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On the Growth and Decay of Discontinuities in Fluids with Internal State Variables

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1. Introduction

Singular surfaces, such as shock waves and acceleration waves, can propagate in materials for which the present stress depends on the histories of the strain and entropy in a manner compatible with a smoothness postulate called the "principle of fading memory", and a theory of such waves of discontinuity exists.[1-5] In that theory explicit expressions have been derived for the velocity and time-dependence of the amplitude of acceleration waves[2] and higher-order waves[5].[6]

Recently B. S. H. Rarity has shown us a manuscript in which he discusses the propagation of acceleration waves in an ideal gas having a single, finite, relaxation time for the exchange of energy between translational and vibrational modes of molecular motion.[7] Using Johannesen's "heat-sink analogy" for the flow of such a gas, Rarity shows that the amplitude of a plane-compressive acceleration wave which has been propagating since time $t = 0$ into a region at rest will

become infinite at a finite time t_c provided the initial amplitude exceeds a critical value which depends on the relaxation time of the gas. Although the method used by Rarity does not yield an expression for the time-dependence of the amplitude, he does obtain a formula for t_c. The existence of a critical amplitude for acceleration waves and the form of Rarity's expression for t_c are completely analogous to known results[2] in the theory of waves in materials with memory. This observation has suggested to us the possibility of developing the theory of materials with internal variables along lines which have been explored in the general theory of materials with fading memory. Elsewhere we have attempted a unified treatment of the thermodynamics of materials with relaxing internal state variables.[8] Here we report calculations of the growth and attenuation of waves in such materials. The emphasis is laid on plane-longitudinal waves advancing into homogeneous regions at equilibrium. The materials considered are fluids for which the specific internal energy ϵ (per unit mass), the pressure p, and the temperature ϑ are determined through constitutive equations when the specific volume $x > \equiv 1/p$, the specific entropy $r\backslash$ (per unit mass), and N internal (or "hidden") state variables $\alpha_1,\ldots,\alpha_N$ are specified at the point. We assume that the material time-derivative of each of the a_i is given by a function h_i of $x>$, $r\backslash$, and the complete internal state $\{\alpha_1,\ldots,\alpha_N\}$:

$$\dot{\alpha}_i = h_i(\upsilon,\eta,\alpha_1,\ldots,\alpha_N), \qquad i=1,\ldots,N. \tag{1.1}$$

No assumptions of linearity are made for either the functions h_i or for the dependence of €, p, and 8 on T]; *X),* or the Ot_j.

Our constitutive equations appear sufficiently general to include as special cases those[9] of theories which explain pressure-volume relaxation phenomena by postulating reactions of decomposition and association; in such applications each a_i becomes the degree of advancement of a chemical reaction. When our theory is applied to gases with finite rates of exchange of energy between translational and internal modes of molecular motion,[10] the number a_i should be interpreted as the fraction of energy in the ith internal mode, and *0* should be identified with the translational temperature (i.e. the "active-mode" temperature).

2. Basic Assumptions

In the present theory a material is characterized by the **following** constitutive equations:

$$\left.\begin{aligned} \epsilon &= \tilde{\epsilon}(\upsilon,\eta,\underset{\sim}{\alpha}), \\ P &- \tilde{P}0\hat{}Sf), \\ \theta &= \tilde{\theta}(\upsilon,\eta,\underset{\sim}{\alpha}), \end{aligned}\right\} \quad <^{2a}> \qquad (2.2)$$

$$\dot{\underset{\sim}{a}} - \underset{\sim}{h}0\hat{}a), \qquad (2.2)$$

where $\dot{\underset{\sim}{a}}$ is the material time-derivative of the internal state vector $a \ll \{a, , \ldots \underline{QL}.\}$. Shearing stresses are absent, and it is assumed that the material does not conduct heat. The response functions $\tilde{e}$, $\tilde{p}$, $\tilde{\theta}$ in (2.1) are not independent, for the second law of thermodynamics requires[11] that $\tilde{\epsilon}$ determine $\tilde{p}$ and $\tilde{\theta}$ through the pressure relation,

$$\tilde{P} - -dj, \qquad (2\text{-}3)$$

and the temperature relation,

$$\tilde{\theta} \ll b\tilde{e}, \qquad (2.4)$$

with d_{υ} and $\hat{}_{\eta}$ denoting ordinary partial derivatives. The second law also requires that £ and $\underset{\sim}{h}$ obey the following internal dissipation inequality:

$$\partial_{\underset{\sim}{\alpha}}\tilde{\epsilon}(\upsilon,\eta,\underset{\sim}{\alpha})\cdot\underset{\sim}{h}(\upsilon,\eta,\underset{\sim}{\alpha}) \geq 0 \qquad (2.5)$$

for all triplets (x, t) of $\mathcal{R}$. Here $\partial_{\alpha}\tilde{\epsilon}(v,\eta,\alpha)$ is the N-vector with components

$$\left(\partial_{\alpha}\tilde{\epsilon}(v,\eta,\alpha)\right)_i = \partial_{\alpha_i}\tilde{\epsilon}(v,\eta,\alpha_1,\ldots,\alpha_N), \tag{2.6}$$

and "·" has its usual meanings for any two N-vectors ξ and ζ

$$\xi \cdot \zeta = \xi_1\zeta_1 + \cdots + \xi_N\zeta_N. \tag{2.7}$$

By an equilibrium state we mean a triplet (x, r, a) such that (i)

$$h(v, \eta, a) = 0, \tag{2.8}$$

and (ii) the solution $\alpha(t) = \alpha$ of the autonomous differential equation

$$\dot{\alpha}(t) = h\left(v_o, \eta_o, \alpha(t)\right) \tag{2.9}$$

is (locally) asymptotically stable. It follows that

$$\partial_{\alpha}\tilde{\epsilon}(v_o, \eta_o, \alpha_o) = 0 \tag{2.10}$$

at every equilibrium state.

We here consider plane longitudinal motions of the type usually studied in shock tubes. Each such one-dimensional motion is described by a scalar function $x = x(X,t)$ giving the location at time t of the material point which has the position X when the body is in a fixed reference configuration $\mathcal{B}$ with uniform mass density ρ^o. It is convenient to identify each material point of the flowing material with its position

in Ω. The velocity v and the specific volume u of X at time t are given by

$$v = \dot{c}>_t x(X,t), \qquad \backslash> - \wedge - d_x x(X,t), \tag{2.11}$$

o

In the absence of body forces and heat transfer by radiation or conduction, the laws of balance of momentum and energy become

$$d_x p + P_o \dot{v} = 0, \tag{2.12}$$

$$\dot{\epsilon} + p\dot{v} = 0. \tag{2.13}$$

It follows from (2.1)-(2.4) and (2.13) that

$$\theta\dot{T} + \langle d_a \tilde{c}).\theta\dot{f} - 0. \tag{2.14}$$

By [a] wave we mean a propagating singularity, i.e. a singularity that moves relative to the material. Let Y_t be the material point, labeled by its position in the reference configuration, at which the wave is to be found at time t. The location in space of the wave at time t is the place

$$y_t - x(Y_t,t). \tag{2.15}$$

Th[e] wave velocity u is just the rate of advance of the wave as seen by an observer at rest:

$$u(t_> - \frac{d}{dT}\wedge t - \frac{\epsilon}{dV}\wedge t\,!\wedge - \tag{2.16}$$

If the material velocity v happens to be continuous across the wave, then we can define a speed of propagation c for the wave by the formula

$$c(t) = |u(t)-v(Y_t,t)|, \qquad (2.17)$$

c is the speed of the wave relative to the material point instantaneously situated upon it. If f(X,t) is continuous in X except for a jump discontinuity at $X = Y_t$, we define the jump in f across the wave to be

$$[f] \overset{\text{def}}{=} f(Y_t,t) - f(Y_t,t). \qquad (2.18)$$

For definiteness we assume that $u(t) - v(Y_t^+,t)$ is positive, so that the wave moves faster than the material immediately ahead of it, and $[f]$ is the increase in f experienced at a given point X at the moment that the wave passes through X.

We shall here assume that x(X, t) is a continuous function of (X, t) for all (X,t) and that all the derivatives of x(X, t), T(X, t), and $\mathbf{f}$(X, t) are continuous at all points away from (Y_t, t) and suffer nothing worse than jump discontinuities across the wave.

We say that a wave is of order N if the N^{th}-order derivatives of x(X, t) suffer non-zero jumps across the wave, while all the derivatives of x(X, t) of order less than N and all derivatives of TJ(X, t) and $\mathbf{a}$(X, t) of order less than N-l are continuous across the wave. Shock waves are of order 1. Waves of order 2 are called acceleration waves, while waves of order 3 or higher are called mild discontinuities.

3. Elementary Properties of Acceleration Waves

Across a wave of order 2 we have

$$[\![\dot{v}]\!] \neq 0, \qquad [\![\dot{v}]\!] \neq 0, \qquad [\![\partial_X v]\!] \neq 0. \tag{3.1}$$

but

$$[\![\dot{v}]\!] \neq 0, \qquad [\![\dot{v}]\!] \neq 0, \qquad [\![\partial_X v]\!] \neq 0. \tag{3.2}$$

For the theory of acceleration waves it suffices to assume that the response function $\tilde{\epsilon}$ of (2.1) is of class C^3 (i.e. 3-times continuously differentiable) while $\tilde{h}$ in (2.2) is of class C^2; it then follows from (2.3) and (2.4) that p and 0 are of class C . Thus (2.1) and (3.1) imply

$$[\![\mathbf{P}]\!] = 0 \tag{3.3}$$

while (2.2) yields

$$[\![\mathbf{M}]\!] = 0. \tag{3.4}$$

Further, (2.14), $(3.3)_2$ and (3.4) imply that $i^\wedge$ is continuous across the wave[13]:

$$[\![\dot{\eta}]\!] = 0. \tag{3.5}$$

Basic to our present subject is Maxwell's theorem[14]: If $f(X,t)$, $d_X f(X,t)$, and $f(X,t) = S_{fc} f(X,t)$ suffer at worst, jump discontinuities

across (Y_t,t) and are continuous functions of (X,t) everywhere else, then

$$[f] = 0 \implies [\dot{f}] = -U[\nabla f], \qquad (3.6)$$

where

$$U(t) = \wedge Y_t . \qquad (3.7)$$

In view of this theorem, $(3.1)_{4\&5}$, (3.4), and (3.5) yield, for $U \neq 0$,

$$[\nabla f] = 0. \qquad (3.8)$$

It follows from (2.1) that

$$\partial_X p = (\partial_v \tilde{p})\partial_X v + (\partial_\eta \tilde{p})\partial_X \eta + (\partial_{\underset{\sim}{\alpha}} \tilde{p})\cdot\partial_X \underset{\sim}{\alpha}. \qquad (3.9)$$

Therefore, by (3.1), (3.8), and the fact that the functions, $d_\eta \tilde{p}$ and $d_a \tilde{p}$ are continuous, we have

$$[\partial_X p] = (\partial_v \tilde{p})[\partial_X v], \qquad (3.10)$$

If in Maxwell's theorem we put, successively, $f(X,t) = v(X,t)$ and $f(X,t) = S_X x(X,t)$, and use $(2.11)_2$, we obtain the following <u>compatibility conditions for an acceleration wave</u>:

$$[\dot{v}] = -U[d_v v] = -\rho U[\dot{\imath}] = \rho U^2 M , \qquad (3.11)$$

To relate U to the speed of propagation c we need merely note that (2.11),

(2.15), (2.16), and (3.7) yield

$$u(t) = U(t)p_0V>(Y_t,t) + v(Y_t,t), \tag{3.12}$$

and, therefore, by (2.17)

$$U(t) = \frac{c(t)}{\rho_o v(Y_t,t)} . \tag{3.13}$$

Thus, by (2.13), (3.10), (3.11), and (3.13), the speed of propagation of an acceleration wave Is given by[15]

$$c(t)^2 = -U^2\wedge\tilde{p}(\wedge,_n,\underset{\sim}{a}), \tag{3.14}$$

with x>, r\, and a evaluated &t the wave at time t.

We assume, of course, that $\partial_v p$ is always negative.

4. The Amplitude of Acceleration Waves

Here we consider an acceleration wave advancing into an infinite homogeneous region at rest in an equilibrium state $(D_{\sigma}T)_o, \underset{\sim}{a}_o)$. Assuming $U > 0$ and taking the reference configuration to be that of the material ahead of the wave, at each time t we have

$$\rho(X,t) = \rho_0 = \rho_0, \quad n(X,t) = n_0, \quad \underset{\sim}{Q}(X,t) = \underset{\sim}{G}, \quad \text{for all } X \geq Y_t, \tag{4.1}$$

where ρ_o is the mass density in the reference configuration. For such a wave, (3.13) and (3.14) yield

$$\overset{\circ}{U} = \overset{\circ}{c} = \text{const.} \tag{4.2}$$

It follows from (3.4), (3.5), and (3.8) that r and a are not only continuous across the wave but in addition satisfy

$$\dot{n} = \dot{\rho} = 0, \quad \dot{\underset{\sim}{g}} = \dot{\underset{\sim}{a}} \quad \text{at } (t = t'). \tag{4.3}$$

Differentiating (2.14) we obtain

$$\dot{\rho}\dot{f} + \dot{n}\dot{s} + \rho(\dot{\underset{\sim}{d}} \cdot \underset{\sim}{O}f + \overline{\underset{\sim}{O}\dot{f}}) \cdot \underset{\sim}{a} = 0, \tag{4.4}$$

and, therefore, by (4.3),

$$\dot{\rho}\dot{\varepsilon} + \rho\underset{\sim}{Q} \cdot \dot{\underset{\sim}{z}} = 0. \tag{4.5}$$

Since $\theta > 0$, and since in the present case (4.1) and (2.10) imply that

$d\tilde{e}_{\alpha} \ll 0$ at the wave, (4.5) yields

$$[\ddot{\eta}] = 0, \tag{4.6}$$

and, from this, (3.5), (3.8), and Maxwell's theorem we obtain

$$[\partial_X \dot{\eta}] = [\partial_X^2 \eta] = 0. \tag{4.7}$$

A convenient measure of the amplitude of a wave of order two is the jump

$$a \ll \pounds >_X p\pounds \tag{4.8}$$

in the gradient of the density. Since we here assume that the material ahead of the wave is in its reference configuration and since dx/dx is continuous across the wave, at the wave a material derivative may be replaced by a spatial derivative, and (4.8) may be written

$$a = [\partial_X \rho] = \lim_{x - y \to t} \partial_X \rho(x,t); \tag{4.9}$$

i.e. the amplitude is just the spatial gradient of the density, evaluated immediately behind the wave. Of course, in general, the amplitude varies in time. Elsewhere, we have derived a general relation[17] for da/dt which reduces to

$$2\frac{da}{dt} = \rho_o [\partial_X \dot{v}] + \frac{1}{c^2}[\partial_X \dot{p}] \tag{4.10}$$

when dU/dt = 0 and $U^2 \ll c^2$, as is the case here. To calculate $[d_v\dot{p}]$ we

first differentiate (3.9):

$$\partial_X \dot{p} = (\partial_\upsilon \tilde{p})\partial_X \dot{\upsilon} + (\partial_\upsilon^2 \tilde{p})\dot{\upsilon}\partial_X \upsilon + (\partial_\eta \partial_\upsilon \tilde{p})\dot{\eta}\partial_X \upsilon + (\partial_{\underset{\sim}{\alpha}} \partial_\upsilon \tilde{p})\cdot \dot{\underset{\sim}{\alpha}}\partial_X \upsilon + (\partial_\eta \tilde{p})\partial_X \dot{\eta} + (\partial_\upsilon \partial_\eta \tilde{p})\dot{\upsilon}\partial_X \eta$$

$$+ (\partial_\eta^2 \tilde{p})\dot{\eta}\partial_X \eta + (\partial_{\underset{\sim}{\alpha}} \partial_\eta \tilde{p})\cdot \dot{\underset{\sim}{\alpha}}\partial_X \eta + (\partial_{\underset{\sim}{\alpha}} \tilde{p})\cdot \partial_X \dot{\underset{\sim}{\alpha}} + (\partial_\upsilon \partial_{\underset{\sim}{\alpha}} \tilde{p})\cdot \dot{\upsilon}\partial_X \underset{\sim}{\alpha} + (\partial_\eta \partial_{\underset{\sim}{\alpha}} \tilde{p})\cdot \dot{\eta}\partial_X \underset{\sim}{\alpha} + \dot{a} - (\partial^2 \tilde{p})\partial_X \underset{\sim}{\alpha}), \tag{4.11}$$

where $(\partial^2 \tilde{p})$ stands for the linear transformation with components $\tilde{d}_{\upsilon}\tilde{d}_{\upsilon}p$, $\tilde{d}_{\upsilon}\tilde{d}_{\underset{\sim}{\alpha}}p$, $\tilde{d}_{\underset{\sim}{\alpha}}\tilde{d}_{\upsilon}p$, $\tilde{d}_{\underset{\sim}{\alpha}}\tilde{d}_{\underset{\sim}{\beta}}p$.

Since η_0, $\nabla\eta$ and $\tilde{a}$ are continuous across the wave and p is of class C^2, each of the coefficients $\partial_\upsilon p$, $\partial_\upsilon \partial_\upsilon p$, etc. in parenthesis in (4.11) is continuous at $Y(t), t$. Hence (4.11), (4.3), and (4.7) yield

$$[\![\partial_X \dot{p}]\!] = (\partial_\upsilon \tilde{p})[\![\partial_X \dot{\upsilon}]\!] + (\partial_\upsilon^2 \tilde{p})[\![\dot{\upsilon}\partial_X \upsilon]\!] + (\partial_{\underset{\sim}{\alpha}} \tilde{p})\cdot [\![\partial_X \dot{\underset{\sim}{\alpha}}]\!]. \tag{4.12}$$

By (2.2),

$$\partial_X \dot{\underset{\sim}{\alpha}} = (\partial_\upsilon \underset{\sim}{h})\partial_X \upsilon + (\partial_\eta \underset{\sim}{h})\partial_X \eta + (\partial_{\underset{\sim}{\alpha}} \underset{\sim}{h})\cdot \partial_X \underset{\sim}{\alpha}; \tag{4.13}$$

therefore, since $\underset{\sim}{h}$ is of class C^1 we conclude from (3.8) that

$$[\![\partial_X \dot{\underset{\sim}{\alpha}}]\!] = (\partial_\upsilon \underset{\sim}{h})[\![\partial_X \upsilon]\!]. \tag{4.14}$$

It follows from (4.1) and (2.18) that here

$$[\![\dot{\upsilon}\partial_X \upsilon]\!] = [\![\dot{\upsilon}]\!][\![\partial_X \upsilon]\!]. \tag{4.15}$$

Since $t) - 1/p$, (4.8) and $(4.1)_1$ imply

$$[\![\partial_X \upsilon]\!] = -p_o'^2 a. \tag{4.16}$$

and by (3.11) and (4.2)

$$\$1 - cp_o^{-2}a. \tag{4.17}$$

Substituting (4.12) and (4.14)-(4.17) into (4.11), we obtain the following differential equation for a(t):

$$\dot{J}| = "\wedge^a + \mu \&Z* \tag{\wedge-18}$$

with

$$a = \frac{v_o^2(\partial_\alpha \tilde{p})\cdot(\partial_\upsilon h)}{2c^2}5, \qquad X\frac{(\partial_\alpha \tilde{p})\cdot(\partial_\upsilon h)}{»_o^2 c^{*2}_\upsilon \tilde{p}}--;\quad . \tag{4.19}$$

Integration of (4.18) yields the following theorem[19]: The amplitude a(t) of an acceleration wave which since time t » 0 has been advancing into a homogeneous region at rest in an equilibrium state (D $_6$r| $_8$0L$_o$) obeys the formula[20]

$$a(t) = \frac{\lambda a(0)}{(\lambda - a(0))e^{M}}f \wedge f_{-} + a(0); \tag{4.20}$$

here X and p. are constants given bjr (4.18) with d$_\alpha$p̃, 5$_\upsilon$p̃, and d$_\upsilon$h evaluated at (x_o, T]$_o$, a$_o$).

In applications we expect to have

$$\backslash i > 0. \tag{4.21}$$

In this case |X| plays the role of a critical amplitude: If |a(0)| < |X|

or if sgn a(0) •* -sgn X then a(t) -» 0 monotonically. If a(0) = X, then a(t) s a(0). On the other hand, if both |a(0)| > |X| and sgn a(0) = sgn X then |a(t)| -» oo monotonically in a finite time t_c given by

$$t_c = -\frac{1}{\mu}\ln\left(1-\frac{\lambda}{\circ_{V\ll/}}\right). \qquad (4.22)$$

For gases we generally have

$$d^{2\sim}p(x\gg,T\mathbf{0}\ \gamma\mathbf{0}f\ ) > 0, \qquad (4.23)$$

and hence, by (4.19) and (4.21)

$$X < 0. \qquad (4.24)$$

ThuSj for a gas, in order to have $\|[d\ p]_{\sim}(t)\| \to \circ\circ$ it is necessary that $[d\ p](0)$ be negative, i.e. that the acceleration wave be "compressive".

One is tempted to suppose that the approach of $|[d\ p](t)|$ to oo as $t \to t_c$ indicates the formation of a shock wave at time $t_{c>}$ but a rigorous proof is lacking. In any event, we see that an acceleration wave moving into a homogeneous equilibrium state in a shock tube can grow into a shock wave only if it is compressive, and this is, of course, just what is observed in practice. Our theory states further that a shock wave should not form unless $-[d\ p]$ is initially greater than $-X$. We do not know whether this critical jump $-X$ has ever been measured with precision, but surely it exists in experience; otherwise the feeblest bursts of sound could grow into shock waves.

5. Mild Discontinuities

The methods we have used to calculate the velocity and amplitude of acceleration waves can be applied without difficulty to waves of order 3 and higher. Here we give results of such calculations.

For the theory of waves of order N we assume that the response functions $\tilde{\mathcal{E}}$ and $\underset{\sim}{h}$ are of class C^{N+1} and C^{N-1}, respectively.

It turns out that, since the materials we consider here do not conduct heat, all waves of order $N \geq 2$ share the common formula (3.14) for their speeds of propagation.[23]

A fundamental distinction between acceleration waves and mild discontinuities arises, however, as soon as we consider reinforcement and attenuation. We may define the amplitude of a wave of order $N > 2$ by the formula

$$a = \underset{\sim}{\lambda}^{N-1} p\mathcal{E}, \tag{5.1}$$

which reduces, when the wave is propagating into a homogeneous region with the properties (4.1), to

$$a = \lim_{x \to y_t} \partial^{N-1}$$

Our main result for such waves is summarized in the following theorem. For the amplitude a(t) of a wave of order 3 or greater propagating into a homogeneous region at rest in an equilibrium state $(x_o, r|_o, \underset{\sim}{a}_o)$ we have

the simple formula[24]

$$a(t) = a(0)e^{""^{Mt}}; \qquad (5.3)$$

the constant p, is again given b£ $(4.19)_1$ with $\tilde{djp}$ and $\tilde{cMi}$ evaluated at $(\upsilon_0, \eta_0, \alpha_{*M_0})$.

Thus, if we assume (4.21), then for a mild discontinuity propagating into a homogeneous region at equilibrium, a(t) -»0 exponentially as t -* a>.

6. Ideal Gases with Vibrational Relaxation

Our general theory covers, as special cases, ideal gases with vibrational relaxation, that is materials for which the constitutive equations (2.1)-_o and (2.2) assume the special forms [10, 25]

$$\epsilon = \epsilon_A(\theta) + a,$$

$$p - RG/X) \tag{6.1}$$

$$\dot{a} \;»\; /c(o,e)U(e)-a], \qquad K > 0$$

with

$$0 = \tilde{0}(\backslash),Ti,a); \tag{6.2}$$

here R is the universal gas constant divided by the molecular weight of the gas, 0 and $\epsilon_A(0)$ are called the active-mode temperature and active-mode energy, while a is called the vibrational energy. It can be shown that for such a material our formulae (4.2) and (4.19) reduce to

$$c^2 - 7R0, \tag{6.3}$$

$$\lambda = -\frac{\kappa C_V}{\upsilon C_A c}\left[\frac{\gamma(\gamma+1)}{\gamma-1} + \theta\frac{d\gamma}{d\theta}\right]^{-1}, \tag{6.4}$$

$$\mu = \frac{(7\text{-}l)/cC_V}{2C_A\gamma}, \tag{6.5}$$

where

$$^{G}A - \overset{d}{55}«A^>' \qquad 7 = 7(0) = \overset{R+}{\cdot T}\Lambda' \qquad {}^{c}V - h'W- \tag{6.6}$$

A

Of course, all the quantities on the right in (6.3)-(6.5) are to be evaluated at the equilibrium state ahead of the wave.

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[10]See, for example, H. A. Bethe and E. Teller, Aberdeen Proving Ground Rep. No. X-117 (1941).

[11]A derivation of the relations (2.3)-(2.5) is given in ref. 8.

[12]Ref. 8, Eq. (7.21).

[13]The analogous result for materials with fading memory is given in ref. 2, p. 277, Theorem 4.3.

[14]Cf. C. Truesdell and R. A. Toupin, "The Classical Field Theories[11], in Encyclopedia of Physics, edited by S. Flügge (Springer-Verlag, Berlin & Heidelberg, 1960). Vol. HI/1, §175.

[15]Theorem 9.1 on p. 292 of ref. 2 contains the analogous result for materials with fading memory.

[16]Cf. ref. 2, p. 297, Remark 11.3.

[17]Ref. 18, p. 245, Remark 2.2.

[18] B. D. Coleman and M. E. Gurtin, Arch. Rational Mech. Anal. 19, 239 (1965).

[19] For the complete analogue of this theorem in the theory of materials with fading memory, see ref. 2, p. 297, Theorem 11.1.

[20] Equations of the general form (4.22) have been derived for Noll's "hygrosteric materials" by J. Dunwoody and N. T. Dunwoody[21], for

materials of integral type by Varley[22] and for more general materials by Coleman and Gurtin[18], and Coleman, Greenberg, and Gurtin[5]. Of particular interest here are Eqs. (4.20) and (4.21) identifying X and μ for materials with internal state variables.

[21] J. Dunwoody and N. T. Dunwoody, Int. J. Eng. Sci. 3, 417 (1965).

[22] E. Varley, Arch. Rational Mech. Anal. 19, 215 (1965).

[23] Cf. ref. 2, p. 325, Theorem 2.2.

[24] A result of the form (5.3) was obtained by Coleman, Greenberg, and Gurtin[5], p. 345, Theorem 5.1, under neglect of thermodynamic influences,

[25] In ref. 8 we discuss in more detail the phenomenological assumptions behind the theory of ideal gases with vibrational relaxation. The number η in (6.2) is the sum of the active-mode entropy and the vibrational entropy.

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1. ORIGINATIN C ACTIVITY *(Corpormlm muthor)*

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On the Growth and Decay of Discontinuities in Fluids with Internal State Variables

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Technical Report

5. AUTHOR^ *(Lmmt nmrnm, tint nmm; InltUI)*

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Gurtin, Morton E.

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
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IS. ABSTRACT

In the non-linear theory of one-dimensional motions of fluids exhibiting mechanical dissipation through th$ relaxation of internal state variables, one can derive exact expressions for the growth and attenuation of acceleration waves and higher-order waves. The methods and concepts used are simpler than those required for analogous probleps in the general theory of materials with fading memory. Here we discuss in detail the time dependence of the amplitude of waves propagating into homogeneous regions at equilibrium.

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