ON THE CATEGORIES OF TOPOLOGICAL ALGEBRA

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In an earlier report [3] we presented a general theory of the categories encountered in general topology and topological algebra. We pointed out that these categories are fibred categories with fibres in the category of complete ordered sets. We considered general properties of such categories, in particular the lifting of functors and of adjoint functor pairs, and some specifically topologically properties such as point separation axioms.

In the present report, we use the language and the techniques of [3] to obtain a general construction of the categories encountered in topological algebra. In fact, we solve a more general problem. Given an operational category $\underline{A}$ over a category $\underline{C}$ and a fibred category $\underline{C}^s$ over $\underline{C}$, we obtain a new category $\underline{A}^r$ which is fibred over $\underline{A}$ and operational over $\underline{C}^s$. $\underline{A}^r$ is also characterized, up to equivalence, by a universal property of pullback type. Specializing to $\underline{C} = \underline{Ens}$, the category of sets, and to a category $\underline{A}$ of algebras, we obtain from the category $\underline{Ens}^s$ of topological spaces, or of limit spaces, or of uniform spaces, a category $\underline{A}^r$ of topological algebras, or of limit algebras, or of uniform algebras.

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The fact that $\underline{A}^r$ is fibred over $\underline{A}$ has an important consequence: all categorical limits and colimits, such as products and coproducts, and projective and injective limits, which exist in $\underline{A}$ can be lifted immediately from $\underline{A}$ to $\underline{A}^r$. This is of course well-known, but a general proof is only possible from a general construction such as the one given here. Thus special liftings of categorical limits and colimits to categories of algebras with topological structure are "discovered" again and again, and sometimes even published.

Another important consequence is this. If free algebras exist in $\underline{A}$, i.e. if the forgetful functor $U : \underline{A} \longrightarrow \underline{C}$ has a left adjoint functor $F : \underline{C} \longrightarrow \underline{A}$, then F can be lifted to a functor $\Phi : \underline{C}^s \longrightarrow \underline{A}^r$, left adjoint to the lifted forgetful functor $\Upsilon : \underline{A}^r \longrightarrow \underline{C}^s$. Special cases of this, e.g. that of topological groups, seem to be well-known, but the author has been unable to produce a printed reference. The general theorem seems to be new.

The construction of $\underline{A}^r$ depends not only on the categories $\underline{A}$, $\underline{C}$, and $\underline{C}^r$, but also on the lifting of the domain and range functors of the operations of $\underline{A}$ from $\underline{C}$ to $\underline{C}^s$. Thus we obtain among our examples not only the usual topological or limit algebras with "jointly continuous" operations, but also topological or limit algebras with "separately continuous" operations.

0. Preliminaries. We shall use the notations, definitions and results of [3], except for some trivial changes such as $\underline{C}^s$ for $\underline{C}^T$, and "left adjoint" for "coadjoint". If a functor $\Phi : \underline{C}^s \longrightarrow \underline{B}^t$ of fibred categories is fibred over a functor $F : \underline{C} \longrightarrow \underline{B}$, then we say also that Φ lifts F. Such a functor Φ is of the form $\Phi(C,x) = (F\,C, \varphi_C\, x)$ for objects, with order preserving maps $\varphi_C : s\,C \longrightarrow t\,F\,C$, one for each object C of $\underline{C}$. These maps have been

studied in [3; sec. 5]; we shall call them the <u>fibre morphisms</u> of Φ. If the fibre morphisms of Φ satisfy $\varphi_A f^* = (F f)^* \varphi_B$ for every morphism $f : A \longrightarrow B$ of $\underline{C}$, then we say that they <u>preserve inverse images</u>.

Consider now two functors $\Phi : \underline{A}^s \longrightarrow \underline{B}^t$ and $\Psi : \underline{A}^s \longrightarrow \underline{B}^t$ of fibred categories, fibred over functors $F : \underline{A} \longrightarrow \underline{B}$ and $G : \underline{A} \longrightarrow \underline{B}$, with fibre morphisms φ_A and ψ_A respectively. Let $\mu : F \longrightarrow G$ be a natural transformation. If $\mu_A : (F A, \varphi_A x) \longrightarrow (G A, \psi_A x)$ in $\underline{B}^t$ for every object (A,x) of $\underline{A}^s$, then these morphisms clearly define a natural transformation $\bar{\mu} : \Phi \to \Psi$ with the property that $P\bar{\mu} = \mu P$ for the projection functors of $\underline{A}^s$ and $\underline{B}^t$. We say in this case that $\bar{\mu}$ is <u>fibred</u> over μ or that $\bar{\mu}$ <u>lifts</u> μ. Clearly, there can be at most one natural transformation $\bar{\mu} : \Phi \to \Psi$ which lifts a given natural transformation $\mu : F \longrightarrow G$, and this natural transformation $\bar{\mu}$ exists if and only if $\varphi_A x \le \mu_A^* \psi_A x$ for every object (A,x) of $\underline{A}^s$. We need the following useful converse of this.

<u>Proposition 0.1.</u> <u>With the notations of the preceding paragraph, every natural transformation</u> $\bar{\mu} : \Phi \to \Psi$ <u>is fibred over a natural transformation</u> $\mu : F \to G$.

<u>Proof.</u> We put $\mu_A = \bar{\mu}(A, \omega_A) : \Phi\omega A \to \Psi\omega A$ for an object A of $\underline{A}$. Then the morphisms $\mu_A : F A \longrightarrow G A$ of $\underline{B}$ define a natural transformation $\mu = P\bar{\mu}\omega : F \longrightarrow G$. For every object (A,x) of $\underline{A}^t$, we have $1_A : (A,x) \longrightarrow (A,\omega_A)$ in $\underline{A}^s$, and thus $1_A\bar{\mu}(A,x) = \mu_A 1_A : \Phi(A,x) \to \Psi(A,\omega_A)$ by naturality of $\bar{\mu}$. But then $\bar{\mu}(A,x) = \mu_A$, and $\bar{\mu}$ lifts μ $\square$.

1. Operational categories. Operational categories were defined in [2], but we give here a new definition which differs from that of [2] in several respects. Every category of algebras is an operational category in this sense, over the category Ens of sets. Another operational category over Ens is the category of compact Hausdorff spaces. Categories of modules, or more generally categories of groups with operators, can be considered as operational categories over the category of abelian groups and the category of groups. Operational categories over a fibred category $\underline{C}^S$ will occur in sec. 2.

From now on, let $\underline{C}$ be a given base category. We define a category over $\underline{C}$ as a pair $(\underline{A}, U)$ consisting of a category $\underline{A}$ and a functor $U : \underline{A} \longrightarrow \underline{C}$. By the usual abuse of language, we shall call $\underline{A}$ a category over $\underline{C}$, the functor U being understood. Example: a concrete category is a category over Ens, the category of sets, with a faithful "forgetful" functor U.

If $\underline{A}$ is a category over $\underline{C}$, then we define an operation on $\underline{A}$ over $\underline{C}$ as a triple (D, μ, R) consisting of functors D and R from $\underline{C}$ to $\underline{C}$, the domain functor and range functor of the operation, and a natural transformation $\mu : D\,U \longrightarrow R\,U$. For $\underline{C} = \underline{Ens}$, this specializes the concept of [2], where the $\mu_A : D\,U\,A \longrightarrow R\,U\,A$ were relations instead of mappings and naturality of μ was relaxed. For this case, our present operations are, in the language of [2], single-valued and everywhere defined. A definition of operations generalizing both the present definition and that of [2] could be obtained by replacing Ens not by a category $\underline{C}$ as in this report, but by a bicategory $\underline{S}$ in the sense of Bénabou [1]. However, we do not wish to pursue this here.

Example 1.1. Let $\underline{C} = \underline{Ens}$, the category of sets, and let $R = \mathrm{Id}\,\underline{Ens}$, If $D\,X = X^n$ and $D\,f = f^n : X^n \longrightarrow Y^n$ for a set X and a mapping $f : X \longrightarrow Y$, then an operation (D,μ,R) on a category $\underline{A}$ over $\underline{Ens}$ is an n-ary operation on $\underline{A}$ in the usual sense. It associates with every object A of $\underline{A}$, with underlying set $U\,A$, a mapping $\mu_A : (U\,A)^n \longrightarrow U\,A$, and every morphism $f : A \longrightarrow B$ of $\underline{A}$, with underlying mapping $U\,f$, satisfies $\mu_B\,(U\,f)^n = (U\,f)\mu_A$. Here, n may be a natural number or a transfinite ordinal number.

Example 1.2. Let again $\underline{C} = \underline{Ens}$ and $R = \mathrm{Id}\,\underline{Ens}$. For a set X, let $D\,X$ be the set of all ultrafilters on X, and for a mapping $f : X \longrightarrow Y$ and $\mathcal{U} \in D\,X$, let $(D\,f)\mathcal{U}$ be the ultrafilter on Y generated by the sets $f(A)$, $A \in \mathcal{U}$. If $\underline{A}$ is the category of compact Hausdorff spaces, then convergence of ultrafilters defines an operation $(D, \lim, R)$ on $\underline{A}$ over $\underline{Ens}$.

Example 1.3. Let $\mathcal{M} = (D^i, \mu^i, R^i)_{i \in I}$ be a family of operations on $\underline{A}$ over $\underline{C}$. If $\underline{C}$ has I-products, then let $D\,A = \prod D^i\,A$ for an object A of $\underline{C}$, and $D\,f = \prod D^i\,f : D\,A \longrightarrow D\,B$ for a morphism $f : A \longrightarrow B$. This defines a product functor $D = \prod D^i$. We define the product functor $R = \prod R^i$ in the same way, and we put $\mu_A = (\prod \mu^i)_A = \prod \mu^i_A : D\,U\,A \longrightarrow R\,U\,A$. This defines a product operation (D,μ,R) on $\underline{A}$ over $\underline{C}$. Thus we may often replace a family $\mathcal{M}$ of operations by a single product operation (D,μ,R).

Definition 1.4. Let $\underline{A}$ be a category on $\underline{C}$, and let $\mathcal{M} = (D^i,\mu^i,R^i)_{i \in I}$ be a family of operations on $\underline{A}$ over $\underline{C}$. We say that $\underline{A}$ is $\mathcal{M}$-operational over $\underline{C}$ if the following two conditions are satisfied.

1.4.1. Every object A of $\underline{A}$ is completely determined by the object $U\,A$ of $\underline{C}$ and the operations $\mu^i_A : D^i\,U\,A \longrightarrow R^i\,U\,A$, $i \in I$.

1.4.2. For objects A, B of $\underline{A}$ and a morphism $g : U A \longrightarrow U B$ of $\underline{C}$ such that $\mu_B^i (D^i g) = (R^i g)\mu_A^i$ for all $i \in I$, there always is exactly one morphism $f : A \longrightarrow B$ of $\underline{A}$ such that $g = U f$.

It follows immediately that the functor $U : \underline{A} \longrightarrow \underline{C}$ is faithful for an operational category $\underline{A}$ over $\underline{C}$.

If $\underline{C}$ has I-products, and if all projections $\pi_{DX}^i : D X \longrightarrow D^i X$, for objects X of $\underline{C}$, are epimorphisms of $\underline{C}$, then one sees easily that $\underline{A}$ is operational for a family $M = (D^i, \mu^i, R^i)_{i \in I}$ of operations if and only if $\underline{A}$ is operational for the single product operation (D, μ, R) of 1.3. Thus we shall restrict ourselves in what follows mostly to categories with a single operation. Our results and proofs can easily be extended to categories with a family of operations; only the notations are somewhat more messy.

Except for the fact that our definition of an operation is more restrictive than that of [2], our definition of an operational category is more general than that of [2]. For $\underline{C} = \underline{Ens}$, the only situation considered in [2], we do not require that domain functors and range functors preserve set inclusions as we did in [2], and we impose no condition regarding subobjects in $\underline{A}$.

2. Operational fibred categories. We combine in this section a fibred category $\underline{C}^s$ over $\underline{C}$, as defined in [3], with an operational category $\underline{A}$ over $\underline{C}$, as defined above, to obtain a "pullback" category $\underline{A}^r$, fibred over $\underline{A}$ and operational over $\underline{C}^s$, with an operation which lifts the operation of $\underline{A}$.

More exactly, we require the following date.

2.1.1. $\underline{C}^s$ is a fibred category over $\underline{C}$.

2.1.2. $\underline{A}$ is a category over $\underline{C}$, with "forgetful" functor $U : \underline{A} \longrightarrow \underline{C}$, and with an operation (D, μ, R) over $\underline{C}$.

2.1.3. Functors Δ and P from $\underline{C}^s$ to $\underline{C}^s$ are given, fibred over D and R respectively, with fibre morphisms δ_X and ρ_X.

2.1.4. The structure morphisms ρ_X of P preserve infima and inverse images (see [3], sec. 2, and sec. 0 above).

The last requirement, while essential for our theory, is not as troublesome as it may seem. For a single operation, P is usually either the identity functor $\mathrm{Id}\ \underline{C}^s$ or a constant functor. In both cases, 2.1.4 is satisfied. If we replace a family $(D^i, \mu^i, R^i)_{i \in I}$, with lifted functors Δ^i and P^i, by the product operation (D, μ, R), then we replace the functors Δ^i and P^i by the product functors $\Delta = \prod \Delta^i$ and $P = \prod P^i$ which lift D and R. If every functor P^i satisfies 2.1.4, then the functor $P = \prod P^i$ also satisfies 2.1.4.

Notations 2.2. For an object A of $\underline{A}$, we denote by r A the set of all $x \in$ s U A such that $\mu_A : (D\,U\,A, \delta_{UA}\, x) \longrightarrow (R\,U\,A, \rho_{UA}\, x)$ in $\underline{C}^s$, i.e. $\delta_{UA}\, x \leq \mu_A^* \rho_{UA}\, x$ in s D U A. We endow r A with the order relation inherited from s U A, and we denote by $\upsilon_A : r\,A \longrightarrow s\,U\,A$ the inclusion mapping. For $f : A \longrightarrow B$ in $\underline{A}$, we define $r\,f : r\,B \longrightarrow r\,A$ by putting $\upsilon_A\,(r\,f) = (s\,U\,f)\,\upsilon_B$. We put $s\,U\,f = (U\,f)^*$, but not $r\,f = f^*$.

Proposition 2.3. For an object A of $\underline{A}$, r A is a complete lattice, closed under infima in s U A. For $f : A \longrightarrow B$ in $\underline{A}$ and $y \in r\,B$, the element $(U\,f)^*\,y$ of s U A is in r A, and thus $(r\,f)\,y$ is well defined.

Proof. For a family $(x_i)_{i \in I}$ of elements of r A, we have

$$\delta_{UA}\,(\bigcap x_i) \leq \bigcap (\delta_{UA}\, x_i) \leq \bigcap (\mu_A^* \rho_{UA}\, x_i) = \mu_A^* \rho_{UA}\,(\bigcap x_i) ,$$

using 2.1.4 and the fact that $\mu_A{}^*$ preserves infima. Similarly,

$$\delta_{UA}\,(U\,f)^*\,y \leq (D\,U\,f)^*\,\delta_{UB}\,y \leq (D\,U\,f)^*\,\mu_B{}^*\,\rho_{UB}\,y$$

$$= \mu_A{}^*\,(R\,U\,f)^*\,\rho_{UB}\,y = \mu_A{}^*\,\rho_{UA}\,(U\,f)^*\,y \quad ,$$

for $f : A \longrightarrow B$ in $\underline{A}$ and $y \in r\,B$ ▯

Theorem 2.4. The data of 2.1 and 2.2 define a fibred category $\underline{A}^r$ over $\underline{A}$ and a fibred functor $\Upsilon : \underline{A}^r \longrightarrow \underline{C}^s$ over $U : \underline{A} \longrightarrow \underline{C}$, with fibre morphisms υ_A which preserve infima and inverse images.

Proof. The first part of the proof of 2.3 shows that $r\,A$ is a complete lattice for every object A of $\underline{A}$, and that $\upsilon_A : r\,A \longrightarrow s\,U\,A$ preserves infima. By the second part of the proof, $r\,f : r\,B \longrightarrow r\,A$ is well defined for $f : A \longrightarrow B$ in $\underline{A}$ by $\upsilon_A\,(r\,f) = (U\,f)^*\,\upsilon_B$. The maps $r\,f$ of the category Cord of complete ordered sets clearly preserve infima and define a contravariant functor $r : \underline{A}^* \longrightarrow$ Cord. Thus the fibred category $\underline{A}^r$ is well defined. The maps υ_A preserve inverse images by the definition of the maps $r\,f$, and thus they are the fibre morphisms of a functor $\Upsilon : \underline{A}^r \longrightarrow \underline{C}^s$ which lifts the functor $U : \underline{A} \longrightarrow \underline{C}$ ▯

Theorem 2.5. The operation (D,μ,R) of $\underline{A}$ over $\underline{C}$ can be lifted to an operation $(\Delta,\bar{\mu},P)$ of $\underline{A}^r$ over $\underline{C}^s$, where $\bar{\mu}$ lifts μ. If $\underline{A}$ is operational over $\underline{C}$ by (D,μ,R), then $\underline{A}^r$ is operational over $\underline{C}^s$ by the lifted operation $(\Delta,\bar{\mu},P)$.

Proof. We have $\mu_A : \Delta(U\,A,\,x) \longrightarrow P(U\,A,\,x)$ in $\underline{C}^s$ for every object (A,x) of $\underline{A}^r$ by the construction of $\underline{A}^r$, and thus μ can be lifted to a natural transformation $\bar{\mu} : \Delta Y \to P Y$, so that the operation $(\Delta, \bar{\mu}, P)$ on $\underline{A}^r$ over $\underline{C}^s$ is defined. An object (A,x) of $\underline{A}^r$ is determined by $Y(A,x) = (U\,A,x)$ and $\bar{\mu}_{(A,x)} = \mu_A$ if A is determined by $U\,A$ and μ_A. If $g : Y(A,x) \to Y(B,x)$ in $\underline{C}^s$ satisfies $(P\,g)\bar{\mu}_{(A,x)} = \bar{\mu}_{(B,y)}(\Delta\,g)$, then $(R\,g)\mu_A = \mu_B\,(D\,g)$ in $\underline{C}$. Thus $g = U\,f$ for a unique $f : A \to B$ in $\underline{A}$ if $\underline{A}$ is operational. But then $(r\,f)\,y = (U\,f)^*\,y = g\,y$, and $x \leq (r\,f)\,y$. This shows that $g = Y f$ for $f : (A,x) \to (B,y)$ in $\underline{A}^r$ iff $g = U\,f$ for $f : A \to B$ in $\underline{A}$, so that $\underline{A}^r$ satisfies 1.4.2 if $\underline{A}$ does $\square$

Finally, we characterize $\underline{A}^r$ by a universal property.

<u>Theorem 2.6</u>. <u>Let</u> $\underline{B}^t$ <u>be a fibred category, and let</u> $F : \underline{B} \to \underline{A}$ <u>and</u> $\Psi : \underline{B}^t \to \underline{C}^s$ <u>be functors, with</u> Ψ <u>fibred over</u> $U\,F$. <u>Then</u> $\Psi = Y\Phi$ <u>for a functor</u> $\Phi : \underline{B}^t \to \underline{A}^r$ <u>which lifts</u> F <u>if and only if</u> $\mu F : D\,U\,F \to R\,U\,F$ <u>can be lifted to a natural transformation</u> $\nu : \Delta\Psi \to P\Psi$. <u>If the functor</u> Φ <u>exists, then it is unique</u>.

<u>Proof</u>. Let Ψ have fibre morphisms $\psi_B : t\,B \to s\,U\,F\,B$. If Φ exists, with fibre morphisms φ_B, then $\bar{\mu}\Phi : \Delta\Psi \to P\Psi$ lifts $\mu F : D\,U\,F \to R\,U\,F$, and $\varphi_B\,y = \upsilon_{FB}\varphi_B\,y = \psi_B\,y$ for an object (B,y) of $\underline{B}^t$. Thus the data determine the fibre morphisms φ_B, and hence Φ, uniquely. Conversely, if μF can be lifted, then $\mu_{FB} : \Delta(U\,F\,B, \psi_B\,y) \to P(U\,F\,B, \psi_B\,y)$ for every object (B,y) of $\underline{B}^t$. Thus ψ_B maps $t\,B$ into $r\,F\,B$, and $\psi_B = \upsilon_{FB}\varphi_B$ for a map $\varphi_B : t\,B \to r\,F\,B$. For $f : B' \to B$ in $\underline{B}$ and $y \in t\,B$, it follows that

$$\varphi_{B'}\,f^*\,y = \psi_{B'}\,f^*\,y \leq (U\,F\,f)^*\,\psi_B\,y = (U\,F\,f)^*\,\upsilon_{FB}\varphi_B\,y$$
$$= \upsilon_{FB'}\,(r\,f)\varphi_B\,y = (r\,f)\varphi_B\,y\ .$$

Thus the maps φ_B are the fibre morphisms of a functor $\Phi : \underline{B}^t \longrightarrow \underline{A}^r$ which lifts F, and clearly $Y\Phi = \Psi$ for this functor[]

3. Complements and examples. We use again the notations of sec. 2.

Theorem 3.1. A diagram $\Delta : \underline{I} \longrightarrow \underline{A}^r$ has a limit or colimit (L,u) in $\underline{A}^r$ if and only if the diagram $P\Delta : \underline{I} \longrightarrow \underline{A}$ has L as its limit or colimit in $\underline{A}$.

Proof. This is [3; 3.4] and its dual, applied to our special case[]

Theorem 3.2. If the functor $U : \underline{A} \longrightarrow \underline{C}$ has a left adjoint functor $F : \underline{C} \longrightarrow \underline{A}$, with front adjunction $\beta : \mathrm{Id}\,\underline{C} \longrightarrow U\,F$, then the functor $Y : \underline{A}^r \longrightarrow \underline{C}^s$ has a left adjoint functor $\Phi : \underline{C}^s \longrightarrow \underline{A}^r$ which lifts F, with a front adjunction $\overline{\beta} : \mathrm{Id}\,\underline{C}^s \longrightarrow Y\Phi$ which lifts β.

Proof. This follows immediately from [3; 5.3] and its proof[]

Example 3.3. If $\underline{C} = \underline{\mathrm{Ens}}$, the category of sets, then the range functor $R = \mathrm{Id}\,\underline{\mathrm{Ens}}$ of an n-ary operation is of course lifted to $P = \mathrm{Id}\,\underline{\mathrm{Ens}}^s$ which satisfies 2.1.4. The domain functor D of such an operation, with $D\,X = X^n$ for a set X, can be lifted to a functor Δ on $\underline{\mathrm{Ens}}^s$, by letting $\Delta(X,x)$ be the set $D\,X = X^n$ with the product structure in $s\,X^n$ induced by $x \in s\,X$. If we apply this to a category $\underline{A}$ of algebras, then $\underline{A}^r$ is the corresponding category of algebras with "jointly continuous" operations, with continuous algebra homomorphisms as morphisms. If $\underline{\mathrm{Ens}}^s$ is the category of topological spaces, or of limit spaces, or of uniform spaces, then $\underline{A}^r$ is a category of topological algebras, or of limit algebras, or of uniform algebras respectively.

Example 3.4. We consider again a fibred category $\underline{Ens}^s$ over sets. Let $(X_i, x_i)_{i \in I}$ be a family of objects of $\underline{Ens}^s$, and let $\pi_i : \prod X_i \longrightarrow X_i$ be the projection mapping. A mapping $f : X_i \longrightarrow \prod X_i$ is called an injection of X_i into the set product if $\pi_i f = \mathrm{id}\, X_i$, and $\pi_j f$ is a constant mapping for all $j \neq i$ in I. The finest structure $x \in s(\prod X_i)$ for which all injection mappings $f : (X_i, x_i) \longrightarrow (\prod X_i, x)$ are continuous, for every $i \in I$, is called the structure of separate continuity of the product $\prod X_i$. Morphisms $f : (\prod X_i, x) \longrightarrow (Y, y)$ of $\underline{Ens}^s$, for this structure x, are "separately continuous" in each "variable" $\xi \in X_i$.

From the domain functor D of an n-ary operation, we obtain a functor Δ' on $\underline{Ens}^s$ which lifts D by letting $\Delta'(X,x)$ be the set $D X = X^n$ with the structure of separate continuity, constructed from the structures $x_i = x$ of $X_i = X$, for $0 \leq i < n$. If we apply this to a category $\underline{A}$ of algebras, then $\underline{A}^r$ is a category of algebras with "separately continuous" operations, again with continuous algebra homomorphisms as morphisms.

Example 3.5. A special case of 3.3 occurs if $\underline{Ens}^s$ is the category of equivalence relations (see [3; 4.5]) and $\underline{A}$ a category of algebras. Then $r\,A$, for an algebra A in $\underline{A}$, is the complete lattice of congruence relations on A, and $\underline{A}^r$ is the category of algebras in $\underline{A}$ with congruence relations.

In 3.3 and 3.4, n can be a natural number or a transfinite ordinal number. If n is finite and $\underline{Ens}^s$ the category of equivalence relations, then the functors Δ of 3.3 and Δ' of 3.4 are the same. If n is transfinite, and $\underline{Ens}^s$ again the category of equivalence relation, then these functors are not the same.

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