Optimal Multiperiod Operational Planning for Utility Systems

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In this paper, the operational planning problem for utility systems is formulated as a mixed-integer linear program (MILP). For multiperiod operation with varying demands for utilities, the optimal choice of units for each period is determined. The objective function accounts for both the operating costs for each period and changeover costs for startup/shutdown of units between periods of operation. A two-stage approach is proposed that requires the solution of MILP subproblems coupled with a shortest path algorithm, resulting in orders of magnitude reduction in computation time as compared to a direct MILP solution using branch and bound enumeration. The computational requirements of the algorithm are linear with respect to the number of periods and global solution of the MILP is guaranteed. Solution of a test problem shows savings of the order of 3% in total annual cost of operation with the main advantage being the simplicity of the proposed plan (few start-ups and shutdowns). The solution method is also extended to the case for ramp function change in demands.

1 Introduction

A common operational feature of utility systems in the industry is varying utility demands. This may be due to changing feed / product specifications in the operation of continuous plants or changes in operations in a processing schedule for batch plants. Utility systems are therefore designed to handle a range of demands because of the uncertain nature of utility demands. A common example is a cogeneration unit satisfying power and steam demands

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which could vary over a given time horizon. For a given demand, a cogeae?-ation unit could satisfy the demand through many different choices of units because of the design capability to handle demand uncertainty. The scheduling of equipment in a utility plant to meet varying demands is an established operational problem (Hobbs [4]). The optimal choice of units would appear to be the lowest operating cost configuration that meets the demands for that period.

However, when demands change in different periods of operation and the optimal choice of units changes, one will normally incur a changeover cost for startup and shutdown of units. The changeover costs link the optimization problem between successive time periods. Thus, the choice of same units for all periods or choosing units based on optimal operating costs for individual time periods ignoring startup and shutdown costs could lead to suboptimal solutions. Solution of the multiperiod operational planning problem is therefore essential to determine the choice of operating units and their operating conditions over the planning horizon.

Utility systems design and synthesis under uncertainty have been studied by Papoulias and Grossmann [8]. Foster [3] reported implementation of a utility planning problem in the industry. In his study, the scheduling of units for operation was accomplished along with manipulation of continuous variables to optimize operating costs for each period of operation. The operational planning of cogeneration systems has been presented by Ito et al. [5] in which the authors have determined the optimal choice of units for a 24 hour planning horizon. Nath and Holliday [7] reported successful implementation of optimization of industrial utility systems using a MILP formulation to decide the choice of units for given values of demands. Kalitventzeff [6] also presented a MINLP problem for management planning of utility networks for chemical plants. However, all the above studies are either based on optimization for a single period of operation or do not include the potential cost of changeover between periods of operation. A review of utility resource planning by Hobbs [4] presents the current problems facing the industry for various planning horizons. As indicated in the article, operations planning over a 1 week horizon is very common under the conditions of varying load forecasts requiring optimal choice of operating units to meet the loads.

In this work, a MILP model is presented for operational planning of a utility system for multiperiod operation. The cost of changeover between periods is accounted for along with operation costs in the model. It is assumed that the utility system can meet all values of demands by optimal choice of operating units and conditions and/or by external purchase of utilities. The choice of operating units (on/off status) or alternative operating modes (e.g. extractive vs condensation turbine) are made using binary 0-1 variables. Given *P* time periods with varying demands, the aim is to determine the optimal operational

planning schedule that meets the demands at lowest total cost.

For a large utility system, the MILP model solution time can be very large even for a single period of operation. For multiperiod operation planning models, the number of discrete variables increase with number of periods. The coupling equations between periods involve only binary variables for representing startup and shutdown costs of units between consecutive periods. Due to this linking between periods, the solution time can increase exponentially with the number of periods, making the problem computationally intractable. Given the numerous available configurations for each period, an exhaustive search of all possible operation plans for *P* periods would be computationally inefficient and impossible. It is therefore of significance to develop a computational strategy for efficient solution of the operational planning model.

The paper is presented as follows. We define the problem in section 2, followed by the model formulation in section 3. The problem structure that will be exploited is described in section 4. In section 5, a decomposition algorithm that relies on use of MILP subproblems and a shortest path algorithm is presented for the operational planning problem. Section 6 describes a modification of the algorithm that only requires partial generation of solutions to reduce computational requirements. Further improvements to the algorithm are proposed in section 7 followed by a description of modifications to the algorithm for extension to the case of ramp function demands in section 8. Finally, example problems are presented in section 9 to illustrate the solution techniques.

2 Problem definition

Given are a fixed flowsheet configuration of a utility system and process conditions for all utility levels, namely, electricity/power demands, steam, water/refrigerant. Also specified are utility conditions of BFW, natural gas etc. required for the operation of the utility plant. A multiperiod scenario is considered, where the utility demands are changing over a given time horizon. The utility demands are constant or may vary linearly within each time period $t = 1 \ldots P$. Also, given are the operating costs and startup/shutdown costs for each equipment. The length of time period may be different for different periods. The operational planning problem is to determine the choice of operation of units for each period that minimizes cost, and that is subject to meeting the utility demands for each period over the entire planning horizon.

The following points need to be taken into account in our problem definition.

(i) Each unit has a minimum and maximum capacity of operation. However, the utility system and its options for external purchases of utilities is assumed to be feasible for all demands in the time horizon (that is, there exists at least one operation mode that can meet any given demand). This can easily be accomplished by considering purchase of electricity and steam from an external source.

(ii) There are startup and shutdown costs for units. Thus, if any unit is shut down for a certain period, there is a shutdown cost, and when restarted in another period, it incurs a startup cost. For example, startup of a boiler involves inspection, cleaning, hydrostatic testing, calibration of instruments, steam line cleaning, valve testing etc. It is also necessary to start up with a dow gradient of temperature with respect with time to avoid thermal fatigue requiring initial operations at low capacity. These contribute towards the cost of startup operation for the equipment [11].

(iii) There is a fixed charge and a variable operating cost for units in addition to changeover costs. The fixed charge is associated with costs that are incurred for the replacement of equipment and that result from wear and tear of equipement during operation.

3 Model Formulation

We assume linear performance equations for simplicity. Nonlinear models can be treated in a similar manner but at higher computational expense. Also, see Papoulias and Grossmann [9] for converting nonlinear equations to linear equations by discretization.

The following sets, variables and parameters are defined.

(i) Indices

n = Unit number.
t = Time period.
p = Utility level for power.
r = Utility level for steam
e.g. High pressure (HP), Medium pressure (MP), Low pressure steam(LP)
b = Utility level for water/refrigerant.
g = Operation mode for unit (e.g. extraction to MP vs LP for steam turbines).

(ii) Sets

U = {n | n = 1... N} is the set of units.
T = {t 11 = 1... P} is the set of time periods.
J_n = {m | Unit n has an input fiowrate from unit m }

O^ {m | Unit n has an output flowrate to unit m }
$S_n gt$ {Variables associated with unit n in operation mode g in period t }
$E_r c$= {g | Unit n operates in mode g }

(iii) Continuous Variables

F_{mnt} =Flow rate from unit m to unit n in period t.
W_{npt} =Power production of level p in unit n in period t
WPpt=Power purchase of level p in period t.
Q_{nri} =Steam production of level r in unit n in period t.
V_{nbt} =Water/Refrig. production of level 6 in unit n in period t

The utilities are positive when produced and negative when consumed in the unit.

(iv) Binary Variables

y_{ngi} = 1 if unit n operates in mode g in period t.
= 0 otherwise.
$yrint$^ 1 if unit n operates in period t.
= 0 otherwise.
zm = 1 if unit n incurs startup cost in period t.
= 0 otherwise.
$zs_n t$ = 1 if unit n incurs shutdown cost in period t.
= 0 otherwise.

(v) Parameters

a_{nm} =Variable cost coefficient for flow from unit n to unit m .
o$_{np}$ =Variable cost coefficient for unit n producing power level p .
a_{nr} =Variable cost coefficient for unit n producing steam level r .
a_{nb} =Variable cost coefficient for unit n producing water/refrig. level 6 .
bnm =Fixed cost for flow from unit n to unit m .
Kp = Fixed cost for unit n producing power level p .
bnr = Fixed cost for unit n producing steam level r .
b_{nb} = Fixed cost for unit n producing water/refrig. level 6 .
c_n = Fixed startup cost for unit n.
= Fixed shutdown cost for unit n.
cp_p = Variable cost coefficient for purchase of power level p.
DW_{pt}= Total demand of power level p in period t.
DQ_{rt}= Total demand of steam level r in period t.
DV_{bt}= Total demand of water/refrig. level b in period t.
h_{mn} =Enthalpy of steam from unit m to unit n in period t.
rj$_n$ = Efficiency of unit n.

We now define the following equations representing the mass and energy balances and operational status, of equipment for the utility system.

(i) Mass bfllftmffi for unit n

$$\sum_{m \in I_n} F_{mnt} — Y\wedge \; r \cdot = Q \; v \wedge - i \quad P \; n = 1 \quad TV \qquad \bullet (1\backslash$$

(ii) Energy balance in each unit n

$$\sum_{m \in I_n} F_{mnt} h_{mn} - \sum_{m \in O_n} F_{nmt}/lnm = E_r \wedge nrt + E_V \wedge npt + E_b \wedge W \qquad (2)$$

$$Vt = l \ldots P, \quad n = l \ldots JV$$

(iii) Logical constraints for operational status of unit in period t

Note that depending on the unit n and its operational mode g in period t, there are associated variables defined in set $S_{ng}t$. Thus

$$\Omega^L_p Vngt \leq Fnmt \leq \wedge^U_F Vngt, \quad F_{nmt} \in Sngt \; Vm, Tl, p, t \qquad (3)$$

$$\Omega^L_W y_{ngt} \wedge \wedge npt \wedge H \wedge t, \quad WnpteS\wedge tVp,!!,\wedge * \qquad (4)$$

$$\Omega^L_Q y_{ngt} \leq Qnrt \leq \wedge Vngt, \quad Qnrt \; 6 \; 5_{np(} Vr, Tl, Q, t \qquad (5)$$

$$\Omega^L_V y_{ngt} \leq Vnbt \leq V^U_V Vngt, \quad Vnbt \; G \; 5_n\wedge_t \; V6, Tl_5, t \qquad (6)$$

where n^),^^ are valid lower and upper bounds respectively.

(iv) Operational status of unit in period *t*

Unit is 'on' if it is operational in any one mode *g*.

$$Vngt \leq yrtnt \quad Vn,g,t \qquad (7)$$

(v) Single Operational mode for unit n in period *t*

Unit is operational in only one mode *g*.

$$\sum_{g \in E_n} y_{ngt} \leq 1 \quad \forall n, t \qquad (8)$$

(vi)Satisfaction of utility demands in period *t*

$$WP_{pt} + \sum_{n} Wnpt \geq DW_{pt} \qquad \forall p, t \qquad (9)$$

$$\sum_{n} Q_{nrt} \geq DQ_{rt} \qquad \forall r, t \qquad (10)$$

$$\sum_{n} V_{nbt} \geq DV_{bt} \qquad \forall b,t \qquad (11)$$

(vii)Startup variables for unit n in period *t*

For each unit n, if it is 'on' in period *t* and 'off' in period *t* - 1, then *Zm,* the startup variable, is equal to one.

$$Znt \geq yrtnt - ynn,t-i \qquad \forall n, t \qquad (12)$$

where yn^o = 0

(viii)Shutdown variables for unit n in period *t*

For each unit n, if it is 'off' in period *t* + 1 and 'on' in period i, then zs_{nt}, the shutdown variable, is equal to one.

$$zs_{ni} \geq yrtnt - ynn_{Ml} \qquad \forall ?i. t \qquad (13)$$

where *yrinf+i* = 0

It may be noted from (12) and (13) that variables z_{nt} and zs_{ni} take only binary values even if they are treated as continuous variables, hence the binary constraint on them may be relaxed.

(ix)Objective function

The objective is to minimize the total cost of operation over all time periods. The fixed and variable cost of operation of each unit for each period and changeover costs between periods are included in the objective function.

$$\min C = \sum_{t}\sum_{n}[\sum_{m}(a_{nm}F_{nmt} + b_{nm}yn_{nt})$$

$$+\sum_p (a_{np} W_{npt}/\eta_n + b_{np} yn_{nt})$$
$$+\sum_r (a_{nr} Q_{nrt}/\eta_n + b_{nr} yn_{nt})$$
$$+\sum_b (a_{nb} V_{nbt}/\eta_n + b_{nb} yn_{nt})$$
$$+ (c_n z_{nt} + CS_n ZS_{nt})]$$
$$+\sum_t cp_p W P_{pt} \quad (14)$$

The MILP model (PI) then consists of minimizing (14) subject to the constraints in (1) - (13). The following points are relevant to model (PI):

(i) Operating costs for each period should weighted by the length of time periods.
(ii) The efficiency of the unit is assumed to be constant over the operating range.
(iii) The condition of constant demand may be relaxed to include ramp function variation of demands. Within a period, the demand is allowed to vary linearly for fraction of time a_t. For a ramp function change in demands, we obtain a parametric MILP. It is assumed that a single configuration will be used for that period in order to prevent many changeovers within that period. The model formulation for that case is explained in section 8.

4 Problem Structure

in order to gain better insight into the mathematical structure of the multiperiod problem in the previous section, consider the following variables for defining a compact representation.

$X_t = \{F_{nmiy}\ W_{npU}\ WP_{pU}\ Q_{nrU}\ V_{nbi}\}$ V n, m,p, r, b
$Y_t = \{ynntiVngt\}$ V fl, Q

$C\backslash = \{a_{mn}, a_{np}, a_{nr}, a_{flb}, cpp\}$ Vn,m,p,r,6
$C_2 = \{bmn, b_{np}, 6_{nr}, b_{nb}\}$ V n,m,p,r,6
$C_3 = \{c_n, cs_n)$ Vn

The MILP problem (PI) given by equations (1) to (14) may then be represented in the following general form.

$$P2: \min_{X_t, Z_t, Y_t} C = \sum_t [C_1^T X_t + C_2^T Y_t + C_3^T Z_t] \quad (15)$$

subject to

$$AX_t \leq b_t \quad (16)$$

$$GX_t + FY_t \leq d_t \quad (17)$$

$$Z_t \geq RY_t - S_1 Y_{t-1} - S_2 Y_{t+1} \quad (18)$$

$$\forall \; t = 1 \ldots P$$

$$Y_t \in \{0,1\}$$

where A,G,F,R,S1,S2 are conformable matrices.

It should be noted that in the absence of changeover costs, equation (18) can be eliminated and the problem can be easily decoupled and solved for each period *t* separately. However, in a realistic problem changeover costs are important, and therefore equation (18) that couples the periods of operation must be included, resulting in a large MILP. The coupling constraints are only in the binary variables Y_t and Z_t (which takes only binary values). The structure of the MILP (P2) for a problem in T periods is shown in Figure 1.

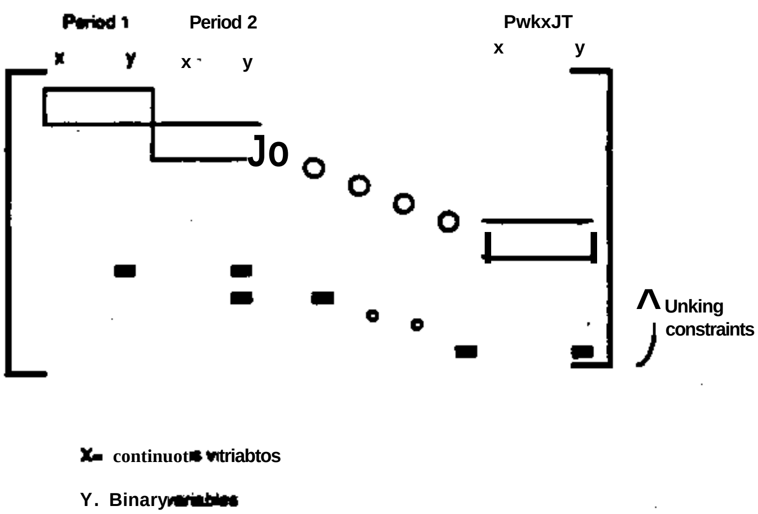

Fig. 1. Problem structure for multiperiod case

Clearly, the size of this problem scales linearly with the number of periods. The computation time would scale exponentially due to an increase in the number of binary variables. Even for a small MILP with 20 units, the solution time for a 12 period problem with 240 binary variables (20 variables per period) could be too large. This motivates the need to exploit the structure of the problem to develop an efficient algorithm. In the following section, a two stage algorithm is proposed for which the solution time increases linearly with the number of time periods. The main idea behind the approach is to remove the linking constraints (18) and solve for each period independently. The effect of the linking constraints on the objective function value is accounted for in the second stage in which the global optimal solution of the problem is determined.

5 Decomposition algorithm for operational planning problem

The proposed decomposition algorithm for the problem exploits the fact that the solution of problem (P2) without constraints (18) is also **a** feasible solution to problem (P2). The following properties are straightforward and are stated below.

Property 1

The solution of the relaxed MILP formulation (MP) is a feasible solution for the problem (P2) where (MP) is formulated as

$$\min_{X_t, Y_t} C = \sum_t \left(C_1^T X_t + C_2^T Y_t \right)$$

subject to **(MP)**

$$\begin{aligned} \mathbf{A}X_t &\le b_t \\ \mathbf{G}X_t + \mathbf{F}Y_t &\le d_t \quad \forall \ t = 1 \ldots P \\ Y_t &\in \{0,1\} \end{aligned}$$

Note that (MP) is a relaxation of (P2) because it excludes the constraints (18). The solution of (MP) is such that any choice of Y_t gives a feasible choice of variables Z_t for (P2). The objective value of (P2) and (MP) may be different and the solution of (MP) would be a lower bound to the (P2) since $C\$ > 0$. Note also that the problem (MP) is separable into P subproblems of smaller dimension and may be solved independently to obtain a feasible solution to (P2). Clearly, the solution of (MP) yields the minimum operating cost configuration for each period t. By including integer cuts to exclude previously obtained configurations (see Duran and Grossmann [2]), it will be possible to obtain the operating costs for all feasible configurations for each period t. Let ϕ^k_t represent the operating cost for the kth configuration in period t. Let the configuration be denoted by Y_t^k and associated continuous variables by X_t^*.

Property 2

Assume all feasible configurations for meeting the demands for a given period t can be enumerated (by choice of Y_t). Let there be Bt configurations in period

t. We may then construct a graph containing J2t &t nodes with operating costs for given configuration as the cost at that node. Create a source s before period 1 and sink o after period P. Join the nodes in any period t with directed arcs to adjacent nodes in period t + 1. *Each node has a corresponding configuration obtained from solution of (MP). The startup/shutdown costs are known from the solution Y_t^k. Let the transition costs from jth node in period t to kth node in period t + 1 be ip(^{k}. Let the cost of the directed arcs be the transition costs (fp$_t^k$. The shortest path between the source and sink is the optimal operational plan for the problem (P2).*

Proof: The graph obtained by the above construction is bipartite with respect to adjacent periods of operations as shown in Figure 2. Each node in the graph represents a feasible solution with the cost of the node representing the operating cost for that configuration. The directed arcs represent all possible transitions from period _t_ to _t_ + 1. Thus, all paths from s to o enumerate the total cost of operation accounting for all possible configurations in each period. The shortest path therefore represents the choice with the least cost of operation through _P_ periods. QED

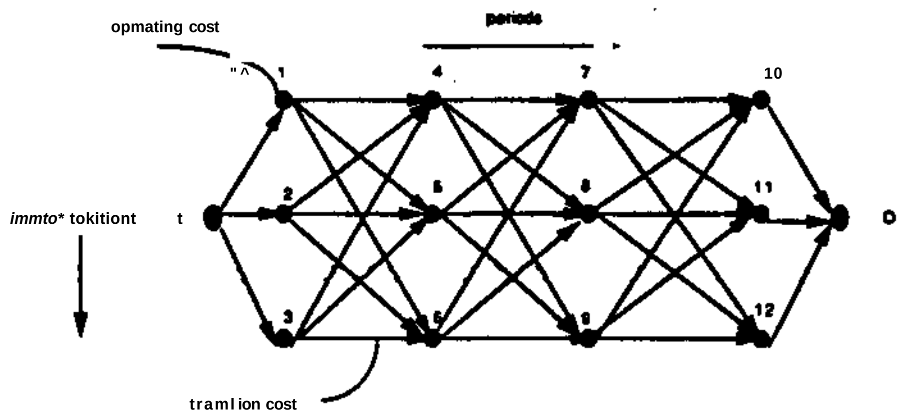

Fig. 2. Directed Graph for shortest path algorithm

It may be noted that if there are TV units, then there are potentially 2^N possible configurations in each period. The complexity of enumerating all configurations for all _P_ periods is $O(2^N P)$. The shortest path algorithm has a complexity of the number of arcs in the graph. Thus, for this algorithm, the complexity of determining the shortest path is $O(2^{2(N+1)}P)$. The complexity of the algorithm is therefore linear in the number of periods as compared to the solution without decomposition which is exponential in the number of periods P. It is evident that enumerating all possible configurations for each period to determine the nodes of the underlying graph could be expensive if the plant has a very large number of units. However, it may be possible to avoid exhaustive enumeration by partially enumerating the nodes as explained in the next section.

6 Modified shortest path algorithm with partial enumeration

It is clear that enumerating or generating all configurations for each period as stated in Period 2 could be computationally very expensive. Furthermore, an exhaustive enumeration does not use any additional information of transition costs between periods. Use of transition cost knowledge would help in avoiding enumeration of **a** significant number of nodes thus reducing the required computation time and reducing the size of the graph. The following observations aid in our development of the modified algorithm.

(i) The operating costs are usually much larger than transition costs between periods,
(ii) The difference in operating costs between any two configurations in the same period is usually large and would be of the order of magnitude of the transition costs between periods.

It should be noted that the above points are not assumptions, but are observations that only aid in the partial generation of nodes which in turn leads to a reduced network for determining the shortest path. The proposed algorithm does not require the above points to be true to ensure optimality of the solution. It simply takes advantage of these facts to reduce the computation.

In order to explain the idea of partial enumeration, consider the following simple example as applied for enumerating the configurations for a certain period T (see Figure 3).

Assume that there are upto 20 possible configurations that can be generated for a given period for meeting the demands. Assume that these configurations are systematically generated by successively solving an MILP to which integer cuts are added to exclude previous solutions. This ensures that the cost of the generated configurations increases monotonically: The procedure for partial enumeration would first determine the configuration with lowest operating cost(OT), which is node 1 with cost of $110,000. The maximum transition cost(TT) from any feasible configuration in period T-1 to T and from period r to T - 1 is then determined. In this case, node 1 has maximum TT of $16,000. The next best configuration is then determined, which is node 2 with OT of $115,000. Clearly, this node cannot be eliminated because it is possible to have a zero transition cost for node 2 in which case node 2 is better than node 1. The maximum TT for node 2 is then detennined at $4,000. The next best configuration is node 3 with OT of $125,000. Clearly, node 3 can be eliminated because even with a zero TT, the cost would be larger than the cost associated with node 2 (which has a sum of OT and max TT of $119,000). Any other configuration that will be obtained will have a higher OT, therefore all nodes after node 2 may be eliminated. Thus, the partial enumeration technique would

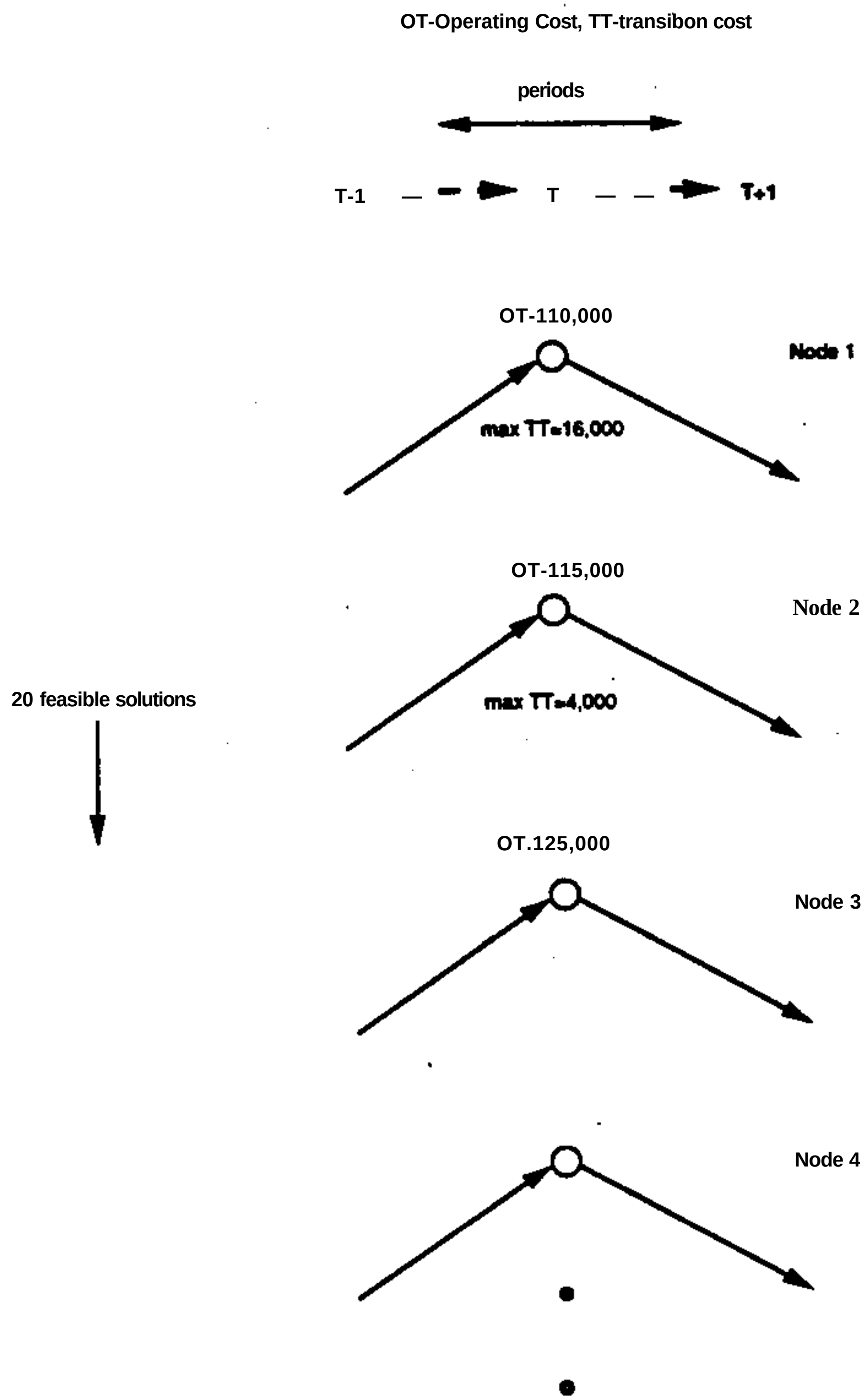

Fig. 3. Illustration for partial enumeration

enumerate only 3 nodes instead of all 20 feasible configurations, resulting in a reduced set of 2 configurations. This idea is formally presented as Property 4 later in this section.

The steps of the algorithm and its associated properties are as follows.

1) For each period $t = 1 \ldots P$, solve the MILP in (MP) with the subproblems (MP_t) which are much smaller in size than problem (P2).

For $t = 1 \ldots P$, solve

$$\min_{X_t, Y_t} \phi_t = \{C1^T X_t + C2^T Y_t\}$$

subject to $\qquad\qquad (MP_t)$

$$\begin{aligned} AX_t &\leq b_t \\ GX_t + FY_t &\leq d_t \\ Y_t &\in \{0,1\} \end{aligned}$$

Thus (MP_t) determines the configuration Y_t° that has the least operating cost ϕ_t for that period t (neglecting all transition costs).

<u>Property 3</u>

The sum $\sum_t \phi_t$ obtained from solving the subproblems (MP_t, $t = 1 \ldots P$) is a lower bound to the solution of problem (P2).

Clearly, the solution obtained from (MP_t) is equivalent to the lowest operating costs for each period with an addition of zero transition cost. Any optimal solution of (P2) would have at least the same operating cost as the one obtained from (MP_t) and an additional transition cost.

Thus, if the solution obtained from (MPt) yields a zero transition cost (calculated from knowledge of Y_t°), then the solution is optimal.

Otherwise, go to step 2.

2) Let t be the count for periods of operation. Initialize $t = 1$.

3) Let k be the count of iterations for period t. Initialize $k = 1$.

4) To determine the maximum transition cost to and from a given time period t, solve the following subproblems (P3) and (P4) defined below. Note that (P3) and (P4) are solved for fixed value of Y_t^{k-1} which is obtained from the previous iteration on k. For example, when $k = 1$, then Y_t° is obtained from step 1. For $k = 2$, the iteration for $k = 1$ provides the data for Y_t^1.

$$\max_{X_{t-1},Y_{t-1},Z_{t-1}} \phi_t^k = [C3^T Z_{t-1}]$$

subject to **(P3)**

$$\begin{aligned} \mathbf{A}X_{t-1} &\leq b_{t-1} \\ \mathbf{G}X_{t-1} + \mathbf{F}Y_{t-1} &\leq d_{t-1} \\ Z_{t-1} &\geq \mathbf{R}\overline{Y}_t^{k-1} - \mathbf{S1}Y_{t-1} \\ Y_{t-1} &\in \{0,1\} \end{aligned}$$

By solution of (P3), an upper bound on the transition cost from any feasible configuration in period t - 1 to the $(k$ - 1)th configuration in period t is obtained.

Similarly,

$$\max_{X_{t+1},Y_{t+1},Z_{t+1}} \phi_t^k = [C3^T Z_{t+1}]$$

subject to **(P4)**

$$\begin{aligned} \mathbf{A}X_{t+1} &\leq b_{t+1} \\ \mathbf{G}X_{t+1} + \mathbf{F}Y_{t+1} &\leq d_{t+1} \\ Z_{t+1} &\geq \mathbf{R}\overline{Y}_t^{k-1} - \mathbf{S2}Y_{t+1} \\ Y_{t+1} &\in \{0,1\} \end{aligned}$$

By solution of (P4), an upper bound on the transition cost from the $(k$ — 1)th configuration in period t to any feasible configuration in period t + 1 is obtained.

It may be noted that

(i) Y_t^{k-1} is known from the previous iteration on k. When $k = 1$, $Y_t^{k-1} = Y_t^0$.
(ii) When t = 1, then Y_{t-1} corresponds to the initial operational status of equipment. When t = P, then Y_{t+1} corresponds to the terminal operational status of equipment.

5) Solve subproblem (P5) which results from adding to (MP) bounding constraints to determine the next feasible configuration for that period.

$$\min_{X_t,Y_t} \phi_t^k = [C1^T X_t + C2^T Y_t]$$

subject to (P5)

$$AX_t \leq b_t$$

$$GXt + FYt \leq dt$$

$$Cl^T X_t + C2^T Y_t \leq [Cl^T Xt\text{-}* + C2^T Y_t^{k\,\prime i}] + ifc* + \mathrm{Vfr}^{i} \qquad (19)$$

$$\mathrm{Vj} = \mathrm{l}\ldots*$$

$$\sum_{n \in M_j} y_{n_{nt}} - \sum_{n \in Q_j} \mathrm{yn.u} \leq |A\wedge|\text{-}1 \qquad \mathrm{Vj«l...ft} \qquad (20)$$

where

Mj = { Unit n | yum = 1 for configuration in iteration *j* }

Qj = { Unit n | yn_{nt} = 0 for configuration in iteration *j* }

Vie {0,1}

It may be noted that

(i) t/£r* and ^£^' are obtained from the solution of (P3) and (P4) respectively.

(ii) For any period t, constraint (19) excludes any feasible solution that has an operating cost greater than the sum of operating cost for any feasible solution *j* (*j* < *k*) and the maximum possible changeover costs between period *t* — 1 to period *t* and the maximum possible changeover costs between period *t* to period *t* + 1. Note that constraint (19) is added cumulatively for every configuration *j* < *k*. This is because for every configuration *j* there is an associated operating cost <fP_t and transition cost V$t- and W+ •

(iii) For any period t, the integer cut in (20) which is also accumulted with each iteration, excludes any configuration obtained in any previous iteration of j.

(iv) Solution of (P5) yields a *reduced set of feasible configurations* for each period t.

If (P5) is feasible, let the solution obtained from (P5) be *X$* and *Yf*.
Set *k* = *k* + 1. Go to step 4.

If (P5) is infeasible, there is no need to generate more configurations for period *t*. Set *t* = t + 1. If *t* = *P* + 1, Go to step 6.
Else , Go to step 3.

6) Construct the following network containing $£_t £$t nodes (see Figure 2). For any period, the operating cost of the jth node is *$*. The directed arcs from any jfth node (*j* < B_t) in period *t* to any *kth* node (*k* < $£_t$+i) in period

$t + 1$ is assigned the actual transition cost $(f?_t^k$. Determine the shortest path from the source to the sink- Let the nodes on the shortest path in any period t be denoted by t* and the corresponding solutions by *XI* and *Y{*. The length of the path is the value of the objective function of (P2) and the optimal solution is *XI* and Y_t\

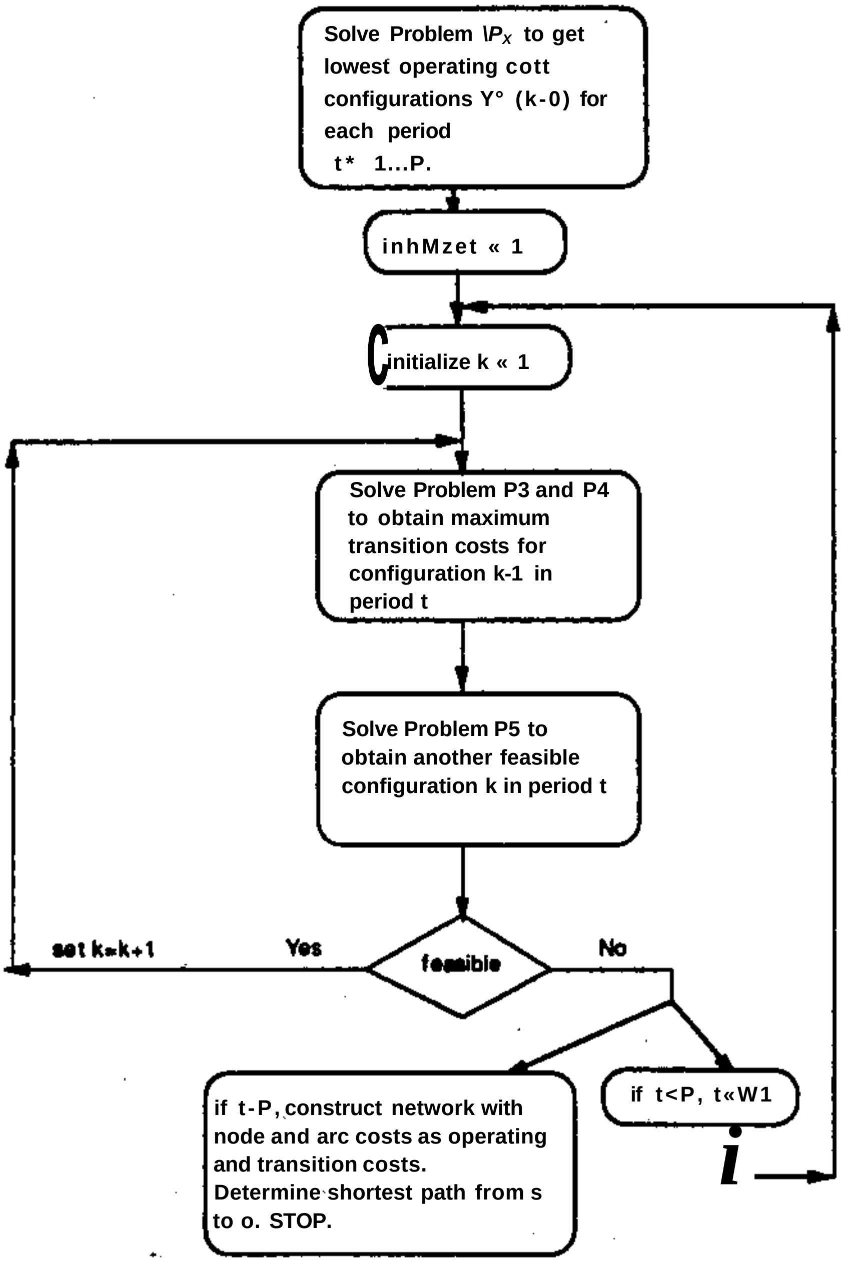

Fig. 4. Flowchart for shortest path algorithm

The partial generation of nodes for the network for the shortest path algorithm relies on the following property associated with constraint (20) in (P5).

Property 4

Any feasible configuration for a time period t with an operation cost <f>$_t$ that satisfies the following equation

$$Cl^T Xt + C2^T Y_t > [C1^T X\}\sim' + C2^T Y_i^{k-i}) + \wedge + \$;* \qquad (21)$$
$$\mathrm{V}\, j = 1 \ldots k$$

may be excluded from the graph as it cannot be in the optimal solution of (P2).

Proof: **The shortest path from any node in period *t* — 1 to a node in period t + 1 passes through some node in period *t* and has an upper bound of**

$$min_j([C1^T X?_-^j + C2^T Y_i^{k''i}) + tfJ+tifc*).$$

Hence any node that satisfies

$$C\backslash^T X_{\%} + C2^T Y_t > [Cl^T Xt\text{-}* + C2^T Y_t^{k'j}] + \$\text{-}* + \quad ti? \qquad (22)$$
$$\forall j = 1 \ldots k$$

will not be on the shortest path from 5 to *o* which is a requirement for optimal solution of (P2). The strict equality is excluded by the integer cut (20). QED.

Thus, constraint (19) in problem (P5) helps to obtain a reduced set of feasible configurations and avoids enumeration of all feasible configurations for each period.

7 Improvements to the algorithm

(i) At the level of solving for the optimal configuration for any period, use of special ordered sets as in (8) and hard logic constraints that describe the relation of the units in the flowsheet may be exploited.

(ii) Due to the order of solving problems (P3), (P4) and (P5) from *t* = 1... P, it is possible to solve problem (P3) for *t* $\geq$ *2* by using the reduced set of feasible solutions in the previous period. This will aid in obtaining a lower value of $\backslash p_t$- which provides tighter constraints in equation (19) for problem (P5). This is possible because the reduced set of feasible solutions is known a priori for period *t* — 1 when iterating for period *t*. Thus, (P3) may be solved with the additional requirement that *Yt-i* 6 {Vi-iii = 1... £$_t$_i} where £$_t$_i is the number of configurations generated for period t — 1.

(iii) A lower bound on the transition costs may be calculated after step 5 from knowledge of Y_t^k. Let T£ be the minimum transition cost from Arth node

in period t from any node in period $t - 1$ and to any node in period $t+1$. Then any node that does not satisfy constraint (23) will not be on the shortest path and may be eliminated.

$$CI^T X^*_t + C2^T Y_t^k + 7? \leq [C\backslash^T Xi + C2^T Yf] + W- + W+ \qquad (23)$$
$$Vj\text{«}l...Jb, \; * = !...\pounds,$$

8 Extension to ramp function demand profile

The solution algorithm for the constant demand case may be easily extended to the case of linear variation in demands. Note that certain demands may be increasing while other demands may be decreasing simultaneously in any time period. We therefore have a scalar parametric, non-monotonic MILP of the form shown below.

$$\min_{X_t, Z_t, Y_t} \; C = \wedge 2_t [CI^T X_t + C2^T Y_t + C3^T Z_t]$$

subject to **(RP)**

$$A.Xt \leq bi + \wedge t$$
$$GA'_t + FY_t \leq d_t$$
$$Z_t \geq RY_t - S1Y_{t-1} - S2Y_{t+1}$$
$$V \quad t = l...P$$
$$Vie \{0,1\}$$

where $p_($ is a vector of variations in b_t (demands), and 8 is a scalar such that $0 \leq 6 \leq$ l.(see Figure 5).

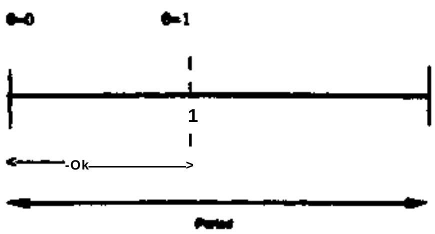

Fig. 5. Ramp function variation for period

Consider a multiperiod case, where the demand in each period varies linearly with time for a fraction a_t of the length of the period t. In order to determine the operating cost for that period, the integral of cost as a function of time is required for the length of the time period. Two possibilities arise.

(i) Within the same period, it is possible to meet the demands for 0 ≤ 0 ≤ 1 using a single configuration (choice of Y_t). The steps of the algorithm remain the same, but the problems (AfP$_t$), (P3), (P4) and (P5) are replaced by *(RP$_t$)*> (RP3), (RP4) and (RP5), respectively, as shown below.

$$\underset{X\wedge fX\wedge ,xt}{\text{pin}} .* = l/2a_t T[Cl^T(X\} + X?) + 2C2^T Y_t] + (1 - *_t T)[Cl^T X^*_t + C2^T Y_t)$$

subject to *(RPt)*

$$\begin{aligned} \mathbf{A}X_t^1 &\le b_t \\ \mathbf{A}X_t^2 &\le b_t + \theta g_t \\ \mathbf{G}X_t^1 + \mathbf{F}Y_t &\le d_t \\ \mathbf{G}X_t^2 + \mathbf{F}Y_t &\le d_t \\ Y_t &\in \{0,1\} \end{aligned}$$

where *X}* corresponds to the solution at $0 = 0$
and X_t^2 corresponds to the solution at $8=1$
and *T* is the length of each period.

The objective function determines the operating cost by integrating over the entire length of the period for varying cost due to change in the demands.

Similarly we have,

$$\max \quad \text{VfL}''^1 = [C2>^T Z_t .A$$

subject to (RP3)

$$\begin{aligned} \mathbf{A}X_{t-1}^1 &\le b_{t-1} \\ \mathbf{A}X_{t-1}^2 &\le b_{t-1} + \theta g_{t-1} \\ \mathbf{G}X_{t-1}^1 + \mathbf{F}Y_{t-1} &\le d_{t-1} \\ \mathbf{G}X_{t-1}^2 + \mathbf{F}Y_{t-1} &\le d_{t-1} \\ Z_{t-1} &\ge \mathbf{R}\overline{Y}_t^{k-1} - \mathbf{S1}Y_{t-1} \\ Y_{t-1} &\in \{0,1\} \end{aligned}$$

and

$$\max_{X_{t+1}^1, X_{t+1}^2, Y_{t+1}, Z_{t+1}} \psi_{t+}^{k-1} = [C3^T Z_{t+1}]$$

subject to (RP4)

$$\begin{aligned} \mathbf{A}X_{t+1}^1 &\le b_{t+1} \\ \mathbf{A}X_{t+1}^2 &\le b_{t+1} + \theta g_{t+1} \\ \mathbf{G}X_{t+1}^1 + \mathbf{F}Y_{t+1} &\le d_{t+1} \end{aligned}$$

$$GX^2_{t+1} + FY_{t+1} \leq d_{t+1}$$
$$Z_{t+1} \geq KY_t^{k-1} - \Omega Y_M$$
$$Y_{t+1} \in \{0,1\}$$

and

$$\min_{X_t^1, X_t^2, Y_t} \ \& = 1/2 a_t T[C1^T(X_t^1 + X_t^2) + 2C2^T y_t] + (1 - a_t T)[C1^T X_t^2 + C2^T Y_t]$$

subject to (RP5)

$$AX_t^1 \leq b_t$$
$$AX_t^2 \leq b_t + \theta g_t$$
$$GX_t^1 + FY_t \leq d_t$$
$$GX_t^2 + FY_t \leq d_t$$
$$\phi_t \leq \phi_t^{k-j} + \psi_{t-}^{k-j} + \psi_{t+}^{k-j} \ \forall j = 1 \ldots k$$
$$\sum_{n \in M_j} yn_{nt} - \sum_{n \in Q_j} yn_{nt} \leq | M_j | -1 \ \ \forall j = 1 \ldots k$$
$$Y_t \in \{0,1\}$$

(ii) If the solution of (RP_t) is infeasible for some t, there is no single configuration that satisfies demand for that period for $0 \leq \theta \leq 1$. In that case, the period should be split up into sections such that for each section., a single configuration is feasible. The primary motivation for this choice is to minimize the transitions within the period of operation. Thus, the largest value of θ that accepts a single configuration can be determined from solution of the following problem $(RP1_t)$.

$$max_{X_t^1, X_t^2, Y_t, \theta} \ \theta$$

subject to $(RP1_t)$

$$AX_t^1 \leq b_t$$
$$AX_t^2 \leq b_t + \theta g_t$$
$$GX_t^1 + FY_t \leq d_t$$
$$GX_t^2 + FY_t \leq d_t$$
$$Y_t \in \{0,1\}$$

Thus, if (RP_t) is infeasible,, $(RP1_t)$ is solved and period t is divided to create two periods of length θT and $(1 - \theta)T$ and the solution procedure is continued from period t.

It should be noted that a limitation of the approach presented in this section is that (RP_t) may not yield the solution for lowest operating cost for that period as there could be a choice with lower operating cost for which the configuration changes at a breakpoint for some value of *0* between 0 and 1. In this paper we do not consider further the possibility of partitioning the time period to identify such breakpoints.

9 Example problems

9.1 Constant demand case

Consider the cogeneration utility plant shown in Figure 6. The utility plant consists of two boilers and two turbines for meeting the demands of HP, MP and LP steam and power demands in two levels. There is also an option to purchase HP steam and power in order to meet the demands. Letdown steam from higher levels is available and turbines have two modes of operation depending on the level of exhaust steam. Each turbine may supply power to any of the two levels with the constraint that only one turbine supplies power to the same level.

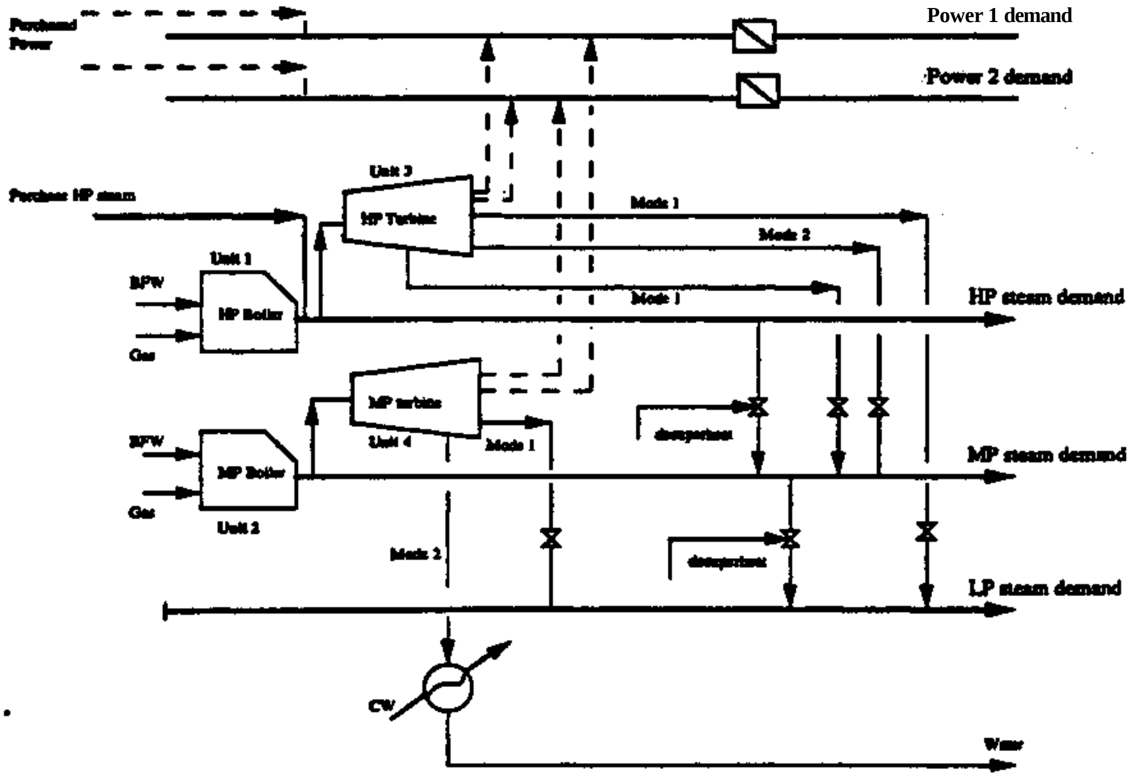

Fig. 6. Example - utility system configuration

The cost data for the units are presented in Table 1. Utility conditions in the plant are presented in Table 2.

TBblel
Cost data for Example

Cost Coefficients for units with associated flow variables

C= Fixed cost+ Variable cost * F

Unit	Flow Variable	Fixed($/yr)	Variable (t hr/ton per yr)
HP boiler (unit 1)	HP steam	90,000	9100
MP boiler (unit 2)	MP steam	40,000	8375
Purchased steam			9700

Cost Coefficients for units with associated power generated

C= Fixed cost + variable cost * W

Unit	Fixed ($ per yr)	Variable ($ /kW per yr)
HP turbine (unit 3)	45,000	25
MP turbine (unit 4)	25,000	21
Purchased Power		300

•For unit 4, additional cost of cooling water in condensation mode is

$760 / yr per ton/hr of steam condensed.

Fixed Startup/shutdown costs for units ($)

Unit	
1	3000
2	3000
3	1500
4	1500

Table 2
Utility conditions for Example problem

Utility conditions in plant			
	Enthalpy kWhr/ton	Pressure Mpa	Temperature K
HP steam	945	4.83	758
MP steam	874	2.07	523
LP steam	762	0.34	412
Extracted Steam	720	.013	326

The tradeoffs for the cost of operation for choice of different units is evident from the cost data. For a given value of power to be generated, the HP turbine is costlier than the MP turbine. However, the amount of steam required by the HP turbine is lower when it is operated in mode 2 when a large value of steam is extracted at LP steam level. This is because of a larger difference in enthalpy between HP steam and LP steam. The cost of producing HP steam is higher than producing MP steam although a lesser amount could be required by the HP turbine. Also, the choice of producing HP steam versus purchase pays off only for a load greater than 150 tons/hr. There is also an option to purchase power at higher cost when the demand cannot be met by the turbines. Clearly, when the demand of power is high and LP steam demand is low, then the HP turbine is a better choice. Conversely, for low power demand, when the LP steam demand can also be met by extraction from MP turbine, then the MP turbine is a better choice. The MP turbine may also be operated in condensation mode which required lesser amount of LP steam but there is an additional cost of cooling water load on the condenser in that mode. Thus, as demand varies for each of the utilities, correspondingly, the choice of operating units could be different based on lowest operating costs.

9.1.1 Example 1

In order to explain the steps of the algorithm, consider a small example comprising of 4 periods of operation. Assume that the option for purchasing power is not available in this example.

Given are the data for the operation of the utility plant for a short term 8 week horizon consisting of 4 periods of operation of equal lengths (Table 3).

Table 3
Data for Example 1

Power Demands for each period (kW)

Power Demands	1	2	3	4
1	6000	3000	5500	3200
2	3000	2100	3025	2400

Steam Demand for three levels for each period (ton/hr)

Level	1	2	3	4
High Pressure steam	49	45	55	50
Medium Pressure steam	45	20	40	45
Low Pressure steam	60	65	50	70

The steps of the algorithm are as follows. The algorithm first determines the configuration with lowest operating costs for each period in Step 1. In this case, there are 12 feasible configurations in each period. The values for the operating costs for each period is provided in Table 4 with corresponding maximum values of maximum transition costs at that node.

It is evident from the solution presented in Table 4, that for the first period only 9 configurations need to be enumerated. This is because for the ninth best configuration, the operating cost is greater the sum of the operating cost and maximum transition cost of node 2. For the second period, however, all the nodes are enumerated in this case. For the third period, all nodes after the ninth node need not be enumerated. Thus, only a partial enumeration of nodes is required, resulting in savings in computation time.

After completing the step of enumerating the subset all feasible configurations that may lie on the shortest path, a directed graph is drawn similar to the one shown in Figure 2. The cost at the nodes are the operating costs and the arc costs are the transition costs that can be determined from the configuration at each node. The shortest path in this example corresponds to a path the includes the second best configuration in each period of operation. The total cost of operation is $413,000 . If however, the best configuration in each period were chosen (corresponding to lowest operating cost), then the total cost is $436,200. Thus, there is a savings of $23,200, which is about 5% of total costs.

Table 4
Solution for Example 1

Costs ($) for feasible configurations for each period

Operating Costs

(Maximum Transition Costs)

Configuration	1	2	3	4
1	109850	85350	107700	106200
	(13500)	(18000)	(18000)	(15000)
2	110000	85810	107750	106475
	(10500)	(18000)	(18000)	(13500)
3	111750	86800	109600	106850
	(13500)	(18000)	(18000)	(10500)
4	115450	86970	114075	106675
	(10500)	(15000)	(12000)	(13500)
5	115950	87000	114600	106800
	(13500)	(12000)	(12000)	(10500)
6	117825	88700	116500	108100
	(13500)	(18000)	(12000)	(13500)
7	118750	89100	117400	108750
	(13500)	(18000)	(15000)	(13500)
8	119600	89325	118300	109250
	(16500)	(12000)	(15000)	(16000)
9	121475	90130	120190	109525
	(18000)	(15000)	(15000)	(16500)
10		91200	130825	110630
		(12000)	(18000)	(13500)
11		92010		111125
		(15000)		(16500)
12		97580		111400
		(18000)		(16500)

*Operating Cost of node 9 in period 1 = 121,475,greater than

Operating Cost + Maximum Transition cost of node 2= 120,500

Table 5
Data for Example 2

Power Demand for each period (kW)

Demand	1	2	3	4	5	6	7	8	9	10	11	12
1	5700	6000	7000	5500	3000	3000	3200	5000	5000	2000	3000	2200
2	3000	3000	3000	3025	2100	3000	2400	4000	5000	2000	3000	2000

Steam Demand for three levels for each period (ton/hr)

Level	1	2	3	4	5	6	7	8	9	10	11	12
HP steam	50	49	43	55	45	50	50	80	80	20	65	20
MP steam	40	45	35	40	20	30	45	70	70	30	50	45
LP steam	50	60	70	50	65	70	70	80	90	100	73	100

The optimal choice of unit in this example is operating unit 3 (HP turbine) and purchase of steam for all 4 periods. This solution is the same as obtained by solving problem (PI) as a single MILP without decomposition.

9.1.2 Example 2

We now consider the same utility plant for a longer time horizon comprising of more number of periods as compared to example 1. Given are the data for the operation of the utility plant for a one-year horizon consisting of 12 periods of operation of equal lengths (Table 5). The option of purchasing power is available in included resulting in a larger number of feasible configurations as compared to Example 1.

The problem (PI) was first solved using a MILP in the full-space of 12 periods. Special constraints (such as hard logic constraints, Raman and Grossmann [10]) were used to reduce the number of possible combinations for branching. For example, one hard logic constraint is that MP turbine is 'on[1] only if MP boiler is 'on'. Also, if there is a startup cost for a unit in a period t, then there cannot be a startup cost for period $t + 1$ represented as

Table 6
Computational results for Example 2

Solution type	Problem size			Solution time	Subproblems solved
	Equations	Variables		CPU seconds	
		Binary	Cont.	(IBM-RS6000)	
MILP	1053	204	721	>6000	single problem for 12 periods
Proposed algorithm	103	17	53	320	369 subproblems

$$z_{n,t} + z_{n,t+1} \leq 1 \qquad \forall n, t \tag{24}$$

Similar constraints may be formulated for shutdown costs.

The solution time for the full-space MILP (using OSL solver on GAMS [1]) was in excess of 6000 Cpu seconds on IBM-RS6000/530 (see Table 6). The solver exceeded the iteration limit and had not arrived at the solution because it had not fathomed all the open nodes in the branch and bound algorithm. However, it stopped at the best integer solution of $1,794,850.

The solution time using the proposed algorithm was 320 Cpu seconds. The solution obtained corresponds to the same optimal solution obtained from the full-space MILP solution. Table 6 gives the data on solution times and the number of subproblems solved in the proposed method. The size of the subproblems is significantly smaller as each subproblem is solved in the space of variables for only one period of operation.

Since the option of purchasing power was included, there are potentially 54 feasible configurations for each period of operation. However, based on the partial enumeration technique, only a small subset of configurations are enumerated (Table 7). More specifically, a maximum of 22% of feasible configurations were enumerated in any period.

The solution obtained for the example problem given in Table 8 illustrates

Table 7
Solution for Example 2 problem

	Number of configurations (subset of all feasible choices) selected for shortest path algorithm											
Periods	1	2	3	4	5	6	7	8	9	10	11	12
Configurations	6	9	9	9	12	12	12	9	9	12	12	12

the need to account for the transition costs to determine the optimal planning strategy. Two cases may be considered for comparison of total cost of operation, as shown in Table 8. Case 1 represents the choice of units based on minimization of operating costs in individual periods (i.e. choose configuration of rank choice 1 for each period which has smallest operating cost). Case 2 is based on choice of equipment that minimizes the total cost that includes the transition costs between periods. It is evident from the results that the optimal configuration in each period need not be the configuration with the lowest operating cost.

When the transition costs are significant as is the case in industrial problems like this example, the choice of operation with a simpler operation profile could result in a lower total cost of operation over the planning horizon. As an example, Table 8 shows the selected configuration for period 2 is the second best in terms of operating costs. This avoids the cost of startup and shutdown of HP turbine after period 1. The ultimate operational plan results in only a startup of 4 units as compared to a choice of 16 startups when periods are optimized independently. For this example, the overall cost of operation is lower by $53,500 which is about 3% of the total cost of operation.

From the solution times obtained, it is also clear that the algorithm results in a significant reduction in computation time. This is a result of the complexity of the algorithm scaling linearly with the number of periods in this example as compared to the exponential increase for the full-space method.

It is also worth noting that the solution under special constraints such as requiring same choice of units for all periods (if required) can be easily accounted for in Step 6 during execution of the shortest path algorithm. For this example, enforcing such a constraint resulted in a solution of $1,800,850. The solution indicated using unit 1 and unit 3 for all periods.

Table 8
Results for Example 2

Period	Optimal choice for independent periods based on operating costs			Optimal choice including transition costs			
	Case 1			Case 2			
	Operating costs (S)	Transition costs (S)	Unit'on'	Operating costs (S)	Transition costs ($)	Unit'on'	Rank choice
1	141200	3000	3, HP	141200	3000	3,HP	1
2	146500	3000	1,3	146700	0	3,HP	2
3	146600	3000	3,HP	146600	0	3,HP	1
4	143625	3000	1,3	143700	0	3,HP	2
5	113550	9000	2,4,HP	114400	0	3,HP	2
6	132225	6000	3,HP	132225	0	3,HP	1
7	140725	6000	2,4,HP	141650	3000	1,3	2
8	199400	9000	1,3	199400	0	1,3	1
9	208825	0	1,3	208825	0	1,3	1
10	114950	9000	2,43P	114950	9000	2,4,HP	1
11	159660	9000	1,3	160900	0	2,4,HP	3
12	126025	9000	2,4,HP	126025	0	2,4,HP	1

HP => Purchase HP steam

TOTAL	Operating + transition cost		
	Casel	Case 2	Difference
Cost ($)	1,848,285	1,794,850	53,435

9.2 Ramp function demand

9.2.1 Example 8

The same example problem was also solved for the case when the demand changes linearly within a period of operation. Each demand may be allowed

Table 9
Data for Example 3 with ramp change in demands

Power demands for each period												
Power demand	1	2	3	4	5	6	7	8	9	10	11	12
1	6510	5925	7150	5260	2675	3045	3225	5250	4960	1575	3200	2055
2	3425	2940	3010	3025	1965	3145	2290	4245	5110	1550	3200	1825

Steam Demand for three levels!for each period												
Level	1	2	3	4	5	6	7	8	9	10	11	12
HP steam	57	48	42	57	43	51	50	84	79	12	73	12
MP steam	46	45	34	41	17	32	47	73	70	24	52	44
LP steam	57	60	71	47	68	70	71	81	91	101	69	104

to vary independently of each other. In this example, the demand was varied in the initial 25% duration of each period, and was kept at a constant value for the remaining length of that period (Figure 7).

The demand data is presented in Table 9.

The solution obtained using the proposed strategy yields a solution of $1,474,100 as compared to a value of $1,501,700 if transition costs are not taken into account. For this example, a single configuration is feasible within each period of operation in spite of the changing demands in that period. The results for this case are shown in Table 10. The solution obtained also shows a simple operation plan which consists of only a single startup of the HP boiler in period 7.

9.3 Expansion planning

9.3.1 Example 4

In this example, the determination of the optimal choice of units for each period of operation is illustrated for the choice of making decisions for capacity expansion.

Consider the same utility plant with the following capacity limitations (Table 11).

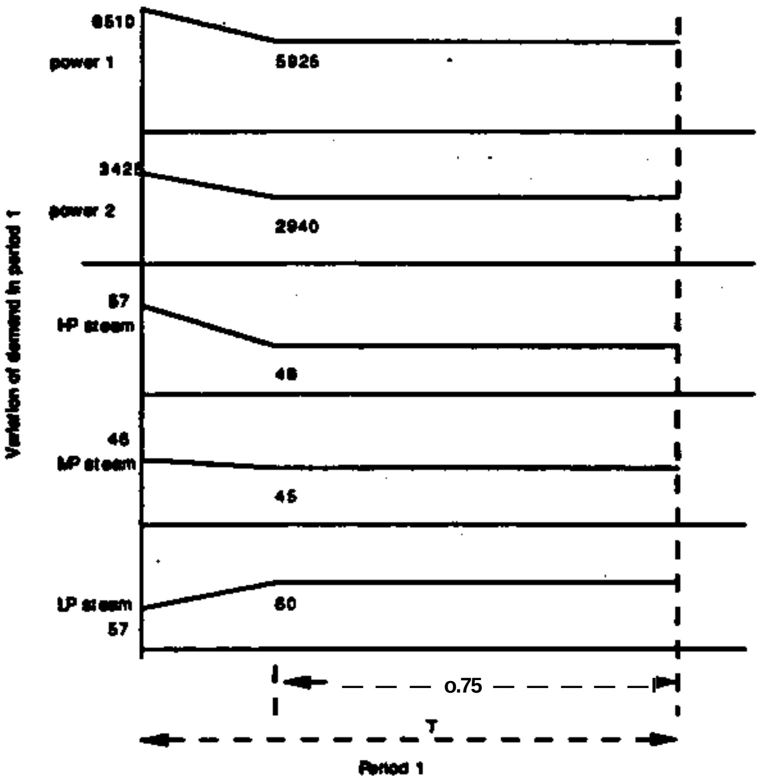

Fig. 7. Example 3 - ramp function change in demands

Assume that a retrofit change in process plant results in an increase in the power demands that cannot be met by the existing utility system. In particular, the new demands are presented in Table 12 for a 4 month planning horizon of 16 periods with a length of 1 week for each period. Batch plants with product campaigns of upto 4 months would be an example where such demand patterns would be common.

The options that were considered to meet the new demands are as follows

(i) Purchase and commissioning of a HP turbine (identical to the existing HP turbine with same equipment specifications and costs).
(ii) Purchase of power from external sources.

The new proposed retrofit configuration for the plant is as shown in Figure 8. If the optimal operation plan includes the choice of utilizing the second HP turbine, then the proposed retrofit configuration will be selected.

*

It must be noted that the configuration of lowest operating cost for individual periods (for even number periods when the power demand cannot be met by existing units) involves operating both HP turbines in parallel. This is because of the higher cost of electricty purchase. However, if the HP turbine is purchased, then for periods when power demands are lower (odd number

Table 10
Results for Example 3 for ramp change in demands

Number of configurations (subset of all feasible choices) selected for shortest path algorithm

Periods	1	2	3	4	5	6	7	8	9	10	11	12
Configurations	2	6	9	9	9	12	12	12	9	9	12	10

	Periods											
	1	2	3	4	5	6	7	8	9	10	11	12
Case 1 (operating costs only)	1,3	1,3	3,HP	1,3	2,4,HP	3,HP	1,3	1,3	1,3	3,HP	1,3	1,3
Case 2 (transition costs)	3,HP	3,HP	3,HP	3,HP	3,HP	3,HP	1,3	1,3	1,3	1,3	1,3	1,3

HP= Purchase HP steam

Table 11
Additional plant data for Example 4

Maximum capacity of HP boiler	100 tons/h
Maximum capacity of MP boiler	50 tons/h
Maximum capacity of HP turbine	13,000 kW
Maximum capacity of MP turbine	8,000 kW
HP steam purchase cost	$3.2 /ton
Investment cost of HP turbine	$45,000 /yr

periods), the configuration of lowest operating cost requires only choice of one HP turbine. This could result in a transition cost in alternate periods for one of the HP turbines.

The problem (Pi) was first attempted by solving a full-space MILP (containing 336 binary variables). The OSL solver failed to obtain the optimal solution after 6000 CPU seconds. The best integer solution obtained was of the order of $5 million. However, using the proposed method, the optimal solution has a total operating cost of $1,601,600 for the 4 month period. The optimal solution was obtained in 1150 CPU seconds. A major reason for the savings was that the MILP for individual periods consists of only 21 discrete variables.

It is interesting to note that the optimal operation plan determines that it is

Table 12
Demand Data for Example 4

Power Demand for each period (MW)

Power	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	**8**	**16**	**8**	**16**	**8**	**16**	**8**	**16**	**8**	**16**	**8**	**16**	**8**	**16**	**8**	**16**
2	**4**	**6**	**4**	**6**	**4**	**6**	**4**	**6**	**4**	**6**	**4**	**6**	**4**	**6**	**4**	**6**

Steam Demand for three levels for each period (ton/hr)

Steam	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
HP	20	20	30	25	25	30	30	25	20	30	25	20	30	20	25	25
MP	105	105	95	105	105	105	100	115	105	105	95	105	100	105	105	100
LP	60	60	50	40	65	50	65	55	70	45	70	65	75	70	80	75

Cost of electricity purchase for each period **S/kW-yr**

Level	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Cost	285	285	285	275	275	275	270	270	270	270	260	**260**	260	**260**	260	**260**

cheaper to purchase electricity and not use the new HP turbine. This implies that it is a better choice to *not* invest in the purchase of a new turbine. In addition, despite the fact that for odd number periods, the cheapest choice is to use HP boiler and HP turbine (with operating cost of $77,800 for period 1), the optimal plan chooses the 5th best configuration with an operating cost of $80,300. Thus, it chooses to operate units 1,2, 3 and 4 in period 1 instead of units 1 and 3. This is justified by the fact that for even number periods when the demand for power is higher, it is better to choose units 1,2,3,4 and purchase electricity to avoid any changeover costs between periods. The optimal operational plan is to use all units 1,2,3,4 and purchase power during all periods.

The number of configurations enumerated in each period is also significantly smaller than the total number of feasible configurations. The maximum number of configurations enumerated was 16 as compared to a possible 34 configurations for each period.

Prom this example, it is evident that the choice of purchasing a turbine that

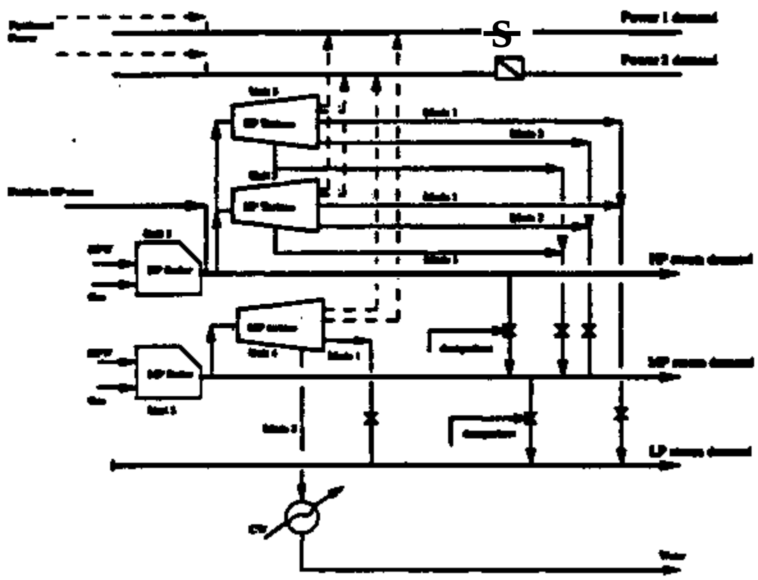

Fig. 8. Example 4 - capacity expansion planning

meets the power demands at a lower cost in individual periods, need not be optimal when the power demands vary and when transition costs are significant. In this example, the choice of purchasing the turbine with changeover between periods would result in an increase in total cost of $42,000. Thus, if capacity expansion were to be undertaken, then solving independently for each time period, it would have resulted in a total cost of $1,643,300 as compared to the $1,601,600 for the case when operational planning for capacity expansion includes transition costs. It is interesting to note that the optimal plan also suggests purchase of electricity in all odd number periods even when power demands can be met by existing units.

10 Conclusions

A multiperiod MILP model for utility system operation planning was presented accounting for operation and transition costs between periods. A new two-stage decomposition algorithm has been proposed for solving large MILP problems. The basic idea relies on constructing a shortest path network by solving subproblems involving only one time period. The algorithm may be applied to problems containing linking constraints associated with binary variables of the form in equation (12). The results have showed that the proposed algorithm can achieve an order of magnitude reduction in computation time in determining the optimal solution of the MILP. The computational time requirement of the proposed algorithm is linear in the number of periods as compared to a exponential increase in computation time for the simultaneous solution of the overall model (P2). Solution of example problems produces upto 5% reduction in total cost of operation when compared to independent solution for each time period. What is more significant , however, is that much simpler operation plans that minimize startup/shutdown of equipment between periods are obtained. The method has also been extended to account for ramp function change in demands and for capacity expansion planning problems. Finally, an example problem was presented to illustrate the fact that the proposed model can be used for performing design retrofit studies.

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References

[1] Brooke A., Kendrick D. and Meeraus A., *GAMS: A users Guide* , Scientific Press, Palo Alto (1992).

[2] M.A.Duran and LE.Grossmann, An outer-approximation algorithm for a class of mixed-integer nonlinear programs, *Mathematical Programming* 36, (1986), 307-339.

[3] D. Foster, Optimal unit selection in a combined heat and power station, /. *Chem. E. Symposium Series* 100 (1987).

[4] B.F. Hobbs, Optimization methods for electric utility resource planning, *European Journal of Operational Research* 83 (1995), 1-20.

[5] K.Ito, R.Yokoyama, S.Horii, Y.Matsumoto and S. Akagi, An Optimal Operational planning model for gas turbine cogeneration plants, *Tokyo International Gas Turbine Congress* , Tbkyo (1987).

[6] B. Kalitventzeff, Mixed integer non-linear programming and its application to the management of utility networks, *Engineering Optimization* 18 (1991), 183-207.

[7] R. Nath and J.F.Holliday, Optimizing a Process Plant Utility System, *Mechanical Engineering* (1985), 40-50.

[8] S.A. Papoulias and I.E. Grossmann, Optimal synthesis of flexible utility systems, *18th IECEC conference* (1983), Orlando.

[9] S.A. Papoulias and I.E. Grossmann, A structural Optimization approach in process synthesis-I, *Computers and Chemical Engineering 7* (1983), 695-706 .

[10] FLRaman and I.E. Grossmann, Symbolic Integration of Logic in Mixed Integer Linear programming techniques for Process Synthesis, *Computers and Chemical Engineering* 17 (1993), 909 .

[11] S.C.Stultz and J.B. Kitto, *Steam, its generation and uses* , Baberton, Ohio (1992).