Relating Network Topology to the Robustness of Centrality Measures*

CASOS Technical Report

Terrill L. Frantz, Kathleen M. Carley

May 2005

CMU-ISRI-05-117

Carnegie Mellon University
School of Computer Science
ISRI - Institute for Software Research International
CASOS - Center for Computational Analysis of Social and Organizational Systems

Abstract

This paper reports on a simulation study of social networks that investigated how network topology relates to the robustness of measures of system-level node centrality. This association is important to understand as data *collected* for social network analysis is often somewhat erroneous and may—to an unknown degree—misrepresent the actual *true* network. Consequently the values for measures of centrality calculated from the collected network data may also vary somewhat from those of the true network, possibly leading to incorrect suppositions. To explore the robustness, i.e., sensitivity, of network centrality measures in this circumstance, we conduct Monte Carlo experiments whereby we generate an initial network, perturb its copy with a specific type of error, then compare the centrality measures from two instances. We consider the initial network to represent a *true* network, while the perturbed represents the *observed* network. We apply a six-factor full-factorial block design for the overall methodology. We vary several control variables (network topology, size and density, as well as error type, form and level) to generate 10,000 samples each from both the set of all possible networks and possible errors within the parameter space. Results show that the topology of the true network can dramatically affect the robustness profile of the centrality measures. We found that across all permutations that cellular networks had a nearly identical profile to that of uniform-random networks, while the core-periphery networks had a considerably different profile. The centrality measures for the core-periphery networks are highly sensitive to small levels of error, relative to uniform and cellular topologies. Except in the case of adding edges, as the error increases, the robustness level for the 3 topologies deteriorate and ultimately converges.

* his work was supported by the Department of Defense and National Science Foundation under MKIDS program 30218466) and the Office of Naval Research under Dynamic Network Analysis program (N00014-02-1-0973). e views and conclusions contained in this document are those of the authors and should not be interpreted as resenting the official policies, either expressed or implied, of the Department of Defense, the National Science indation, the Office of Naval Research, or the U.S. government.

Keywords: social networks, data error, centrality, robustness, sensitivity, simulation

Table of contents

I. Index of Tables

II. Index of Figures

1. Motivation

By its nature, social network analysis is burdened by the underlying complexity of both the underlying subject matter and its data collection procedures. While analysts have made substantial progress in developing techniques to analyze the data that they have collected, measurement error in social network data remains an omnipresent problem (Marsden, 1990). In addition to other intrinsic complications, social network measurement error arises from the inherent ambiguity of human-informant reliability—unintentional (Freeman, Rommey, Kimball, & Freeman, 1987; Killworth & Bernard, 1976) and intentional (Carley, 2003)—, and the intricacy of collection-instrument design.

In spite of this widely recognized problem, analysts continue to infer a great deal from the error-prone network data. Of course, analysts derive important quantitative measures of the network using the data that they have. Unfortunately, the accuracy of their analysis may in fact be bounded, or at least restricted, by the accuracy of the underlying data they prudently rely upon. Subsequently, analysts and particularly consumers of information may in fact be making misguided judgments based on mistaken analysis derived from flawed source data. Given the subsequent presumed-error in the network measures, one can only contemplate how misguided past analyses may in fact be and what the impact on subsequent actions may have been.

The research herein is broadly motivated by the wide recognition that measurement error is truly ubiquitous in social network data; yet we have little understanding of the actual impact on the critical quantitative measures we rely upon for our analysis. Irrespective of the past calls for the study of the impact of this problem (Marsden, 1990), even vague attempts to address this quandary are rare. While an unsuccessful literature search may brand the matter as *terra incognita*, there have indeed been a mere handful of pertinent articles published on the subject. Certainly, further exploration is warranted.

Our research aims to increase the knowledge of the impact of erroneous source-data on network measures. Specifically, we seek to understand the robustness of network measures of *centrality* relative to the network's topology. We project that, given the known characteristics of a social network—including *a priori* estimate of error characteristics—, analysts may ultimately be able to quantify the impact of these errors on centrality measures and adjust their analyses accordingly. Ultimately analysts may some day harvest more accurate information obtained from the likely *true* network rather than from the erroneous *observed* network

2. Introduction

The term *robustness* as it pertains to social networks has two related, albeit different connotations. The robustness *of a network*, is concerned with the reliability (Kim & Médard, 2004) and continued functioning of a network following an intervention. Post-9/11, this is particularly in the context of a destructive attack—purposeful (Tsvetovat & Carley, 2005) or accidental—on the nodes or connections in a network. The robustness of a network is particularly relevant in communication-type and flow-oriented networks. The purpose for understanding robustness of a network has more of a management of the network connotation.

Another connotation of the term robustness—the one in which we are primarily concerned herein—is the robustness *of the measures* of a network. When associating the term to a measure, the meaning has more of a statistical connotation. Studying the *robustness of a measure* of a network can also be referred to as conducting a sensitivity analysis on the measure. In keeping with the terminology of the most-recently published research in this area, in lieu of using the term *sensitivity*, we too will use the *robustness* term, although the terms can be used interchangeably.

A measure is *robust* if a slight perturbation in its input produces a slight change in its output. Robustness is clearly desirable in a measure, as input data is seldom without error, and robustness implies that the measure's output for the true data (the input data without error) is nearly equal to that of the data with error.

Further to investigating the robustness of a measure, our attention is drawn to the robustness of *measures of centrality* of a network because the notion of centrality is one of the first (Moreno, 1934) and foremost measures social analysts concern themselves with. A network-actor's centrality is often associated with the level of their prestige and power relative to other actors, which is a key question of even the most basic actor-level social network analysis. At the overall network level of analysis, or sub-group level, identifying the group of actors with the highest values in the various measures of centrality is also a common activity.

There are numerous perspectives on centrality in a network as evidenced by the multitude of measures formulated and substantiated in literature. Freeman's (1979) seminal essay in the first volume of *Social Networks* seemingly introduced the core concept to the social network community. Particular to this study, we scrutinize four specific measures of centrality chosen because of their prominence in network analysis. We focus on: degree, closeness, betweenness, and eigenvector centralities. (These four specific measures are also core to a prior robustness study whose methodology we follow closely—see our Methodology Section for more information.).

Consistent with the foundational importance of centrality to social network research; to date, past studies of robustness—again, as we regard the term—have focused exclusively on measures of centrality as opposed to any other group of or individual measures. Although the total number of robustness-specific studies is somewhat limited, there are both empirically-based and simulation-based studies to consider.

One approach an analyst may take to address inherent measurement error is to use comprehensive statistics (Frank, 1971) or one of several sampling techniques (Erickson & Nosanchuk, 1983; Frank, 1978; Frank, 1981; Galaskiewicz, 1991; Granovetter, 1976) on the observed network to make "reasonable, if not excellent" (Galaskiewicz, 1991, p. 347) estimates of the actual centrality measures for the true network.

In a recent, combined meta-analysis and simulation study utilizing empirical social network data obtained from 8 independently conducted studies, Costenbader and Valente (2003) analyzed 11 measures of centrality for their robustness to simulated network data error[2]. They concluded that under "some circumstances" (p. 305) analysts may still use measures calculated from data that has missing information. They continued however, "The results of this study should be interpreted with caution..." (P. 305) and warned of limitation to the generalization of their findings.

Another combined case study—specifically, a 16,726 node collaboration network—and simulation by Kossinets (2005) also investigated data error by applying random error to empirical data. Kossinets found that errors such as those resulting from boundary specification (including only a subset of relevant nodes) can significantly alter network-level statistics such as average degree centrality, clustering, and other measures.

[2] Actually, Costenbader and Valente (2003) drew repeated random samples (subsets) from the existing network data in order to investigate sampling techniques (Rothenberg, 1995). We posit that their approach is congruent with investigating data set error.

In this study we seek to develop more generalized findings that specific case-based studies allow. To accomplish this, we can take advantage of unlimited computing power to randomly sample both the true network and the observed network as opposed to prior studies using an empirical data set as the true network and sampling only the observed from that.

The Borgatti, Carley, and Krackhardt (in press)—which is a template for the methodology of this study—explored robustness of four centrality measures (degree, betweenness, closeness and eigenvector) in random graphs and had several findings including: (a) Measure accuracy declines predictably with increasing error, (b) the measures have a similar robustness pattern and level, (c) the type of error (node / edge) has little affect on the robustness level, and (d) increasing density reduces accuracy of the measures, except for edge-addition where accuracy increased.

As is often the case, initial studies on networks are conducted on the uniform random network model attributed to Erdős and Rényi (1959). Following tradition, Borgatti, Carley and Krackhardt conducted their research using ER graphs. We extend on their work by exploring the relevance of a network topology is on the findings of Borgatti, Carley and Krackhardt. Since social networks are rarely of the uniform random network variety, we investigate the robustness of measures of centrality under different network topologies. The three topologies we study are: uniform, cellular, and core-periphery (Borgatti & Everett, 1999).

Cellular networks are characterized by consiting of a collection of distributed, but sparsely connected, tightly-coupled cells, a.k.a. groups, that are often small and (if functional) operate independent of one another and can be somewhat self-similar in their form. Core-periphery networks are those that have a single primary cohesive core group that is sparsely tied to a periphery of others that often are not ties to others beyond those in the core.

3. Method

This study borrows its methodology directly from the experiments recently conducted by Borgatti, Carley and Krackhardt (in press). To evaluate the robustness of four measures of centrality, they conducted a multitude of experimental trials using simulated relational data in the form of uniformly-random networks, i.e., Erdős and Rényi (1959) uniform graphs. For each replication, they generated a *true* network that was effectively randomly drawn from the complete ensemble of possible networks based on several control parameters. This *true* network was then systematically perturbed (effectively drawing randomly from the realm of all possibilities), resulting in a corresponding *observed* network, i.e., the *true* network with simulated measurement errors. The value differences between the corresponding centrality measures for the network-pair where then evaluated. The experiment was controlled under a factorial design in which they varied several parameters that characterized the generation of the *true* and *observed* network-pairs.

Herein, we duplicate the prior design and expand on the analysis by introducing *network topology* as an additional control parameter, i.e., independent variable. In essence, we simply add an additional dimension to the factorial design which allows us to investigate the relationship between network topology the robustness of the centrality measures. We control for network topology by systematically varying the generation of the *true* network across these three network forms: uniform, cellular, and core-periphery.

Note: Readers familiar with the design of the Borgatti, Carley and Krackhardt (in press) experiment may choose to pass over, or merely skim, the remainder of this section as we provide similar methodological information; although herein, somewhat different terminology is employed and different aspects of the design are accentuated.

3.1 Six-Factor Full-Factorial Block Design

The overall approach incorporates a six-factor (8 × 5 × 4 × 3 × 2 × 2) full-factorial block design totaling 1,920 independent trials. The factors we control for, i.e., control variables, and the respective number of values (in parentheses) for each are: network topology (3), network size (4), network density (8), error type (2), error form (2), and error level (5). Given a factorial design, a particular trial is characterized by a combination of the six control variables, each systematically assigned one of their respective possible values. For example, one specific trial is described as "topology=cellular, size=100, density=30%, error type=node, error form=remove, and error level = 10%."

Emanating from this factorial design, at the end, a six dimension *results matrix* is ultimately assembled. Each dimension corresponds to one of the six control variables. Each element of the results matrix contains a group of summary statistics, e.g., mean-average, that is calculated using data values from the set of replications underlying that trial. It is this completed results matrix that is the principal focus of the results analysis we present later in the report.

Each trial is replicated 10,000 times under identical conditions, i.e., using the same assigned values for the six control variables. Each replication is an experimental unit conducted entirely independent of any other. The outcome of each replication is a set of values for basic centrality measures which ultimately contributes to the trial's robustness summary statistics for which it is a part.

3.2 Control Variables

For the purpose of our discussion, we segregate each of the 6 independent control variables into one of two classes; either the: (a) Network Class, or (b) Error Class. Each variable is classified according to its connection to either the construction of the initial true network, or to creating the perturbations leading to the formation of the observed network.

Table 1. Independent variables and assigned values differentiating trials

Independent Variable	Number of Values	Assigned Values
Network Class		
Topology	3	uniform, cellular, core-periphery
Size[a]	4	10, 25, 50, 100
Density[b]	8	1%, 2%, 5%, 10%, 30%, 50%, 70%, 90%
Error Class		
Type	2	node, edge
Form	2	add, remove
Level[c]	5	1%, 5%, 10%, 25%, 50%

[a] specifies the number of nodes in the true network
[b] specifies the number of edges in the true network relative to its number of nodes
[c] specifies the number of edges or nodes—added or removed—relative to the original true network

The control variables in the Network Class are: topology, size, and density. Variables in this class represent a defining characteristic of the true, a.k.a., *orignal,* network. Possible values for the network-topology—referring to the network-level structure—are: uniform, cellular, and core-periphery. Possible values for network-size—referring to the number of nodes making up the network—are: 10, 25, 50, and 100. Possible values for the network-density—referring to the edge density of the overall network—are: 1%, 2%, 5%, 10%, 30%, 50%, 70%, and 90%.

The control variables of the Error Class are: type, form and level. Each variable in this class refers to the manner in which the true network is systematically perturbed. Recall, the original *true* network is randomly changed (within systematically specified parameters), thus creating the new *observed* network. Possible values for the error type are: node, and edge. Possible values for the error form—referring to the addition or removal of the error-types, i.e., nodes or edges—are: add, and remove. Possible values for error size—referring to the percentage of nodes or edges, being added or removed—are: 1%, 5%, 10%, 25%, and 50%.

3.3 Network-Pairs

Each replication within experimental trials involves a single, unique network-pair. Each network-pair consists of one *true* network and one *observed* network. The *true* network pertains to the "truth" of the network under study, while the *observed* network pertains to data in reality collected for that same true network. Using this network-pair paradigm provides an opportunity to identify and inspect the precise differences between the true and the observed at the detail level (we can identify specific nodes and edges that are changed, or in error) and according to any measures from either case.

We give rise to the true network by randomly drawing a single unique network from the set of all possible realizations that can be constructed from the specific characteristics designated by the control parameters (topology, number of nodes, density, etc) for the trial. This network is labeled the *true* network for this network-pair. An exact copy of this *true* network is then perturbed according to other control variables for the experiment (error type, form and level). This true-but-now-changed network is labeled the *observed* network. More detail to the creation of these networks appears later in this section.

3.4 Measures of Centrality

We evaluate network centrality as a generalized concept by assessing four specific node-level centrality measures which are instrumental to most social network analysis. For each node in a given network (true or observed), we gauge: degree, betweenness, closeness, and eigenvector centralities.

To calculate the four values for each node, we make use of ORA (Carley & Reminga, 2004), which is network-statistics software that is established in the network analysis field. After the various values have been calculated for each of the nodes; for each of the four measures, ORA provides a ranked list of nodes, ordered from the node with the highest value to the lowest.

This process results in four separate ordered lists of nodes for each network in the network-pair. We refer to the ordered-lists formed from the true network as the *true centrality* list and those from the observed network we identify as the *observed centrality* list. Using the respectively paired true and observed centrality lists, we can calculate the congruence of the four centrality measures across the network-pairs, i.e., the corresponding true and the observed networks.

3.5 Calculating Congruence

From respective true and observed centrality lists for each network-pair, we calculate the congruence in five ways: (a) Top1, (b) Top3, (c) Top10%, (d) Overlap, and (e) R-Squared. These each provide an indication of the congruence between the true and the observed networks in terms of their four centrality measures. If both of the centrality lists making up a centrality-pair are identical in all aspects, they would be perfectly congruent. Three of the congruence measures are binary values while two are real values from 0 to 1, inclusive.

- *Top1* is a binomial value that reflects whether (=1) or not (=0) the top-rank node in the true network is also the top-ranked node in the observed.
- *Top3* is a binomial value that reflects whether (=1) or not (=0) the top-rank node in the true network is one of the top-3 nodes in the observed network.
- *Top10%* is a binomial value that reflects whether (=1) or not (=0) the top-rank node in the true network is also ranked in the top 10% (of nodes) in the observed network.
- *Overlap* is a real value between 0 and 1 (inclusive) that reflects the extent to which the top 10 nodes in the true network match the top 10 nodes in the observed network. The formula for this ratio is: $N(T_t \cap T_o) / N(T_t \cup T_o)$.
- *R-Squared* is the squared value of the Pearson correlation between the true and the observed centrally measures. For the R-Squared value, nodes not found in both the true and the observed networks are excluded from this statistic.

For each trial (each consisting of 10,000 replications) basic summarization statistics of the congruence measures are recorded, including: minimum value, maximum value, average value (the arithmetic mean), and standard deviation. These summary statistics are determined for each of the measures of robustness and represent a quantitative perspective on the congruence of the centrality measures for a given experimental trial.

3.6 Determining Robustness

A measure is robust if a small change in its input value(s) produces only a slight change in its output value. In this study we ponder the robustness of the measures of network centrality; that is, given different levels of error in the input data, how much difference is there in the values for the centrality measures.

To determine and quantify robustness, we investigate the summarized congruence values vis-à-vis the combinations of values for the control variables, Network class or Error class. We form tables and corresponding line graphs to facilitate analysis, from which we draw our findings and conclusions.

3.7 Generating the True Network

We apply the Network Class control variables to specify the *true* network for each replication, which is statistically independent of one another. This involves assigning one value —based on the specific trial for which the replication is affiliated with—to each of the three characteristics: network topology, network size, and network density; to characteristic the specific population of networks from which the true network is essentially randomly drawn.

In practice, we actually generate the drawn network by using software written specifically for this study that follows the algorithm(s) described in Frantz, Airoldi, and Reminga (2005). Using the topology-specific generation procedure, we ultimately embody the true network in a DyNetML (Tsvetovat, Reminga, & Carley, 2004) formatted data file (optionally held in CPU memory, or persisted via a disk file).

The network size control variable is assigned a single value from this list: 10, 25, 50, and 100. This refers to the exact number of nodes making up the true network. We limit ourselves to these smaller-sized networks as they correspond in order of size with many social network studies whose data is collected through survey or some subject response vehicle, as opposed to a study that can collect data in an automated form.

The network density control variable is assigned a single value from this list: 1%, 2%, 5%, 10%, 30%, 50%, 70%, and 90%. This value is used in a formula to determine the number of edges in the network.

Following the network topology control variable, we characterize the topology for each true network as one of three forms: uniform, cellular, or core-periphery. "uniform" is an Erdős and Rényi (1959) graph (also called a "Bernoulli" graph, among other names) that begins with a fixed number of nodes (herein based on the network size control variable) and systematically evaluates each possible node pair to determine if that specific potential tie between the two nodes will actually exist or not—based on a fixed probability. In our experiments, this probability is fixed to a value according to a formula based on the network density control variable.

The "cellular" network is further characterized by a parameter provided to the software, called "*mean cell size*" that we fixed with a value of 6. This is the average number of cells making up a single cell in the network, which is tied together as a complete clique.

The "core periphery" network is further characterized by a parameter provided to the software called "*alpha*" that we set with a value of 6. The number of edges a node has is proportional to its attribute vector value. In a network with N total nodes, node k's attribute vector value = $(10*N / (k+1)^{alpha})$.

3.8 Generating the Observed Network

To generate data for the observed network of the network-pair, the true network is purposely perturbed. Depending on the Error Class parameters designated, we add (or remove) nodes and edges from the true network dataset to create a new, observed, dataset. Within the parameters, these perturbations are random following a well defined heuristic, described below.

In the case of adding nodes, new nodes are added according to the number specified by the error-level parameter (values: 1%, 5%, 10%, 25%, and 50%). To determine the number of new nodes the error-level parameter is multiplied by the value of the network-size parameter (values: 10, 25, 50, and 100). For example, in the case of a true network with 100 nodes, to be perturbed by adding nodes to the level of 5%, we would add 5 nodes to the true network to generate the observed network. Further, edges are also added accordingly to the new nodes to prevent adding simply a set of isolate nodes. To add these edges, for each new node, another node is randomly selected from the existing nodes and a corresponding number of edges are added to the new node to match the number of the sampled node. The alter edges are ties to randomly selected other nodes that do not already have an edge to the new node.

In the case of adding edges, new edges are added according to the number specified by the error-level parameter (values: 1%, 5%, 10%, 25%, and 50%). To determine the number of new edges, the error-level parameter is multiplied by the number of edges in the true network (this determined by the network size multiplied by the density). As in the case of the edge addition step when adding nodes, already existing edges are not strengthened.

As with the true network, the observed network is represented in DyNetML (Tsvetovat, Reminga, & Carley, 2004) and stored in computer memory and persisted on computer disk.

3.9 Procedure

While the true networks are considered to be drawn uniformly at random from all possibilities given the Network Class variables, in practice the true networks are actually *generated* according parameters consisting of the Network Class variables and computer code applying algorithms according to the desired network topology. The result is the same regardless of the actual heuristic used, i.e., a network with the expected values of the Network Class variables is presented randomly from all the possibilities constrained by values of the Network Class variables.

- Repeat for each value (*n*=3) of network topology
  - Repeat for each value (*n*=4) of network size
    - Repeat for each value (*n*=5) of network density
      - Repeat for each value (*n*=2) of error type
        - Repeat for each value (*n*=2) of error form
          - Repeat for each value (*n*=6) of error level
            - Replicate 10,000 times for each trial
              - Generate true network based on Network Class control variables
              - Perturb the true network to create an instance of an observed network based on Error Class control variables, creating a network-pair
              - Compute centrality rankings lists for both the true and the observed networks
              - Calculate pair-wise congruence values for each of the four centrality ranking lists
            - Calculate summary statistics for the congruence values for each of the four centrality measures (*n*=10,000): minimum, maximum, mean, standard deviation.
- Report summary statistics across all experiment blocks (*n*=1,440)

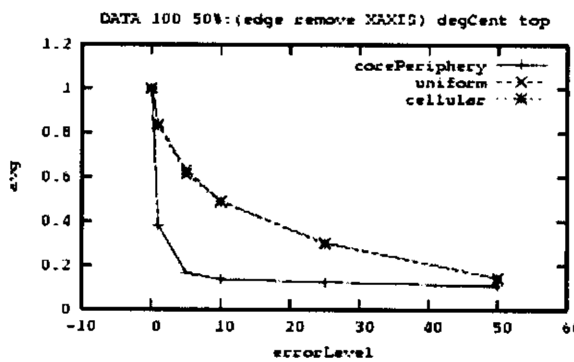
Fig. 1. Pseudo code summarizing the factorial block procedures of the methodology

4. Results

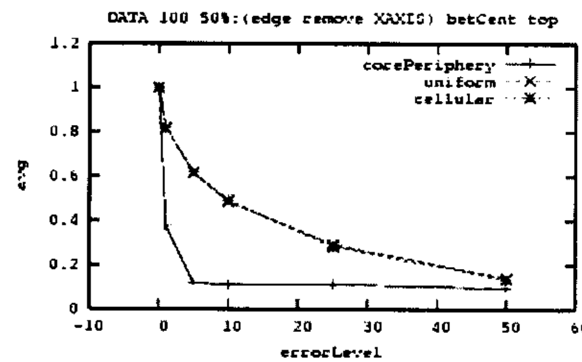
We report five specific observations stemming from our review of the entire set of results data, although we present only from the perspective of the case of the100 nodes with 50% density (100/50%) true networks. While in this section we present only selected charts chosen specifically to augment the five observations, the entire set of result-data tables for the 100/50% true network experiments are provided in the Appendix and the data for all experiments are available from the authors.

We present the detail of our observations in the case of a single true network to keep the text undemanding. We consider this approach suitable since the 100/50% case provides a neutral configuration for comparing the sensitivity of the centrality measures; Borgatti, Carley and Krackhardt (in press) establish that when evaluating robustness 50% density is an impartial point relative to the type of errors (node/edge add/remove).

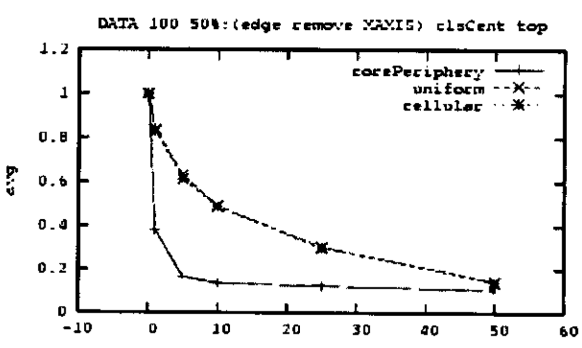
Our first observation is the prominent similarity of the accuracy scores across the four centrality measures, regardless of the experiment parameters (true network and errors control variables). Throughout the experiment under the same true/observed network parameters, degree centrality, betweenness, closeness, and eigenvector centrality, all showed remarkably similar values for measure accuracy (with one exception to be pointed out later). In every instance of the combination of topology, size, density, error-type and error-form, robustness profile across the measures is comparable. As expected, the actual accuracy values for a congruence measure (top1, top3, top10pct, etc.) differed, but, as we observed, their corresponding values were consistent across each of the four centrality measures.



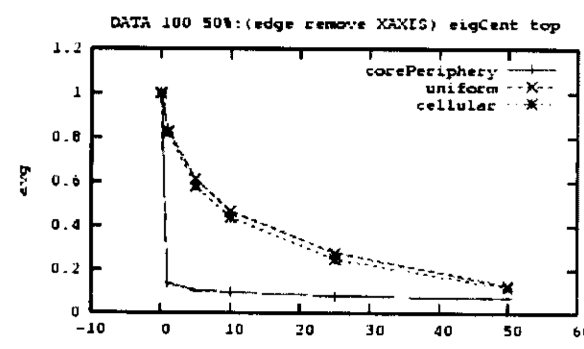
(a) Degree Centrality



(b) Betweenness Centrality



(c) Closeness Centrality



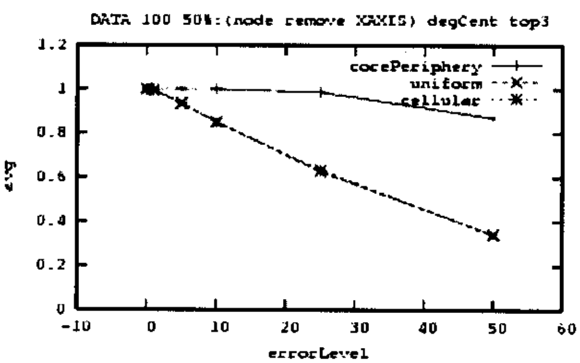
(d) Eigenvector Centrality

Fig. 2. Cross-cut of four centrality measures showing comparable accuracy profiles.

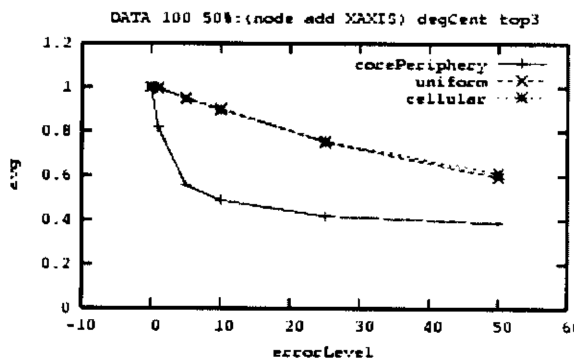
As an example, Fig. 2 shows this phenomenon for the 100/50% network with edge-remove errors for the *top* congruency measure. Each line on the chart represents the accuracy between the true and the observed network for a given topology at different error levels. The similarity of the four charts can be readily seen, which provides a simple visual to this phenomenon. It is easy to also recognize in Fig. 2 that the uniform and cellular lines lie atop one another; this will be discussed separately later.

To further keep the text undemanding, we will also limit our presentation of results from this point forward to those of the degree centrality only. Degree centrality is certainly the least complicated of the measures to think about; as provided by the first observation, all observations about degree centrality can be equally ascribed to betweenness, closeness, and eigenvector centrality measures (again, with one exception to be pointers out later). Readers should consider degree centrality as a proxy for the other centrality measures.

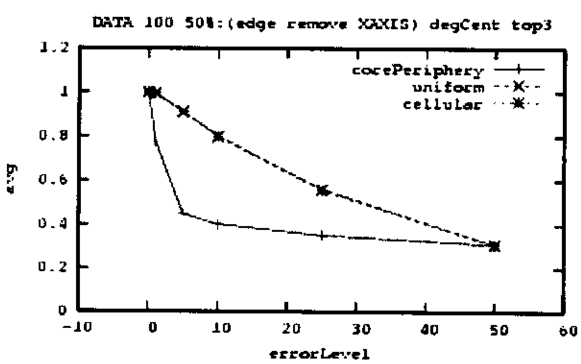
A second observation, as noted briefly above, is that the uniform and cellular topologies have nearly identical accuracy results from every view of the data. When looking at any plot of the results, the graphic representations of the two most often lie atop one another. In many cases the average accuracy values are identical, with numerous exceptions, but the differences are usually limited to only a tenth of a percentage point. Figure 2, used to support the first observation, also provides an example of the duplicity of the results for the uniform and cellular topologies. In some instances the two do display as separated lines, albeit very slightly. For this reason, we will continue to show the three topologies on graphs, but will generally discuss uniform and cellular as a united pair.



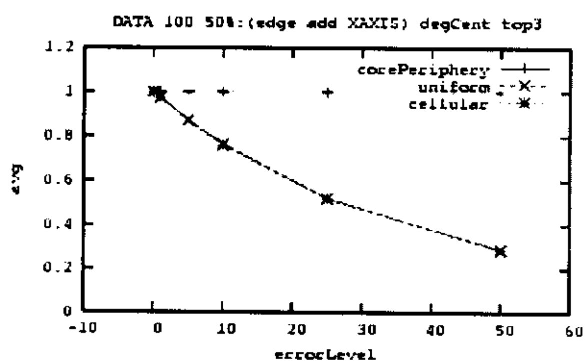
(a) Node Remove



(b) Node Add



(c) Edge Remove

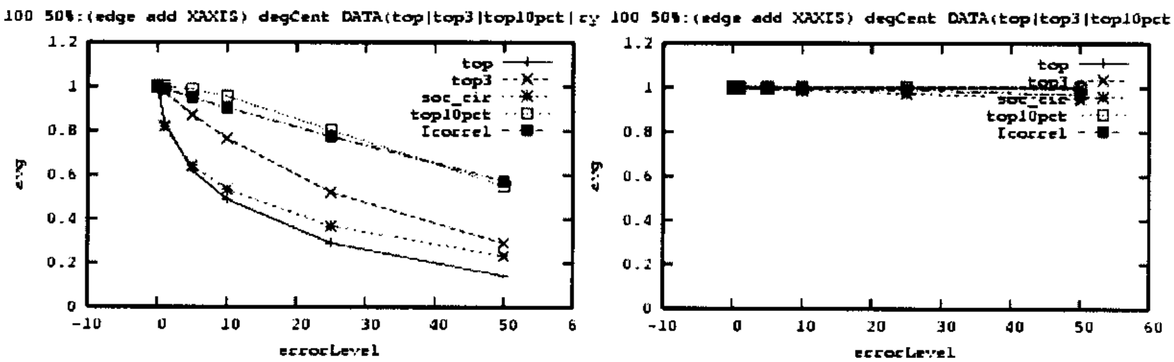


(d) Edge Add

Fig. 3. Degree centrality showing average accuracy of top3 congruency measure

Our third observation is that, in two of the four error type/forms, the core periphery topology *consistently* shows significantly more accuracy than that of the uniform and cellular topologies. In the specific cases of node-remove and edge-add errors, core periphery has much more accuracy that the other topologies when any error is introduced. In these cases core periphery seems to have highly robust measures of centrality, seemingly immune to network data errors. Conversely, in the contrary cases of node-remove and edge-add, core periphery has markedly less accuracy as soon as *any* error is introduced.

Figure 3 shows an example of the distinctive accuracy profiles of core periphery and uniform/cellular topologies for the degree centrality / top3 congruence measure. Removing nodes has little impact on a core periphery network and is hardly noticeable until levels of 50% error. Similarly, adding edges has little effect, if any, on the core periphery networks. However, removing edges has dramatic affect on the degree / top 3 measure immediately upon any introduction of error as does the adding of nodes. The profile for the uniform/cellular networks is noticeably opposite of the core periphery network although the slope of the accuracy line is somewhat consistent for all four error type/forms. Notice, again, that uniform and cellular lines lie almost exactly atop one another in this set of graphs.



(a) Uniform Topology (b) Core Periphery

Fig. 4. Plots of degree centrality showing average accuracy for edge-add errors.

Our next observation, the fourth, is that core periphery networks show extremely high levels of accuracy for edge-add errors. The accuracy is to the extreme that the errors appear practically inconsequential. Figure 4(a) shows the accuracy for the uniform and cellular networks deteriorating monotonically as error level increases. To the contrary, as Fig. 4(b) shows, the average accuracy levels, as error level increases, are consistently well above 0.95 for each of the congruence measures. In Fig. 4, each line is a different congruence measure: top, top3, top10pct, overlap (labeled soc_cir) and R-Squared (labeled Icorrel), pertaining to the degree centrality measure.

Further to the fourth observation, one particular case is relevant to point out. While the accuracy levels across the four centrality measures deviate from one another, Fig. 5 shows the accuracy of the eigenvector measure for core periphery networks as being uncharacteristically different from the other centrality measures. This is in stark contrast to consistency of the measures in other scenarios.

The fifth observation is that the core periphery network with node-add or edge-remove errors, is highly sensitive to small errors relative to uniform/cellular networks sensitivity at the same error levels. As shown in Fig. 6, the accuracy of top and top3 congruency measures drops sharply with errors of 1-5%. At the 10% error level, the accuracy has already approached an apparent asymptote level, while other topologies have more of a linear trajectory under the same conditions. Core periphery (plot b) shows significantly less robustness than the uniform topology at particularly small levels of error. Each line is a different centrality value: top, top3, top10pct, overlap (labeled soc_cir) and R-Squared (labeled Icorrel).

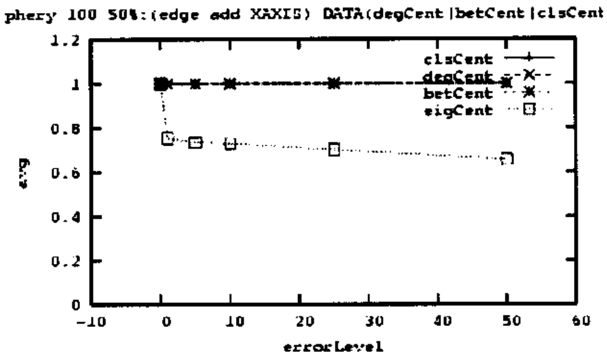
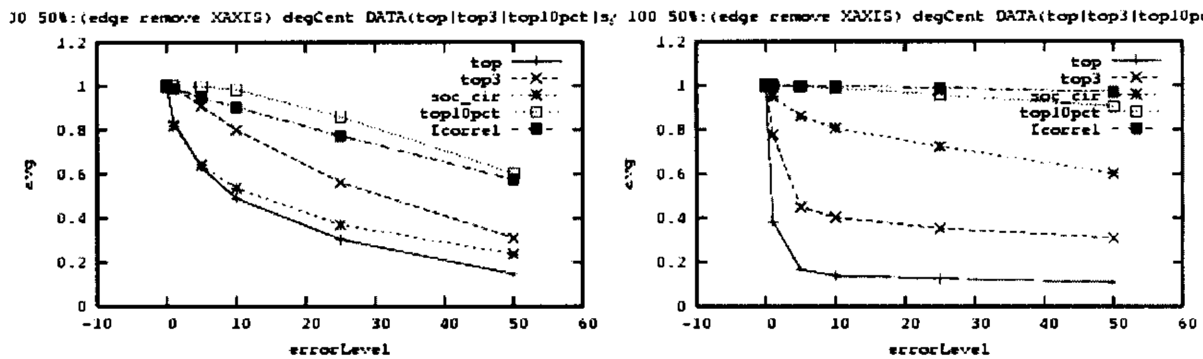


Fig. 5. Average accuracy values for core periphery (100/50%) network with edge-add errors



(a) Uniform Topology (b) Core Periphery

Fig. 6. Accuracy profile for degree centrality by congruence measures for uniform and core periphery

5. Discussion

The observations arising from our experiments provide a window and give insight into identifying a connection between a network topology and the robustness of centrality measures. Our observations are both consistent with prior published research and further, they present early evidence of a relevant relationship between topology and the measures' robustness. In short, we believe that there may indeed be an *important* relationship—albeit as yet, an ambiguous one—between network topology and the robustness profile of common centrality measures. Through our experiments, it appears that while uniform and the cellular topologies have *nearly identical* robustness profiles, we found that the core periphery topology has a *very different* robustness profile that is clearly distinct from that of the other two topologies considered.

In the remainder of this section, we discuss the new-found evidence to substantiate our claim. As we will provide only our speculation in an attempt to explain our observations, each observation warrants its own targeted and separate study to confirm and to fully understand reasons for and the dynamics of the particular observed phenomenon. Our observations—first introduced in the Results Section—pertaining to the robustness of centrality measures as are listed here:

1. Robustness among the four centrality measures is similar (with a partial exception)
2. Robustness of uniform and cellular is nearly identical
3. Robustness of core periphery differs greatly from uniform/cellular:
   - Higher robustness for core periphery with node-remove or edge-add errors
   - Lower robustness for core periphery with node-add or edge-remove errors
4. Extreme robustness (high) is found in core periphery with edge-add error
5. Extreme robustness (low) is found in top1 and top3 congruence measures of core periphery with node-add or edge-remove error

Our opening observation—that the accuracy for each of the four centrality measures is similar over different levels of error—is consistent with a finding of the Borgatti, Carley and Krackhardt (in press) study. Our observations augment and strengthen their finding by showing the same phenomenon they discovered, albeit across the differing topologies. In general, this phenomenon seems to hold regardless of the particular topology being studied.

However, we observed a notable exception to this; a difference exists in the case of eigenvector centrality for a core periphery network. In this particular case, unlike the other centrality measures, the eigenvector measure found to be extremely sensitive. For very small error levels it was much less robust than the other measures given the same true network and error parameters, which is in conflict with other circumstances.

By its design, a characteristic of the eigenvector measure is that it minimizes the influence of the near-isolates of a node; the measure gives more weight to the global centrality. By adding equally distributed random edges to the network, statistically the global centrality of nodes may be more quickly impacted than the local because there are likely more *distant* nodes than *local* to any given node. In the case of core periphery, with its clique-like core and sparely tied periphery, it follows that other centrality measures (degree, betweenness and closeness) would be impacted less in this scenario because of their mathematical characteristics.

We surmise, therefore, that the topology of a network affects the comparability (the consistency or inconsistency) of robustness profiles across different centrality measures.

The consistent and near-exact similarity of robustness profiles for the uniform and the cellular topologies was a surprise to discover and at first is rather difficult to explain; deeper investigation into this finding is certainly warranted to aid in adequately explaining it. We suspect this phenomenon may arise from our method of generating the cellular network which may be producing a network very similar (patterns of the edges between the nodes) to the random networks, formed using different generation algorithms.

If true, this suggests that networks generated using all-together *different* algorithms, under certain parameters, may in fact results in *identical* networks, and thus are indistinguishable from their source algorithm, leading to similar robustness profiles. Further, the highly differentiated robustness profiles for the core periphery network provides clear evidence that topology can impact the robustness of centrality measures and its generating algorithm is different from the uniform and cellular algorithms.

The extreme levels (high and low) of robustness in particular cases and measures for the core periphery network provide evidence that the topology combined with the type of error has a significant affect on the robustness profile. The profiles for the centrality measures in the case of uniform and cellular networks has a smooth shape, while the core periphery, depending on the particular measure, has a smooth and an abrupt shape.

We surmise that a network's topology itself is not a sole and decisive factor in the determination of robustness of centrality measures; the similarity or dissimilarity between the generative characteristics of the network topology and the manner in which a measure is formulated influences the comparability of robustness profiles across different centrality measures.

6. Future Research

As provided earlier, the motivation for this study is the recognition of a major problem inherent with social network data, that is, the omnipresent measurement-error that is embedded in the data we so carefully collect, and subsequently analyze; in short, the data we depend upon is known to be erroneous. We have been warned of the possible implications and understand some causes (Killworth & Bernard, 1976), but only now we are beginning to systematically identify and quantify the implications. There remains an abundant need for additional research into the robustness of network measures since the problem of mistaken data may be a factor for a long time. In this section, we put forward three suggestions for future research.

First, while we have shown that topology has an affect on the robustness of centrality measures, there is the next question about the precise extend to which each of the many different topologies and their variants distinctively affect the robustness profiles. In the guise of network topology labels, subtle differences in the methodology for generating a given network may possibly result in diverse robustness levels. Perhaps it is a characteristic of a topology (thus a family of topologies) that matter, not a specific topology itself. For example, there are many ways to generate a core periphery network; each variant needs to be explored and individually related to a specific robustness profile.

Second, as we and others have openly acknowledged, errors in observed social network data most likely are not truly *random* in nature. Early research, such as this, specifically toward investigating robustness have been limited to research based on random error as opposed to more realistic, systemic or non-randomly influenced errors in the data. One notable exception to this is the Marsden (1990) study which examined both random and non-random errors. Certainly, this makes the research much more complicated, but the community will be rewarded with theories based on richer scenarios.

Third, it should prove invaluable to analysts when they have statistically valid confidence levels and error bounds applicable to their specific observed network. Such quantities may possibly be based on the known parameters and characteristics of the observed network combined with the *a priori* true network information and error characteristics. To date, analysts are constrained by using measures determined only from the observed network, thus are being limited to working with descriptive statistic only. The analysis of networks will take a huge leap forward when confidence levels can be assigned to collected data that will ultimately lead to including p-vales with the statistics we calculate from observed data.

7. Conclusion

Our experiments and analysis of the data leads us to the conclusion that a network's topology is, in fact, related to the robustness of the centrality measures. We have shown that, in at least one specific case (core periphery versus uniform and cellular), different topologies can have distinctive, or conversely in another case (uniform versus cellular) nearly identical, profiles of robustness for common measures of centrality. We have reported evidence that there is a profound difference in centrality measures' accuracy for core periphery networks vis-à-vis accuracy for uniform and cellular networks, leading to the conclusion that when considering the robustness of centrality measures of a network, topology matters. Since our findings are entirely new to the research community, we call for more research into this phenomenon. Understanding the impact of mistaken observed data in social network analysis is critical to accurately projecting results of quantitative analysis to the qualitative assessment of a social network.

8. Appendix

The four tables herein provide complete results for an entire experiment involving a true network of 100 nodes with a density of 50%. Similarly-detailed tables for networks of the other size (10, 25, and 50 nodes) and densities (1, 2, 5, 10, 30, 50, 70, and 90%) are available from the authors.

Average Accuracy Values for 100 Node, 50% Density Network— Node Remove

Measure	% Error	Degree Centrality Uniform	Degree Centrality Cellular	Degree Centrality Core Periphery	Betweenness Centrality Uniform	Betweenness Centrality Cellular	Betweenness Centrality Core Periphery	Closeness Uniform	Closeness Cellular	Closeness Core Periphery	Eigenvector Uniform	Eigenvector Cellular	Eigenvector Core Periphery
Top 1	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.87	.87	.99	.86	.85	.99	.87	.87	.99	.86	.88	.71
	5	.68	.68	.95	.67	.67	.95	.68	.68	.95	.66	.67	.56
	10	.55	.56	.90	.55	.56	.90	.55	.56	.90	.52	.56	.41
	25	.34	.36	.75	.34	.34	.75	.34	.36	.75	.32	.34	.16
	50	.16	.16	.51	.15	.15	.51	.16	.16	.51	.14	.16	.06
Top 3	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.99	1.00	1.00	.99	.99	1.00	.99	1.00	1.00	.99	.99	.84
	5	.93	.94	1.00	.93	.93	1.00	.93	.94	1.00	.92	.92	.77
	10	.85	.85	1.00	.85	.85	1.00	.85	.85	1.00	.82	.84	.71
	25	.63	.63	.99	.62	.63	.99	.63	.63	.99	.59	.61	.51
	50	.34	.34	.87	.33	.33	.87	.34	.34	.87	.31	.33	.30
Top 10%	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.98	1.00	1.00
	10	.99	.99	1.00	.99	.99	1.00	.99	.99	1.00	.96	.99	1.00
	25	.91	.91	1.00	.90	.90	1.00	.91	.91	1.00	.85	.89	.99
	50	.65	.64	1.00	.64	.62	1.00	.65	.64	1.00	.59	.62	.97
Overlap	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.84	.84	.98	.84	.84	.98	.84	.84	.98	.84	.86	.98
	5	.69	.69	.91	.68	.68	.91	.69	.69	.91	.67	.68	.91
	10	.58	.58	.83	.57	.57	.83	.58	.58	.83	.56	.58	.83
	25	.39	.38	.62	.38	.38	.62	.39	.38	.62	.36	.39	.62
	50	.21	.21	.35	.21	.20	.35	.21	.21	.35	.20	.21	.35
R-Squared	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	.99	1.00	1.00	1.00	1.00	.99	1.00	1.00
	5	.97	.97	1.00	.97	.97	1.00	.97	.97	1.00	.94	.97	1.00
	10	.95	.95	1.00	.94	.94	1.00	.95	.95	1.00	.89	.95	1.00
	25	.86	.86	1.00	.85	.85	1.00	.86	.86	1.00	.77	.86	1.00
	50	.70	.69	.99	.68	.67	.99	.70	.69	1.00	.60	.70	.99

Average Accuracy Values for 100 Node, 50% Density Network— Node Add													
		Degree Centrality			Betweenness Centrality			Closeness			Eigenvector		
	% Error	Uniform	Cellular	Core Periphery	Uniform	Cellular	Core Periphery	Uniform	Cellular	Core Periphery	Uniform	Cellular	Core Periphery
Top 1	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.87	.88	.53	.86	.86	.52	.87	.88	.53	.87	.89	.27
	5	.72	.72	.24	.71	.70	.14	.72	.72	.24	.71	.71	.12
	10	.62	.62	.19	.61	.61	.11	.62	.62	.19	.61	.61	.11
	25	.46	.46	.15	.45	.45	.11	.46	.46	.15	.46	.45	.10
	50	.33	.34	.14	.32	.32	.10	.33	.34	.14	.33	.32	.10
Top 3	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	.82	.99	1.00	.80	1.00	1.00	.82	1.00	1.00	.50
	5	.95	.95	.56	.95	.94	.38	.95	.95	.56	.95	.96	.35
	10	.90	.90	.49	.89	.89	.33	.90	.90	.49	.89	.90	.34
	25	.75	.76	.42	.74	.75	.32	.75	.76	.42	.75	.76	.32
	50	.60	.61	.39	.59	.59	.31	.60	.61	.39	.60	.60	.31
Top 10%	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	.99	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	.99	.99	.99	1.00	1.00	1.00	1.00	1.00	.99
	25	.96	.96	.99	.95	.95	.98	.96	.96	.99	.96	.97	.98
	50	.88	.88	.98	.87	.87	.95	.88	.88	.98	.88	.88	.96
Overlap	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.85	.85	.99	.85	.85	.98	.85	.85	.99	.85	.87	.97
	5	.72	.72	.96	.72	.72	.94	.72	.72	.96	.72	.72	.90
	10	.64	.64	.93	.63	.63	.91	.64	.64	.93	.64	.64	.85
	25	.51	.51	.88	.50	.50	.84	.51	.51	.88	.51	.51	.80
	50	.41	.41	.82	.40	.40	.77	.41	.41	.82	.41	.40	.76
R-Squared	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	.99	.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	.98	.98	1.00	.97	.97	1.00	.98	.98	1.00	.98	.98	1.00
	10	.95	.95	1.00	.95	.95	1.00	.95	.95	1.00	.95	.95	1.00
	25	.89	.89	1.00	.88	.89	.99	.89	.89	1.00	.89	.89	1.00
	50	.81	.81	.99	.80	.80	.97	.81	.81	.99	.81	.81	.99

Average Accuracy Values for 100 Node, 50% Density Network— Edge Remove													
		Degree Centrality			Betweenness Centrality			Closeness			Eigenvector		
	% Error	Uniform	Cellular	Core Periphery	Uniform	Cellular	Core Periphery	Uniform	Cellular	Core Periphery	Uniform	Cellular	Core Periphery
Top 1	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.84	.83	.38	.82	.82	.38	.84	.83	.38	.83	.82	.14
	5	.63	.61	.17	.61	.62	.12	.63	.61	.17	.61	.58	.11
	10	.49	.49	.14	.48	.49	.11	.49	.49	.14	.47	.44	.10
	25	.30	.30	.13	.29	.28	.11	.30	.30	.13	.28	.25	.08
	50	.15	.14	.11	.14	.14	.10	.15	.14	.11	.13	.12	.07
Top 3	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	.99	.78	.99	.99	.77	1.00	.99	.78	.99	.99	.37
	5	.91	.91	.45	.91	.90	.36	.91	.91	.45	.89	.88	.32
	10	.80	.80	.40	.80	.79	.34	.80	.80	.40	.77	.75	.29
	25	.56	.56	.35	.55	.55	.33	.56	.56	.35	.52	.49	.26
	50	.31	.31	.31	.30	.30	.29	.31	.31	.31	.28	.25	.21
Top 10%	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.98	1.00	.96
	10	.98	.98	.99	.98	.98	1.00	.98	.98	.99	.94	.97	.93
	25	.86	.86	.96	.85	.85	.98	.86	.86	.96	.81	.80	.85
	50	.60	.60	.91	.58	.58	.93	.60	.60	.91	.54	.53	.71
Overlap	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.82	.82	.95	.82	.81	.96	.82	.82	.95	.81	.80	.84
	5	.64	.64	.86	.63	.63	.90	.64	.64	.86	.63	.61	.73
	10	.54	.53	.81	.53	.52	.85	.54	.53	.81	.51	.50	.66
	25	.37	.37	.72	.36	.36	.77	.37	.37	.72	.35	.33	.53
	50	.24	.24	.60	.23	.23	.67	.24	.24	.60	.21	.20	.38
R-Squared	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.99	.99	1.00	.99	.99	1.00	.99	.99	1.00	.98	.99	1.00
	5	.95	.95	1.00	.95	.95	1.00	.95	.95	1.00	.92	.94	1.00
	10	.90	.90	1.00	.90	.90	1.00	.90	.90	1.00	.83	.89	.99
	25	.77	.77	.99	.76	.76	.99	.77	.77	.99	.68	.73	.96
	50	.58	.57	.97	.56	.56	.96	.57	.57	.96	.47	.51	.87

Average Accuracy Values for 100 Node, 50% Density Network— Edge Add

	% Error	Degree Centrality Uniform	Degree Centrality Cellular	Degree Centrality Core Periphery	Betweenness Centrality Uniform	Betweenness Centrality Cellular	Betweenness Centrality Core Periphery	Closeness Uniform	Closeness Cellular	Closeness Core Periphery	Eigenvector Uniform	Eigenvector Cellular	Eigenvector Core Periphery
Top 1	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.84	.84	1.00	.82	.81	1.00	.84	.84	1.00	.83	.80	.75
	5	.62	.62	1.00	.60	.60	1.00	.62	.62	1.00	.61	.56	.73
	10	.49	.49	1.00	.49	.49	1.00	.49	.49	1.00	.47	.41	.73
	25	.29	.29	1.00	.28	.29	1.00	.29	.29	1.00	.28	.21	.70
	50	.14	.14	1.00	.14	.14	1.00	.14	.14	1.00	.13	.07	.66
Top 3	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.98	.98	1.00	.97	.97	1.00	.98	.98	1.00	.97	.96	.85
	5	.87	.87	1.00	.86	.87	1.00	.87	.87	1.00	.85	.82	.83
	10	.77	.76	1.00	.76	.76	1.00	.77	.76	1.00	.74	.68	.82
	25	.52	.52	1.00	.51	.52	1.00	.52	.52	1.00	.49	.41	.80
	50	.29	.29	1.00	.28	.29	1.00	.29	.29	1.00	.28	.17	.76
Top 10%	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	.99	.99	1.00	.99	.99	1.00	.99	.99	1.00	.97	.97	1.00
	10	.96	.95	1.00	.95	.95	1.00	.96	.95	1.00	.93	.91	1.00
	25	.80	.80	1.00	.79	.80	1.00	.80	.80	1.00	.76	.68	.99
	50	.55	.54	1.00	.54	.55	1.00	.55	.54	1.00	.53	.37	.96
Overlap	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.82	.82	1.00	.81	.81	1.00	.82	.82	1.00	.81	.79	1.00
	5	.64	.64	.99	.63	.63	.99	.64	.64	.99	.63	.59	.99
	10	.53	.53	.99	.52	.53	.98	.53	.53	.99	.52	.48	.98
	25	.37	.37	.97	.36	.36	.96	.37	.37	.97	.35	.30	.96
	50	.23	.23	.95	.23	.23	.92	.23	.23	.95	.22	.16	.92
R-Squared	0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	.99	.99	1.00	.99	.99	1.00	.99	.99	1.00	.98	.99	1.00
	5	.95	.95	1.00	.95	.95	1.00	.95	.95	1.00	.92	.93	1.00
	10	.90	.90	1.00	.90	.90	1.00	.90	.90	1.00	.85	.87	1.00
	25	.77	.77	.99	.76	.76	.98	.77	.77	.99	.70	.70	.99
	50	.58	.57	.97	.56	.56	.94	.57	.57	.98	.52	.44	.96

9. References

Bolland, J. M. (1988). Sorting out centrality: An analysis of the performance of four centrality models in real and simulated networks. *Social Networks, 10,* 233-253.

Borgatti, S. P., Carley, K. M., & Krackhardt, D. (In Press). On the robustness of centrality measures under conditions of imperfect data. *Social Networks.*

Borgatti, S. P., & Everett, M. G. (1999). Models of core / periphery structures. *Social Networks, 21,* 375-395.

Carley, K. M. (2003). Dynamic network analysis. In R. Breiger, K. Carley, & P. Pattison (Eds.), *Dynamic Social Network Modeling and Analysis: Workshop Summary and Papers.* Committee on Human Factors, National Research Council, pp. 133-145.

Carley, K, M., & Reminga, J. (2004). *ORA: Organization risk analyzer.* Carnegie Mellon University Technical Report, CMU-ISRI-04-106

Costenbader, E., & Valente, T. W. (2003). The stability of centrality measures when networks are sampled. *Social Networks, 25,* 283-307.

Erdős, P., & Rényi, A. (1959). On random graphs I. *Pubicationes Mathematicae Debrencen, 6,* 290-297.

Erickson, B., & Nosanshuk, T. (1983). Applied network sampling. *Social Networks, 5,* 367-382.

Frank, O. (1971). *Statistical Inference in Graphs.* Stockholm: Försvarets Forskninganstalt (Research Institute of National Defense)

Frank, O. (1981). A survey of statistical methods for graph analysis. In S. Leinhardt (Ed.) *Sociological Methodology,* pp. 110-155. London: Jossey-Bass.

Frantz, T., Airoldi, E., & Reminga, J. (2005). *Methods for generating experimental complex social networks.* Carnegie Mellon University Technical Report, CMU-ISRI-05-118

Freeman, L. C. (1979). Centrality in social networks conceptual clarification. *Social Networks, 1,* 215-239.

Freeman, L. C., Rommey, A. Kimball, & Freeman, S. C. (1987). Cognitive structure and informant accuracy. *American Anthropologist, 89,* 310-325.

Galaskiewicz, J. (1991). Estimating point centrality using different network sampling techniques. *Social Networks, 13,* 347-386.

Granovetter, M. (1976). Network sampling: Some first steps. *American Journal of Sociology, 81,* 1287-1303.

Kim, K., & Médard, M. (2004). Robustness in Large-Scale Random Networks. IEEE INFOCOM 2004, March 2004, Hong Kong. Retrieved on 30 May 2005 from http://web.mit.edu/minkyu/www/doc/Infocom2004.pdf

Kossinets, G. (2005). Effects of missing data in social networks. Pre-print to *Elsevier Science.*

Marsden, P. V. (1990). Network Data and Measurement. *Annual Review of Sociology, 16,* 435-463.

Moreno, J. L. (1934). *Who shall survive?* Washington, DC: Nervous and Disease Publishing.

Rothenberg, R. B. (1995). Commentary: Sampling in social networks. *Connections, 18,* 104-110.

Tsvetovat, M., Reminga, J., & Carley, K. M. (2004). *DyNetML: Interchange format for rich social network data.* Carnegie Mellon University Technical Report, CMU-ISRI-04-105

Tsvetovat, M., & Carley, K. M. (2005). Structural Knowledge and Success of Anti-Terrorist Activity: The Downside of Structural Equivalence. *Journal of Social Structure (JoSS), 6,* Article 2. Retrieved 30 May 2005 from http://www.cmu.edu/joss/content/articles/volume6/TsvetovatCarley