Quality Innovation in Design and Manufacturing:
An Economic Model

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EDRC24-103-93

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Abstract

Due to several possible production modes, an industrial output may have different quality levels. Production processes and quality assurance plans are chosen and adjusted, generally as a lagged reaction to customers[1] input and competitors' strategy. Different techniques based on cost benefit analysis have existed to assess beforehand the overall benefits to society of such decisions; however, these techniques do not necessarily provide any insight as to the resulting influence on corporate profits. This paper presents a context in which social and corporate optimum can be compared from an engineering perspective. The reasons for a manufacturer to improve the quality of its products are studied under several market conditions. The incentive is the strongest in a competitive environment where the benefits of quality innovation are twofold: quality innovation increases consumers' demand and allows the manufacturer to keep more substantial profit margins.

Introduction

More than ever, industrial quality is one of the critical elements in the fierce competition between manufacturing companies. From an engineering standpoint as well as from a management perspective, quality is a challenge that cannot be ignored: a lack of competitiveness can often be attributed to quality problems. In these cases, the first logical approach is to question manufacturing practices. An abundant literature deals with systematic methods to improve quality at zero or low cost (*e.g.,* Freedman et al., 1948; Deming, 1950; Shewhart, 1986; Taguchi, 1986; Phadke, 1989). However, this paper addresses a different problem that may arise and has received much less attention: several alternate production modes can often be employed to manufacture the same product, resulting in costs and quality levels that are significantly different. From this perspective, the choice of a production mode, and thereby of a quality level, consists of trading off cost for quality. An economic analysis is required to estimate the benefits of decisions like choosing a tighter manufacturing process or screening a production. We approach the problem from an engineering perspective by determining product quality levels that are likely to result in commercial successes, based on the intrinsic characteristics of the products. Marketing or advertising issues will not be considered.

1. Product value to society

Experience has shown that decisions about manufacturing quality cannot rely on simple accounting practices: the gains and losses when dealing with quality are not expressed directly in terms of dollars. Generally, the costs involved can be easily estimated, but the benefits are somewhat intangible (*e.g.,* consumers[1] satisfaction). More often than not, a change in consumers' satisfaction is only detected by a change in the number of sales at a period well after the decision has been made. Consequently, the profitability of a manufacturing decision involving quality is at best uncertain and there is a need to better assess the impact of this decision. More precisely, it is necessary to figure out beforehand what quality level a product should have to be a potential commercial success.

Economic theory provides a method to estimate the value that consumers attach to a product with given characteristics (Mishan, 1975). Considering a given market, let us assume that the consumers are ranked according to their willingness to pay for the product. We denote by P(n) the maximum price that the n-th consumer is willing to pay (see Figure 1). An aggregate measure of the value that consumers attach to the product is

$$\int_{Jo} P(n)\,dn \,,$$

where N is the size of the market segment (total number of potential consumers).
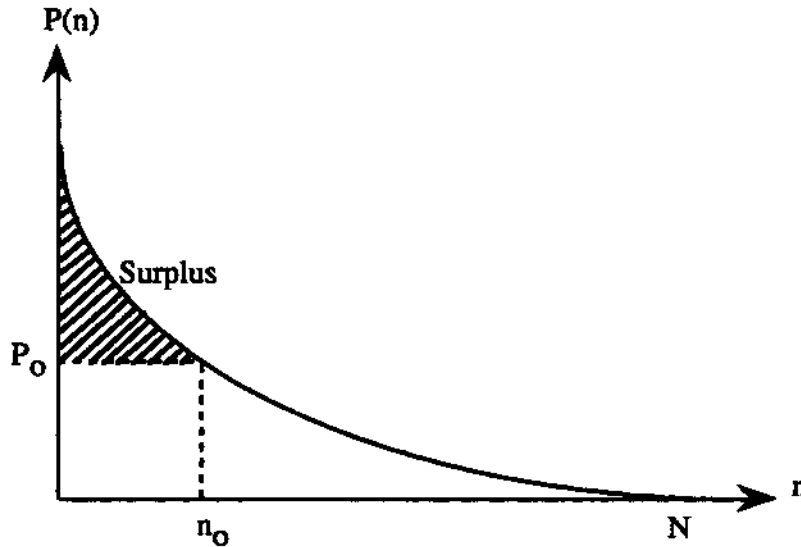

Figure 1: Willingness to pay for a particular product

Suppose the actual price of the product is P_0. Only n_0 consumers will buy the product. The difference between what they are willing to pay and what they actually pay is the consumers' surplus (S). This surplus measures the satisfaction provided by the product:

$$S = \int_{o}^{n_o} P(n)\,dn - noP_0.$$

The total value of the product to society, or welfare provided by the product (W), is obtained by adding the manufacturer's profit (sales revenue minus cost) to the consumers' surplus:

$$W = \int_{0}^{n_o} P(n)\,dn - n_0P_0 + n_<(P_0 - C] \bullet \int_{Jo}^{o} P(n)\,dn - noC$$

where C is the manufacturing cost.

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2. Concept of quality loss

Taguchi relates quality to the overall well being of society: *"Quality, both tangible and intangible, may be defined in terms of the loss imparted to all people after a product leaves the hands of an enterprise"* [Taguchi, 1978, p.1]. Taguchi defines the quality loss as the expense resulting from *(i)* the variability of the function of the product, and *(ii)* the possible harmful side-effects of the product [Taguchi, 1986, p.2]. Example of losses to be considered are the average maintenance cost or cost of repair. Along with this conception, a more accurate measure of welfare should consider the whole service life of the product and therefore, include the quality loss L:

$$W = \int^{n_p} P(n)\, dn - n_o[C + L] .$$

(1)

Taguchi uses the quality loss to determine the amount that a manufacturer should spend on quality control. A good illustration is the setting of manufacturer's tolerances, as opposed to consumer's tolerances [Taguchi, 1986, p.21]. A manufacturer should set an internal tolerance on his product in such a way that a product be reworked when the cost of rework is lower than the loss society would incur if the product were not reworked. This principle corresponds to a maximization of welfare as define in equation (1). Since welfare maximization is not necessarily compatible with the objective of a profit maximizing firm, corporate decision makers are often reluctant to follow the method.

A different approach is generally used in economics to include quality considerations in the computation of welfare. Quality is defined in terms of value to the consumers. It is viewed as a characteristic influencing consumers' willingness to pay. The quality of a product is represented by the scalar q, although a vectorial approach does not fundamentally change the method. As in [Spence, 1975], consumer's surplus can be written as

$$S = \int_0^{n_o} P(n, q)dn - n_o P_o,$$

which leads to the following expression for welfare:

$$W = \int_0^{n_o} P(n, q)dn - n_o C(q).$$

Taguchi criticizes this approach on the grounds that it makes any engineering application impossible because of the wide range of values expressed by consumers [Taguchi, 1986, p.2]. However, this difficulty can be avoided by considering only a market segment, for which the

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values expressed by consumers are more homogeneous, and can be aggregated more easily. Thus, we assume that for a particular market segment, consumers' values are identical and can be expressed by a function $V(q)$. $V(q)$ represents the highest price that consumers are willing to pay for a product of quality level q. For this market segment, welfare has a more simple expression:

$$W = n_o V(q) - no C(q).$$

Again, welfare maximization is achieved by minimizing the sum of the manufacturing cost inside the plant and the loss of value outside the plant. However, the influence of q on corporate profits is still not clear.

3. Social optimum

In this section, we give a set of conditions under which a quality level maximizing welfare is also profit maximizing for the firm. Without loss of generality, the quality level of a product can be described by a quality indicator q such that $0 \le q \le 1$. The highest possible quality is achieved when q is zero *(e.g., zero defect, or no significant deviation from expected performance)*. A value of unity for q corresponds to the lowest quality level *(e.g., 100 % defective products)*. The cost, C, of manufacturing the product studied is a decreasing function of q, and denotes the lowest possible production cost associated with an output with quality level, q. We assume $C(q)$ to be differentiable and convex; this situation is likely to occur when only one process is available for the manufacture of the product (Cagan and Kurfess, 1992).

Let us consider a situation where a product is manufactured at a quality level q, and sold to consumers for a price p. We restrict our analysis to a market segment where consumers are indifferent between an increase in the quality level of the products and a price reduction: we do not consider the case where consumers require a precise quality level because of particular uses of the product. We also suppose that there is no income effect: no limit exists on the price that consumers are *able* to pay (as opposed to the price they are *willing* to pay). We model consumers' behavior as a maximization of the surplus $S(p, q) = V(q) - p$ (Smith, 1986), and manufacturers' behavior as a maximization of the profit $7c(p, q) = p - C(q)$. Both objectives are functions of the independent variables p and q. However, the sum $S(p, q) + rc(p, q)$ is only a function of q:

$$S(p, q) + \pi(p, q) = V(q) - C(q).$$

Therefore, the sum of the two satisfaction measurements depends exclusively on the quality level. $V(q) - C(q)$ can be seen as the value created (denoted by $Vc(q)$) by the manufacture of a product. The benefits of this value creation are shared by the manufacturers and the consumers: q determines the amount to be shared, and p decides of the allotment of this value.

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Assuming V(q) differentiable and strictly concave, Vc(q) is a differentiable and strictly concave function over the compact set [0,1], therefore it has a unique maximum at q*(see Figure 2).
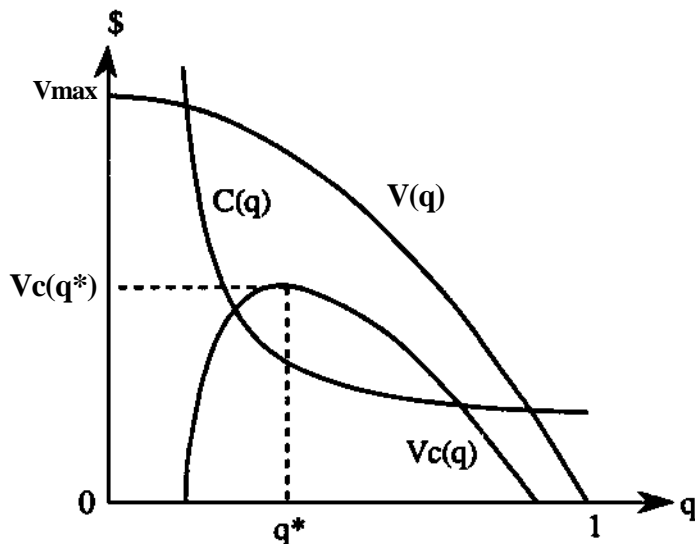

Figure 2: Quality level maximizing the value created.

Proposition: Let us denote by q* the quality level that maximizes S(p, q) + rc(p,q). q* is the only Pareto efficient quality level, *i.e.* given a price p and a quality level q, unless q = q*, it is possible to find p* such that, by manufacturing the product with a quality level q* and selling it at the price p*, either the consumers[1] surplus or the manufacturers' profit is increased, and none of them is decreased.

Proof: In a situation where the quality level is q and the price is p, the consumers' surplus is V(q) - p, and the manufacturers' profit is p - C(q). If the quality level is too high (q < q*), then let us choose p* = V(q*) - V(q) + p. The consumers' surplus is unchanged, and the manufacturers' profit becomes p* - C(q*) *Le.,* V(q*) - V(q) + p - C(q*). Because q* is defined as the value that maximizes S(p, q) + rc(p,q), necessarily, V(q*) - C(q*) ≥ V(q) - C(q). Substituting V(q*) - C(q*) by V(q) - C(q) in the expression of the profit reveals that the profit is greater than V(q) - C(q) - V(q) + p = p - C(q), which is the original profit. In other words, the manufacturers are better off and the consumers are not harmed. The case where the original quality level is lower than q* can be handled in a similar way; in the resulting situation the consumers are better off and the manufacturers are not harmed. A similar reasoning also proves that q* is a Pareto efficient quality level.

The concrete meaning of the existence of q* is that there is a quality level that maximizes the potential commercial success of the product. The actual success depends also largely on marketing and advertising efforts. However, if a product is sold, the quality level q* maximizes the consumer's surplus and the manufacturer's profit resulting from the sale. From an engineering perspective, q*is the level at which the product should be designed and manufactured.

4. Case of the monopolistic firm

We consider a market dominated by a single enterprise, manufacturing and selling a single industrial product. Let us determine the profit maximizing values of the price p and the average quality level q of the product. If N is the consumers' demand and C the cost of production of a single product (function of q), the total profit % of the firm can be expressed as

$$rc_T(p,q)=N[p-C(q)].$$

It is assumed that the firm can make accurate predictions as to the consumers' demand and can generate a supply to meet that demand. We assume the demand to be an increasing function of the consumers' surplus S (S = V(q) - p):

$$N = F(S) = F(V(q)-p).$$

Under the assumption of differentiability for C, the profit maximizing values have to satisfy the following optimality conditions:

$$\frac{\pi_T}{dp} = -F(V(q) - p)[p - C(q)] + F(V(q) - p) = 0, \text{ and}$$

$$\frac{\pi_T}{dq} = V'(q)F(V(q) - p)[p - C(q)] - F(V(q) - p)C(q) = 0.$$

Combining the two equations sets the quality level at the value of q satisfying

$$[V'(q) - C'(q)] = 0,$$

which also corresponds to the maximization of the value created. Under the assumption that consumers' demand is a function of consumers' surplus, the profit maximizing quality level for a monopolist is as expected, the Pareto quality level.

To determine the profit maximizing price, an additional assumption has to be made on the demand function. Following (Cook & DeVor, 1991), we set a limit on the number of products that can be sold (a V(0)), and assume a linear demand function:

$$N(p, q) = a [V(q) - p].$$

Under these conditions, price should be set half way between value and cost: manufacturer's profits and consumers' surplus are equal. The value created is equally shared between the manufacturer and consumers.

It is dubious that the quality level of a production can be adjusted as easily as its price. In most cases, major changes in the production processes are required, and retooling may be necessary. Therefore, the model provides an indication as to the quality level towards which it is suitable to go. It can be noticed that no matter what the quality level is, the model recommends that the value created per product be equally shared by the manufacturer and the consumers. This is an engineering decision that will have longterm benefits to the manufacturer; short term market influences are assumed irrelevant.

5. Competitive case

5.1 Modelization

The competitive case (oligopoly), where a limited number of firms (n) are competing on a given market to mass produce and sell a given product, can be modeled similarly to the monopolistic case. We assume that the products sold by different firms are similar by their functions, but have different prices and different quality levels. To be able to compare the results with the monopolistic case, we also model the demand function for firm i (N^pj, q_j)) as a linear function of the consumer's surplus (Sj) it offers:

$$Ni(Pi, qi) = \alpha i [V(qi) - ft] = a_i S_v$$

Moreover, the maximum number of products that can be sold should be $\alpha V(0)$, as defined in in the monopolistic case. It is obtained by extrapolating the overall demand function for maximum quality when price goes to zero:

$$\sum_{i=1}^{n} N_i(0, 0) = \sum_{i=1}^{n} \alpha_i V(0) = \alpha V(0).$$

This imposes the following constraint on the coefficients of the demand functions:

$$\sum_{k=1}^{n} \alpha_i = \alpha.$$

(2)

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The only additional effect that must be incorporated in the model is a coupling between the demand functions of the firms: the sales volume of a given firm depends on the sales of the other firms. To this effect, the coefficient a_i should reflect the competitive position of firm i with respect to the other firms. Consistently with the simple approach adopted in the previous section, we choose a_i proportional to the ratio of the surplus offered by firm i to the sum of the surpluses offered by all the firms:

$$\alpha_i = K \frac{S_i}{\sum\limits_{k=1}^{n} S_k}.$$

Using (2) reveals that K should be equal to a; the complete expression of Nj becomes

$$N_i(p_i, q_i) = \alpha \frac{S_i^2}{\sum\limits_{k=1}^{n} S_k}.$$

Under these assumptions, the profit of firm i is given by:

$$\pi_{Ti}(P_i, q_i) = a \frac{S_i^2}{\sum\limits_{k=1}^{n} S_k} [V dq O - S_i].$$

This formulation introduces a coupling between the variables related to a firm and that of the other firms. It models the impact of competition on the profit of an individual firm. Consistently with the monopolistic case, the "main effect" is a linear function of the consumers[1] surplus offered, with perturbations due to the existence of competitors (see Appendix 1). The model assumes that an increase of the average consumer 's surplus results in an expansion of global market demand. Cook and DeVor (1991) offer an alternative model which assumes a fixed size market. This is discussed further in Section 7.

5.2 Profit maximizing decisions

We will not consider the case where a collusion between the firms artificially maintains high prices, since this case is not very different from a monopoly situation. We study the case where each firm defines its optimum quality level and price, taking its competitors' quality levels and prices as given and fixed. This setting is similar to that of the Cournot model [Cournot, 1897], where the variables are prices and outputs. Profit is maximized when (see Appendix 2)

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$$C'(qi) - V(qi) = 0.$$

The optimum value of qj is once again the Pareto optimal solution. The profit maximizing surplus for firm i is now expressed by:

$$Si = V(qi) - pi = \frac{1}{4}I_{\cdot}V_c(qD - 3\times Ji + V[v_c(qD - 3 aj^2 + 16 V_c(qi) O\{ J,$$

(3)

where Gj is the sum of the surplus offered by the other firms. It can be seen that when $c_{\dot{x}}$ is zero, the profit maximizing price is equal to that of the monopolistic case. Furthermore, $S_{\dot{x}}$ is an increasing function of the sum of the surplus offered by the competitors. Figure 3 represents the variations of Sj as a function of c_v In a competitive situation, a firm offers a larger surplus than in the monopolistic case. Consumers can receive as much as two thirds of the value created in the case of strong competition (versus only one half in the monopolistic case). The well known benefits of economic competition for the consumers appear in this model as in previous models based on price and output
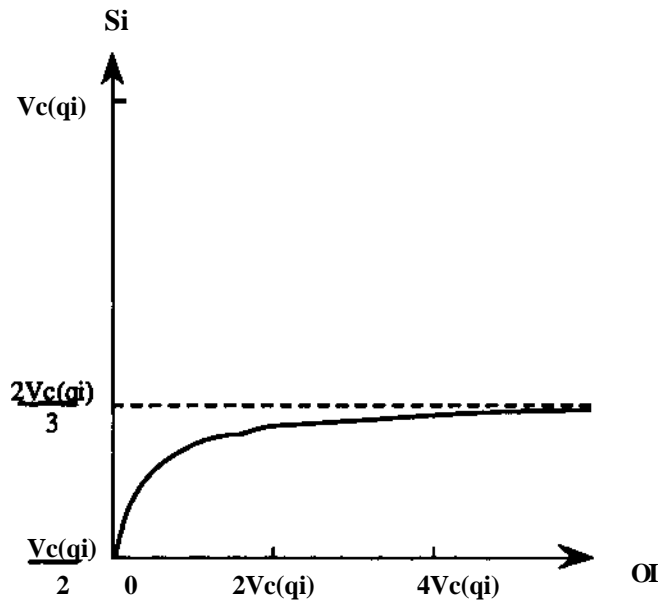

Figure 3: Firm i profit maximizing surplus as a reaction to the surplus offered by its competitors

6. Incentive for quality innovation

The improvement of an existing manufacturing process or the introduction of a new process induces a change in the cost versus quality curve. A cost reduction may occur over some quality range, or a quality level that was not feasible before becomes available. Generally, such a manufacturing innovation impacts mainly the high quality levels. When the cost reduction occurs

for quality levels higher than the optimum quality level, q* is shifted to the left (see Figure 4). Depending on the magnitude of the required investment to update the manufacturing equipment, it may be profitable to increase the quality of the products. This section studies the consequences of the quality innovation in a competitive environment.
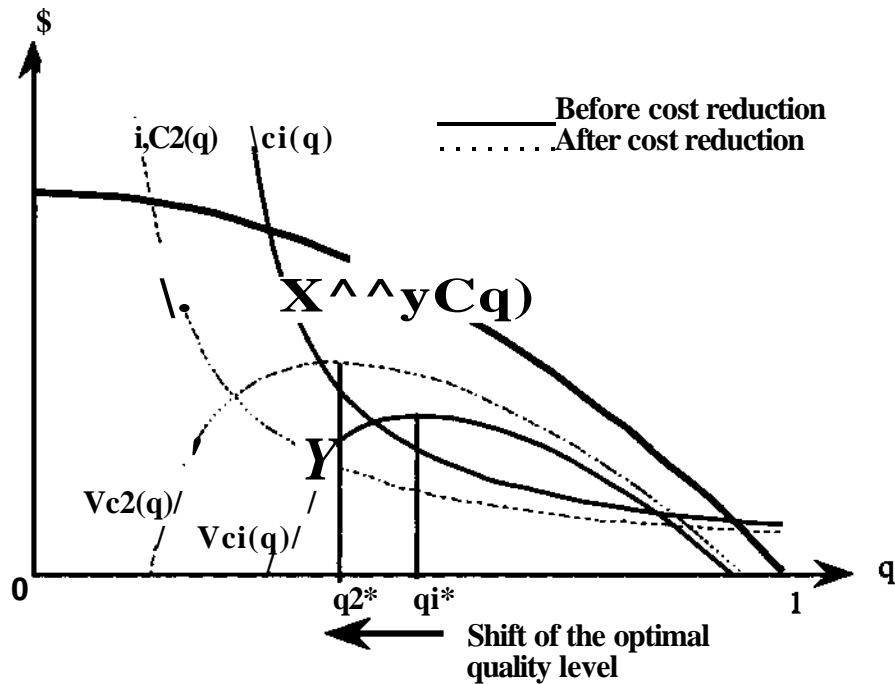

Figure 4: A cost reduction induces an improvement in the optimum quality level.

Let us assume that, in the context of the previous model (section 5), one of the competitors has been able to reduce its cost function, and takes advantage of the reduction to increase its quality level. Because of the complexity of the expressions involved we will not give exact analytical solutions but rather, we will focus on how the situation is changed compared to the case where all the competitors create the same value. We only consider the equilibrium of the system, that is to say the point where no firm has any incentive to change its prices. Equation 2 shows that Sj is an increasing function of Vc(qi) as well as of a^ consumers always benefit from the quality improvement performed by one of the competitors. However, the expansion of market demand is mainly to the benefit of the firm that has improved the quality of its products. Moreover, it can be shown from equation 3 that the ratio of the surplus offered over value created decreases for the firm with the highest value created (see Appendix 3):

$$\frac{S_.}{c(q_i)} \text{ is a decreasing function of } \frac{Vc(q_i)}{\sigma_i}.$$

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This indicates that the business conditions for the more advanced firm are more favorable since it can reduce the proportion of value created it has to give to the consumers. In fact, the more advanced the firm is, the closer its pricing policy gets to that of a monopolist

As found in the previous sections, producing at another quality level than q* is a waste of resources. It hurts firm profitability in any kind of market, but even more so in a competitive market where there is a strong incentive to improve processes to create more value to escape from a competition based on prices.

7. Discussion

A good illustration of the model developed above is the manufacture of computer chips, where the impact of process improvement on product quality level is particularly clear. The quality of a chip can be assessed in many ways; one important measure is the number of functions available on the chip (Wilson *et ah*, 1980): for a given state of technology, there is a minimum lithographic feature size, and therefore a maximum number, p, of components that can be implemented on the chip per unit area. Therefore, the quality of the chip is roughly inversely proportional to its area A. The manufacture proceeds as follows: the chip is replicated as many times as possible on a silicon wafer of area, S. Due to several factors including the amount of dust in the room and the accuracy when positioning the wafer, a certain number of defects occur on the wafer with density, D. Under a few simplifying assumptions (Jaeger, 1989), the yield of the process is

$$Y = e^{-DA}.$$

Since the products are inspected before being shipped to the customers, the yield impacts directly the cost per working chip: if C_o is the processing cost per wafer, the cost per working chip, C, becomes:

$$C = \frac{C_o\, A}{S\, Y} = \frac{C_o\, A}{S}\, e^{DA},$$

If n is the number of functions (n = Ap), the cost per working chip, as a function of n, is

$$C = \frac{C_o\, n}{Sp}\, e^{(nD/p)}.$$

This expression shows that if p (density of functions on the chip) is increased or D (density of defects on the wafer) is decreased due to process or design improvement, the impact of the cost reduction is higher for large values of n and therefore for high quality levels. This illustrates the relationship between process improvement, cost reduction and quality increase, as explained in

Section 7. The global market expansion that is still occurring in the computer chip industry is probably not completely explained by quality improvements since product diffusion also plays an active part. However, the frenzy of innovation that has taken place is a striking example of the double effect of quality innovation: increase of the overall market demand, and improvement of the competitive position of a firm.

For more mature products like in the automotive industry, the dominant effect of a quality innovation is more of a competitive position improvement than an overall market demand expansion. Then, a "zero-sum game" modelization as in (Cook and DeVor, 1991) may be more appropriate (e.g., a given number of products are sold in a given market segment, and quality differences between competitors only influence market shares).

8. Conclusion

In this paper we use a classic economic framework to assess the value society attaches to the quality of a product based on engineering considerations. This approach allows us to examine the concept of quality loss from a societal and corporate perspective. Under a set of assumptions on consumers* behavior, an optimum quality level is defined. The existence of an optimum explains the propensity of manufacturing companies to produce at the same quality level, within a given market segment. This clustering results in a competition on prices, from which consumers benefit. Under these business conditions, we show that most of the value created is transferred to the consumers (low prices). We view quality innovation as an opportunity to escape from this situation. Thus, by creating more value, a firm increases the demand for its product and is able to have a pricing policy much closer to that of a monopolist. This "weak monopoly" is probably only temporary since others firms are likely to follow.

Appendix 1

The function used to model consumers[1] demand for firm i is

$$D_i = ct \frac{\hat{}}{\sum\limits_{k=1}^{n} S_k}.$$

In this expression, the consumers surpluses offered by the various firms have the same order of magnitude, therefore $\dfrac{S_k - S_i}{S_i}$ is small compared to 1. Di can be rewritten as

$$a \frac{\hat{Sf}}{nSi + \sum\limits_{k=1,\, k \ne i}^{n} [S_k - S_i]} \,,\ \text{or}\ \frac{aSi}{n} \frac{1}{\left[\ln,\ \frac{1}{n} \sum\limits_{k=fk,\ne i}^{n} \frac{[Sik \cdot S0]}{Si} \right]}.$$

Since for small x

$$\frac{1}{1+x} \simeq 1-x,$$

D_i can be approximated by

$$D_i = \frac{aS_i}{n} - \frac{a}{n^2} \sum\limits_{k=1,\, k \ne i}^{n} [S_k - S_i].$$

Appendix 2

Setting the partial derivatives of $rc^{\wedge}$ to zero yields the following equations:

$$\left[S_i - 2 \sum\limits_{k=1}^{n} S_k \right] [p_i - C(q_i)] = - S_i \sum\limits_{k=1}^{n} S_k, \tag{4}$$

and

$$\left[S_i - 2 \sum\limits_{k=1}^{n} S_k \right] [p_i - C(q_i)] = \frac{-Si\ C(qi)}{V'(q_i)} \sum\limits_{k=1}^{n} S_k. \tag{5}$$

The left hand side of (4) must equal the left hand side of (5), therefore

$$C'(qi) - V(_{qi}) = 0.$$

From (3), we can extract the expression for the surplus:

$$Si = V(qi) - pi = \frac{1}{2}[v_c(qi) - 3Oi + V[v_c(qi) - 3 \langle jj|^2 + 16\, V_c(qi) \langle^*].$$

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Appendix 3

From equation 3, we get

$$\frac{S_i}{VdqO''} = \frac{1}{4}\left[-3\frac{\sigma_i}{V_C(q_i)} + \sqrt{\left[1 - 3\frac{\sigma_i}{V_C(q_i)}\right]^2 + 16\frac{\sigma_i}{V_C(q_i)}} \right].$$

Since the function f such that

$$f(x) = \frac{1}{4}\left[1 - \frac{3}{x} + \sqrt{\left[1 - \frac{3}{x}\right]^2 + \frac{16}{x}} \right]$$

is a decreasing function of x, $\dfrac{S_i}{x_W^{*} \tilde{}}$ is a decreasing function of $\dfrac{V_C(q_i)}{O_i}$.

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