the implementation

This subsection considers the following aspects: validation, program control, representation, and efficiency. First, in order to verify that Studnt is close to the original, Appendix Egives the results of test runs on 27 problems as given in the original publication. All of Studnt's answers are acceptable approximations to the solutions produced by STUIDENT. These tests used all of the Ps of Studnt except: S65, T3, T6, T7, T19, T20, D1, D9, D13, D14, D65, D67, D75, D87, P8, P28, P65, M30, M50, C5, C50, C52, V21, A3, A15 (that is, 25 out of about 260). There is no essential difference between these $P_{s}$ and $P_{s}$ that were actually used for the tests, so that this deficiency is not serious.

Programs written in Psnlst must use data signals to provide control, as is the case in all PSs. Several features of Psinst are useful in coordinating control signals. The main one is its stack memory, :SMPX, which is a temporary memory that effectively orders new elements of the Working Memory by their recency of assertion. Ps are selected for firing on the basis of this recency order, with those using the most recent data selected first, and with others pushed down in the stack until all the consequences of the newer data have been considered. The recency order is specified by the left-to-right order in RHSs of $\mathrm{P}_{s}$, such that the left-most assertion is considered to be the most recent. If a data instance is re-asserled at some time after its initial assertion, it is given a higher position in the recency order, corresponding to its most recent assertion. This re-assertion is analogous to data rehearsal in other systems. Another Psnlst feature is that when a $P$ is selected for matching, it may fire more than once, as opposed to firing once, allowing other Ps to be examined relative to the new data from that firing, and then returning to consider other possible matches that were available at the time of the original match. That is, all possible firings occur, in arbitrary order, before proceeding. Thus a set of Ps representing steps in some process can be working on more than one input element at a time, with multiple firings giving the appearance of paraliel sequencing on the inputs.

In Studnt, control passes in various flexible ways between: $S P_{s}$ and $T, A, D$, and $F$ $P_{s} ; P$ and $C, M$ and $V ; C$ and $R ; M$ and $R ; R$ and $P$. The $I P_{6}$ are evoked by most other groups. Appendix D gives a picture of the changes in control. The recursive nature of the parsing process, that is, the maintenance of the tree structure of the chunks, is encoded in the labels attached to chunks as they are split. Strict control sequencing is exhibited in the initial scan processing ( $S P_{s}$ ), in the splitting of chunks ( $P 10$ ), in the variable comparisons ( $V \mathrm{P}_{\mathrm{s}}$ ), and in the answer-building ( 85 ). That is, the $\mathrm{S}, \mathrm{V}$ and B Ps use specific signals to perform definite sequences of steps in fixed orders. The chunk-splitting process orders the chunks by attaching to each a numerical priority, and then processing according to that, resulting in the appearance of a stacking mechanism. The sequencing of the main scan, with control passing from $S$ to (and from) $T, A, D$, and F Ps makes use of the stacking mechanism of :SMPX to order the consideration of process initiation and completion signals, which are emitted simultaneously by S Ps. That is, an S Pemits both
an initiation signal and a complation signal, with the initiation signal processed immediately and the other stacked in :SMPX for consideration after everything relating to the initiation signal has been completed. Many looping processes were noted: P20-P29, C20-C22, two in the R's, V!5-V60, several in the A's, and two in the B's. A loop can easily maintain tight control by using a special signal which is asserted first in its actions, and which is only used by other $P_{s}$ in the same looping process. The mechanism of re-asserting data to cause re-examination at some later point is used twice, in P 10 and in 85 . Multiple firing of Ps is used to advantage in three places, V25, A63, and A67, and special care is taken to prevent it in $V 55$. In $V 25$, for instance, a new variable is compared to all previous ones, with the set of previous ones considered all at once instead of serialiy. In summary, we see that in an environment without conventional control primitives it is straight-forward to achieve a variety of flexible control facilities.

The unstructured Working Menory of Psnist is intimately connected with Studnt in two ways. The number of items in the memory is much larger than is efficiently stored in the linear Working Memory of other PSs. The range of Working Momory size for the Test2 example is from 115 to 321 items (these are initial and final figures, since no intermediate values are known, but no significant differences are expected for more accurate monitoring). The final memory size for Test16, the biggest test, is 765 . The :SAPPX mechanism narrows the focus of attention to a small portion of this mass, but even :SMPX becomes relatively large. For instance, the maximum number of :SMPX entries for Test2 is 126, but this is probably much larger than the number of distinct memory items that are referred to, since a data item occurs in many entries. Very little effort was made to limit the memory size, since the interpreter is capable of handling such magnitudes efficiently. Thus, these figures should not be taken as representative. The second effect of the Working Nemory is that it is more general and more cumbersome than the special string representation used in STUIDENT, but the benefit of making everything more explicit counteracts that minor difficulty, as we see in Section C.

The execution times of the tests given in Appendix $E$ are in the range from 2 minutes to 20 minutes, with the average around 5.6 minutes (on a PDP-10 computer). This is within an order of magnitude of what would be considered reasonable times for these tasks as performed by humans. One might expect a computer with the limited knowledge that STUDENT has to do an order of magnitude better than that, so that PSs seem not particularly speedye. Two things might easily make this order of improvement: more efficient implementation of the interpreter, and some way of compiling Ps (they're run interpretively at present). Also, the efficiency limitation may not be as serious as it appears, because one might argue that as more knowledge is added, little is added to total run time, since the number of applications of Ps in doing particular task would not necessarily go up significantly. This assumes that not much is added to the time required for selection of the next $P$ to fire. This is reasonable based on limited experience so far, which inclicates that the ratio of examinations to firings is fairly low. (Humans probably have no problem with huge amounts of knowledge because of some parallelism in the recognition-selection process.) It also may be that new knowledge would interact only slightly with existing knowledge, so that there would be little interference with the

These times are in the right range for humans; the only STUDENT figure is that it took less than a minute (on a 7094) to do the age problem TEST6, which Studnt does in about 7.5 minutes, about a factor of $20-30$ slower.
selection processes. That is, things that are relevant to present $P_{s}$ would only rarely be relevant to new oness. Memory usage is on the average about 95 K 36 -bit words. About 35 K of that is devoted to the Lisp and Psnist interpreters.

6 This is similar to the problem space closure concept in Newell and Simon (1972), chapter 14, pages 819-820.

Studnt

## C. The Knowledge in Studnt

The primary results presented in this section are based on viewing Studnt as the result of a knowledpe encoding process. Philosophically this view is similar to MCCarthy's Advice Taker proposal (1958), which laid out a plan for a general program that could modify its knowledge and its internal working procedures in accordance with advice given externally. The details of McCarthy's proposal were expressed with reference to a systematization of comimon sense knowledge as declarative statements in predicate logic, whereas the present approach expresses knowledge informally in unrestricted natural language and has a PS program as its target. That is, Studnt is analyzed as if it were the result of the assimilation of a large number of knowledge statements (KSs) in natural language. These $K S s$ are shown to interact with each other to form the encoding of the knowledge as a PS.

The general strategy taken here is appropriate when viewed in the framework of a knowledee acquisition approach to Al. This general approach consists of several steps: a precise formulation of the knowlesdge that it is necessary or desirable for an Al program to have; a suitable programming language, interpretable by a computer, for the ultimate expression of knowledge as procedures and data; and some way to bridge the gap betwenn the external representation and the internal (procedures and data) representation of the knowledge. This is to be contrasted with a knowledge generation approach, which I believe is implicit in approaches using mechanical theorem-proving techniques, perhaps; inspired by McCarthy's Advice Taker. Knowledge generation takes knowledge in the form of axioms and operates on it according to inference rules, in the hope that knowledge sufficient to produce intelligent behavior will result. A generation approach does not distinguish the three steps above, in part because the internal and external representations are the same; also it is not concerned with exhibiting a full body of knowledge, but rather with finding an adequate basis for generation. Since the generation approach has not yet been successful, the present approach is proposed as an alternative. Since it is a first approximation, some aspects have been alluded to, illustrated, and circumscribed, but it remains; informally (and vaguely) expressed. Expressing the knowledge precisely in any language (natural or artificial) is no small endeavor, and it is an activity that has not been carried out at the present scale by any previous work. The use of unrestricted natural language in the present work will be justified below (Section C.11).

At present, a computer program for the knowledge encoding process does not exist, although no insurmountable difficulties in constructing such a program can be foreseen. Rather, the knowledge has been oblained by an analysis (also not computerized) that represents a dual of encoding knowledge, namely, by a knowledge extraction process. The extraction is based on the meanings of the predicates that compose Studnt's Ps. Although the KSs were obtained analytically by an extraction process, it has seemed most natural to express them as if for use in encoding. Of course, Studnt is the result of an encoding process, but there is no basis for saying what the author had in mind during that original encoding, since accurate records were not kept.

The KSs fall quite readily into three major classes, which will be referred to as the $N$ class, the Q class, and the $Z$ class. The $N$-class statements ( Ns ) contain all of the taskoriented knowledge, for instance, knowledge about how arithmetic expressions are
represented in natural language, how to recognize a specification of which variable is to be solved for, how to transform idioms, and so on. Nost of the description of Studnt in the preceding section is at this level, loosely speaking. To organize this knowledge, we will use and augment slightly the concept of problem space (Newell and Simon, 1972, chapters 3 and 14), and we will refer to $N$ statements as being at the problem space level.

Q-class statements (Qs) deal with implementation knowledge. These define terms used at the problem space level and provide a collection of programming techniques suitable for the requirements of the problem space. The Qs are stated in a sufficiently general way to be useful in conjunction with other problem domains than Studnt's domain and with other programming languages besides Psnist.

The $Z$ class of statements ( $Z s$ ) deal with Psnlst control constructs, namely the special control features of Psilst that affect the actual form of the Ps. The present analysis neglects other Psinist features such as syntax and the properties of $P$ conditions and actions; this level is suppressed because of its straight-forward, routine nature.

In addition to the three classes of KSs that comprise the abstract content of actual Ps, a fourth, concrete component is central to the analysis: the predicates, which are the probiem-specific prograniming constructs. The knowledge extraction process is entirely dependent on the predicates' meanings (see the preceding section) for forming the KSs. The knowledge encoding process as presently formulated takes the predicates as given, and uses them at the appropriate (near-finai) step in building the Ps. The predicates are the basic expressive primitive for all the KSs, so that their meanings span the three classes ( $\mathrm{N}, \mathrm{Q}$, and Z ).

The division of $\mathrm{K} 5 s$ into Ns , Qs, and Zs raises some interesting questions relating to what kinds of KSs might be necessary to augment Studnt's capabilities and relating to what might happen to the contents of each class as shifts to other programming languages, other lask domains, and so on, are considered. But the division has also led to the hypothesization of a more general model of knowledge acquisition. The model puts the $N$, $Q$, and $Z$ components into a larger framework, and indicates the location of some interesting topics for further work. It is used to display the interdependencies of those three classes, it makes more explicit what other knowledge is needed to complete the knowledge encoding process, and it allows questions about the origins of the Ns, Qs, and Zs to be posed. In particular there are interesting questions relating to the formation of the problem space that is the basis of Studnt. Finally, the model of knowledge acquisition makes contact with work by other researchers.

This section commences by presenting a model that can be used to give an overview of the Ns; the model describes the knowledge at the problem space level abstractly, and provides a basis for determining the relationships of various subsets of KSs. A definition of problem space is included in that discussion. Section C. 2 goes through the knowledge encoding process for a particular P, illustrating haw KSs interact and how contact with Studnt predicates is made. The interactions of KSs in forming a selection of other Ps is given in Section C.3, illustrating the uniformity of the encoding process over all of Studnt, and raising the question of "bugs" that became evident. The encoding process is summarized in Section C.4. We then shift the focus to the division into Ns, Qs, and Zs, giving abstract characterizations for the $\mathrm{Qs}_{s}$ and Zs to parallel the model given in Section
C.1; other aspects of the division are discussed at the same time. Section C. 6 returns to the topic of knowledge extraction, the preceding subsections having laid a foundation for the necessary details. The more global view provided by the hypothesized knowledge acquisition model is elaborated in Section C.7. The last four subsections, Section C. 8 through Section C.11, give conclusions, comparisons to other approaches, considerations with respect to understanding systems, and foreseeable problems in extending this work.

## C.1. Characterizing the content of the knowledge statements

The Ns are the class of KSs that deal with the knowledge in Studnt at the problem space level, namely knowledge about the task environment and how to deal with it (problem space is defined more precisely later in this subsection). In other words the Ns are a mixture of process-independent facts about the domain of algebra word problems and of knowledge about specific methods and control sequences that can be used to coordinate the application of the domain facts to produce appropriate problent-solving behavior. They are a mixture because they are what is immediately extractable from the Studnt $P_{s .}$. As we will see below in discussing the model of knowledge acquisition (Section C.7), the consideration of pure task environment knowledge is one level removed from the problem space level, and in any case the problem space level cannot be bypassed, as that model is presently envisioned.

To provide an overview of the $\mathrm{N}_{\mathrm{s}}$ and to establish a vocabulary of elements and relations, wo propose a model, in the following sense. A model is a coherent body of objects and relations that represents some more complex structure, in such a way that manipulations (relations) on elements of the model correspond to manipulations (relations) on elements in the modelled structure. A model generally abstracts, suppressing some elewents and relations and thus emphasizing others. In this sense flowchart is model of the control flow of a process.

The model of the Ns gives a global overview, grouping the Ns according to their more global function. For instance, key terms in the model (for instance, "chunk") are defined at some point, have relations to other terms, are manipulated or transtormed, and so on, in ways that are clearly specified in the model. For more detail, the model provides pointers into the actual subsets of Ns. The presentation of the model at this point should help the reader to place the $N s$ that occur in the following subsections in perspective; the model is also essential to the identification of this level as the problem space level. The model is central to the knowledige encoding and knowledge extraction processes, but in ways that are difficult to pinpoint given the informal stage of the present analysis. That is, the use of such a model was evident at many places while the analysis was being done, but a clear picture of its use did not emerge; it probably will not do so until the processes are automated. We will discuss this further below.

The model of the Ns consists of statements a. through p. below. The objects in square brackets, such as [NS6-NS10, NS13], are sets of KSs that are elaborations of the associated model statement. The KSs are listed in full in Appendix $F$, and they will be discussed further in the subsections following this.
a. Input: a sequence of "words", each occupying one "position".
b. Output: a set of "equations" composed of "expressions" consisting of. algebraic variables (domain: real numbers), real constants, and common arithmetic operators; a list of specific variables whose values in the solution of the set of equations is sought, with an optional "answer unit" in terms of which the answer is to be expressed; a set of assumed equivalences between sequences of words that stand for algebraic variables. [N:31-NB3].
c. For every sequence of words there is a desired (canonical) form, to which the sequence is transformed. [NT1-NT32, NM9-NM11].
d. A word may belong to one of several classes of words; other operations that depend on the word may use its class membership properties. [ND1-ND14, NM13].
e. The operations of transforming the input sequence and assigning words to classes are correct only if done in particular order relative to each other and within the word sequence; this sequencing is achieved by the "initial scan". [NS1-NS5, NS11, NS12, NS14, NP2, NC15].
f. The sequences are broken into "chunks" according to membership of words in particular word classes, and according to interrelationships between the words in these classes.
g. The first subdivision into chunks is based on membership of boundary words in a set of classes distinct from the classes that determine further subdivisions. [NS6-NS10, NS13].
h. Further subdivision of the chunks is conditional on certain class memberships, i.e., there are two ways of proceeding from the first subdivision. [NS4, NS5)].
i. Under the first kind of further subdivision, the chunks are subdivided according to the properties of words of the "operator" class, and according to relative positions of these, as determined by a "scan", with each resulting chunk associated with the operator which formed its boundary as an "operand"; when a chunk is subdivided, the chunk membership property of the operand parts is changed by "renaming". [NiP1, NP3, NM1, Nh48, NC1-NC10, NC15, NR11, NR2].
j. One class of words requires a chunk to be rearranged in specific ways before it can be subdivided into variables and operators; i.e., "verbs". [NW2-NM7, NM12].
k. The resulting chunks and operators are then arranged as expressions in a tree structure (the tree structure is thus also determined by class memberships of its operators); such a tree structure with the operator "EQUAL" at its top node is an equation. [NC11, NC12, NC17].
I. A chunk that can be subdivided no further is termed a "variable"; variables which have similar word-sequence structure are assumed to refer to the same algebraic variable; similarity is determined by a set of rules; a variable may also refer to some previous expression. [NP4, NC14, NC16, NV1-NV18].
m . The second lype of subdivision is determined in ways specific to particular word configurations; its result is the second output component, i.e., the list of variables to be solved for, termed "FVs". [ NF 1. . N: $\mathrm{N}: 9$ ].
n. An imput sequerice that is recrignizably of a particular ciass, "age problem", undergoes special transformations in addition to those normally applied in cearhine the desired (canonical) form for sequences of words [NM1-NA11, NA13].
o. In an age problem, ceitain serments of text may be copied from one position to another, ciependent on class mentberships or on the presence or absence of paricular word sequences. [NA12, NA14NA17].
p. The result of reftain of the above operations is that certain estimates of the size of the space of possible outputs can be made or adjusted. [NI! MIIO].

The concepl of problem ?pace arose out of the need to describe the space in which human problem solving activilu: take place (Newell and Simon, 1972, p. 59). In particular, it is essential to be able to describe the possibutities for the behavior, rather than being limited to describing only the actual behavior. As uriginally formulated (Newell and Simon, 1972, pp. 810-811) a probleni space has five components: (1) a set of elements, each representing a state of knowledse aloout a task; (2) a set of operators that produce new elements from existmg ones; (3) the initial element; (A) the desired element or set of elements, to be reached from the initial element by applying operators; (5) the total knowledge available, which ranges from temporary dynamic information to long-term reference information. This can be seen to be simar to a general formulation of the heuristic search method (see, for instance, Newell, 1969), but there are differences. In human problem solving, a set of invariant features that are restrictive compared to heuristic search hold for problem spaces: the set of operators is small and finite (or finitely generated); a new knowledge state is produced every few seconds or so; and backup (the set of elements that can be returned to) is very restricted. Also, as we will illustrate below, the Newell and Simon definition allows the existence of plans that can give varying amounts of direction to the search. The instantiation of the problem space concepl for Sludnt presented below has ordered components (3) and (4) before (1) and ( 2 ) it has combined (1) and (5), since there is in Studnt no need for any distinction in knowledge states; and it has added two components (e' and f') whose presence will be further discussed below. The following gives Studnt's problem space by referring to the model of the Ns above.
a'. The inilial state of knowledge is statement a.
$b$ '. The problem or desired state is $b$.
c'. Elements, or knowledge states: the partially processed input string + all of the internal symbol structures pertaining to the problem.
d'. The operators, which produce new elements:
i. initial-scan operator set: transformations, dictionary tags, and segmenting: statements $c, d, f, g, n$, and $o$.
ii. FV-segmentation operator: $m$.
iii. parsing operator: scanning and splitting chunks, building expressions: $\mathrm{f}, \mathrm{h}-\mathrm{k}$.
iv. variable-matching operator: I.
$e$ '. Plans: $e$; sequencing implicit in $g-m$.
f'. Monitoring transitions to new knowledge states: p.

Two features of this problem space description deserve closer attention. First, something needs to be said to justify the size of the operators chosen, since the operators are sets of $P_{s}$. Studnt fortunately has a set of $P_{s}$ that monitor the knowledge state as major new information comes in, the I Ps, providing a natural dynamic boundary for the operators. To brisfly reiterate the function of the l's, they are connected with measuring the size of the space of possible outputs at any point in the process of solution. For insitance, at some point, we may know that there are two equations, five operators, and four distinct variables, which determines a finite number of possible outputs (the task of Studnt being to reduce that number to one). Further support for this division into operators comes by assuming 50 milliseconds for each Working Memory action of the process, and then computing the time this gives for each dymamic operator segment. The result (neasured on a typical example) puts the time within the three- to five second range observed by Newell and Simon (1972) for comparable problem space operators in general human problem solving. In particular, on the problem TEST2, the $P$ I3 fires albout 30 limes, and there are about 2100 Working Memory actions, giving 70 actions between firings of $13 ; 70 \times 50$ milliseconds $=3.5$ seconds. These figures are approximate, and actually only about two thirds of 13 's firings are meaningful as operator boundaries (it fires more than once at some boundaries), but this still gives five seconds as the result.

The second feature of the problem space that needs to be discussed is the existence of plans, point e' (e. in the model). A plan is some kind of explicit control that guides the applications of operators (Newell and Simon, 1972, pp. 822-823). At one extreme of planning in this sense is a specific algorithm that is guaranteed to achieve the desired result. The main plan in Studnt is the initial scan, which rigidly controls the order of application of the operators by moving a scan pointer along the input string from left to right. A plan controls the ordering of the operators in the initial-scan operator set. If we remove the sequencing assumptions in these plans, we get a process with more of a heuristic search structure, with various orderings tried according to some search scheme, and with some way of ordering the resulting end products in order to pick the best. Some search is necessary as is illustrated by the phrase " 30 per cent of". "Of" is changed to the operator "times" if preceded by a number, and " 30 per cent" becomes ".30". Clearly two different results obtain depending on the order of testing for "per cent" and "of preceded by a number". An interesting problem for further research is the transition from a planiess process to the final Studnt, and in particular, whether plans are added bit by bit, with processing taking advantage of pieces of plans wherever possible, and searching otherwise. To investigate this further, the PS formulation, with all control explicit in the data sitate and in $P$ conditions, seems more suitable than standard control structures. Formulating Studnt as a problem space in this way serves to organize the model (at least, for purposes of exposition), it points out interesting research questions, and it makes contact with other research in problem solving that will be discussed in Section C.7.

## C.2. Knowledge interactions in forming a production: S13

We now present an example of the knowledge encoding process as it is envisioned for an important Studnt $P$. The implied form of the encoding process, however, is not nearly as imporlant at this stage as the KSs themselves and how they can be seen to interact. The following brisfly introduces the process, postponing a more exact discussion until examples are presented.

The knowledge in a $P$ is built up around a particular KS, its principal KS. The $P$ results as parts of the prinsipal KS refer to subjects of other KSs, thereby causing them to interact with it, defining its terms and elaborating the conditions under which it applies. A particular N interacts with other Ns to give the total intention of the P . Qs and Zs are then added as required to define terms, to provide specific techniques, and to make contact with the control structure of the underlying language. This process will now be illustrated by examining 513 in detail. In case the reader loses the overall structure of the following details, the material is summarized in Figure C. 1 at the end of this subsection, and Section C. 3 gives a summary in a different form.

S13 is a $P$ that controls the initial scan of the input problem, invoking the transformation process and doing sone bookkeeping on the string elements scanned.

```
S13; "TF SCAN" :: TGSCANFINR(X) & LEFTOF(X,Y) & NOT ISDELIM(X) & ISSCANCHUNK(C)
                & CHUNKLEN(L)
m TFSCAN(Y) & TFSCANFIN(Y) & INCHUNK(X,C) & CHUNKLEN(L+1)
    & N:GATE(1,5) & NOT TGSCAN(X);
```

    where N:GATE(1,5) : NOT TGSCANFIN?(X) \& NOT CHUNKLENKL)
    The principal KS for 513 is NSILe:
NS 11. THEE INITIAL. SCAN PROCEEDS FROM LEFT TO RIGHT IN THE PROBLEM STRING, PERFORMING THE FOUR FUNCTIONSO\& AT EACH POINT IN TURN, AND ADOING EACH WORD SCANN:D TO THE CURRENT CHUNK.

The first phrase brings in Q4:
Q4 THE PROCESS OF SCANNING INVOLVES MOVING A SCAN POSITION FROM AN OLD POSITION TO A NEW ONL:

To determine the old position, use is made of TGSCANFIN?:
TGSCANFIN?(p) completion of initial scan processing at p, ready to move scan pointer.
The new position is determined by using Q8 which brings ineee LEFTOF:
Q8 PROBLEM STRINGS AND SUBSTRINGS ARE SEQUENCES OF WORDS, READ FROM LEFT TO RIGHT, WITH EACH WORD DIRECTLY TO THE LEFT OF THE WORD FOLLOWING IT.

LEFTOF(p1,p2) p1 is directly to the left of p2.
This has determined everything relevant to the old position of the scan pointer,

[^1]represented by the first two L.HS conjuncts in S13. At a knowledge level that is suppressed here, it is understood that "old" would imply something in the condition (LHS), whereas the "new" refers to something in the action side of the $P$. What actually goes into the action side for the new pointer position depends on parts of NS11 that will be taken up later, after the interactions from what has been done so far have been discussed.

Now, the initial scan does not always proceed unconditionally, as stated by NS 12 :
NSI. 2 WHEN THF: END OF A CHHHK IS SCANNI:D, THE CHNNK IS COMPLETE, AND THE INITIAL. SCAN IS INTERRUPTED FOR TH: CHUNK SPLITTING PROCESS.

This interaction results, by indirection, in the third LHS conjunct. First there is an association to NG\%, which detines how the end of a chunk is recognized:

## NS7 WIHN A PERIOD WITH A DELIMITER TAG IS SCANNE:D, THE END OF THE CURRENT CHUNK HAS BEEN REACHED, IF THE: CHUNK IS NOT AN FV CHUNK.

Using the meaning of ISDEL.IM, we get the third conjunct:
ISDEAIM ${ }^{\prime}(\mathrm{p}) \quad \mathrm{p}$ is a delimiter.
Here, a choice was made on whether the ISDELMM argument should be $X$ or $Y$, that is, whether to interrupt the scan before or after looking at the delimiter of the chunk. The choice of X, namely the element just passed, follows from consideration of Q14 (which the knowledge encoding process would consult every time such a condilion were tested):

Q1. 1 DURPMG A SCAN PROCESS, WHEN A CONDITION IS STATED IN TERMS OF THE POSSIBLE OUIPUT OF SOM价 PROCESS THRT IS APPLIED AT EACH SCAN POINT, THFE TEST FOR THAT COHDITION AT A PARTICULAR POINT SHOULD BE DEFERRED UNTIL THE SCAN HAS PASSED THE POINT.

In this case, one example of a relevant Studnt Iransformation is stated by NT25:
NT25 ", ANI." TRANSFORMS TO "PERIOD".

We now proceed to the second phrase of NS11, which refers to performing four functions in turn. This is elaborated by Q5:

Q5 APPLYING A NUIMBER OF FUNCTIONS IN TURN MEANS TO APPLY THE. FIRST, AND WHEN THAT IS DONL:, APPLY THE: SECOND, AND SO ON.

So we need to know what the first function is:
NS 1 TH: FIRST FUNCTION OF THE INITIAL SCAN IS TO APPLY TRANSFORMATIONS AT EACH POINT IN THI: SCAN.

Since we're doing a sequence of functions, we look at:
© This kind of imperative language is typical of expressing $K S s$ as if to an encoding process.

Q15 WHEN A SEQUENCE OF ACTIONS IS TO BE PERFORMED, MORE FLEXIBILITY IN AITERING THE COUJRSE OF THAT SEQUENCE OBTAINS BY BREAKING IT INTO SEPARATE STEPS, EACH REQUIRING AN INITIATE SIGNAI. ANID HAVING A COMPI.ETION SIGNAI.; THS BREAKING INTO STEPS IS ESPECIALLY USEFUL FOR
 THE SEQUENCE ARE ACTUALLY EXECUTED.

This gets us to the use of TFSCAN and TFSCANFIN:

| TFSSCAN $(\mathrm{p})$ | signal to initiate check for string (ransformations at $p$. |
| :--- | :--- |
| TFSCANFIN(p) | signal completion of TFSCAN at $p$, ready for next step in the scan <br> process. |

We use two signals because of:
Q24 WHIEN THERE ARE: MANY MORE WAYS OF COMPIETING A PROCESS EVOKED BY AN INITIATE SIGNAI. THIAN WAYS OF INITIATING IT, THE COMPLETION SIGNAI. SHOULD BE EMITTED AT TH: SAME TIME AS THE INITIATE SIGNAL, IN SUCH A WAY THAT THE: INITIATE SIGNAI. IS EXAMINED FIRST.

Since the order of consideration of these two insertions is critical, we must make use of:
72. THI: FIRST TVO RIGHT HAAII. SIDE INSERTIONS ARE ORDERED AT THE TOP OF :SMPX; WIII:N IT IS DESIRED TO DO ON: THING FOLLOWED BY ANOTHER, ORDER THE "INITIATE." SIGNAI.S ACCORDINGI.Y.

So, now we have the first two conjunets of the RHS.
The final phrase of NSII deals with noting that each word scanned is part of the current chunk. This cannot be unconditional, because of an interaction with NS 10:

NS 10 THE PERIOD AT THE END OF A CHUNK IS NOT INCLUDED AS PART OF THAT CHUNK OR ANY OTHER CHUNK.

This associates first to NG7 (see above), which says we're testing on "period". By the same reasoning as used before, this exclusion also has to be done after the scan on a position is done, so the NOT ISDELIM lest serves a double purpose. To add to the current chunk, we need to know whal it is:

ISSCANCHINNK (c) c is currently being scanned; it is not an FV.
This is the fourth LHS conjunct, and the act of noting is taken care of by the third RHS conjunct, which uses:

INCHHNKL $(p, c$ ) element at $p$ is in $c$.

* The Qs at times express qualitative goals like flexibility and efficiency, rather than simply giving absolute direction.

The use of ISSCANCHINK allows us to clean up a loose end regarding the use of NS7. We must verify that in fact the end of the chunk has not been reached, and the NOT ISDELIM will work, provided this isn't an FV chunk; the definition of ISSCANCHUNK guarantees it.

This takes care of the central action with respect to NSII. It remains to consider some other associations which are related but are less essential to the main process. NI7 has to do with scanning, in fact, with the number of words scanned:

NI7 TH: LENGTH OF TH: PART OF TH: PROBLEM AS YET UNSCANNED CHANGES EACH TIME A NEW OPERATOR, EQUATION, OR PERIOD IS SCANN:D, AND IT CHANGES BY THE NIJMF3ER OF WORDS SCANIU:D SINCE TH: LAST CHANGE OR SINCE THE BEGINNING OF THE PROBLEMS.

CHUMALEN is the counter:
CHUNIKLEN(n) current length of the current scan chunk is $n$.
To change a counter, we need the old value in the LHS, with the new value as part of the RHS. QG requires us to delete the old value of the counter:

## Q6 WIIEN A VAI.UE OF A COLNTTER IS CHANGED, THE OLD VALUE SHOULD BE REMOVED.

This gets the sixth RHS conjunct.
We have not mentioned the fifth and seventh RHS conjuncts, whose purpose is to erase old scan signals. The appropriate KS :

Q3 FOR STORAGE EFFICIENCY, PROGRAM SEGMENTS THAT RESPOND TO SCAN SIGNAILS OF THE "COMPLETION" TYPE SHOULD ALSO REMOVE THE CORRESPONDING "INITIATE" TYPE:, AS WELL. AS REMOVING THE USED "COMPPIETION" SIGNAL, IF IT IS POSSIGLE THAT NO PROGRAM SEGMENT RESPONDS TO THE INITIATE SIGNAI.

There are other KGs that deal with the initial scan, which would be examined, but rejected, in the process of building S13.

NS2 THE SECONID FLUMCTION OF THE: INITIAL SCAN IS TO APPLY AGE-PROBLEM TRANGFORMATIONS, IF TH: PROBLEM IS AN AGE PROBLEM, AT EACH SCAN POINT.
NG3 TH: THIRD FUNETION OF THE INITIAL SCAN IS TO PUT DICTIONARY TAGS ON WORDS AS EACH WORD IS SCANIN:D.
NS4 THE FOURTH FUNCTION OF THE INITIAI. SCAN IS TO CHECK FOR A NEW HIGH PRECEDENCE WITHIN THE CHIUNK BEING SCANNI:D, IF THAT CHUNK IS NOT AN FV CHUHUK AS EACH WORD IS SCANMS:D.
NG5 THE FOURTH FUNCTION OF THE: INITIAL SCAN IS TO APPLY THE FV TRANGFORMATIONG, IF TH: CHINHK BEING SCANNI:D IS AN FV CHUNK, AS EACH WORD IS SCANMM:D; AN FV TRANSFORMATION IS ANY OPERATION THAT DEAL.S WITH THE DETERMINATION OF FV CHIHNKS.
NS6 A CHUNK THAT STARTS WITH A WORD THAT IS A QWORD IS AN FV CHUNK.
NS§. THE: FIRST. CHUNH TO BE SCANNH:D STARTS IMMEDIATELY TO THE RIGHT OF THE LEFT ENID OF THE: PROBLEMS STRING.

NG9 WHIN TH: ENID OF ONF: CHUNK IS REACHED, ANOTHER BEGINS IMMEDIATELY, UNIESS THE: RIGHT END OF THE PROBLEM STRING HAS BEEN REACHED.
NS 1.3 TH: I.AST CHUNH: IN A PROBLEMA IS ALWAYS AN FV CHUNK.
NS? through NS5 are rejecled because they deal with functions of the scan other than the first. NS6 and NS13 are rejected because the QWORD tag is the result of the third scan function, and is thus unavailable. NS8 is relevant, and interacts with NSIL to produce another P, S10. NS9, NS10, and NS12 (the last two were displayed previously) do not add to the action because of the exclusion of their conditions with the third LHS conjunct.

Figure C. 1 summarizes the interactions between the KSs that form S13 as described above. Each arrow represents an interaction, with its origin at the KS (or predicate, in one case) that initiates the interaction by requiring further elaboration.

## C.3. Summaries of inferactions for selected productions

This subsection gives summaries of the formation process for a representative set of $P_{s}$. Since each summary lists only a $P$ and its principal $K S$, the reader must refer to Appendix $F$, which lists the $K S s$ in full, in order to follow the detail.

Each summary starts out with a listing of the $P$ and its principal KS. If the $P$ has any macros, their expanded form is given. The body of the summary is organized into "sentences", delimited by ".", broken into segments delimited by ";". A sentence represents closely intervelated processing, with each segment dealing with the determination of a set of conjuncts of the P. The conjuncts are referred to by labels such as "L1" and "R3", which stand, respectively, for "first L.HS conjunct" and "third RHS conjunct". In counting in RHSs, EXISTS conjuncts are ignored. Lines giving macro expansions also give labels for the conjuncts in []'s to aid in determining referents of labels for the conjunctions containing the macros. Within segments, " $\delta$ " is used to indicate "interacts or combines with", a binary operator on KSs; "->" is used for "associates to". " $\&$ " has a higher binding power than " $\rightarrow$ ", i.e, $a \& b \rightarrow c \& d$ is really ( $a \& b$ ) $\rightarrow(c \& d$ ). These are, of course, to be interpreted loosely. Each sentence has as subject its first element; segments that start wilh " $\delta$ " or "->" implicitly have an occurrence of the subject.

The summary of 513 appears first, so that the reader may become accustomed to the notation on familiar material. The meaning of "excitatory interaction" is explained below.


Figure C. 1 Knowledge interactions in forming S13

Summary for S13:
S13; "TF SCAN" :: TGSCANFINR(X) \& LEFTOF (X,Y) \& NOT ISDELIMKX) \& ISSCANCHUNK(C) \& CHUNuLEN(L)
$\Rightarrow$ TFSCAN(Y) \& TFSCANHIN(Y') \& INCHUNK(X,C) \& CHUNKLEN(L+1) \& N:GATE $(1,5) \&$ NDT TGSCAN(X);
where N:GATE $(1,5) \equiv$ NOT TGSCANFIN2 $(X) \&$ NOT CHUNKLEN(L) [R5, R6]
principal (model statement e.):
NG ! 1 THE: INITIAL. SCAN PROCEEDS FROM LEFT TO RIGHT IN THE PROBLEM STRING, PERFORMING THE FOUP FUICTIONS AT EACH POINT IN TURN, AND ADDING EACH WORD SCANIH:D TO THE CUIRRENT CHUNK.
first phrase: Q4 $\rightarrow$ L1; Q4 \& Q8 $\rightarrow$ L2;
excitatory interaction: NSI.2 $\rightarrow$ NS7 $\rightarrow$ L3;
Q14 \& NT25 (\& others) $\rightarrow$ arg of L3.
second phrase: Q5 $\rightarrow$ NS1 \& Q15 \& Q24 \& Z2 $\rightarrow$ R1, R2.
third phrase: L.4, R3;
excitatory interaction: NS 10 -> NS7 \& L. 4 def'n $\rightarrow$ L. 3
(again, arg as above).
Q4 $\rightarrow$ NIT $\rightarrow \mathrm{L} .5, \mathrm{R4} ; \mathrm{QG} \rightarrow \mathrm{RG}$.
Q3 -> R5, R7.

The following summaries are given to indicate the uniformity and general applicability of the above knowledge encoding process to all of Studnt's Ps. T12 is a typical initial-scan transformation $P$, with much simpler structure than S13. M10, C60, and C75 deal with the process of breaking down chunks into operators and operands, and then putling the compleled expressions together to form an equation. F60, F70, and F75 illustrate the processing of one type of FV form. These examples illustrate the application of over half of the Qs , and introduce twenty new Ns .

The examples also include three "bugs" which were discovered by the knowledge analysis (see C75, F70, F75). These are bugs from the standpoint of the analysis, not defects in the actual output of the program. The first involves having two Ps with overlapping conditions, where a combination of the two into one is more appropriate, and is dictated by the analysis. The second bug is an inconsequential incorrect ordering of RHS assertions. The third seems more serious, since it is an omission of updating the element that denoles which chunk is the current scan chunk. However, its bad effects are cancelled by the failure of other Ps to check for or make use of that information. A more general discussion of the types of bugs encountered in the process of doing the knowledge analysis is below, Section C.4.

Summary for T12:

```
T12; "TWICE..>TWO TMAFS" :: TFSCANHV?-1) & STRINGEQ'(TWICE),X,Y)
    m MODLEN(1) & EQ2(V?-1) & WORDEQ(V?-1''2)
            & NOT WOROEQ(V?-1,'TWICE) & STRINGINS(`'TIMES),V?-1,Y)
            & N:GATE(AL.L,-2);
    where STRINGEQ(`(TWICE),X,Y) = IEFTOF(X,V'-1) & EQTWICE(V?-1)
            & L.EFTOF(V?-1,Y) [L2,L3,LA]
            STRINGINS(`(TIMES)V?-1,Y) : EXISTS(T1) & LEFTOF(T1,V?-1)
                        & EQTIMES(V?-1) & WORDEQ(V?-1,'TIMES)
                    & LEFTOF(V?-1,Y) [R5-R8]
            NEGATE(AL.L,-2) \equivNOT TFSCAN(V?-1) & NOT EQTWICE(V?-1)
                            & NOT LEFTOF(V?-1,Y) [R9, R10, R11]
```

principal (model statement c.):
NT12 "TWICE" TRANSFORMS TO " 2 TIMES".
NT12 $\rightarrow$ L3, R2, R3, R6, R7; (checks other NT's, by Q11, but no effect); Q8 $\rightarrow$ L2, LA, R5, R8; Q12 $\rightarrow$ RA, R10, R11.
"Iransforms to" $\rightarrow$ NSI ..> L.1; Q7 $\rightarrow \mathrm{R} 9 ; \mathrm{Q9} \rightarrow$ args of R2, R3, R4; NIS $\rightarrow$ R1 (order determined by NIIO \& Z1).

Summary for M1O:

\& NOT EQMULTIPLIED(A2) \& NOT EQDIVIDED(A2) \& NOT EQINCREASED(A2) $\Rightarrow$ NEWEQN $(X) \& \operatorname{CSPLIT}(C, X, X) \&$ HASOP(C,'EQUAL) \& NEGATE(2);

$$
\begin{equation*}
\text { where NEGATE(2) }=\text { NOT HASIS(C,X) } \tag{R4}
\end{equation*}
$$

principal (mostel statement i.):
NCA A CHUHK WITH A HIGHEST. PRFECEDENCE OPERATOR MARKED, EXCEPT "SQUARF" AND "SQUARPED" , IS SPLIT INTO TWO NEW CHUNKS, WITH THE LEFT END OF THE LEFT CHUHK THI: SAME AS THE ORJGINAL, RIGHT END OF THE LEFT CHUNK THE WORD DIRE:CTI.Y TO THE: L.EFT OF TH: PHRASE REPRESENTING THE OPERATOR, LEFT END OF THE: RIGHT CHUNK DIRECTLY TO THE RIGHT OF THE PHRASE REPRESENTING THE OPERATOR, ANIJ RJGHT ENID OF THE RIGHT CHUNK AT THE RJGHT END OF THE ORJGINAI. CHUHNK.

NCA $\rightarrow$ L1, L2; \& NH1 \& NC5 $\rightarrow$ R3; \& Q16 $\rightarrow$ R2.
string in condition $\rightarrow$ Q11 $\rightarrow$ inter with $\mathrm{NCl} \rightarrow$ L.4, L5, L6; \& Q8 -> L3.
"equal" in NM1 $\rightarrow$ NC12 $\rightarrow$ NIL $\rightarrow$ R1 (order by NilO \& Z1).
"split" in NC4 $\rightarrow$ Q13 $\rightarrow$ R4.

Summary for C60:
C60; "SPLIT CHUNHK" :: CSPLIT(C,LOCL,LOCR) \& LEFTOF(X1,LOCL) \& LEFTOF(LOCR,X2) \& LABELU(C,N, ${ }^{\prime}$ ) \& MXCPRIOR(M)
$\Rightarrow \operatorname{EXISTS}(C L, C R) \&$ N:WPL.OP(C) \& RRENAMME(X2,C,CR) \& L.RENAME(X1,C,CL)
\& LABELU(CL, $N+1, C\rangle \& \operatorname{LABELU}(C R, N+1, C) \&$ HASCPRIOR(CL, $M+2$ ) \& HASCPRIOR(CR, $M+1$ ) \& MXCPRIOR( $M+2)$ \& CHUNKENDL(X2,CR) \& CHIUNKENDIR(X1,CL) \& NEGATE $(1,2,3,5)$;
where NEGATE $(1,2,3,5) \equiv \operatorname{NOT}$ CSPLITT(C,LOCL,LOCR $) \& \operatorname{NOT}$ LEFTOF(X1,LOCL) \& NOT LEFTOF(IOCR,X2) \& NOT MXCPRIOR(M) [R11-R14]
principal: NCA (see above)
NCA $\rightarrow$ NC5 \& Q16 \& Q8 $\rightarrow$ L1, L2, L3.
"new chunks" -> NR1 \& Ni2? \& Q53 -> NC15 -> Q19 -> R2, R3.
NC5 $\rightarrow$ Q2O $\rightarrow$ L4, R4, R5, L5, RG, R7, R8.
renaming $\rightarrow$ Q21 $\rightarrow$ R9, R10.
operator placed in expression $\rightarrow$ NII $\rightarrow$ NILO \& Z3 $\rightarrow$ RI, order of R1, R2, R3.
Q18 -> R11. Q17 -> R12, R13. Q6 -> R14.
Summary for C75:
C75; "FINISH SEEA =" :: I.ABFLU(C,N,P) \& I.ABELF(C1,M,C) \& LABELF(C2,M,C)
\& HASOP(C,X) \& SATISFJES(X,X EQ EQUAL) \& HASCPRIOR(C1,PR1)
\& HASCPRIOR(C2,PR2) \& SATISFIES2(PR1,PR2,PR1 ? ${ }^{(G G R E A T ~ P R 2) ~}$
\& SATISFIES(M,M EQ 2) \& HASEXPR(C1,Y) \& HASEXPR(C2,Z)
$\Rightarrow$ N:WREFEXPR(C1) \& HASEXPR(C, $\langle X, Y, Z>) \&$ LABELF(C,N,P) \& NEGATE(1);
where N:GATE(1) = NOT LABELU(C,N,P)
[R4]
and $\langle X, Y, Z\rangle$ converts to the LISP expression (LIST X Y Z)
principal (model statement $k$.):
NC1 1 AN EXPRESSION IS A TREE STRUCTUIRE OF THE FORM ( $\mathrm{a} b \mathrm{c}$ ) WHERE a IS THE OPF:RATOR, b IS THE: TREE EXPRESSION FOR THE LEFT OPERAND, AND o IS THE SAME FOR THE RIGHT OPERAND.

NC11 -> LA, L1O, L11, R2.
"tree structure" $\rightarrow$ Q20 $\rightarrow$ NC5 \& NC17 $\rightarrow \mathrm{L} 1, \mathrm{~L} 2, \mathrm{~L} 3, \mathrm{~L} 6, \mathrm{~L} 7, \mathrm{~L} 8$, L9, R3; Q33 -> R4.
"left operand" $\rightarrow$ NC14 $\rightarrow>$ L5, R1. (conditional, others are C70, C78.)
(in the given KS framework, NC12 should also be included; reason
for its absence is related to the growth of the program: C70-C78 were not split into the three conditions originally, so that C90 was necessary.)

Summary for F 60 :
F60; "FIND FV" :: EQFIND(X) \& FVSCAN(X) \& ISSCANFV(C) \& CHUNKENDL(X,C) \& L.EFTOF $(X, Y)$
$\Rightarrow$ CHUNNKENDL $(Y, C) \&$ RTANIJPERGOING(C) \& NEGATE(2,4);
where N:GATE $(2,4) \in \operatorname{NOT} \operatorname{FVSCAN}(\mathrm{X})$ \& NOT CHUNKENDL $(X, C)$ [R3, R4]
principal (model statement m.):
NFS A SENTENCE WHICH STARTS WITH "FIND" HAS FV CHUNKS STARTING AFTER THE "FINID" AND GEPARATED BY "ANIJ" , AND IT ENIJS WITH "PERIOD".
$\mathrm{NH} \mathrm{B} \rightarrow \mathrm{L}, \mathrm{L} 4$.
"FV" -> NS5 -> L2, L3; transformation -> Q7 -> R3.
"find" adjacent to start $\rightarrow$ Q8 $\rightarrow$ L.5.
removal of "find" $\rightarrow$ Q23 -> L4, R1; last phrase of Q23 $\rightarrow$ R4;
\& Q10, inhibiled by NF 9 .
separator after chunk $\rightarrow$ later in scan $\rightarrow$ Q22 $\rightarrow$ R2.
Summary for F70:
F70; "\&-. ." :: FVSCAN(X) \& RTANIPERRGOING(C) \& EQPERIOD(X) \& LEFTOF(W,X) $\Rightarrow$ ISVARCHUNK (C) \& CHUTHENDR(W,C) \& FVSCANEND(X,C) \& NEGATE(1,2);
where $\operatorname{NEGATE}(1,2)$ : NOT $\operatorname{FVSCAN}(X) \& \operatorname{NOT}$ RTANDPERGOING(C) [R4, R5]
principal: $N \cdot \Omega$, see above.
NF\& \& Q22 -> L2. L3. "FV" -> NS5 -> L1.
"period" -> NF"2 \& Q23 \& Q8 $\rightarrow$ L4, R2;
NS1O \& NS11 \& Q14 inhibits NOT INCHUNK(X).
end of FV $\rightarrow$ NF $3 \rightarrow$ R1; $\rightarrow$ Q16 $\rightarrow$ R3; $\rightarrow$ Q18 $\rightarrow$ R5;
$\rightarrow$ NS12 -> Z2 $\rightarrow$ order of R1, R3 (bug: R2 should be after R3).
transform $\rightarrow$ Q7 -> R4.

Summary for F75:

> F75; "\&*. \&": : FVSCAN( $X$ ) \& RTANDPERGOING(OC) \& EQAND( $X$ ) \& L.EFTOF $(W, X) \&$ LEETOF $(X, Y)$
> $\Rightarrow$ ISVARCHUNK (OC) \& CHHNKENDR (W,OC) \& EXISTS(C) \& NEWFV(C) \& ISFV(C) \& RTANIJPERGOING(C) \& CHUNKENDL(Y,C) \& NEGATE(1,2);
> where N: $\operatorname{GATE}(1,2)=\operatorname{NOT} F V S C A N(X) \& \operatorname{NOT}$ RTANIJPERGOING(OC) [R7, R8]
principal: Nf: 3 , see above.
NF\& \& Q22 - L L L L3; \& Q25 \& Q8 \& Q23 -> L4, L5, R2. "FV" $\rightarrow$ NS5 $\rightarrow$ L1.
separator $\rightarrow$ NF $3 \rightarrow$ R1; $\rightarrow$ Q25 \& Q1 \& Q16 $\rightarrow$ R4, R6; $\rightarrow$ Q22 $\rightarrow$ R5; Q25 new chunk $\rightarrow$ N11 $\rightarrow$ R3.
(bug: missing ISSCANFV update, apparently a serious bug,
but it works ok because other $P_{s}$ don't check)
transform $\rightarrow$ Q7 $\rightarrow$ R7; R5 \& Q34 $\rightarrow$ R8; end of chunk $\rightarrow$ NS $12 \rightarrow$ R1 before R4.
NIL $\rightarrow$ NILO $\rightarrow Z 3 \rightarrow$ order of RHS, except bug, should be R3, R1, R4, R2.

## C.4. Summary comments on the details of the analysis

This subsection discusses in a more general way the knowledge encoding process revealed in the examples just given. Then, there is a short discussion of the bugs that were detected in carrying out the analysis for all of Studnt. The reader will need to refer to Appendix $F$ to follow the examples used as supporting evidence.

The knowledge encoding process starts out with statements that are close to the alsstract model characterization of the target process. That is, particular KSs are selected to be principal KSs on the basis of their plan-like nature, as opposed to being simple assertions of facts. For example, among the NS's, NS 1-5, 8, 11, 12 and 14 are used as principal KSs, while NG6, 7, 9, 10 and 13 are not (actually the inclusion here of NS6 and NE9 needs to be qualified, see below). Similarly, NC2, $4,6,7,10$ and 11 are the NC's that are principal. It is evident from these examples, however, that it may be impossible in general to decide which KSs can be principal without fully working out the interactions, to see how the KSs stand in relation to each other. Note that model statement g. (Section C.1) is elaborated almost entirely by non-principal $K$ Ss. This may indicate that the structure of the model can be helpful in distinguishing principal from non-principal. Another common feature of non-principals is the use of phrases like "whenever": NM12, Nh13, and NF9 are examples.

Once a principal KS has been chosen, interactions of three main sorts occur: definitional, excitatory and inhibitory. A definitional interaction is an interaction in which one KS defines a term in another. We have seen a definitional interaction in the use of Q5, dealing with sequential application of functions, which is further elaborated definitionally using NSI, ultimately obtaining conjuncts R1 and R2 of S13. An excitatory interaction is an
interaction between $K S$ s that results in additional specific conditions for the application of the principal KS, e.g., NG12 interacts with NSIL to result in conjunct L3, a condition element thal excludes the normal scan processing when a delimiter is seen. An inhibitory interaction, on the other hand, is one that suppresses elements of $\mathrm{Ps}_{\mathrm{s}}$ an illustration is the interaction of Q10 and NF9 in the summary for F60 above, which suppresses rearranging scan pointers on the removal of "find" from an FV chunk.

The Q KSs interact according to the definitional type of interaction, above, and perform two other types of function: erasing unneeded Working Memory items and adding programming techniques. These three broad types of Qs are discussed further below, but at present we consider how they come to be applied. Erasing Qs are applied after other interactions have been completed, and the application is fairly direct from their statement. For instance, Q6 applies in the 513 example to delete the old value of the counter when a new value is computed. The prorraming-technique $\mathrm{Qs}_{\mathrm{s}}$ are more central to the process, as is illustrated by the episode which results in conjuncts R1 and R2 of S13. NS 11 speaks of performing some actions in sequence (paraphrasing freely), so that Q15 is directly applicable, along with $\mathrm{Z2}$, by virtue of stated application conditions. The justification of Q24 is not nearly so direct, involving aspects of the process which are more problematic. That is, it assumes knowledge of a non-local sort, namely that there are many transformations (NT's). It also is complicated by being cast in PS-like terms, so that perhaps it should be classed as a $Z$ not $Q$. These issues will be discussed further below, and need not detract from more general considerations of how Qs and Zs come into the interaction process, as intended by the use of the $\$ 13$ episode above. The $Z \mathrm{KSs}$ interact in ways similar to the programming-technique Qs .

The process of selecting principal KSs and carrying out interactions can be viewed as a variant of a goal-subgoal scheme, where a goal might be to form a $P$ from some $K S$, with subgoals generated during the interactions and stacked for later consideration (cf. a similar organization, "contingency planning", in Buchanan's (1974) automatic programming system). These subgoals arise when interactions are discovered which require KSs to be considered as principal KSs, which might not have otherwise been considered as such. Ps that result can be termed subsidiary Ps, One example of a subsidiary $P$ whose "principal" occurs elsewhere as a non-principal is S65, with principal NS9 (this is, in fact, one of the Ns listed previously as exemplary non-principals). Another class of subsidiary Ps responds to store-recompute decisions, whereby some aspect is computed by the subsidiary $P$ and stored as a data element to avoid repeating the computation. For example, S 60 is built around NS6, which is more assertive than plan-like and thus would not ordinarily be a principal KS. Certain kinds of programning techniques require coordination of more than one $P$. The primary example of this is looping, which requires a set of $P_{s}$ representing the body of the loop and another set representing its termination. In this case a goal-subgoal organization could be used to keep track of the disjoint pieces of program.

Analyzing the Ps from the standpoint of the KSs in them has resulted in the discovery of bugs, of the following five varieties: (1) omission of updates to data structures that turned out to be redundant (for example the group, taken together, F75, S15, V25 fi); (2) failure to delete properly (C2, F50); (3) RHS ordering not correct, with some assertions not important to order placed before ones whose order is important (F70, F75); (4) separation of $P_{s}$, where combination is possible (C75, C90); (5) awkward
combination of $P_{s}$, where separation would result in less complexity in $P$ conditions ( $S 17$ 535 could be re-organized). The first lype, although occurring only once, seems to be the most serious (it was discussed in Section C.3). The primary reason that the particular exarmple didn't resull in errors by the program is the redundancy of the Working Memory, that is, the Ps that processed the partially erroneous data did not check it for complete consistency. The redundancy is due to the overly cautious nature of the problem space plans, which dictated the structures to be built during initial scan (NSII), and the tack of the checks on the data structures is due to insufficient tendency of the Ns (in this case, NV6) to be associated with checks on data consistency.

The basic issue here seems to be that in analyzing how a program ought to be written within the present scherne, and in comparing that with the actuat program, the actual program falls shorl of expectations in ways that can not be tolerated in the output of some automatic programming procedure. That is, an automated procedure to produce programs in the present scheme would need to (and could be expected to) exercise more caution in such situations, producing programs as close to being correct as is possible. A further consideration is that the result of the bug's presence is that not everything is explicit. That is, effects of changes to the program would not have been noticed at locations where no checks occurred but things were by detault assumed in good shape. In general, this is a bad practice, since PSs are capable of the desired explicitness, and should exploit it. It is clear that the second, third and fourth types of bugs are similar to the first in these respects. The fifth class of bug is really a matter of programming techniques that might have been used to result in less complex conditions, and in general, fewer $P_{s}$, since separating conditions into distinct sets of $P_{s}$ makes the possibilities additive insitead of multiplicative. In some places in the program this principle was applied, but the applicalion was not uniform. The knowledge encoding process is expected to involve some search in investigating interactions of $K S s$, in order to decide between allernative expressions of program segments.

## C.5. Further characterizations of the knowledge statements

We now return to the lopic of the partition of KSs into the $N s, Q s$, and $Z s$, which was introduced at the beginning of this section. The coherence of the N class has already been demonstrated by presenting a model for the $N s$ and by associating that model with a problem space formulation of Studnt's problem solving. The Qs and $Z s$ do not appear to be coherent enough to construct a model at this time; the structure of the $Q$ and $Z$ knowledge will only emerge after a fuller set of such statements has been determined. This subsection will group the Qs and $Z \mathrm{~s}$ into some broad categories, and then discuss the $N \cdot Q \cdot Z$ partition with regard to substitutibility of other such sets of KSs for the present ones, modularity of knowledge, and augmentation of Studnt and how it affects the various classes of KSs.

The Q KSs can be divided into 3 broad types: definitional [ $\mathrm{Q} 4,5,8,25,53$ ], erasing $[Q 3,6,7,12,13,17,18,(23), 31,33,34,37,(38), 39,41,46,(51)]$ and progranming techniques [all the resi]. Some of them have secondary meanings which belong in a class other than the primary one, and this is indicated in the preceding and following lists by enclosing in parentheses. We have seen above that some of the Ns are also of a definitional type, so that we must distinguish between the two as follows. Definitions that
are problem space dependent, e.g. "the end of a chunk is the delimiter, period", are classified as Ns. Qs are intended to be just the opposite, since they define entities that can be encountered in many task environments, such as strings and scanning.

The Qs can also be characterized by primary topic, as follows:
a. Sequencing, applying functions, communication between processes, use of signals $[5,15,16,18,19,24,28,37,(38),(39),(41), 42,(49)$, 50, (51)].
b. Stanning $[3,4,14,22,(26),\langle 31\rangle, 44,48]$.
c. Transformations on strings $[7,9,10,(26), 31,42,(52)]$.
d. Numeric: counting, ordering, and finding maxima $[6,26,27,29,30,32$, 40].
e. Clean-up operations, attribute erasure $[13,33,34,38,39,41,51]$.
f. Strings $[8,11,12,(22), 43,52]$.
g. looping $[(21), 36,45,46,49]$.
h. Structures: tree, linear, splitting linear ones, separators, renaming [20, 21, 23, 25, 53].
i. Initialization [1, 2, (27), 47].
j. Use of a dummy as a place-holder [35].

Topic a. is the topic which may appear to have the most dependence on PSs, so that something more is required to justify any claims for generality. That topic's Qs are stated in terms of processes with two kinds of associated signals, initiation and completion, with the former emitted by the evoking process, and the latter by the evoked one. Signals are taken to be entities that can be processed, cancelled, and conditionally emitted. The crucial assumption is that signals can be emitted to be processed in a particular order, that is, that many can be emitled simullaneously, with processing of those in some specified order. This last assumption is the attribute that is most difficult to justify as appropriate to a non-production-system context. Further study will reveal if this is a major difficulty or not. Three of the KSs in particular are offensive in regard to possible scope limitations: Q15, Q24, and Q40. The first two use the signal order attribute just mentioned. Q40 specifically mentions "multiple firings", which is recognizable as referring to firings of $P s$. But the statement is referring to a more general concept, that of synchronizing the results of asyuchronous processes, so that the choice of words may be questionable, but the concept maintains the desirable degree of generality. One further point is that the erasure component of $Q s$ is not at all necessary (at least, visibly) in languages which automatically discard local memory contexts, or which don't require explicit data signals for control primitives.

The Zs can be grouped into five topics:
a. Order in RHSs of $\operatorname{Ps}[1,2,3,11]$.
b. Re-assertion of instances, use of : $\operatorname{SMPX}[4,7,8]$.
c. Peculiarities of the match, especially its being keyed to new data [5, 6].
d. Contradictory actions possible [9].
e. Specific control of looping [10].

The following model of Psinst, although not fully general, suffices to explain the content of the $7 s$. Psinkst is a PS interpreter in which $P_{s}$ detect conditions in an associative unstructured Working Memory. As a result of detecting conditions, specific actions are performed, consisting of additions to and deletions from the Working Memory. The Working Memory at any moment is partitioned into new cata and old data, where new data are elements that have not been processed relative to specific Ps to which the elements may have relevance, i.e., Ps whose conditions may become true as a result of the elements. For a condition to be considered true, at least one element of it musi match a new data element. The order in which new data elements are processed with respect to relevant $P_{s}$ is determined by a stack, :SMPX, and the order of eiements in the action sides (RHSs,) of Ps determines order of placement in the stack. Elements which may have become old becone new again by repeating their addition to the Working Mernory (referred to as re-assertion). Each data element's first element is its predicate, and clements of the Working Memory are grouped by predicate. Predicates can be declared to be nonlluents, in which case data elements with those predicates never have the new sitatus, i.e., no :SMPX entry is made for processing conditions relevant to nonfluents. Predicales are fluents, if they arte not nonfluents.

Of the sel of Zs three are related to the issue of whether there is some non-local knowiedge in the $P_{s: ~}: 75,76$, and 78 . That is, these seem to rec;uire that one $P$ knows what actions some olhers are performing, and perhaps how they're sequenced. This in fact is not the case, with one exception which can be avoided. $Z 8$ is similar to Q11, in that it requires knowtedge of other KSs, and need not be dependent on actual Ps. 25 and 26 are alike in that they can be handled in a very local manner, although one use of 26 actually has a more global scope. Thal is, when a $P$ wants to exclude firing again on data, part of which it has already processed, it can emit a signal specific to itself which indicates this, or if can include in its condition some part of its action which can be used for such an indicator. The use of 76 ( $P$ V 5 ) that violates localness (and which can be fixed in the former way) assumes that one signal it emits ultimalely results in the change which is used in its condition to exclude spuribus action later on.

Three aspects of the way the KSs have been partitioned indicate a wider applicability for the model and molivate the particular boundaries chosen. First, the division into $N_{s i}, Q s$, and $Z_{s}$ is intended to be such that other analogous sets of KSs could be substifuted with no interaction with statements in the other sets. For instance, we might want to use the Qsi and Zs in conjunction with knowledge about solving logic puzzies, or we might want to program STULDENT in a different language. It turns out that this ideal is altained strongly in only one direction. For instance, changing to a different problem space woutd nol affect the statements in the $Q$ and $Z$ sets, although the sets would probably need to be expanded with additional elements to meet different demands on technique. A change in the underlying programming language would not necessarily affect the Qs and $N s$, although it is often the case that such changes come about in order to adapt fully to the available language facilities. In the case at hand we have two instances of this kind of language dependence. In the comparison above between STUDENT and Studint, we saw how the change in language affected some of the plans in the problem space. We have also seen above how PS concepts may have weakly influenced how the Qsi are stated. The clean substifutibility of sets of statements at the $N$ level is really the most important and desirable form of substitutibility, since in a larger knowledge acquisition context, the other forms of change would never occur.

The second aspect of the $\mathrm{N} \cdot \mathrm{Q} \cdot \mathrm{Z}$ division is the issue of modularity of knowledge. A body of knowledge is modular if it has internal coherence or rich internal inter. connectedness while relations to external knowledge are significantly fewer. Modularity is useful because it allows a body of diverse knowledge to be decomposed into units (modules) larger than primitive elements, making it more manageable and allowing structure to be made evident more easily, Individual KSs are hardly modular: they interact to a large extent with other KSs. But they do have a certain orderliness with respect to the containing knowledge structure as represented by models. So instead of individual KS modularity, we have model-mevel modularity, of two types. Within a model, there may be a partition that allows some relatively independent part to be taken as a unit and perhaps replaced as a unit. An example of this might be a major change to the way similarities of variables are determined (model statement I., Section C.1). The model as a whole might be taken as a unit and replaced. For instance, a shift to a different problem space might occur. The considerations raised above in connection with substitutibility apply to this case. This approach to modularity is speculative, and it depends on the exact form taken by models when the knowledge encoding and extracting processes become actual programs.

The third aspect of the way the KSs have been partitioned deals with augmentation of the sel of Ns, rather than the larger operation of completely replacing it. One clearcut case of augmentation already exists in Studnt, namely the age-problem heuristics (A Ps ). There are 19 Ns (all of the $N A^{\prime}$ 's plus $N S^{2}$ and NiD6) that are age-problen-specific, 11 such Qs $(Q 26,31,42-44,47-52)$, and one $Z(Z 8)$. That is, those $K S s$ were added to extend Studnt to the new set of tests (Test6, 9 and 10). The A Ps themselves use three Ns, 13 Qs and six Zs that are used elsewhere in Studnt, which indicates small $N$ overiap but large $Q$ and $Z$ overlap. When we consider the age problens solved, we see that the $A P_{s}$ were only alocut 87 . of the total number of $P$ firings, indicating a large overlap in processing with other problems. The conclusion from this is that augmenting the given framework to include a new class of problems can easily be seen as extending the knowledge sets involved, with a majority of new KSs in the N class. As long as the augmentation doesn't require major new kinds of processing (as sketched above, Section C.5), it can rely to a large degree on existing mechanisms. In fact, the original STUDENT design (and consequently Studnt's design) is such that the age problem augmentation was relatively easiy to do, but this doesn't detract from the present conclusions, because the class of augmentations of the same type is large. Augmentations of a more difficult type (as defined in Section C.7) might have less $Q$ and $Z$ overlap.

## C.6. The knowledge extraction process

So far, our discussion has been oriented towards viewing Studnt as the result of a knowledge encoding process, but as stated in the introduction to this section, the knowledge was extracted from Studnt by an analysis. The primary attribute of the knowledge analysis is the many-many mapping between KSs and Ps, and to justify this we need to re-examine the knowledge extraction process.

Since the reader already has some faniliarity with S13, we can use it as an example of how the form of KSs emerges from its content. We review what each conjunct contributes as follows:

L1: finished with initial scan at $x$, ready to move pointer.
L2: $x$ is to the left of $y$.
L3: $x$ is not a delimiter.
LA: current scan chunk is c .
L5: current tength of scanned chunk is 1 .
R1: starl transform check at $y$, the new scan pointer.
R2: finish transform check at $y$.
R3: $x$ is in chunk $c$.
R4: current length of scanned chunk is now $1+1$.
R5: negate L1.
R6: negate L5.
R7: remove old scan-check signal for $x$.
From this description, we can sketch how the knowledge contained in 513 can be read off directly from the surface structure of the P. NS1L is composed of three phrases, two of which derive from L1 $+\mathrm{L} 2+\mathrm{R} 1$, the third from $\mathrm{L} 4+\mathrm{R} 3$. The first cluster says essentially that the scan is updated, left-to-right, and then the transform check is started. The second says that $x$ becomes part of the current scan chunk. These elements fit together in such clusters by virlue of shared variables, $x$ and $y$ in the first case, $c$ in the second, and by virtue of predicates with similar meanings. In the formation of NS1.1, QA and Q8 have been abstracted as scparate definitions, since they are recognizable as potentially useful in many places. An exception to the scan process is given by L3, by virtue of its negative sign, so that it is known that some knowledge has interacted by specifying some incompatible action under the negated condition. From knowledge of the abstract model of the process, that negated condition is evidently an instance of the end of a chunk, so that NS 12 is hinted at, using the definitional KS NS7. A further refinement of LS is that its argument, $x$, carries some information, since without other considerations, $y$ would appear to be equally possible (of course, an arbitrary choice might have resulted in $x$, but we must look first for some other justification). How that information is elaborated should be clear from the analysis of 513 that was carried out in detail above. Interestingly, the argument $x$ of L3 provides a link to two actions, and the interaction with NS 12 resulls only in the use of $y$ in R1 which is linked to $x$ by L2. It appears again in R3, so that another interaction is evident, this time having to do with adding elements to chunks, KS NS10. Another feature that can be read off from the $P$ is the update of the length of the scanned chunk, with argument \& linking L5 and R4. This link is expressed by NI7. Finally, the last three RHS asserlions, R5-R7, are deletions, and lead to the formation of the appropriate QKS .

So, reading off what a $P$ does gets a set of propositions, which are then taken singly as KSs, or, if several are so interdependent that they cannot stand alone, they are grouped as one KS. Support thal some cluster is a meaningful grouping is gained from occurrences in many Ps, resulting in a certain econonly of expression as the analysis is extended. The question of why the many-many mapping is obtained thus reduces to why the size of the P is what it is. S 13 is the size it is because a certain number of things have to be done as the scan progresses, and they must be done before the process goes on. There is a good reason why it is less than elegant in operation if it is broken down into its component parts, with each a separate $P$. If each $P$ did the thing stated by a single KS, the various Ps would be obliged to check each other's output, and at times to
force retractions of certain actions. For instance, in 513 , without explicit interactions with NS 12, a signal would be emitted as if the scan were to continue, but that signal would be intercepted and delayed while the chunk spitting process were done. As things actually are, that condition is recognized before any signals are emitted, and behavior adjustment occurs: appropriately. Breaking up a $P$ into smaller ones would thus require extra KSs for the additional control. Cleally there is an optimum with respect to minimizing the number of $K S 5$. Of course, matching overhead and efficiency would be affected by this change in organization, but that is a secondary concern at the moment. On the other hand, making Ps contain more KSs does nol pay because one then has to multiply $\mathrm{Ps}_{\mathrm{S}}$ in order to get all of the logical combinations of conditions. For instance, if three Ps perform one stage of a test, and four others perform another stage of the test, combining Ps might require as many as twelve Pss (where seven had sufficed) to handie all possible paths throught the two lest stages.

Figure C.2 illusitrates the many-many mapping between $N s$ and $P s$, for the $S P_{s}$, restricted to MS's. (NIP's, NI's, Qs, and Zs are not shown; S20, S25, S30, and 540 use NP3, while $513,515,540,560$, and 965 use NI s).

Distributional data for the KSs over $\mathrm{Ps}_{s}$ supports the size that was chosen as a unit $K S$. This data is derived mostly from Appendix F, which gives the Ps that use each KS, and which has at its end a table that gives distribution frequencies for $P_{s}$ having specific numbers of Ns, Qs, and $2 s$. The rest of the data comes from an inversion (not included) of that appendix, which gives the $K S s$ associated with each $P$.

For Nu, nearly a majority ( 59 out of 154) are used in only one P, somewhat fewer are used in two (33), and fewer still in three or four (14 and 3, respectively). Ns that are used in more than four $P_{s}$ are less numerous, with frequencies at or near zero. There are extremes, however: NIJO is used in 70 Ps (the maximum), and some others that are heavily used are NII, NSI, NS3, NDIS, and NIS. For Qs and $Z s$ the distribution in frequencies is about the same (10) for uses in each category for 1 to 3 Ps , down to around 3 for 4 to 9 uses, and then at or near 0 , with the maximum number of uses 105 for Q 8 (other heavily used KSs: 72, Q12, 71, Q7, and Q18). Thus the distribution of Q and $Z$ uses is somewhat flatter and more spread out than for the Ns:, which is in accord with their being more generally applicable than the Ns:. The high frequencies for low numbers of uses supports a unilary properly for KSs, as opposed to compositeness. The many-many mapping of KSs to $P_{s}$ is supported as follows. There are about 55 Ps for each frequency dass for 1 to 4 $K$ Ss in each of the $N$ and $Q$ chasses (accounting for a total of about 220 Ps ). This means, for instance, that about $55 \mathrm{P}_{\mathrm{s}}$ have 2 Ns and about 55 Ps have 2 Qs , though not necessarily the same 55 Ps. There are 3 Ps with only one KS (Mn0, V10, and A77), and 20 Ps with only 2. There are about 10 Ps for each frequency class for 5 to 8 KSs in each of the $N$ and $Q$ classes, and the other $K S$ frequencies are near $O$ ( $S 40$ has the maximum of 19 , with close runner-ups: C60, F75, F15, M55, M50, M $30, \mathrm{M} 20$, and F35).

With respect to principal $K S s$, a majority of $K S s$ that are principal are principal for only one $P$. But only about 100 Ns are principals, so that some serve as principal for more than one P. One way this is possible is illustrated by NS11: it is principal for S10, 513, and 515 , each of which elaborates a case of its use under different conditions. NDI (and other ND's) are composite, defining a set of words to be members of the same word class at once rather than (unconcisely) making a separate statement for each membership


Key: Direct uses are solid lines, weaker interactions, broken ones.
Figure C. 2 The mapping of NS's to S Ps
assertion. NF 1 is not strictly a conpound statement, but F5, F15, and F20 each use a subpart of it as their principal component.

We now summarize the ways in which the various kinds of KSs can be extracted from Ps, based on the experience with the full Studnt analysis. As in the above example, the Nss are determined: by combining the meanings of predicates; by comparing the LHS and the RHS, using common variables; by the occurrence of NOT in the LHS, indicating an excitatory interaction. Determining the exact content, however, of Ns and Qs does require some kind of collection of several cases of use, so that an appropriate generalization can be made, for economy of expression. Also it must be deternined in a non-immediate way just which terms are to be handled by definitional sorts of KSs, and whether those definitions are Ns or Qs. But these considerations really only apply when the reading is starled from scratch, and once the basic terminology for PS is established, the
determination process is much easier. To determine the Qs of the definitional and erasure types is quite straightforward: erasure knowledge is based on occurrences of negated lemplates in the RHS, and definitional knowledge can be assumed whenever there is some gap between terms in $N_{s}$ and predicates. To determine programming techniques, the following clues are used: presence of signals; ordering of signals in the RHS; presence of data lhat is elsewhere used in a parlicular way (Q28, Q42); particular type of predicate (e.g., Qf6); re-assertion (Q42). For the Zs, we have the following: order of the RHS; reassertion; seemingly silrange condition elements, for instance P-specific ones. Wilh respect to the use of RHS order in determining $Q s$ and $Z s$, something more must be known than local considerations, since Psingt does not have an explicit notation for which of the RHS elements really do have an important order relative to each other. This "something more" is simply closeness to the principal KS of the $P$, or closeness to the problem space plans that are directing the processing. In general, only the first few elements, or in most cases just the first one, have an ordering constraint, with the rest being don't-care's.

## C.7. A model of knowledre acguisition

The process of knowledge encoding lits into a model of knowledge acquisition along the following lines. An arlificial intelligence is seen as an entity with capability for gathering picces of information, which are used in formulating behavior patterns organized as problem spaces. A piece of information by itself is insufficient to produce appropriate behavior. Ralher, it must be assimilated or understood by having it fit into models that have been previously acquired or that are built up by a problem-solving process. This process of understanding consists of first expressing the new information in terms that overlap with some problem.space. level model and then allowing the information to interact as illustrated above to form new Prules. This broad model goes along with the view that intelligence is increased by increasing the ability to select a particular behavior out of all the possibilities in a given situation. In the PS model, selectivity is increased by adding rules and by correspondingly increasing the complexity of $P$ conditions. This growth in selectivity can easily be seen as growth in a discrimination net (see Rychener, 1976, or Hayes-Roth and Mostow, 1975) in which each condition element is taken as a node in the network. A match to a $P$ condition then corresponds to finding a path in the network to a terminal node, at which are stored the elements corresponding to the action side of a $P$.

Figure C. 3 illustrates the components of the model. Each box in Figure C. 3 represents some body of knowledge, either as an abstract model or as a specific set of detailed facts. Boxes in solid lines have already been discussed, along with the processing indicated by the arrows that results in the Ps. Boxes in broken lines are parts of the process that are hypothesized, but are insufficiently elaborated at present to permit further specification. The figure shows static data dependence; i.e., it indicates that knowledge in one box is used in forming the knowledge in the other. It doesn't indicate anything, for insiance, about how a knowledge encoding process would access the various bodirss of knowledge dynamically, nor does it include the knowledge extraction process. Except where arrows merge, interaction of knowledge (as illustrated in Section C.2) occurs within the boxes, e.E., Ns; with other Ns:. The arrows show, rather, how a body of knowledge forms by development or elaboration from other knowledge (e.g., box 4 to box 5), or how such developments merge in a largely additive way to form a body of knowledge ( 6,7 , and 8 into 9 ).


Figure C. 3 The model of knowledge acquisition: Bodies of knowledge

Some of the broken-line boxes are not expected to present much difficulty, namely 12, 15 and 16. The others represent more difficult problems than what has been solved so far. Boxes 1-3 are where much of the real high-level problem-solving takes place, namely in the precise formulation of the task environment and in the construction of the problem space within which dealing with that environment is possible. It is during that
process of formulation and construction that the intelligence is added which results in part in the "plan" portion of box A, that portion which directs the application of operators in the problem space. The specification of box 13 requires a process of concept-formation, which results in the set of predicates and their meanings which were taken as given in the above analysis. The creation of the elements in box 6 is possibly more complex than is indicated. It is conceivable that programming techniques are not simply a collection of facts, but rather are a capabilify in the form of more general knowledge and procedures which on demand can generate the particular instances of programming know-how which are the Qs in the above analysis.

With respect to Figure $C .3$ it only remains to point out some examples for ew of its parls. The connection between boxes 12 and 6 is unused in the formation of most of the Qs, and we have discussed above for Q15, Q24 and Q40 some of the problematic aspects of this connection, and how they might be resolved. The connection between 12 and 17 reflects the fact that a few of the predicates are oriented towards the structures used in the Psilist PS. One example is the HASCPRIOR predicate, which assigns to each chunk in a Studnt problem a priority. If a stack data structure were available, these numerical values could be done away with, since the result is a stack-like ordering of the chunk processing. Another example is the set of predicates which are used to keep track of the tree structure of the arithmetic expressions. In a Lisp environment, for instance, the recursive nature of function calling would encode the same concepts. Finally, it should be pointed out that boxes 10 and 12 may have enough in common to be merged into a single body of knowledge, although with the present limited objectives their distinctness can be maintained.

The major component of the lask environment (box 1) is the method to be used. Studnt's method is a variant of the Dintch method (Newell, 1969), where the "form" against which inputs are matched is expressed as a grammar, a set of rules capable of generating all possible forms to be matched. The grammar itself is not implemented as a generator of forms (top-down) but rather as a recognizer, a bottom-up precedence-based parser. The transformations that Studnt applies to bring the input to a recognizable form correspond to normalizations that are somelimes done by template matching procedures, to get inputs into suitable form for a given set of templates. Even if we take the method as given, there is still a significant amount of problem-solving to arrive at Studnt's problem space as described by the absitract model in Section C.1. Studnt divides a task into two parts: processing the input to arrive at a form suitable for the matcher and the matching (parsing) itself. To get the first part, a problem-solver must form such ideas as: transformations on strings; classes of words; marking word classes with tags; organizing the process as a left-to-right scan; organizing the input string as a series of chunks with delimiters and operators as boundaries; and so on. The match has two distinct components, the parsing process and the variable-identification process. The parsing uses: the concept of chunks; the system of operator precedences, which must be extracted from ordering relations noted in the task environment somehow; properties of FV-specific words; and sio on. Studnt's variable-identification process, which is applied after a structure has been parsed, is not itself a parser but consists of a rather weak collection of equivalence rules, but even this rudimentary process uses: a left-right scan of variables to be identified; rudimentary pronoun reierent substitution; and specific equivalence rules.

The phrases above referring to left-to-right scan bring out once again (cf. Section
C.1) another feature of the requirements of problem space formation: the addition of plans. Plans take the place of exploratory (backtracking) search, so that their appearance in a solver's problem space is of importance. It remains a significant problematic aspect to determine how they're added.

To summarize, the problem-solving involved in forming the problem space is of an ill-structured nature, requiring concept-formation and plan-formation processes that are poorly understood at present (but see the discussion below in Section C. 9 of the work of Hayes and Simon, 1973). Because the problems in studying the problem space formation process in more precise terms appear formidable, we should look for supporting evidence, and in particular we can question two aspects of the Studnt problem space: is it the correct problem space and can it be arrived at by other means. Concerning the correctness, there are three viewpoints: the human problem-solving viewpoint, the AI program viewpoint, and the implementation viewpoint.

The first view deals with whether there is support for the model from human problem-solving studies. Paige and Simon (1966) considered exactly this question, and their conclusion was that humans' basic problen space is like STUDENT. They went on to consider informally a set of augmentations of the basic problem space, suggesting that STULIENT could accommodate at least some of those augmentations. The Paige and Simon paper did not consider the protocols relating to the basic problem space in sufficient detail to support or contradict the finer details of the STUDENT model, such as its system of operator precedences, but it is safe to assume that no gross differences were evident.

The AI program viewpoint considers the question of whether Studnt (or STUDENT) can be extended comforlably to the real task, namely problems chosen without care to simplifying the language. My informal examination of a set of 33 problems from a collegelevel algebra text (Rosenbach et al, 1958) can be summarized as follows: none of them are directly solvable, five could be solved by easy extensions, 14 by harder extensions, and 14 by extensions of major difficulty. By easy extension 1 mean addition of simple idiomatic transformations. By harder extension 1 mean adding specialized knowledge to solve problems in particular domains of discourse, such as problems dealing with coins, interest, and mixtures (chenical solutions and alloys), and adding more context dependence to certain idiomatic transformations and pronoun referent replacements. By extensions of maior difficulty, I refer to: problems requiring elaborate semantic models to create the set of equations, that is, where sonte inference is required to derive necessary relations from given information (e.g., certain complex rate-distance problems, for which a diagram is an essential part of a human's solution); problens requiring elementary knowledge of points, lines, and curves; problems calling for symbolic solution as opposed to numeric; problems requiring solving a previous problem with different numeric values; and problems requiring operations on relations, such as reversing the role played by two variables. This last class of extensions also has the property that a problem solver that is an extension of Studnt would spend more of its computing effort in the extension than in the basic Studnt mechanisms. This is not the case, I believe, for the first two classes of extensions. This assertion can be supported by results obtained with respect to examining the age-problem heuristics as an extension of Studnt, within the present Studnt, which is discussed in more detail below in considering the extension as an addition of KSs (Section C.8). The ageproblem extension is of the harder extension category. From this breakdown of how Studnt might be extended, we can take some support for the present problem space formulation.

The implementation viewpoint concerns itself with the problem of implementing the given version of Studnt, which has been solved in (at least) two cases. If the model of formulating the problem space, given the task environment, and then encoding that problem space as a program, is approximalely correct, then the problem solving involved is of a particularly high order, especially in comparison to the state of the art in AI. But since it is likely that the conceplual structures we find in the finished programs correspond to the problem space organization that aided in their implementation, we have: still further support for the correctness of the present formulation.

This lasit topic ties in with the second aspect of the problem space formulation that we might question, namely whether the given problem space can be arrived at by some other means. In particular, can it be arrived at by a simple specialization process on previously-tearned natural language processing? Has simplifying the input domain and building up a problem-solving process from scratch added unnecessary complexity? Given the lack of evidence on this, in parlicular with respect to more capable Al programs, we can only offer a few speculations, remaining within a human problen-solving viewpoint. Perhaps humans, in solving this class of problem, do not rely on plans as much as on weaker searcli.like methods. Thus the plan-formation aspect of the problem space formation process may nol need to be explained. It is necessary, in addition, to consider the role of teaching and imitation as aids in the process (and perhaps teachers and authors of lexts could benetit from the Al formulation). But certainly the concept-formation process is only pushed temporarily out of sight by saying that the problem space used is a specialization of some familiar capabilities. That is, the concept formation took place somewhere during the arising of these capabilities, although its occurrence over a longer period of time may make it, ultimately, more easily explained.

## C.8. Conclusions on the knowledre analysis

The knowledge analysis has shed light on the essential aspects of how knowledge is encoded in PSS, and thus takes a definite position on how PS programs are wrilten, augmented and refined. A PS program starts out as (parlial) encoding of knowledge stated in terms of some problem space. Ordinarily, the program is then tested, and defects come to lieht as a result of interactions that were not considered in the original encoding. The new interactions may be dealt with by forming new KSs which are then considered as additions, or they may correct oversights in processing that produced the original. For knowledge to be added, it must first be stated in terms that make contact with the problem space in which the program is formulated (or with an abstract model at the problem space level). Then there must be consideration of the ways the new piece of knowledge can interact with the given ones. In determining those interactions, the explicitness of expression, allowing knowledge content to be easily read as explained above, is instrumental. Replacement or modification of knowledge requires a similar consideration of interactions. It is important to emphasize that in this formulation, program behavior can not be augmented by simply adding $P s$, as is the case in some rule-oriented systems, because Ps here are encodings of more than one KS. This is the case because of the conceplual structuring, provided by the problem space (model). The circumstances allowing simple addition of rules are those where the plans in the problem space are lacking, so that some method of heuristic search among possible behavior sequences is undertaken. This allows the addition of knowledge in its pure form because at the higher level there is very weak structure, and no basis for determining any interactions.

The analysis has demonstrated the directness of encoding of problem space knowledge, by virlue of the ratio of Ns to the other types of KSs. With $154 \mathrm{Ns}, 53 \mathrm{Qs}$, and 11 Zs , it is apparent that the Nss predominate, and that the control knowledge specific to PSs is quite minimal. These figures do not include the very low-level PS syntactical knowledge, for two reasons. That knowledge is fairly constant over the entire set of $\mathrm{Ps}_{\mathrm{s}}$, and it is sufficiently simple that it quickly becomes automatic for the programmer, requiring little attention during the programming process. Not only is the encoding of knowledge direct, with little knowledge required to bridge the gap between a high-level problem space description and the actual language, but it is also the case that the size of each programming unit is small in terms of number of KSs : on the average, each P contains 2.88 $N s, 2.86$ Qss, and 0.65 Zs . It is asseried here that the above analysis indicates that litlle other knowledge needs to be considered beyond these 6.39 KSs . The explanation for this is that the structure of the problem space has satisfactorily co-ordinated its component KSs.

Because, with this framework of encoding, it has been possible to consider knowledge at rather general tevels, it is appropriate to view it as the beginning of a comprehensive model of knowledge acquisition. It takes an explicit position on what knowlestge is (at the natural language level, but not at a more formal level), it proposes mechanisms for its incorporation into some existing body of knowledge, and it exhibits the result of assimilation of knowledge, namely the Ps. It is interesting to point out that other experiments have indicated how $P$ conditions can be stored as an EPAM-like (Feigenlsaum, 1963) discrimination network (see Hayes-Roth and Mostow, 1975, Watermen, 1975, and Rychener, 1976). The prosent formulation also indicates how processes of problemsolving and concept-formation enter into knowledge acquisition. It takes a clear position on the difference between knowing and understanding some piece of information, namely that knowledge is not understood fully until its interactions with other knowledge have been considered according to the knowledge interaction process hypothesized here.

Asi a model of knowledge acquisition, this approach may contribute to the automation of learning or of incremental addition of knowledge to a PS program. Going further, it may suggest a different mode altogether of expressing PS programs, namely natural language (or at least some language that expresses knowledge in a way similar to the KSs, orthogonal to the $\mathrm{Ps}_{\mathrm{s}}$ ), and in a more limited implementation, would constitute a powerful "programmer's helper". Along these lines, it can be noted that the division into Ns, $\mathrm{Qs}_{\mathrm{s}}$, and Zs would perhaps remove the burden of specifying programing techniques from the programmer. Also, variations in programs would result from variations in the set of predicates used by the program in constructing programs. That is, the predicates form a conceptual base for the programming system to work with, which might best be determined interactively.

The three subsections that follow contain some fentative conclusions from this work, and attempl to s.tructure its extension, its development, and its application to other areas, First, we compare this approach to related work and point out how this approach might be used to restructure those results. Then we consider Studnt as an understanding system and propose some ways that a knowledge encoding analysis can be used to measure varidus dimensions of understanding. Finally, further research that is essential to supporting this analysis will be discussed.

## C.9. Comparisons with olher approaches

It is difficult at this time to compare our results with other approaches to encoding knowledge, because no other sludies have taken a sufficiently similar approach. However, we can point out features of interest as viewed from this approach, and indicate further studies that might be undertaken to this end. The reader is cautioned that some topics are raised in a very cursory fashion, with the intention that these may deserve further consideration based on this initial exploratory examination. This subsection is primarily intended to sketch how this work seems to relate to other approaches.

A very interesting comparison can be made to another PS organization, Newell's (1973) PSG. This comparison is based on thorough knowledge of that system, but not on a detailed jmplementation of some program in PSG. The commonality of PSs indicates that we should only have to look at the corresponding Zs. PSG is a PS interpreter in which Ps detect conditions in a linearly ordered Working Memory (STM). As a result of detecting conditions, specific actions are performed, consisting of adding, deleting, modifying and reordering the elements of STM. When more than one $P$ condition is true at the time of recognition, that $P$ is allowed to fire which uses STM elements closest to the front of STM. The detailed comparison is as follows (cf. the $Z$ model given above, Section C.5):
a. Order in RHS and order of examination of Ps: very similar to Psnlst, except order in the RHS is reversed; in PSG, the last (rightmost) RHS insertion is at the front of its STM.
b. Re? assertion in Psinst corresponds to data rehearsal (the NTC action) which brings elements to the front of STM.
c. Matching and the problem of spurious $P$ firings: it is possible to put elements in front of other elements, so that the others don't take part in matching, but PSG has no new-old distinction on STM elements; thus some (ad hoc) unknown memory structuring must be used to prevent spurious firings (e.g., renaming data elements, which retains the information but changes the set of sensitive conditions).
d. Problem of contradictory actions: either non-existent because of the order of actions, with deletions getting done before insertions generally, or it must be handled in the same way as in Psnlst.
e. The control of looping is the same for both systems.

This comparison of PSG and Psnlst does not deal with all of their differences, because it is limited to the control mechanisms only, and because the control mechanisms that have to be considered are limited by the domain determined by Studnt. Our conclusion is that PS control issues are essentially the same in both systems, increasing our confidence that our assertions about $P S$ have some general validity.

With respect to more conventional languages, a couple of points can be made as motivation for more detailed studies. The step size of PSs compares quite favorably to a simall recursive l.ISP function. That is, a $P$ and a recursive lambda expression have simitar size, expressive power and isolation in terms of knowledge content. LISP, however, generally suffers from the "subroutine interaction problem", since knowledge interactions are not carrised through to the extent allowed by PSs. The size of programming unit is much smaller than an Al.GOL block structure, where the assumptions at some point in a
program depend on a lexically very large extent, with each inner nested block inheriting knowledge assumptions from its outer containing blocks. If we were to attach assertions at various points in an Algol program corresponding to KSs that are assumed at those points, then places that are nested in several block levels would have all the relevant local assumptions plus those of all the containing levels. For a $P$, the KSs that hold are determined locally. Thus a PS program has knowledge distributed more or less uniformly over its parts where an Algol program shows wide variations in density of knowledge. Proving correctness of a conventional program is done by attaching assertions to a flow charl and then following the flowchart sequentially, verifying assertions at a point in the context of accumulated asserions from the flowchart traversal, whereas for a PS, verification can be (it is claimed, to be supported by further research) much more localized, with no need to deal with control flow. The knowledge encoding approach poses the question of proving correctness of programs as the process of determining the following features: the knowledge content; whether the knowledge is correctly encoded, i.e., whether all relevant interactions have been exploredi and whether the knowledge is correct with respect to the given task environment.

If we are to compare PSs to Panner-like languages (see Bobrow and Raphael, 1973) it is essential to point out that at the $Z$ level, these languages have a pattern-goaloriented implicit search, which may have targe ramifications on how the other knowledge levels are formulated. A more general question to be answered is how the encoding of knowledge as Planner theorems is diflerent from encoding it as Ps. An attempt at making a system flexible in terms of augmentation was done by Winograd (1972), and the result (unpublished) was that to add certain kinds of knowledge, some other knowledge of the internal workings of the program was necessary. In other words, more than just an N.like statement, with pure problem space content, was necessary. Charniak's (1972) systematization of a body of knowledge relating to children's stories would have to be reformulated from a problem space viewpoint, in order to make comparisons. This is made more difficult because there is a lack of explicit statement as to exactly what that body of knowledge consisted of. A good deal of discussion by Charniak was based on the body of knowledge without getting down to a strict separation of the knowledge from various interesting issues related to it.

A recent study by Hayes and Simon (1973) investigates the process of extracting problem-space-related knowledge from the instructions for a problem-solving experiment. This involves studying protocols of human behavior, and attempting to model the processes as a computer program. The program assumes a particular form for the problem space: the GPS (see Newell and Simon, 1963) form of heuristic search with means-ends analysis. The program thus addresses the area dealing with boxes 1 and 2 in Figure C.3; its output is a set of task environment "statements" that have a form suitable for input to a GPS-like problem solver. Although the work covers only a small portion of knowledge acquisition as outlined albove and makes strong assumptions about the desired form of the problem space, it serves as a useful base point for further work along the lines of the acquisition model and especially for the problem space formation process.

Finally, we compare the present approach to Sussman's (1973) model of skill acquisition. The model (Hacker) deals with the knowledge used in constructing problem.solving procedures in a toy blocks world. There are several categories of Hacker FACTs (its version of KSs): one deals with details of the toy blocks world, giving attributes of
pre-defined blocks operators, for instance preconditions for the PUSH operator; a second is programming techniques, which deal with the particular model of problem-solving being used; two others deal with Harker's "critics' gallery," a body of knowledge about bugs in procedures; the fifth is a program library, wilh procedures that have been used for previnus problems; and the sixth, a "notebook" with comments on programs in the library. Fared with a problem, Hacker uses the appropriate bodies of knowledge to build a first approximation to a procedure to solve the problem. By executing that procedue in a "careful" mode, bugs in the procedure are uncovered, the critics' gallery proposes a solution, and another atlempt is made to execute the procedure. An example of how this works treats the problem of writing a procedure to build a tower of blocks. An initial procedure might do fine until it tries to pick up a block with another block on top of it. At - that point, the critics' gallery might propose inserting a line of code before the pick-up operation, to ensure that no block is on top of the block to be picked up.

Sussman did not gather together his FACTs and comment on them as a group, but by my count, there are 12 blocks-world $\operatorname{FACT}$ s, 16 programming techniques, and 8 critics gallery FACTs. FACTs relevant io a particular procedure are not all combined at once as envisioned in the present knowledge encoding process, but it is necessary to put together a first approximation to a procedure and then execute it to see what happens. Thus, it is not the case that knowledge can be extracted from Hacker's procedures by an analog of our knowledge extraction process. To find out in detail the properties of a library procedure that was constructed previously, in order to modify or generalize it for a new problem, it has to be executed and its behavior monitored. Also, if the result of careful execution is new knowledge, that knowledge is not incorporated into the procedures for generating programs, so that it would be used appropriately for future problems, but rather it becomes a new entry in the crifics' gallery, and can only be used to patch up bugs in carefully-executed procedures. In principle, there seems to be nothing to prevent the critics' pallery from growing to very ummanageable and inefficient proportions, especially with the possibility of critics' being formed to correct other critics' actions.

Sussman's Hacker approach takes a definite and more or less traditional stand on the issue of modularity of knowledge, whereas the proposal here represents a different approach. Hacker's KSs are kept in pure form as FACTs and grouped conceptually into modules that are daimed to be substitutible or interchangeable for modules dealing with other problem domains. The Studnt approach (ideally, given that the present model can be implemented) is that individual $K S s$ are not kept in pure form but only in the encoded form. The encoded form, however, is sufficiently explicit that the statements can be recovered, at least enough to compute further interactions. Modularity is still maintained at the problem space level. Thus the PS trades explicitness of representation for individual statement modularity. Just how the PS approach as proposed here would be worked out in detail is still an open question, and will be discussed below in connection with problems for further research.

## C.10. Understanding and intelligence in Studnt

In order to approach issues related to understanding, intelligence, generality, and similar topics, we adopt the understanding dimensions approach of Moore and Newell (1973). Moore and Newell define understanding by saying that a system understands some
piece of knowledge if it uses it whenever appropriate. They propose eight dimensions along which understanding systems are to be evaluated: (1) representation of knowledge; (2) action, the conversion of knowledge into behavior; (3) the assimilation to the internal structure of external (task environment) structure; (4) the accommodation of the internal structure to external structure (which includes learning, incorporation, or acquisition of new knowledge structure); (5) directionality, the structure that initiates and guides processing toward specific ends by appropriate use of knowledge; (6) efficiency; (7) how the system responds to error; (8) depth of understanding, an indication of how effectively knowledge is brought to bear whenever appropriate. Studnt takes $P_{s}$ as its ultimate knowledge representation, and the interpretation of $P_{s}$ as the means of obtaining action. The rollowing paragraphs discuss (3) through (8) in turn.

Assimilation will be posed as a question of whether Studnt adequately encodes all of the KSs. That is, a KS is said to be assimilated when its encoding in Ps has been effected. For Studnt, this question is mapped into determining whether all of the interactions of the knowledge have been correctly considered. Evidence that the program can solve problems that require various subsets of its KSs is at best only indirect support that it understands the knowledge. We must postpone a definite determination of Studnt's degree of assimilation of its knowledge until more concrete progress is made in automating the knowledge encoding process, thereby making more definite the meaning of interaction. The best possible estimate at present is based on taking the number of uses of KSs in $\mathrm{Ps}_{\mathrm{s}}$ as the number of interactions (roughly 1650) and taking the number of "bugs" discovered in the process of the analysis (about 50), to get $97 \%$. Even though this figure is suspect because it reliss on the accuracy of my own judgment as to what is correct for the 50 bugs found (in general, a more knowledgeable encoder is necessary, to judge the result of an encoding process either directly or through behavioral tests), it illustrates a measure of assimilation based on the knowledge encoding approach.

The accommodation dimension raises questions with regard to how the Studnt structure can be augmented to expand its area of performance. As discussed above (Section C.5), on a sample of 33 problems not given to Studnt, 5 (157) would require easy extensions 0 to Studnt, 14 ( $42.5 \%$ ) require harder extensions, and 14 ( $42.5 \%$ ) require extensions that present major difficulties. The first two classes (57.57) could be reasonably said to be within the range of Studnt's ability, while the rest require such radically different approaches as to be beyond Studnt, in the sense that the "Studnt" nature of a program to solve them would be diminished relative to the total program. Thus Studnt might be said to be $57.5 \%$ accommodating. These figures are, of course, based on this author's judgment of problem difficulty. They are suspect also due to the sample chosen: it is indeed a significant problem to determine what set of problems to examine. Studnt can solve a class of problems of unlimited size, and there are classes outside its reach that are also unlimited. The cautious appraisal of the 57.57 figure would be that it illustrates a possible methodology for measuring acconmodation, dependent upon the knowledge-encoding approach (as it is used to evaluate the knowledge necessary to effect the accommodation), but that a great deal more research is necessary in order to support both the general approach and the specific measurement obtained.

[^2]In keeping with the statement at the beginning of Section C.7, I would identify directionality with intelligence. This accords with the view expressed by Newell and Simon (1972, chapler 3, pp. 88-89) that the inteligence of a problem-solver is related to the difficulty of a problem for that solver, as evidenced by its search behavior. That is, the more directed the search is, implying examination of fewer irrelevant alternatives, the more intelligent the solver. Studnt's intelligence cannot be measured by examining its search behavior, because the only sense in which it does search is that it constantly progresses toward completion by scanning, chunking, and building expressions. We can, however, examine qualitatively the knowledge that directs the constant progress, and comment on how it might be possible to formulate its limitations by studying the space of problem spaces. Studnt's intelligence is embodied in the plans it uses. These plans are inflexible, prescribing specific actions in specific orders. According to the model of knowledge acquisition presented above, this intelligence is acquired during the problem space formation process, and if the intelligence is limited, it is due to limitations in the problem space. As Newell and Simon point out, if the problem space were richer, allowing the direction of processing to be based on more appropriate discriminations (as required by the task environment), a problen solver (Studnt) would have greater potential intelligence. A more exact understanding of the space of problem spaces for solvers of Studint-like problems might allow Studnt's intelligence to be measured relative to other programs. Such a measure might be based on an analysis of knowledge in the form of plans contained in such problem solvers, especially if the body of knowledge forned by taking the union of all such sets of plan knowledge is a coherent whole.

On efficiency, the main point we can make is that since knowledge is encoded procedurally as $\mathrm{Ps}_{\mathrm{s}}$, with only the Icmporary state in Working Nemory, the interpretation sub-issue has little impacte The interpretation sub-issue is that if many levels of interpretation of knowledge are required, the factors of extra computing time required at each level multiply (cf. the difference in running a program compiled, interpretively, or on a simulated computer). In parlicular, while Studnt is solving a problem, it is not the case that it musi search to find the implications of some piece of knowledge or to decide how two items of information must interact. This apparent efficiency is at the cost, perhaps, of an expensive knowledge encoding procedure; this cost will only be known after further research.

In the general category of error, the knowledge analysis leads to the consideration of how to assign blame to particular KSs for some faulty behavior. This approach says that the error is not localized in particular $P_{s}$ but rather is due to faulty (incorrect) KSs or to failure to consider interactions between $K S$ s; thus an error may be due to the contents of a set of Ps . In diagnosing and correcting an error, it is clear that the processes of knowledge extraction and knowledge encoding are essential. We can speculate that not only will the contributing, $K S s$ have to be known, but that some relative reliability measure on KSs might be useful (reliability perhaps determined by successful use on past problems), in deciding on corrective action. For the present Studnt, there is a computation of the contribution of particular $K S s$ to the total behavior. The listing of the $K S s$, Appendix $F$, gives the $P s$ in which each $K S$ is used, and the actual TESTs in which each KS is applied by virtue of some $P$, which incorporates it, firing during the TEST. For instance,

[^3]it is clear that the almost all of the NS's (initial scan) are used for all the TESTs, whereas each of the NT's (transformations) is used in a small subset, where the subset varies according to which NT is examined. On the whole, the NT's, the ND's, some Nh's, some NC's, the NF's, the NA's, 14 of the Qs, and 78 are used only in subsets of the TESTs, white the other $\mathrm{Ns}_{\mathrm{s}}, 39$ of the Qs, and the Zs (except $\mathrm{Z8}$ ) are used in all of the TESTs.

To measure. Studnt's depth of understanding within the knowledge-encoding approach, it is necessary to consider whether all knowledge interactions are properly worked out. For instance, it might be possible to construct an example that uses knowledge in Studnt in such a way that Studnt fails to apply it appropriately. Such an example has not yet been found, but that doesn't rule out the possibility entirely. (This task is much more difficult than finding problems that use knowledge that Studnt doesn't have at ail, or finding problemis where Studnt's knowledge is inaccurate.) The kinds of interactions that are worked out are perhaps determined by the problem space, so to find a proof or counterexample, it may be necessary to have an exact and full understanding of how interactions are related to the problem space (more is said on this in the following subsection).

## C.11. Directions for further research

The analysis of the knowledge in Studnt has provided a framework for posing further research questions relating to four major areas: (1) verifying the analysis by automating the knowledge encoding process; (2) testing the extendibility of the model by adding knowledge that extends the domain of solvable problems; (3) testing the substitutibility of the model components by trying to apply the analysis to other programming languages; (4) testing the applicability of the overall model of knowledge acquisition by similar analyses of Al programs for other task areas. We have already presented some directions to go on question (4), in Section C.9. Topics (2) and (3) depend to a large extent on progress with respect to (1), either using PSs or some other programming language. The following paragraphs speculate on the central issues to be resolved in attacking question (1).

An immediate question relating to automating the analysis is the choice of language for the KSs. One approach is to analyze the KSs themselves for underlying semantic structure, in order to determine the kind of mechanical translation that needs to be done to express the knowledge in a directly assimilable form, or in order to design a more suitable formal notation. Natural language was sufficient for the purposes of the present first approximation at a model of knowledge, and its use obviated the need to do a design of a formal language at the same time as the analysis was being done. Certainly it is not necessary to have a language more powerful than natural language, but rather it may be necessary to use a language that places less burden on the processor in filling in implied relations and objects. Any use of an arlificial or formal language faces another problem: how to guarantee that the formal language has a systematic basis, or that it is possible to decide how to express some idea, for inslance with or without making ad hoc extensions to the language. Sussman (1973) and Charniak (1972) both expressed knowledge in formalisms directily usable by their (partially hypothetical) programs. But they in fact ignored the theory of construction of these formal assertions, and in many cases simplified and altered them for human readability. (These two are emphasized in preference to
"pure" predicate calculus; formulations for the reason that the predicate calculus approach has not been practically applied to such task areas to date.) In other words the systematization of expressing the knowledge is inside the head of whoever is using it and is thus for purposes of analysis; elfectively lost. Also the parts of the programs that make assumptions about input form are scatlered, rather than collected into a language interface. Using natural language, on the other hand, necessitates building sorne translation program, but that program can then be inspected, presumably, and the theory of construction of formal representations of knowledge that it embodies can be extracted and made explicit.

The analysis of the KSs, either with a view towards using an artificial language for further work or as the actual interface to the encoding process, will require advances in the present sitate of the art. The most promising approach at this time may be to use ideas similar to those of Hayes and Simon (1973). Their approach, which was successful in analyoing the task instructions for a problem-solving experiment and which derives from an approach to aulomaled protocol analysis (Waterman and Newell, 1973), is based on loosely processing the natural language input, attempting to make connections with known forms, but otherwise ignoring parls of the input that cannot be parsed (the parser is designed to react flexibly to such noise).

As an adjunct to the actual automation of the process, it might be useful to test how much of the scheme can be used by humans in writing PSs. It is reasonable to look for a strategy of making explicit the knowledge to be encoded, at the same stage in the programming process that is occupied by a top-down "structured programming" strategy wilh a more conventional language. This would divide the programming into two stages, one involving the clear formulation of the body of knowledge to be encoded, and the other involving the problem-solving neecessary to complete the PS encoding.

The representation of the KSs internally is another major unsolved problen. The moin aspect of this is the question of duality of representation: is it necessary to keep both the procedurally-encoded knowledge as it exists in the $P_{s}$, and something corrosponding to the individual $K S s$ ? It seems essential that knowledge be kept available for interactions arising sonte time after its initial acquisition. A fact might even be made use of for construcling and revising many different problem spaces, in addition to aiding the addition of knowledge in closely related areas. As sketched above, it seems plausible that a program could determine the knowledge in a $P$ by examining it, given the meanings of the predicates, and given an overall understanding of the problem space. It might be possible to aid this process considerably by encoding the $P$ LHSs as a discrimination net, and then using the net to discriminate, and to study the interactions of, the KSs themselves. Thus the net would simullaneously represent the desired duality, with one interpretation being used to match conditions of $P_{s}$, and another interpretation, based on predicate meanings, to regenerate the knowledge content of Ps. This adds to the design considerations for representing Ps as a discrimination net, and provides more motivation for pursbing that topic furiher.

Several questions can be formulated with respect to the various components of the above analysis. First, it might be necessary to refine the decomposition into Ns, Qs and Zs that was developed above, since automating may add requirements to the structuring of the statements. The process of determining which $K S s$ are to be taken as principat ones
needs more exact specification. It might be fruitful to investigate the question of how to generate the predicates, which would involve trying to characterize predicate meanings in a general way, as well as the question of how to refine this concept structure to fit the needs of the specific implementation. This aspect would involve, in advanced form, the examination of the Ps' structure to determine which subsequences of conditions would be more suitably expressed as single predicates, perhaps making decisions as to whether some predicate could be computed once instead of being recomputed on demand, or vice versa. Finall', the question of whether Qs need to be kept as a body of statements (either explicit or implicit, depending on the solution of the duality problem) or whether there might be some method of generating techniques from more abstract statements, by some kind of problem-solving process with knowledge of functional aspects of programming.

The process of how the KSs interact to form the Ps needs to specified much more carefully. Particularly important is to break them down in such a way that their associations and inter-relations with each other are clearer. The knowledge about Psinst syntax at the lowest level, which wasn't considered here, would probably be encoded directly in the P-building processes. The process of applying the KSs of the Q and Z type requires recognition of conceptual structures that are not well understood at present. For instance, there would be a general set of criteria for recognizing a situation where knowledge aloout looping techniques can be applied (some of these situations are explicit in the Qs at present, but the statement of a general set of them, and how they're applied, remain as open problems). How the $N s$ interact raises the most interesting questions, which are difficult to approach at the present informal stage of the analysis. The model for the Ns (or the problem space that it represents) seems to provide a rich interconnecting structure for the basic objects that are described by the Ns. This structure allows some kinds of interactions and development to take place, and prohibits others. For instance, the modet makes a clear distinction between chunks that represent arithmetic expressions and chunks that represent the find-variable (FV) specifications for a problem; processing done on arillimetic expressions is by this distinction determined to be unnecessary on FVs. Since this kind of dependence of interactions on the containing model (problem space) structure was not central to the analysis of Studnt, it may be that it begins to have imporlant effects only on more complex task domains, but it may be that the dependence will become evident as the analysis is automated.

Further research must be dirrected towards supporting the idea, implicit in the formulation of the knowledge acquisition model, that knowledge can be compartmentalized in varidus models. One interesting problem is to make explicit the model of pure task environment knowledge (box 1 in Figure C.3), and similarly another is to produce a pure formulation of the problem-solving methods. The use of models to replace the loose absitract descriptions provided for the Qs and Zs (Section C.5) is an important topic to pursue. The Q model must include functional goals like flexibility and efficiency, which are evident in some of the Qs, but which are at present isolated and unrationalized attributes.

The higher-level components of the model of knowledge acquisition, dealing with the formulation of the particular problem space given the nature of the task environment, introduce a very interesting set of research problems. As detailed above, there may be a significant amount of problem-solving and concept-formation in this process. This involves, for instance, the recognition that arithmetic operators form boundaries for portions of text, and that the operators can be processed by techniques used for phrase-structured
grammars. Given some weak-method formulation of the problem space, such as some way of using heuristic search, the addition of the problem space plans used above constitutes an interesting learning problem.

The relationship of $P S s$ to the overall knowledge acquisition model needs to be empirically determined. That is, a convincing case needs to be made that PSs can adequately represent the wide variety of procedures and data that have historically been used in Al programs. For instance, can PGs be used to represent semantic networks, and inferences of the type that have been achieved by using backtracking search? On a more general level, it would be useful to characterize the varieties of knowledge, and how knowledge is encoded and manipulated, for the full range of past Al systems. It may turn out to be the case that the class of programs whose knowledge fits into the present framework is limited. Whether this is the case might be determined by analyzing other PSs using the present methodology. A particular area of current interest is the problem of representing uncertainty of knowledge sources (Shortliffe, 1974) and of learning and gencralizing from real environments (Becker, 1973). At one level of description, more generally applicable $\mathrm{P}_{\mathrm{s}}$ are ones with more general condition elements, but the process of acquisition and creation of more general knowledge for forming those elements needs a greal deal of elaboration.

The present analysis has tried to elucidate as many aspects of the knowledge encoding process as possible, withou becoming committed to all amount of further work that would be impossible in the scope of the present paper. The fact that the analysis includes details for the entire Studnt program supports the basic conceptual structure of the model, and allows certain important conclusions to be drawn about how knowledge is encoded in PSs. It is suggested that this level of detail is appropriate for the other sludies of knowledge encoding oultined above. Further detailed research into the effectiveness of the model for use in an automated knowledge system is best postponed until more basic questions with regard to the use of PSs as a language have been investigated (see Rychener, 1976).

## D. Summary of Conclusions

Our conclusions from this study can be separated into those from the implementation itself and those from the knowledge analysis. Studnt adequately solves 27 tests that were done originally by STUUENT. Interesting features of program control as achieved by the PS are: the use of explicit data as control signals; the use of data elements to imitate a recursive (hierarchical) parsing of the inputs, and to build the tree-structured output expressions; the use of Psinist's :SMPX to sequence and coordinate processing; and the use of Psinlst's multiple-firing capability in processing sets of items. The internal Working Memory representation of Psinst embodies a choice for generality as opposed to the conciseness and ease of manipulation of a special-purpose string representation. The Working Memory is at least an order of magnitude larger than other known PS architectures can hande efficiently. The time efficiency of Studnt is quite reasonable for an interpreted language, and is less than an order of magnitude slower than a human on the same task. Studnt differs from STUDENT in the gross organization of the processing, doing a single left-to-right scan over the input to achieve what STUDENT did with several sets of rules applied in sequence, each of which made multiple scans of the input seeking various patierns. The two implementations use roughly the same number of rules, with Studint's rules having more complex conditions and actions due to the data representation.

The primary aim of the knowledge analysis is to examine in detail the knowledge in Sludnt and how it is encoded in the Ps. The knowledge is expressed as 218 naturallanguage statements of three broad categories, with the concept of problem space forming the organizational structure of the category comprising the majority of the statements. Each of the three classes of KS is described by an abstractly stated model, for which individual KSs are instantiations of detail. The S 13 example illustrates the nature of the interactions of many knowledge statements in forming one of a set of related Ps. The mapping between $P s$ and $K S s$ is many-many, due to the number of actions performed conveniently by a single $P$ and diue to the convenience of expressing $K S s$ economically. This economy is in the sense of being usable for interaction in a variety of ways, thus gaining more contribution to the total Studnt program per KS. Data on the distribution of KSs over the full set of Studnt $P_{s}$ give further support for the size of knowledge unit chosen and for the many-many nature of the mapping. An average $P$ is the result of combining 2.88 KSs of the problem space type, 2.86 task-independent programming techniques, and 0.65 slatements dealing with PS control. The mapping between problem space and Ps is fairly direct, given thal of the 218 statements used, only about one fourth are programming techniques, with $5 \%$ of the total dealing with PSs. Thus the encoding process deals mostly with the addition of problem space knowledge. A brief look at a case of augmentation within Studint indicates that most new knowledge is of the problem space category, with large overlap in the other categories. The knowledge analysis was developed entirely from the explicitness of $P$ conditions and actions, allowing the knowledge to be read off in a systematic way.

The form of the knowledge analysis led to the hypothesization of a more comprehensive model of knowledge acquisition, as might be realized using PSs as a basis. The major problem of the formation of problem spaces from less structured task environment knowledge can be formulated in this model. This involves advances in the state of the art in problem-solving and concept-formation. Within the model, the process
of programming in PSs is seen as a knowledge-encoding process, where the explicitness of PSs is used to advantage in debugging and augmentation. The decomposition of the knowledge into problem space versus programming techmques is promising in terms of being able to build up a set of standard techniques which would effect the encoding of numerous problem spaces of diverse sorts, amounting to substitubility of the various knowfedge models. The utility of the model is based on being able to automate the knowledge.encoding process, which depends on being able to process the natural language statements, determine the knowiledge content of existing, Ps, and carry out the interaction process. The model thus raises numerous questions for further research. Techniques being developed in protocol analysis and in aspects of human understanding, exemplified by the work of Hayes; and Simrm, may provide a basis for the natural language processing involved.

Comparison to other approaches, especially Sussman's Hacker model, brings out the position of PSs vis a vis modularity of knowledge. The models of the KSs are modular, but the PS encoding is an explicit representation of the full extent of possible interactions among the statements. Thus the encoding is at the extreme position of a modularity dimension, with access to the knowledge in a modular way dependent on explicitness.

There are several benefits from positing a level of knowledge between its expression as knowledge aboul a task environment and its expression as Ps. KSs as exemplified here are closer to problem-space..level models than are Ps. There is significant problem solving, namely finding the interactions of $K S s$, in making the translation from KGs to Ps. There is also problem solving, of a different sort, in forming the problem space from knowledge of the lask environment and knowledge of methods. The separation of problem space knowledge from programming techniques and lower-level PS knowledge is promising with respect to applying known techniques to new bodies of problem space knowledge, with a minimal need for re-shaping the problen space to fit the available techniques.

Measures along the understanding system dimensions of hoore and Newell are suggested by the knowledge analysis. A (very tentative) figure of 977 for Studnt's degree of assimilation is based on taking the successful encoding of a $K S$ into a $P$ as a unit of assimilation. The kinds of problem Studnt could do, based on its present knowledge and on the knowledge required to extend its performance to other classes of problems, gives an estimate of $57.5 \%$ for Studnt's degree of accommodation (this is based on crude sampling but points oul how the knowledge analysis approaches the question). The present approach suggests a way that depth of understanding and erfor might be handled using KSs as units contributing to a particular solution, but at present nothing more precise can be said. The figures given above are not to be taken as precise measures, but rather as indicalive of the potential fruitfuness of the overall approach.

We started out this study of STUDENT by asking questions related to its intelligence and understanding, from the viewpoint of an analysis of AI programs. What has developed is an elaboration of the use of models and particularly of the concept of problem space. Intelligence is seen as knowledge in a problem space, in the form of plans, that guides the application of other knowtedge as a solution is sought. The plans in Studnt have been explicilly pointed out, and a betler understanding of Studnt's use of the match method has been reached. What Studnt understands is made manifest in the 218 KSs , along with our
abstract characterizations of them. Further work to verify and extend the analysis will tell us how applicalsle it is. The details must be verified by deepening the formalization and by automation. The breadih of scope of the model will be realized from studies at a level comparable to the present study, on a wide variety of AI programs.

## D.1. Acknowirdpements

The initial motivation for undertaking an analysis of Al programs and for the use of a PS for STUDENT as a means to that end was provided by Allen Newell. He also made many useful comments on the work as it developed. Any ideas that seem to lack adequate scientific support or any failures to consider the right issues relating to the problems discussed remain the author's responsibility.

Studnt

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## Appendix A. Short Summary of Psnlst Features

## A.1. System architecture and production format of Psinlst

A production system (PS) is a set of conditional rules, productions ( $P_{s}$ ), that represent changes to a symbolic model of a situation along with conditions under which those changes are to be made. A production system architecture (PSA) provides: a Working Momory (WIM), which contains symbol structures representing the dynamic state of the situation being modelled; a Production Memory (PK1) which contains the $\mathrm{P}_{\mathrm{s}}$; a particular control mechanism known as the recognize-act cycle, by which $P_{s}$ are repeatedly expecuted or fired - a P that is recognized to have its condition satisfied with respect to WhM contents is fired by having its actions performed, whereupon the cycle is repeated using the new contents of WM (WM is updated by the actions of the $P$ that is fired); and a set of conventions or ordering principles by which a single rule may be selected from the set of rules that are recognized to be satisfied by the contents of Wh during any recognize-act cycle.

The Psinst (PS analyst) is a PSA, as follows. Wh is an unordered set of data items called instances. Each instance is an ordered list of two or more elements, where the first element is a member of a set of constant atoms called predicates, and where succeeding elements are either atoms or list structures - list structures however are opaque, their internal structure not being accessible to the recognition mechanism of the PSA. Instances are considered to be grouped logether in the WM according to their predicates. PM is an uncrdered set of $P s$, each consisting of a left-hand-side or LHS (the condition part) and a right-hand-side or RHS (the action part). The form of LHSs and RHSs will be discussed below. The recognize act cycle consists of a match of the LHS to WM, resulting in bindings for variables contained in elements of the L.HS. A firing then uses those bindings to create Wh instances according to the elements of the RHS. Two features of the match are unusual. First, all possible matches are found, and a firing occurs immediately for each match. That is, within a single recognize-act cycle, many firings of the same production may occur. Serond, a match must include at least one data instance that is new with respect to the $P$ that is matched, where new is defined as having entered WM after the previnus firing of the $P$. The artion part of a recognize-act cycle consists of adding or deleting WM instances, and of oplionally making changes to PM using ADDPROD and other special operators explained below.

The way Psimst orders satisfied Ps to select one for firing (this is the fourth PSA component) is by ordering events that occur during the action part of the recognize-act cycle. This is done by using a stack memory that records, for each WM change, the set of Ps that might become satisfied as a result of the change. The stack meniory is called :SMPX, slack memory for production examinations. More recent Wh changes are stacked on top of older ones, so that $P_{s ;}$ satisfied by more recent changes are guaranteed to fire, if satisfied, before Ps using older changes. The order of recency of changes with a P firing are determined by the order of conjuncts within the P's RHS. This ordering principle leaves two selection orders unspecified: if more than one $P$ using the same WM change is satisfied, one is arbitrarily chosen to fire and the other is pushed down in :SMPX by the changes made by the selected $P$; if a $P$ fires more than once in a recognize-act cycle (more
than one match is found for the P), the firings are done in an arbitrary order. With respect to the former arbitrary choice, if one $P$ is to be selected before another one that uses the same WM change, the LHSs of the two Ps must explicitly be mutually exclusive. That is, it is the user's responsibility to distinguish between don't-care and necessarilyordered situations. Given the :SMiPX mechanism for ordering $P$ firings, the recognize-act sycle can be summarized as follows: a change occurs to Wh, resulting in :SMPX entries; starling from the top of :SMPY, $P_{\mathrm{s}}$ are matched until a P condition is found to be satisfied; the actions of the satisfied $P$ are executed, resulting in stacking up new entries in :SMPX; and so on.

The following is a Pstrlst production that appears in a PS that models a hungry monkey in a room with some bananas, as the monkey recognizes its hunger and tries to reach for the bananas.

```
H1; "HUNGRYY" :: HUNGRYY(M) & ISMMNNKEY(M) & ISBANANAS(B) & LOC(B,X,Y,H)
    #> GOTO(M,X,Y) & REACH:OR(M,B);
```

The name of the P is H .1 , its comment is "HUNGRY", and the remainder of the P gives the LHS and the RHS, separated by " $\Rightarrow \gg "$ ". The LHS is a conjunction of templates for WM elements; each template is a predicate followed by a list of variables. When a match succeeds, each variable is bound to a specific token from the Wh instance corresponding to the template. H1 would match a situation in which the instances (ISMONKEY MNK-1), (HUHGGRY MNK.-1), (ISBANANAS BAAN-1), and (LOC BAN.-1 1-1 J-3 K-2) are present, to produce two new instances, (GOTO MiNK-1 1-1 J-3) and (REACHFOR MNK-1 BAN-1), assuming, say, that the (HUNGGRY MNK-1) instance is a new one. $M$ is bound to MNK-1, $B$ to BAN.-1, $X$ to 1-1, and so on. NNNK-1 is a loken for the monkey, BAN-1 for the bananas in the room, $[-1$ for a spatial location along the $X$ coordinate axis, and so on. The GOTO and REACHIOR instances become instigators of further action, if PS to model the corresponding real actions exist and if other conditions in the model are appropriate.

## A.2. Fealures of Psinlst programs

The notation for $P_{s}$ in $P_{s i n} s t$ is a subset of the Mlisp language, or rather a special interpretation of Milisp expressions (see Mlisp, by D. C. Snith, a Stanford AI Lab report, available at CMU). A PS consisls of one or more modules, each of which is represented as an Mlisp EXPR consisting of a BEGIN ... END block. Each module consists of optional declarations, followed by a list of labelled Ps. A $P$ is simply a disjunction of an optional comment string and two conjunctions, the first conjunction being the L.HS, the second, the RHS. A special function is used to translate these conventions into the format used internally by Psinist.

The following presents novel syntactic features that are encountered in reading Psnlst programs:

| \%. | - the Mlisp comment character; text between 7 's is ignored. |
| :--- | :--- |
| $"$ | - used to quole Lisp S-expressions |
| ; string constant delimiter (for instance, Psinlst comments) |  |
| $\Rightarrow \quad$ | - a semicolon is used after a P name and to separate Ps |
| $\Rightarrow \quad$ - this symbol separates LHSS of Ps from RHSs |  |


| : | - used to separate Psinlst comment string from associated LHS (is DEFIN:'d to be OR) |
| :---: | :---: |
| ? | - Misp character -quote character; must be used for characters that have special Alisp meanings: For instance, V?-1 is an identifier, not " $V$ minus 1 ". |
| 8 | - AND |
| <> | - Mlisp syntax for (LIST |
| ก | - Mlisp syntax for Lisp APPEND function, for joining two lists |

Summary of notation for $P_{s}$ :
name ; "comment" :: L.HS => RHS ;

The following comments explain other special features of Psnlst programs, but only to the extent necessary for easier reading of the programs. Examples of these features are to be found by the reader in specific PS s.

Macros: certain things that look like predicates are really macros, expanding into a sequence of predicales with arguments; these are usually expanded at load time, by user - detined Lisp programs.
NOT specifies "absence of" when it precedes LHS conjuncts; it denotes deletion when it precedes RHS conjuncts; in L.HSs it may also precede a nested conjunction, NOT( ...), in which case the conjunction is matched as if it were an L.HS, and if it succeeds the LHS match fails; these negated conjunctions may be nested, that is, they may contain nested conjunctions (see also EXISTS, below).
NEGATE is a buill-in macro thal specifies which of the LHS conjuncts are to be negated in the RHS, by number, or by using ALL; if negative integers follow ALLL as an argument, it means "Al.L but" the instances specified by the negative integers; for instance, N:GATE(3) would stand for NOT ISBANANAS(B), in the above example.
SATISFIES, SATISFIES2, SATISFIES3 are special predicates for testing values of variables during the match, using lisp predicates; the numbers 2 and 3 are the number of variable arguments (SATISFJES takes one).
VE: $Q(x, y)$ is equivalent to SATISFIES2 $(x, y, x$ EQ $y$ ), ie equality.
VN: $Q(x, y)$ is equivalent to SATISFIES2 $(x, y, x$ NF:Q $y$ ), ie, inequality.
Conjuncts in RHSs may use arbitrary expressions as arguments, to be EVAL'd as Lisp expressions during the $P$ firing process. (Misp includes Algol-like arithnetic expressions.)
NONF:LUENT(p) declares $p$ to be a non-fluent, that is, an insertion of an instance of predicate $p$ into the Working Memory does not cause any Ps to be matched for possible firings keyed to that insertion. In other words, no entry is made to :SMPX for that change.
REQUIRE $(a, b, c, \ldots)$ dechares that $a, b, c, \ldots$ are required modules of the PS whose main module contains the declaration.
PSMACRO( $f 1,12, \ldots)$ dectares files to be read to define user macros.
DCNMD $\{1, f 2, \ldots$ ) declares files to be read as command (CMM) files.
EXISTS in an RHS causes creation of new objects whose names are extensions of the arguments of the EXISTS; those objects are then used in the remainder of the RHS to form instances.
EXISTS in an LHS must be in a nested expression of the form NOT( ... ; its function then is
to tocally dectare its arguments as variables, causing them to be initialized to NIL for the mateh that follows, within the (... ).
OELAYEXPN[D(X) where $x$ is some macro call: this specifies that the macro is not to be expanded when the $P$ is inserted, but during the actual firing of the $P$; this is only used when the predicales of the RHS depend on values not known until run time; it can not appear in ths's.
ADDPROD(prod,prec,comnt, ihslist,rhslist): primitive for adding a $P$ (named prod) with comment comnt; Hislist and rhslist are lists representing new LHS and RHS; the prec argument is either a $P$ name, indicating that prod is to be placed after it, or is taken to be the name of a new module of which prod is the first P; ADDPROD causes assertion of (ADJPPRODP prod).
REPPPROD(prod,comnt, thelist, rhsilist): replace comment, LHS, and RHS of prod as indicated; asserts REPPRODF(prod).
REPLHS(prod, ihsilist): replace LHS of prod as indicated; asserts REPLHSP(prod).
RE:PRHS(prod, rhstist): replace RHS; asserts REPRHSP(prod).
RE:PCOLNNT(prod,comnt): replace comment string; asserts REPCONNTP(prod).

## A.3. Features of the Irace output

- TOP LEVEL ASSERT - the initial starting assertion, typed by user.
! . a P fired
number following ! - the firing was the number'lh
P.name followed by '-' then number - the number'th firing of the $P$
"sitring" - the comment string associated with the P
USINE: ... - instances from the Working Nomory used in matching the LHS
(xxx . yyy) ... - assignment that was made for the match: $x x x$ was assigned the value yyy, etc.
INSERTTING ... - the insertions and deletions made by the RHS
( : SA例Y.... number ) - a display of :SAMP after firing; number is length of :SNAPX; each entry is enclosed in [J's
EXAMANING... - gives the name of the $P$ and the key insertions causing the examination
/IRY .. means that a non-fast $\cdot$ ail examination is being done; fast-fail is a quick check on whether any posilive predicate has no instances, before the full-fledged match is tried (formerly /NIF)
WARNING ... - appears when an instance is inserled or deleted but was already present or absent, respectively
*+ - appears for a warning for an instance insertion
*.. - appears for a warning for an instance deletion
If the RHS inctuded ADOPROD, REIPPROD, REPCONNT, REPLHS, or REPRHS, a message is printed before the INSERTING line.

PSHPR:AK comment AT ... - a break in execution; user interactions consist of commands in ()'s; the system responds with output dependent on the command, or with "ok"; (OK) is trped by the user to resume execution.

The above appear on a full :DVERBOS=4 or :TVERBOS $=4$ trace; the following are modifications for lesser traces:
the P-firing, message is all on one line most of the EXAMAINING message disappears; only the $P$ name remains; if /TRY occurred, only the / appears (in case of verbosity 1 , not even $P$ names appear)
mest of the WARNING message disappears - only the *'s remain the USING and INSERTING lines disappear the messages from AlDIDROD et al drop out break messages, commands, and possibly their outputs disappear

After execution, typically a DUMAP occurs (delimited by "OUMP"), followed by the output of PERFEVAI.:

Run time for the present RUN invocation A small table of figures:

EXAM is the number of examinations of $P_{s}$
TIRY is the number of non-fast-fail (/TRY) examinations
FIRE: is the number of $P$ firings
WMACT is database (Working Memory) actions: insertions + deletions
$E / F, E / T, T / F$ give ratios of the first three
the line following the numbers gives an average time figure for each of the relevant numbers in the preceding line (divides total run time by each of the numbers)
Detail on Working Memory changes; "NH:W OBJECTS" are those created by EXISTS
Maximum length attained by :SMIPX
CORE: gives current available LISP core, plus amount used in current run
:ACTS - a list of the major artions in the current core-image
TRACE - a list of Ps that fired, in the order that they fired
FIRE:D $\times$ OU'T OF ... - gives number of distinct $P$ s that fired

Studnt

STUONT APPENDICES

ENT:

ORDER OF GROAPS OF PQOUHIIONS: (STDDMCRVFA甘1X)



```
7. EXMANATION GH MMSRSS:
```





```
    WORDINS(WN;ww)
        &Qw(w(vV) Wokn givV:Ww)
    STRIAESTNS(IMA GIICO)J:FR)
```







```
        * STCIMA:TMRS((AN ND CO)NOST)
```



```
        | ( NDMARK{ST) & (NDWAMRK(ST)
$
```

NONF( UF NITIEFIOF WORTS Q):





 - CHINEIEN(S)



 - Not FVECAN(X):




SIR: "TG SCAN" = IHASCAME INY.








$\Rightarrow$ TGiscant laz(x) ; RE (iall( 1 ):
535: "SCAN FY" - [SSCCNTINRXI 3 ISSCANTV(C)












 - NE (iATI (IA) \& NOT INSCANF V(CM:) \& NOT TGSCANF JNZ(V.):

-) ANSWTRTIUJLO(F) * NE (iATf( 1 ):

I PACE 3. TRANSI ORUATIONS:

## 


 - (EfTOt ( $\times 2$ ) S MEGATI(All):












 \& WRRDEQVO. 1.1 TMESI NOI WORTX QV. I.TER)


TG: "LESS IHAN. YESSTHAN" : IFSCAMV'. I) SIRIMA Of (ICSS IHIAN)X.Y)








 - MK (ATITALL.2.3):


 \& Ne (iATt(All. 7) \& NOT WOME giv. 1.7wol:









 onecintrinll. 7 ):
 6 WhRC Q(Y,WW) SATIST IS S(WW.(MUMRDO WW)] 6 IEf TOF (YI)


?VAETHVMOLLARSY?









 - NEGATF(ALl):
\& OERCENTY $\therefore / 100.0 \vee$ ?
T17: "HOW MANY. HOWM" = TFSCAN(V?-1) \& STDIMEF QC(OIOW MANY)X.Y)





 \$ NOT EQIS(X)


 - Not \{aIS $(x)$




 - Not t ANDDOJH $\mathrm{F}(\mathrm{C})$

 - ISSCANKSHAKK(C)



Tク5: "



T7SO: ",






 8 EDMMAER $Q(Y)$


 * Nor WRHNE C(xitxcef RDS):

737: "BY.:15" : $1 f$ SCAN(x) \& factr(xi) ) invistc)





 - NOT If ASCAN IN(SW):

157: "IF OUI LEN": TFOUIIEN(XYR) * CMINX(tN(N)

ENO:

* pace a - dicijonaky iags:

DI, "IESSTHON OP?" = 1








DIA) "EXPONENI (\#) : It
 * Nol Wokv w(x.? ? ? )


 - Not WCHM LX:(PRR);
































- NOT CGMANY(Y)




CNO:


## $\therefore$ FACE 5 - POECCOFNC: SCAN 3

## EXRR SIUNO: HIFGIN

D1: "VRAFREC": 1SUfRTXX) - 4ASPREC(X9):
F2: "1S FREC" = \{Q1SXX IS1S(x) WHASFGEC(KB\}:









- NHT EXISIS(C7M7) B UASCPRICR\{CTMフ)

- NOT (EXISTS(C3M3) \% HASCPRIOR(C.7M3)

- Nol Mat CSCAND(<3)

 - NOT me CSCANO(CO)
- NOT(EXISTSIC (M) \& HASCRRIOR(CIMI)
- SATIST IE SP(HOMIMO TEGRFAT MSI) NOT MRECSCANO(C:1))
- Nor (EXISTS(C7M7) \& HASCPRIOR(C7,M7)

- PRECSCAN( $(O X)$ HICHMELC(CO.OX)


- NOT CHINKP NOR(XC)
$\because$ PRE CSCAN(C,W) HITYIVREC(CMX) * WEGATf(12):
223: "PREC SSAN ON" = PRECSCANTC.K) \& HIGHREC(CN.Y) \& HASFREC(XM) - NOT SATIST IISP(MN:(CREATCRN MN)) E LEFTO (K.W) - NOT CIRMKF MDR(Y.L)
- PaE CSCAN(C.W) NECATE (1):
 - LEF FOF $(x, W)$ NOT CHONE NDR( $\times, C$ )
$\because$ MRE CSCAN(C.W) WEGATf( 1$)$ :


- HIGWMREC(CMX) WRECSCANDTC) NE GAT( (I 2):


$\therefore$ PRECSCANDSC) NE GATP( 1 ):


```
    - CHINK! NMI (Y.C.
```

    \(\because\) PRECSCANTHC) \& ME (iATt ( 3 )
    

 $\because$ HASIStCX) \& Ne (in? (?):










 - 1 Ahe 1 LK (C.NJ)


## END:



## 








































A5I A ค
MAO: "HASYE RB Dft : HF:SVERSTC.V)
 of 1rs.fnink(a, ()$)$


$\therefore$ ISCPANK(C) \$ WOT MRE CSCAM)(C) NOT HASPREC(V.9) \& NE GATF (1):
 - HASVERESC,V) S LEETO (VN) O WTRUX XN,WN)

- SATISE JES(Wv. (AXIMRERE WN))



- LAKY LUKCM9) MASCPRIOR(C.O)
 3 (AUKLUKNMPI HASCPRIOR(CND)


- NEGATI(1A.56, .10 .14 .15$) 8$ NOT HASPREC(Y,9):
₹ M50: All .AIR.V.Nv.X.A7I ...A7R


s HASVERHSC.V) \& (EFTOR (VN) A WCRD QN.WN)
S SATIST IESRWN (NUMAR(RF WN))

 - \{HCRINX(AC)\}
\& LAKFL(IGMP) HASCPR1GR(C.O)



- STRIMATHST(IS)VN) \& LEFTO (N.Y) HASCREC(IAS)






 - NECATH(5):
 - 1sscancinine\{c)


 - SATIST If Siww' (MOMBE.DP WW))
 s Michtr(l):
 * NOT SATISTIES\{WW (NIMAER R WW)) ISOPI(K)
- NOT ISC $1(\mathrm{X})$ :

END:
? PACF 7 . CHINK SPLITIJNE: RE F ORMINK, AND RE NAMINK; ?
EXPR STIKR(): H\& GIN



 - $10 \mathrm{OH}(7)$

 5 LET TOF (Y, I) \& EQYY(l)

- NEWRETOP(PLUS) \& CSF( $11(C \times 2)$ ) UASCPAC, PLUS):





 \& LEF TO ([XOMODXMV) \& CHINXENDR(DXIMY.CtI) \& 4 QZ(MRAV)


 3 CHMNKT NOM（SG）



 －Néのでくへしい）

 －NE Cin？（ 1 ）：



















C55，－OPO EARK＂＝HASENOS（ $x$ ）WORTY $\alpha(x \times w)$










 －HAST XVR（C I，Y） 4 सRST XPR（C7）






C78：－











－JES（QN（C，X）：








 －NOT（HINWK NOR（W CK．）\＆NOT CHINXI NDR（W MK）





## 

（ND）

IPACE 8 VAR IOENT TESIS：
IXO SIUY（T：REGINDSGACRO（STIONM）：





．HASEXPR（CE）NE GAT（（1）；


$\therefore$ VARC（ EAM $\mathrm{PP}(\mathrm{C})$ HAS（ XRR（C．X）ME GATf（2．5）：

 －THISTFSTEO（C）NE CGATf（2）：
 ＊NOU（EXISTS（C）ISREFEXPR（C7）
．）THISTESTED（C）OMGATf（2）：



－I QVADRE MDC（ I）\＆NE GATl（ 1 ）；




 －NEGAT（I）：

 －Nol CHMKENDR（YC？）


 © NOI CHINKE NOR（Y，C）




 －（Ef TO $[Y I]$ \＆WORD $(x / \times W)$



－IGHHMEIEST（C．ICT．W．Y）：MEGATE（1）：


 －（eftol（x．w）





 －t QMIMHERS（W）CIFINKE NOR（WLI）
$\therefore$ COVANCHJNK（C 1G．？）a MEGAT（（1）：

 －cienky nda（wCi）
$\because$ CQVANCHINK（C．IC7）A MEGATC（1）：
 －CMANET NOR（XC）






 s'










END:

2 PASE 9 IVESAMAJAR:

## 























 - LEf $10+\left\{\begin{aligned} 3\end{aligned}\right.$





 - LE! 10 (v.才)


 - Mffall !nt! :
 s (EtIO $(x, y)$









(ND:
電
: FAGE 10 - AGE PRCRY.EM MFImISTJCS :

AI: "ASK SCANI": ASCAN(F) \& FQAS(Y) \& LESYO(X,Y) \& EQOLDN




.- ACFFRCH(F) ACEREFCNI(1) \& NECATI(1):






6 DFI AYF YRND(STRIMEINSI (IN (C\& NSYM) YI ARS ?, IN (CENSYM) YFARS) $\times 2$ ) ) s wiciatr(all):



- DEIAYFYRNDSSTRINEINS( (GGENSNM) YEARS AGO ?. (CFENSMA) YFAKS $A(: 0) \times 1)$ - negaticall):

 s Veciatt (l) ?):


 8 NSCintf(ALL.Z):

 - MGAIf(nll):















- MCO!

- MOXEN(-1) \&EFTK (XZ) \& NE GATI(ALL. 2 ):

- LEFTO (IV) LESTOF(V.W)
a Nols (GQulle(r) \& EAE(l) towtraniv))





-AC! COMK\&D(X,YD:DW) 3 NEGATf(13):


 * $\mathrm{NGENTL}(1):$
 - (ETTK (ONIO WNDXEPFW)
- ACF OPNEFD(X.YP) PW ) NEGAT( 1 ):

AA3: "AGE OP CORL." = ACFOPAKPO(XYDI.) SATISFIES(LLENGTIMU) EQ 3) - LEFTOF (OT) : LQ ROM(O) EQNOW(F) WORIX DXPSW)

- ACHOPNE OXXYPS. (PW) \& NEGATI(1):






```
        - W(M) CXA \(A\) (M)
```




```
        - wamy exrsiwn
```







```
        - THEJR(OU( \(\mathrm{K}, \mathrm{Y}\) ADI) \()\)
```









```
        - IEf Tide (w....) o mol tas(w)
    "ACFPEf Crwint
```


















```
    - ACtIRONCO (Y, viv X):
```



```
        * wornc wader
```





```
    \(\because\) mernemlintithi) \& irolliol Avies,
```



```
        - Nor ftiscant jax
```



- Asiloseron (r. rw Xl:

- wichuc cooow)
















- SATIST IIST(MZNNY TOIESSN)

- HASf XMR(V7f2) NOT(EXISTMR) HASRERR(f.3R)))
\& NOl(IXISTS!Q) MASRERE(ER))
- (H)ILCREDR(V) ANSWTRH2IILOT(r):



s Iff TOF (X,Y) \& WORD $X Y, W$ )
$\rightarrow$ WCOL\{CT(VL,Y) HASRtPR(EL W) W) NEGATf(12):
ENO:
: tace 17. 19toprons s
(xCR STUJ):I)ETN


\& SEACES:ITS(O.FQNS.'TVS:OPSSAPS)







 \$ SNACESTIENN. I] NEGATI(12):












 8 MReRMSS(OIO703) PRNKVA日S(YIV7V3.va)


 b NET SAIISIEStoofQ EQUAA)

 - sfacesiten(n)




- MWSITE (X) ERCHVAUS(VIVZV3,VA.1) \& WE (GATI (ALI)

 - मRCREVS! 157 - SATISTIfS(f Tf 2 EQO)







CNOSNO.


## 









END:

## 










WHEN MALY W/AS AS of O AS ANN IS NOWF?

END:







 (IND IHE MSUKI (GRICS P.) ):
 4 Yf AOS MGO?



END:


 3 TIMESAS ODASHSII?






HOW MANY :GONS ( ORIAL E 1 (ATHOM 7 ) )
END:










 IS ECOMIIfSPREHEKIQ ?.FIBA IIE TIME IT TAKES

TO TEAVEL (RCM MLW YORY TO (OS ANKELES EY JE? ) ):
(ND:




 TIF. SlMM OT TEF. NUMESR DF, FCINDS O AIMCMOS IN THE DOX AND


IIF COST CK ALMONTS KR POUND OI ALMONON IS 7 ? ?
 (IN) IHE COST OF TIF AI.MONDS ZN TIF, BOX ANO WE COST © TIE RECANS IN IFF BOX: ?.J):
 AND ONE MINUEQ IS IF! ANGER IHAN THF OTHER NLMAER?. FIND Hit I wo Mmerers ?.) ?:
 IS NTIES FYR GALION $\rightarrow$. THF DISTIMAI DE TWE EN HOSTON AND NE W YORE
 IRIF REI WEFN NEW YORK AND EOSTON? ) ):

ENO:

## 

 1S THE OVFRHE AOCOSTM US THE RIGNJM: COST FOD FACHISRSON








 ARMY IACHMONIHIS $\% 150$ TIMES TIE MUMFER OF INJFORCIFD SODIERS?
14F SUM Of THELAKIER AMOANT ANO WE FAY FOR RESERVES EACH
 IH RUSSSIAN AREAY HAS AND IIK MIMERC OF UNIFONMED SCIDIERS It HAS ?.1):
 DIHEFTMCK BETWEEN THE TWO MusitRS?
 FIM) TH I WO Mmetes ?.) ?:

## (NO:


X 27 : If one of tif: MMAERS IS CONSI CIIIVF TO THE OHER NIMAER? (JNJ 1 IE TWC MWRERS ? ) ):









## END;



 j):








END: CNO.

Acrmant C. CRESS.REELREMCI OF SIUONI PREDICAIES
XDEF OF STIIONT MREOS

Ac: comp
LHSUST S AEGM AET AER AES
RHSUSLS AEJ AG3I AEG AET - AET 7
act coinde in
LHSHSLS MEA Ag8 AEg

Af: CONPESM
llisusl S AE\%
RHSUSE 5 AES - Afi AER AE9
actor
Llisust SA3R
RTKUKES AZA RZE ATR
acterivio
LHSLIStSAAI MAT AAB
RUSIIS!S A38-NAI A42 - A07 M83 -A43
acr rosisem
1HEUISIS AE. 3 AR
RUSUSIS AR1 ART AR3 ARS
aber rem
(HEUSISSTE O5S 057 A61 A6?
NESTfOLSI7

ACEfroncol
tHSUSI S A7.3 A75

AGSict
LHSLISTSA5: AGBA71 AK
MESTO A5.1 ASSAE31
RHSUSKS AGO
AG! Del Coth
LHSUSISA8.3 A6.31
RHSUUStSAET. A63 - A63I
ACEDECNI
LHSUSIS AfR
QuSUS:SA:ADA3 AGA -RGA
ANSINITCR
145usis (13
RUSUSIS 04 - 13
ANEWTRHUTIO
(HSUSES 01
RHSUSES $\$ 70$ H:
ANSWIRAMIRU?
LHSuStS 115

AsicAN
HSSUSISATA7A3

0Wllotere
HISUS! $\$ 16$
RHISUSIS 15 - 86
CHICOINTED
L"Suss S vab
NESTEO - V55

CHIFSIEO
IHSURES VG5 VED VES
R-4SUSES V5 V55 -V55-v60 -V65
CrMINEFDD.
 F OCFASFSO \$ FOO A7A ARE A78 E6
NESTEO 151 V


F7!
CHIQKINCR




CHMEKItN
thstusis Si3 T50 15717

CSHII
1HSUSTS C6O
RHSUSISMSOC7C5CRCIOCI7C72C2FC52Cs5 -C60
OFFODLIST
RHSUSES If1IIS
ENDAMARK

EQCHENKTEST



COJ：


EQVAMIHEMK

NEST（5）V7A V）！
RHE！s！size vicr var val
EQVAWDEMO
t HSUSE 5 V7！V76．

FVIST
2 HSUSES 42

F VE：CAN


FYSCANEND
1 HSUNT S SG5 570

WASCPRJOK



HASE XIRR



HASIS
1HEUSIS MICCTCSC．
prsists S sus mio
HASOP


HASODO
LHSUSis C55



RHSUSLSF50 675
HASOD？
f Hesusiscio
RHEUSE S FAO C 10
HASDRE




いHSUSE S HE
NEST「D！Ib

HASTSQLIARF
LHSUSi S C15C17
RHELISTS FOS－CIf C． 17
HASSQLIAMEO
lHSUSES C：5OC5？

HASUOPCIRNK
trsust svo 10
RHSUSE S C 1 ！C Com
HASVFRE
IHSUSE S M7O M3O MAD A！
PHISUSES 530 MPD सA．30 MAO M！O M
HICTIPRE：


FA5 F5O DE＇5－F70 PIS，
1FDELETE
LHSUStS 125
NE：TEDT 179.13
RHSUSt 5 Tフy
INCHINK



－R2 RA－RA RE，FFF F＇I EK RERG：R9
ISANSTMI！
LHSUIS： 513
RHESUSE S： 40413.613

ISCRININK


1SDETJM
IHEUSES SI3 SAO
RHSESSTS 19 t
ISt ON
unslis！sc90
151 v
LISIISISGRO37
NSTETM H7
RHISUSIS SGOF 15F75－67
151s
llacust 507
015 T SI 5038
Ission
HSHKS 68 FO
RHSUSLS 013010015017018019
1SOFI
Hisusis Fa mb maij m7s
RUS！
15 coc
1HSUSIS53
R4SUS：SDS 03
1SCESSON
LHSusIS AE．1 Nej
RHSUSIS OAI DAA OATDSO O53 AGI
ISNLIRA

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NE WPL VAN
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HASUSIS 113
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| 03． 1 | ＊ | 0．） |
| P3－1 |  | P． 1 |
| 5－6－1 | ＊ | $5 . \ldots .$. |
| $12 \cdot 1$ | ＊ | 1.1 |
| 55.6 | ＊ | $\bigcirc .1$ |
| DST： 1 | － | 0． 1 |
| Pre 1 | － | r． 1 |
| S．9．－ | ＊ | 5．．．．．．．．．．．11 |
| 106－1 | ＊ | 1．1 1 l |
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| 112－1 | － | 1.1 |
| S1\％－11 | － | S．．．． 4 |
| DS－1 | ＊ | 0．1 |
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| 5093 | － | S．．．．．．．．．．． 17 |
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| 05－2 | － | 0.1 |
| P5\％ | ＊ | P． 1 |
| $530 \cdot 1$ | ＊ | 5.7 |
| 111－1 | ＊ | 1.1 |
| S1P10 | ＊ | $5 . . .4$ |
| D11－1 | ＊ | 0.1 |
| M65－1 | ＊ | M． 1 |
| P5．3 | ＊ | $0 \cdot 1$ |
| 53072 | ＊ | S．．．．．．．．．．．．11 |
| Ta）－ | ＊ | 1.1 － 1 |
| 51：－24 | ＊ | ¢． 1 |
| Dsfez | ＊ | D． 1 |
| S．S－18 | ＊ | 5.8 |
| P35－2 | ＊ | P． 1 |
| Mf 51.2 | ＊ | H． 1 |
| 6．64－3 | ＊ | （．） |
| P6－3 | － | F．．．．．．．．．．．．ir |
| P1it．4 | － | r．．．．．．．．； |
| CS－1 | ＊ | 1．．ir |
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| P1י5－5 | － | b，．${ }^{\text {a }}$ |
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| 19，6 | $*$ |  |
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| Cis－ | － |  |
| （6） 11 | ＊ | p.......s |
| P16． | － | ．．．．${ }^{\text {a }}$ |
| US．6 | ＊ | U．．．．．．．．．．．．．．．．．．．．．．il |
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| $\mathrm{CT}-3$ | ＊ | C．．． |
| P6－15 | － | F．．．． 4 |
| Plide | ＊ | ＇．．． 3 |
| U5．－ 7 | ＊ | ＇1．．．．．．．．．．．．．．．．．．．．．．．．．．．ts |
| P15－4 | ＊ | י．．．． 5 |
| 5 L － A | ＊ | $5$ |
| ［79－3 | － |  |
| 517.35 | ＊ | 5.1 |
| D81－1 | ＊ | 0.1 |



S．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．
$\begin{array}{ll}1 & . \\ 0 & \ldots\end{array}$
．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．
M． 3
c．．．．．．． 80

f． 5
B．．．．．．． 19
1 ．．．．．．．．．．．．．．．．．．．．．．．．．． 8 z

## Averonolix E. RESLX IS: OR 27 TESIS

TEらい!



FVI JS1 (PH.1 ((1)AN. J)) )


 (CR. 2 VAR. 2 ) (CR. 3 VAV.A)

RUN TJME 1 MJN, 73.2 SEC

```
EXAM TRY f]0r whact l/f [/\ T/N
1RJ5 050 735, 937 7.81 A.S$: 1.71
0.0A!.3 0.j%5 G.3!, Or,Y8% SICAV;
```

557 INS! RTS 3RO URLE1FS 2G WANNIHFSS 31 NW OHMC. 15
MAX: EMAPX LENGTHI I:A5

FIRf. 0 G9 OUl Or 750 phens

## 1E.3T2







(YAR. 1 (2)) (VAN. 5 (1HREE)) (VAV. F ( $(0.5)$ )
FYLIST (PEI.I (CVAK. 1 VAL. 3) $)$





 (CR.5 (1 IMES VAV.G VAU. 3)) (CR.E.VAL.3)

RLW TJME A MIH, 3E.3 ST:

|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4445 | 29 | 559 | 2116 | 8.81 | 367 | 7.71 |

$0.057 \mathrm{O}, 270$ 0.502 0.1.47 StCAVG

MAX sidex IEAKill lís


IfST3






FVI IST (PH. 1 ( (VAR. 1 l ) )
EQYAKCFINNK (C.a Cl.1)



(CR. 1 VAN.2) (CH. 2 VAN. 3) (CN. 3 VAN. 1)
WUN TJME $1 \mathrm{MJN}, \mathrm{T} 7.5 \mathrm{Sic}$
EXAM TRY 4 ISE WMACI F/F i/t T,F
2919 E.04 786 1189 10.1 A83 2.04


MAX :SWOX IENKTTH 105

FJPED 67 MIT (F 76.0 PRMS
TESTA



 (C. 7 (fotiat VAR. 3 YAM. 5)


rvist frg. ( (fyar. 1))
EOVALCHANK (C-3(Cl-1)(Cl -A CR-7) (Cl-5 CR-3)



(CA-1 († IMES VAR A (EXFY (TIMES VAR-2 VAR-3) YAR. A ) (CR. 2 VAK. A)


RLW TJME 5 M IN. 7.50 STC

| EXAM | Por | 1 lef | WMACT | F/F | $t / T$ | 1/\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4700 | 1080 | 557 | $2 ¢ 92$ | R.51 | 3.19 | 7.66 |
| 0.0638 | 02804 | 0.546 | 0.138 | SrCA |  |  |
|  |  |  |  |  |  |  |
| MAX SWMFX LENSTT1 163 |  |  |  |  |  |  |
| COAF (fREE.FLH) : 3613.756 ) LSt 0 (6472.776) |  |  |  |  |  |  |

10ED 96 out 6260 mecos
15915



(C. 7 (f QIIAL VAR. I ( 1 IME 5 VAR. 5 VAR. 2 ) $)$

(VAQ.a (erocl AQ)) (VAR 5 (7))
FVI IST (PH-I (f'AE. 7 VAR. 1)


( $(6.7$ (f.QIAI VAR. 1 ('IMES YAR. 5 YAK. 2 )) (C. 3 VAR. 7 ) (C. 4 VAR. 1$)$



RIN TIME 3 MBN. SO.I SEC

```
EXAM TRY fje wunct E/F E/T T/A
\(\begin{array}{lllllll}3689 & 970 & 0.85 & 183! & 7.89 & 3.73 & 7.17\end{array}\)
\(\begin{array}{lllll}0.0623 & 0.231 & 0.192 & 0.176 & \text { SfC AVG }\end{array}\)
```

1مל7 INSIRTS 776 UFLETES 5G WARNJWES $\$ 7$ MEW OHJF CTS
WAX:SHEXLEAKSTH1:
CORE (fRFE.5(H): (A475, R99) USEO (5560.633)
F Jat 0 R9 OUI or 760 PROSS
Tfsis
(GMHEY IS TWICE AS ODAS ANN WAS WHEN MADY WAS AS OID AS ANN IS NOW. It MARY 1S 74 VรADS OID. WHW OLD IS ANN 7)
1SE ONIC. I (EQLIAL VAD. 1 (T JMES VAR. 2 (MIMNSS VAR. 3 VAQ. A) IM)

HASNEPR (VAV. 1 (MARY S AGF)) (VAR. 2 (2)) (VAR. 3 (ANW S AGF)) (VAR.A (FS5OB))
(VAg. 5 (2A))
FVI IST (PR. 1 (ival 3) 3 )



$(C l-1$ VAR. 1$)(C 1.2$ VAR. 2 ) (Cl. 3 VAR. 3 ) ( $(1 . \operatorname{A}$ (MJMUSS VAR. 1 VAR. A) )
(C: 5 VAR. 1) (Cl . G VAM. 1) (CR. 1 (1 IMES VAR. 2 (MINUSS VAR. 3 VAK. A) )

(CR.6 VAR.5)
RUN TJME 7 MIN .75 .3 SEC
EXAM TRY $\mathcal{E}$ TRY WMACT E/F $E / T \quad T / F$
555616.77 GRG 2811 R.10 $3.31 \quad 2.44$
$0.0801 \quad 0.266 \quad 0.639 \quad 0.158$ SrC AvG
1590 INSIRIS 1721 DELETES IOC WARNIMES B3 NEW ORJFCTS
MAX :SMSXLENKITH INO


TIST]














FUN TIMF AMIN．TI．F SH：
EXAM TRY＇jFF Whati 1 If it Tit
$0585 \quad 1317 \quad 578 \quad 7059$ \＆ 0.8 3．n！ 7.08
0.05970 .707 0．51n 0．131 SLC AVf；

MAY：：SMFY．LएNGTH；131


1FSIR






FVI IET（ FH －I（（VAL 5 ）$)$
EQVALKHILNK（R－3（a．es）



（CR． 3 \｛1（ME 5 VAU．A VAU．Si）（〔．A．A YRU． 5 ）

EXASI IRY IJF．WHASI E，T（i＇t $7 / F$
3003 हлA 358 lass 8．51 $3.50 \quad 7.39$
O．OFAS 0.279 0．5n母 0．333 st C Avf；

MAD：：SMFXIENC，11：1：！



TEsig



（C． 7 （f QUAL（TM LES V







（ $(1.6$ VAL． 6 ）（ 20.1 （1 TAES VAV． 3 （M］MSSS VAD． 3 VAD．A）））




```
EXAM TRY IJFY WMMCI FAT IGT 1/F
EB36 707% 77) 3@%V E%% 3.17 76%
```






```
FJRFO 173 0U1 ON 2FOP年OOS
MEST10
```




``` 97．（ IM Hut．SACt ．）
ISEQN（C．！（EQUAL VAR．I（1 JMES VAR． 7 VAL 3））
```



HASEFES（VAL． 1 GIUL S FATHEDS UNCCE S AGE．）（VAR． 7 （ZI）

FVI 151 （CH．）\｛（VAP．5）\})

（能を．（i））（c9．9（t－6）
HASI XIR（C．－1（IQUA）VAR． 1 （T TMES YAR． 2 VAR．3）））




（CR．1（I TMFS VAR． 7 VAR．3））（CS．）VAR．3）
（CR． 3 （1 IMFS VAU．A（PLUSS VAN 5 VAK．P））（CR．A VAR． 7 ）
（CR．5（PN USS VAR 5 VAN．7））（CR．G VAC．2）（CR． 7 VAR．B）
（CR．E（CUSS YAM． 3 YAQ． 3 ）（CR．9 VAR．（4）
PLON TIME 10 MSN． $78 . A$ SEC

```
IXAM TRY FIEF WMACI ENF E/T T/A
8776 7807 1170 03.33 7.01 2.93 7.39
0.0%69 0.274 0.535 0.105 SfCAVG
```


MAX ：SMF X LENGTH 135

F JEFD II3 OUIT＊ 259 PREOS
1FST：I
（tIOH HAS TWICF AS RANY FISHI AS MARY HAS GUPDJES IF MARY HAS 3 GIPPJES ，HOW MANY f $15 \cdots$ DOES S ICM HAVE 9 ）

HASPEAR（VAV I（2））（VAQ． 2 （IHF MUMELR OF FISI TOM IUASU）
（VAR． 3 （HIE MMHER Of G（PFFISS MAKY HAS））（VAR． 1 （3））

EOVAVCHINK（C－3（R．2）（CX－ 3 CR．1）


（C4．1 VAN． 3 ）（CD． 2 VAK． 2 ）（ $(S .3$ VAK．A）

REN TJMF $A$ M TN 1.97 S［C

```
EXAM lRY flRE WMACT &TF E/T TR
3311 108, 006 16,77 8.73 3.09 7.66
0.0726
```

9AG JNSIR1S GRI DELE TES AI WARNIMESS GI NEW ORFCIS
MAX：SWFX LENGTH IST

FIRF 0.90 OUT OF 280 FRCOS
testip
（II I SMAN EQUALS G IMPERS．AND IFATHOM EQIALS EFEET，HOW MANY SFANS \｛ Q（HALS I FATHOM 7$\}$
ISE ON（C． 1 （FQUIAL（T IME S VAR． 1 VAR－ 7 ）（T JMES VAR－ 3 VAN．A）$)$
（C． 7 （EqIIAL（I IMES YAR．I VAR．5）（T IMES．YAR． 6 VAR．7ll）

（VAR．G（6））（VAR． 7 （FOOT））
FVISST（FH．J（（VAN 5））
ISANEINII（STAN）
EQVAVCINK（C－3（R．5）（Cl－5C： 7 ）
HAS\｛ XCR（C．－1（EGUAL（TJMES VAR． 1 VAR．2）（IJMES VAR． 3 YAR．A）））



（CR． 1 （ 1 LMES YAU． 3 VAU．A） （（CR． 2 VAR． 2 ）（ $($ CR． 3 VAR．A）
（CR．A（IJMES VAR G，VAR．J）（CR． 5 VAR． 5 ）（ $(R .6$ VAR． 7 ）
RIN TIMR A MIN． 55.3 SEC
EXAM TRY IIRE WMACT E $N$ t $/ \pi$ T $A$
$\begin{array}{lllllll} & 7905 & 977 & 1807 & 6.34 & 3.13 & 2.05\end{array}$
$0.102 \quad 0.319 \quad 0.355 \quad 0.164$ SEC AVG
1010 INSTRTS 76.7 DKLETES 59 WARNIMES 56 NE W OHAE CTS
Mhx ：SWex LENGTH 171
CORE（IREC SLKい：（5371．972）USEO（5044．60a）
F JREORJ OUI OF 259 FRONS
TFSTIS

 THFY YIAVf 3$)$



FVI JST (PEt. I (fVAL. b))


 (Cs. 2 VAK. 3 ) \{(CU. 3 VAU.A)

DUNTTMA TMIN. IRS Sit




MAIX: SMRX LENGTH ITS


## FIREO RGOUT 760 FREMS

## iEsila

(CIHE MUMFER OF SIUOINTS WHO FASSED THE AOMISSIONS IEST IS IOTECCENT OF THE
 SUCCESSFIH (ANDIISAIES IS 77 , WHAT IS IIE MIMHIER OF STIDENIS JH THE HICM scricor 37




FVI IST (FH. 1 (NAE 3) )
EOVARCTHINK (C. 3 ( 6,2 )

 (CR. 2 VAU. 3) (C.W. 3 VAU. 5 )

RLIN TJM\& $3 \mathrm{MlN}, 73.5$ ©

| EXAM | TRY | F10\% | WMACl | (/r | $1 / 1$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a\%c\% | 1/46 | nıa | 180.3 | 16.6) | 0.11 | 7.58 |
| 0.0137 | 0.178 | Ons | 0.113 | St C, |  |  |

10A5 INSARIS Tris Delet
MAX :SMFXLENSiTH ing


itsin







(VAR. 6 (PCNR))

FVI 1ST (\$H. 1 (IVAV. 7W)

HASE XRR (C. 1 (EqIIAI VAR. 1 (I IMFS YAU. 7 VAU. 3 ) )


\{CR. 1 (1 IMES VAU. 2 VAU. 3 )\} (C4. 2 VAK. 3)
 (CR.5 VAR.6.)

RUN TIME OMJN, IA. SLC

```
EXAN TRV PIRE: WMAC, 1,F I/T 1,%
5011 l:37 513 70.41 9.77 0.05 7.01
0.050t: 0.206 0.097 0.172 SHC AVG
```


MAX ENNOXLENGTH INA

FIRED S4 OUl OF 75 FROUS
TESTIG
 BOX AND IH COST O THE FECANS IN THE FOX. FOR A (ARGE BOX ISISS COS? ? 8.3.5. THF WEICNI, IN POMNOS, A AOX OF MIXPOMITS IS IHE SIM OF

 FER FONO OF ALHONOS IS \$1, ANO THE COST C PECANS ItR FOINO OF FECANS
 CHCANS IN IFE EHOX
ISI GN IC. 1 (TOLIAL VAR. 1 (AI USS VAR. 2 VAU. 3) )
(C. 7 (t QIAA VAQ. 1 ( 1 lMES YAR.A VAR. 5) $)$
(C. 3 ( $\{$ QUAL VAQ. 6 (FX USS VAQ. 7 VAD. B) $)$ )




(VAU. 7 (iHIE COST The MIMONDS IN IFF ROX)



(VAL R \{lIS MIMFI D Of FOINOS O PECANS IN TIE EOX:) (VAR 9 (3))
5 II ISI (FH. 1 ( (VAR. 2 YAR. 3)) )


HASI YRR (C. 1 (EQUAA! VAR. 1 \{PI USS YAR. 2 VAR. 3) H
(C. 7 (f qUAL VAR. 1 (TIMES YAR.A VAN. 5 )) )
(C. 3 (EQUAAI VAR. 6 (FI USS YAR. 7 VAR. 8 I)
( 0.5 (EQUAL TXICTIf NI YAR. 2 VAR. 7 ) (I IMES VAR. 10 VAR. 5 ) )








( $\mathrm{CR} . \mathrm{B}$ (1) ME S VAR. 10 VAR. 5 ) (CR. 9 VAR. 7 )
RUNTIME 20 MIN. GZOSSC

EXASH TRY \&IDF. WMACT Eff E/T TA

$0.0 \times 6.7$ 0.23n 0.570 0.178 StCAVG

MAX: :SMFX LENCTH: 135

FIRFO too OHT Of 25R SRONS
1!5117


ISI QN (C. I (EQUAL (II USS VAR. I VAR.2) VAR.3))

 (VAR.A(16))
FVI IST (CH.I (UVAR. I VAR. 7 ) $)$

HASf XCR (C-I (EQUAL (TIUSS VAR. I YAR-2) VAB.3))




RIN TJME 2 MJN : Ta. 9 SEC
EXAM TRY $F$ IRE WMACT E/F E/t T/F
35.21877394 1610 8.94 4.01 2.23
0.04970 .109 O.A4A 0.10 SECAVG

939 JNSIRIS 675 OFLETES AG WARNIMES 55 NEW OBJFCTS
MAD: : SWOX LENKTH ISt



TESTR
I(IUE GAS CONSIMPIION OF MY CAR IS ISMIIES PER GALLON. 1HF DISTAMCE BETWEEN (FOSTON AM) W: $w$ York IS 250 MJLES , WHAT IS THE MUMPER OF GALIONS OF GAS USCOON A IRIF DE TWEEN NE W YORX ANO HOSTON ?)
ISEON (C I \{IGLIAL VAR. I (TIMES YAL. 7 (QUIOTI(N) VAR. 3 VAR.A) $)$ )
( C .7 (EGIAL VAR. 5 (T JMES YAR. 6 VAk. 3 )) )
HASREFR (VAR.) (HE GAS CONSIMPIION OF MY CAR)) (VAR. 2 (15)) (VAR. 3 (MILE))

(VAL. 1 .(250))
(val. 7


EQVAVLHINE (CU.E.C. 3)










MHD: :SMF $\times 1$ +NGill In



## Ifstig












(VAL. 7 (OO)) (WhC. 5 : $: 10)\}$
FVI IST (rA. 1 (fVAL. 7 YAU. A) )








RUN TIME 7 MIN A5, : St


0.0505 0.1\%: $0.0+8$ D.175 sit Avt,


CORE (FREF.F(HL):(F.O72 - 1695) USID (11330. (617)
F 106.093 otit on 260 rheos
15ST70





 THE N!MEICD OF INJF ORUFD SND) RAS II HAS A)


(C. 4 (EQUAL (IM ISS: VAD. 7 VAL. A) (I IME Si YAV. 9 VAU. $f$ ) ))

HASEEPR (VAU. 1 (G.)



 (VAU. 9 ( $n$ 5000))
FVI $15 T$ (rel. I ( (V)N. 2 VAN. 3)) )

(C.R-5 CR. 7) (CR.F. CV. 1)






( (Q. 1 VAK. 3) (CR. 10 VAK.A) (CR. 11 VAP. 6) (CR. 7 VAK. 7 )



RUNTSME IA MIN. 71.5 SF. C .
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75:83 JRERRIS 223A DFLETES 178 WAKNIMKS I! 2 NEW OHJCTS
MAX SWer: leNKTHIINE

FIRTO 99 OUl or 758 SRONS
tistrl





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ISAM TEY itcy wUACT it t/T T,F
47AT 1331 5. 239 2258 R.3] 3.5G 7.34
$\begin{array}{lllll}0.0418 & 0.149 & 0.349 & 0.0279 & \text { SECAW; }\end{array}$
179R INSIRTS OGO TEIETES 79 WARNJEGS 70 NEW OARCTS
MAR siufx liNKity lal

F105.0 93 OII 260 PROOS
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1S( ON (C. 1 (EQUAL (MUSS VAR. \{ VAU. 2 ) VAR.3))
(C. 7 \{EQIAL VAG. 1 (CI US VAR. $A$ VAR. 7 II)

(VAR.A(1))
FVISI (FA. 1 (fvac. I VAR. 7 m )

HASCXR (C. ) (ERUAI (TUUSS VAR. 1 VAQ. 7) VAR.3))


(CR. 2 VAR. 7 ) (CQ. 3 (MUS VAR. A VAR. 7 ) (CR.A VAR. 7 )
RLW TIME 3MJN, 3.A7 SKC
EXAM IRY FIMF WMACT E/T F/T T/
3C.86 OAR $007 \quad 1670 \quad 9.17$ 3.89 2.36
0.1948 0.193 0.136 0.113 SICAVG

MAY SXISX LENCTHI 138

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IS! ON (C-I (EQIAL (TMUSS VAR. I (PI USS VAR. 7 VAR. 3)) VAK.A))

(C. 3 (fQuAL VAR. 3 truss vak. 1 VAv. 2)")


FVI IST (PR. 1 (VYAR. 1 YAR. 2 YAR.3)


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    (CR.E.C:7)(CH.EC: 3)
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    ((R.2(P) \SE VAU.) VAU.3))(CN. 3 YAU.3)
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O.05nt; 0.3%% 0.A17 0.117 SEC.Avf;
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(C-2 (fQUAL VAQ. 3 frilfe vas 1 VAv. TD)













(CR.7VAK. P) (CR.O.VAU.3)



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0.0:3% 0.1F% 0.^17 0.10% Stc, Avi;
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 HASEEPR (VALU. 1 (C)) TVAM. 7 (H)) (VAG 3 (D)) (VAG. A (1)) (VAG. 5 (3))

 (CR.7 CQ.3)




(CQ. 1 (FI US (T JMF S VAU. ? VAU.3) VAD. A) (CH. 7 VAR. A) (CR. 3 VAU. 3) (CW. A VAV. 5)
(CK.5VAL. 3) (C.U. \&. VAU.A) (CU. 7 VAN. 3)

RINTIMAK. $\triangle$ MIN. TIRSSC

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ExNN 12Y 'ff% WMAC: &,# &/T 1/%
378% 1009 519% 106.9 6.33 S.26 1.9A
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(C.) (EQIIAL VAQ. : VAR. 5)

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(VAQ.5!7)
fVIIST (rg. ( (fyak. 2)) )
EQVANTHINK (C. 3 CR-3) (CA. 0 ( 1.3 )



(C0. 7 VAR-3) (C0. 3 VAQ. 2 ) (CN. A VAR.5)
RUNT TIME 3 M IN. 36.A SEC

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0.0531 0.150 0.071 0.117 SECAVG
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MAX SUDXIENGTH IFJ
CORE (TREFFIR1): (0895.913) (ISCO (5587. F35)

FIREOR OUI © 2ROMRMS
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 WIMT IS THE WFIGHI © TIE SHIPS CARGO ग)

(S. 7 (FQLIAL VAQ. A (TIMFS YAE. 5 VAR.3) )

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(VAQ A (IIS NET WE ISNi]) (VAR.5 (15000))

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CVAlt'IINK (CR.O (R 7)
HASI XPD \{C J (EQ IAL VAR. I (TIMESVAQ. 2 VAQ. 3) ))


(CR. 3 \{1 TMFS YAQ. 5 VAR. 3)\} (CR.A VAR.3)

RUN TIME 2 MIN 97.3 SCC

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ExaM TPY EiPF WMACT E/f E/T T.f
3430 857 30.) 1550 0.07 4.05 7.74
G.cag9 0.202 a.n.2 0.111 SFC AVG
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CORS (IREEFUL) (5574. 1037)(1510 (A90g. 511)
FIDTO 79 OUT Of 760 IhOOS
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[^0]:    - This is being done by James Gillogly, as part of a Ph.D. thesis, in preparation.
    * PS is used to abbreviate production system in this paper; PSs is its plural; $P$ will be used to abbreviate production, plural Ps.

[^1]:    - Ns are given labels of the form $\mathrm{N}+$ initial of a P group + number + occasionally a letter. These are defined by separate KSs presented below.
    Some of the connections between KSs and between KSs and predicates may require free interpretation and detective work on the part of the reader. It is beyond the present scope and purpose to be more precise.

[^2]:    - Perhaps easy extensions are more properly considered to be assimilation, since they require little structural change.

[^3]:    - We will ignore whether Ps themselves are interpreted or compiled, given the understanding-system level of this discussion.

