Optimally Directed Truss Topology Generation Using Shape Annealing

G. Reddy, J. Cagan

EDRC 24-101-93

Optimally Directed Truss Topology Generation Using Shape Annealing

Giridhar Reddy
Graduate Research Assistant
and
Jonathan Cagan
Assistant Professor
Department of Mechanical Engineering

Technical Report # EDRC 24-101-93
Engineering Design Research Center
Carnegie Mellon University
Pittsburgh, PA 15213

December, 1992

Abstract:

This paper presents a technique for the generation and optimization of truss structure topologies based on the *shape annealing* algorithm. Feasible topologies for the truss are generated through a shape grammar, in an optimally directed manner, using simulated annealing. The algorithm can incorporate stress and buckling constraints as well as assembly and manufacturing constraints, during design topology generation.

Introduction:

The advent of computer methods of analysis and design have propelled intensive research in the field of structural optimization for the last three decades. The optimization problem is to minimize an objective function, such as weight or cost, subject to structural constraints such as stress constraints , displacement constraints and geometric constraints such as the location of the load, anchor points and obstacles. The method for solving this problem usually consists of the following steps:

1) Derive a topology for the structure.

2) Optimize the topology. This step normally consists of the following four parts:

   a) Choose the variables for the design problem, those being only the sizing variables or the sizing variables and the shape variables such as the coordinates of the nodes for truss structures. The former is called the *sizing optimization* problem and the latter is referred to as the *shape optimization* problem.

   b) Choose the objective and constraints to be used in the optimization problem. The objective may be weight or stiffness and the constraints may be stress, displacement, buckling and dynamic constraints. The designer could also use geometric constraints such as presence of obstacles in between the load and anchor points.

   c) Model the structure with finite elements.

   d) Optimize the objective over the constraints on the shape and sizing variables.

Step 2, shape optimization of a given topology, is rather well understood although not necessarily easy. Step 1, the generation of the actual topology, is a difficult problem with little previous research; much previous work has used empirical knowledge. This paper will focus on step 1, the design of truss structure topologies. In particular, we propose a rigorous way of generating optimally directed truss topologies.

Related Literature:

Research in optimization of skeletal structures can be classified into several categories depending on the number of load cases considered, on the kind of constraints used, and on the use of geometry as a variable. Interested readers are referred to a survey of the field of optimization of skeletal structures by Topping [1983]. Topping writes in his survey "Most algorithms are formulated to either remove members, or move joints and sometimes both in a design procedure which seeks to minimize weight or cost. But rigorous methods incorporating algorithms which consider the possibility of introducing new members and nodes during the optimization procedure have yet to be developed." Nearly all the structural optimization techniques developed until now can only optimize the structure for a given topology and are not capable of generating optimal topologies or integrating topology generation into the optimization procedure. The solutions derived by these techniques are limited by the topology chosen.

In the last decade some researchers have started to look at the problem of generation of initial topology. Shah [1988] has used a heuristic approach to generate the initial topologies for structural shape optimization. This method is based on three sets of rules: global rules, matching rules and axes polygon constraints. Global rules are general guidelines for structural synthesis. Matching rules match the requirements of the present design to something designed in the past and thus determine the class of structures the present design should belong to. Axes polygon constraints help generate a large number of arrangements within the class of structures as determined by matching rules. Rogers, *et al.,* [1988] also use a knowledge based system

(STRUTEX) for the initial configuration of the truss given the location of load and anchor points. Lakmazaheri and Rasdorf [1990] have suggested the use of predicate logic and theorem proving for synthesis of trass structures. All of the above methods divide the problem into two parts, the topology generation part and the shape or size optimization part. They try to generate a topology for the given set of loads and anchor points using some heuristics and only then use size or shape optimization. None of these methods integrate the topology generation and shape optimization routines and hence have not been very successful in optimal generation of trass structures. These methods and all traditional expert system-based methods are limited by the extent of the knowledge base the designer has input into the system and thus are unlikely to generate any novel designs.

One technique in the literature that has been successful at structural topology layout is the homogenization method for solving generalized structural layout problems by Bends0e and Kikuchi [1988]. The homogenization method is based on the use of an artificial composite material with microscopic voids. An initial domain provided by the designer along with boundary conditions is discretized into finite elements. The effective properties of the composite material are determined in terms of the density of the material *{i.e.,* sizes of the holes) by applying homogenization to the model of a unit cell with a rectangular hole. A given amount of the composite material is then optimally distributed with density as the sizing variable. The result obtained is interpreted as defining a shape. Depending on the material constraint they derive an optimal topology that can vary from trass-like topologies to closed solid shapes. Papalambros, *etal.,* [1990] have incorporated the homogenization method of Bends0e and Kikuchi into an integrated structural optimization system (ISOS). This system starts by generating information about the topology for the structure using the homogenization method. Next this topology information is processed and interpreted using vision techniques. Finally, the system performs detailed shape and size optimization of the derived topology using standard optimization techniques. The homogenization method is computation intensive and requires expert knowledge to decipher the image produced by the algorithm. Currently only stress constraints are integrated into the homogenization process; buckling is only considered in the final shape optimization stage. The method has obtained significant success in generating optimal topologies for truss structures; however, the limitation of the method comes when non-physical constraints addressing manufacturing and general design concerns are considered.

In this paper we propose to use an algorithm based on the *shape annealing* algorithm as proposed by Cagan and Mitchell [1993]. This method combines the advantages of optimization-based techniques, production system-based techniques, and parametric properties of shapes to generate new, optimally directed topological configurations.

Background:

Shape Grammars:

Stiny [1980] introduced shape grammars as a formalism for shape generation. In the field of architecture, shape grammars have been successful in generating various types of artifacts. Examples include villas in the style of Palladio (Stiny and Mitchell [1978]), Mughul gardens (Stiny and Mitchell [1980]), prairie houses in the style of Frank Lloyd Wright (Koning and Eizenberg [1981]), and suburban Queen Anne houses (Flemming [1987]). Stiny defines a *shape* as limited arrangements of straight lines in a Cartesian coordinate system with real axes and an associated euclidean metric. Boolean operations of union and difference as well as the transformation properties of translation, rotation, reflection, scale, and composition are defined on these shapes. Distinguishing information about an individual shape can be assigned through *labels*. Labels can be used to specify a shape or particular side(s) of a shape as well as non-geometric information. For example, the active sides of a magnet may be specified or the mating side of a part of an assembled product may be specified. A set of grammatical rules called *shape rules* are defined on the set of shapes. These shape rules map one shape onto a different shape. Only modifications specified by these shape rules are permitted in the process of shape generation. The result is an algebra of shapes and a grammar formalism from which languages of shapes can be derived.

A *shape grammar* has four components: a finite set of shapes (S), a finite set of labels (L), a finite set of shape rules (R), and an initial shape (I). In general, a shape rule can be defined as:

$$(S,L)^{-} \xrightarrow{R} (S,L)^{+},$$

where a rule from the set of shape rules (R) acts on a set of shapes in S and labels from L, (S,L)", to form a new set of shapes and labels, $(S,L)^{+}$. One example of a shape rule is shown in Figure 1. The shape in the left hand side is a triangle. The label "•" on the triangle indicates the side at which mating can take place. In the right hand side is the result of application of the rule. A new triangle has been added onto the mating side creating a more complicated shape composed of two triangles.

Shapes can be parametric in that the geometric scale of the shape can be varied. Grammars describing such shapes are called *parametric shape grammars.* Parametrization of shapes can increase the scope of shape grammars enormously and model shapes that could not otherwise be realistically modelled.

Shape grammars differ from traditional production systems in that the resulting shapes may illustrate features different from those modeled in the rules themselves. Stiny [1991] discusses the emergent properties of shapes where new features emerge from a shape derived by a sequence of applications of shape rules. In Stiny's formulation emergence occurs because shapes are stored as *maximal lines,* the longest straight line formed by the union of overlapping

lines (Stiny [1980]), where subsets can model new shape features. In the method we introduce in this paper, emergent properties of shapes result from the optimization analysis.

Spillers [1985] suggests the use of grammars for dealing with the problem of structural connectivity and the generation of an initial topology. In this paper we present a method based on shape grammars for generation of structural topologies in an integrated system for structural topology generation and shape optimization.

Simulated Annealing:

Simulated annealing is a stochastic optimization technique which has been demonstrated to solve continuous problems *(e.g.* Jain, *et al.*$_9$ [1990]; van Laarhoven and Aarts [1987]; Cagan and Kurfess [1992]; Cohn, *et* a/., [1991]) and discrete problems *(eg.* Kirkpatrick, *et al.* [1983]; Jain and Agogino [1990]). Introduced by Kirkpatrick, *et al.*$_y$ [1983], the simulated annealing algorithm is based on Metropolis[1] Monte-Carlo technique (Metropolis, *et al.*$_9$ [1953]). This method derives its name from the analogy of annealing of metals. In the process of annealing metals the metal is heated to a high temperature and cooled gradually until it reaches its minimum energy state. At high temperatures the metal molecules are at a highly random state and as the temperature is reduced the randomness reduces until they reach the stable state. The rate of cooling affects the quality of the annealing process. Corresponding to the natural temperature, an artificial variable called *temperature* is created which reduces gradually with time. This temperature reduces according a cooling schedule called the *annealingschedule.*

In simulated annealing, a feasible state si, is randomly selected and the energy (*i.e.*, objective function) corresponding to the state, E_{S1}, is evaluated. A different energy state s2 is then

generated and evaluated to E^- For objective minimization, if $E_{S2} < E_{S1}$, then s2 is accepted as the new solution state. If $E_{S2} > E_{S1}$, then acceptance depends on the probability function

$$Pr(s2) = e^{-\left[\frac{(E_{s2}-E_{s1})/T}{Z(T)}\right]},$$

where T is the temperature and Z(T) is a normalization factor. The probability defined by this function reduces with reducing temperature and resultantly the randomness in the algorithm diminishes with temperature. Convergence of this algorithm to the global optimum for the continuous case, under certain rigorous conditions, was proved by Lundy and Mees [1986]; however, global optimality is not guaranteed in practice. Interested readers are also referred to a book on simulated annealing by van Laarhoven and Aarts [1987].

Shape Annealing:

The generation of novel designs requires the generation and exploration of a large design space. Shape grammars can be used to generate designs by applying different sequences of rules from the grammar. Cagan and Mitchell [1993] propose that the search of the possibly infinite design space resulting from a shape grammar can be accomplished with the concepts of simulated annealing. Their *shape annealing* algorithm selects and applies shape rules in an optimally directed manner. Given a current design state, an eligible rule is chosen from a shape grammar and applied to the design. If the new design produced does not violate any constraints, it is evaluated and sent to the Metropolis algorithm. The Metropolis algorithm compares the new state to the old state and determines whether to accept the new design according to the principles of simulated annealing (it is accepted if better and accepted with a probability if worse). If the rule violates any constraint then the old state is maintained as the current one.

A series of rules when applied successively to an initial design can drive it to a local minima. To move away from these local minima the algorithm requires the presence of rules to backtrack; *i.e.*, rules should be able to be applied in either direction. Thus, in the set of rules there exists a corresponding subtractive rule for every additive rule. This allows shape annealing to explore large design spaces.

Truss Generation:

We propose to use the shape annealing algorithm for the design of truss structures. Truss structures consist of several interconnected truss elements. Due to stability reasons, the truss elements often connect to form a set of interconnected triangles. A shape grammar approach to this problem with triangular shape elements is thus a natural solution. Some of the possible shape rules, corresponding to addition/removal of a triangle to/from the existing shape, are illustrated in Figure 2. Each of the rules can be executed either way; *i.e.,* they act both as subtractive and additive rules. The design can backtrack from a local optima due to the presence of these reversible rules. Again the label in each rule indicates how the rule will be applied, although not necessarily against the labelled face as in rules 2 and 3.

The method presented here seeks to divide the problem space for truss generation and optimization into two spaces, the *shape* space and the *artifact* space with a one to one correspondence existing between them. The manipulation, modification and generation of the design is done in the shape space while the analysis and evaluation of the design is done in the artifact space. The topology of the truss is generated in the shape space using shape grammars, and mapped into the artifact space. The design in the artifact space is then evaluated using finite element and mathematical programming techniques. The result is passed back into the shape space and is accepted or rejected based on the evaluation of the objective function and the

annealing temperature. This cycle is repeated until the solution converges to a good (optimally directed) design.

In this implementation of the concept, an initial triangular element is placed at about equal distance to all the load and anchor points, Figure 3a. In the next step, one of the shape rules is selected and applied to the present design and a new design in the shape space is derived. In the shape space, a design consists of the load, anchor points and a set of interconnected equilateral triangles. Figure 3b illustrates a design in the shape space after a number of iterations have occurred. Next, the nearest node in the configuration of triangles is determined for each of the load and anchor points and the triangles connected to that node are stretched so as to move the node to that load and anchor point. At the end of the *stretch* operation, a truss capable of transmitting the load from the force to support points exists; a valid design as exemplified in Figure 3c is produced. The design is now viewed from the artifact space and is ready for analysis and parameter optimization.

The algorithm is shown as a block diagram in Figure 4. The first step is to input the requirements to be met by the truss. The shape rules remain the same as shown in Figure 2; this grammar can be used to generate a truss for any given requirements. Selection of an appropriate annealing schedule is a primary requirement for good convergence of the simulated annealing algorithm. Huang, *et al.*[9] [1986] describe a way of selecting an efficient annealing schedule; however, a simple schedule as found in Cagan and Reddy [1992] is currently used in the implementation. The shape annealing algorithm is initiated by choosing the initial shape and modifying it by selecting and applying one of the rules in the shape grammar. If the resulting configuration does not violate any constraints the design is moved to the next step in the algorithm as shown in Figure 4, else the new configuration is rejected and a new rule is selected and applied. The design is then transformed from the shape space to the design space using the stretch operation before being evaluated.

The only information a design at this stage has is its topology, the shape and sizing variables being assigned default values. The trass structure is modelled as a set of trass elements to facilitate its finite element modelling. The stiffness matrix $^{f}k^{f}$ for a trass element is a 2x2 matrix

$$k = \frac{AE}{L}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

where A is the cross-sectional area of the element, E is Young's modulus of elasticity and L is its length. The stiffness matrix of an element is conceptually similar to the stiffness coefficient of a spring, but in a trass element there can be two independent displacements resulting in a 2X2 stiffness matrix.

The stiffness matrix for each trass element is calculated and assembled to form the stiffness matrix for the trass structure, $[K]$. The set of equations given by

$$\{F\}=[K]\{D\},$$

where {F} is the vector consisting of external forces acting at all the nodes and {D} is the vector of the global displacements of the nodes due to the forces, are solved for the displacements. This global (for the whole trass) displacement vector is used to calculate the elemental displacement vectors d for each trass element, d consists of four displacement components as shown in Figure 5: u^1, u^2, v^1, and v^2. As we are dealing with only trass elements, $(v^1 - v^2)$ is defined to be zero. The stress a in a trass element can be found by using the formula:

$$a = E\,(u^1 - u^2)/L;$$

positive a implies tension and negative *a* implies compression.

The stress constraints, yield and buckling, can then be calculated. As the truss element cannot support bending, we can assume their end joints to be pin joints. We use Euler's formula for buckling of pin-jointed elements. The stress and buckling constraints are:

$$\text{Yielding Constraint:}\quad abs(a) - Gyp < 0,$$

$$\text{Buckling Constraint:}\quad a + (7c^2\, E\, I) / L^2 > 0,$$

where Gyp is the yield stress and I is the moment of inertia of the cross-section.

The modified method of feasible directions as implemented in DOT (VMA [1990]) is employed for optimization of the truss structure, although other optimization techniques can be utilized. In the optimization process, the objective is to minimize the weight with respect to sizing (area) and shape (nodal coordinate) variables subject to stress and/or buckling constraints. The weight of the optimized truss structure in the artifact space is taken as the evaluation of the design in the shape space. If the cross-sectional area of an element gets small enough, then the element is removed. Note that parametric and emergent properties of shapes are derived directly from the optimization analysis.

If the evaluation of the topology generated, *current_design*, is less than that of the present design in the shape space then we accept it to be our new design in shape space. If the evaluation is higher then we accept it based on the probability function defined by the annealing temperature. The probability of accepting a design with higher evaluation decreases with reducing temperature as dictated by simulated annealing. Next, the design is routed back to the *select and apply shape rule* block in Figure 4. This loop is executed until convergence is achieved or until the temperature reduces to zero. The design to which the algorithm has converged or the best design produced during the annealing process is taken as the optimally directed design for the given set of forces and anchor points.

Examples:

We have applied shape annealing to two problems illustrating different advantages of the algorithm. In the first example we verify the results of the shape annealing algorithm with results from the literature. In the second, we demonstrate the algorithm's ability to generate alternative superior concepts to those previously considered. Both examples seek to find a topology with minimum weight.

Example 1:

For the specifications as shown in Figure 6a, Papalambros, *et al.*[9] [1990] generated an optimal truss, Figure 6c, using their homogenization method. The shape annealing method was applied to the same set of load and anchor points, the resulting structure generated is shown in Figure 7a. It can be easily observed that the shape annealing topology as shown in Figure 7a is very similar to that generated by the integrated structural optimization system of Papalambros, *et al.* In fact the similarity is more apparent when compared with the optimum material distribution diagram they obtain from homogenization, Figure 6b, before refining the design to that shown in Figure 6c. Note that the same refinement heuristics as employed with homogenization could be used with shape annealing. In these examples, buckling has not been considered. If, during topology generation, buckling is included in the shape annealing method along with the stress constraints, the optimally directed design obtained is as shown in Figure 7b.

Example 2:

The specifications and the topology for the standard 10-bar truss problem are shown in Figure 8a. The stress levels in the truss are constrained to be in between -25000 psi and 25000 psi and the density of material is taken to be 0.1 lbs/in^. The 10-bar truss configurations after sizing and shape optimization without buckling constraints are shown in Figure 8b. The optimally directed structure obtained by the shape annealing algorithm if only stress constraints are used is shown in Figure 9a. Comparison of weights shows an improvement over the 10-bar topology; the truss generated has a mass of 1477 lbs while the 10-bar truss has a mass of 1546 lbs on shape optimization and 1588 lbs if only sizing optimization is done. If buckling constraints are also considered, the optimally directed structure as generated by the shape annealing algorithm has a weight of 3225 lbs and is shown in Figure 9b. This compares well with the weight for the design found by shape optimization of the original 10-bar truss with buckling constraints, 4399 lbs, and the design produced by shape optimization of the truss in Figure 9a (whose topology was derived from shape annealing without buckling constraints) with buckling constraints added, 3550 lbs.

Discussion:

The shape annealing method of truss generation is an evolutionary design process. It starts off with a design consisting of a single triangular element. This design could be infeasible, *i.e.*, it may not connect to all the load and support points. The algorithm builds up the design from this initial state by consequent addition and removal of triangular elements. In the examples presented above only the stress and buckling constraints are used but we could easily add displacement constraints and area constraints. Manufacturing requirements and assembly requirements, such as the angle between the truss elements, and any additional expert

knowledge about the design can also be included into the design process by incorporating them into the shape grammar as shape rules. This method combines the advantages of optimization based methods and knowledge based methods. The existing knowledge about design, manufacture and construction of truss structures can be used in the design process as shape rules or as constraints to be negotiated by the annealing algorithm. Concurrently, the shape annealing method can produce an optimally directed truss structure for the given requirements, as seen in Example 2. It is probable that a structure produced by the shape annealing method is the global optimum for simple structures as illustrated in this paper, but it can not be proven at this time.

In practice it is quite probable to have interfering objects between the load points and the anchor or support points, as illustrated in Figure 10. The final design must be optimally directed and at the same time not intersect the obstacle. If we incorporate these geometric constraints into the gradient-based shape optimization routine (the method of feasible directions), the design space becomes non-convex and the method gets stuck in a local optima. A simulated annealing approach allocating penalties for constraint violations is being pursued to solve such problems. A similar approach can improve the run time of the overall algorithm by formulating approximations to the shape optimization process.

Presently the algorithm has been investigated for only two-dimensional truss structures. One immediate direction for future work is to test the application and use of the algorithm for three dimensional truss structures. Two-dimensional and three-dimensional elements, for example plate and brick elements, can also be made use of in the process of generation and optimization of structures for a variety of applications.

Conclusions:

Historically the bottleneck of truss structural design has been the problem of choosing a topology for the structure before its being optimized. In this paper an evolutionary design process, shape annealing, is proposed to generate and optimize truss structural topologies. This method can also be extended to include design of other structures.

Acknowledgements:

The authors are grateful to the Engineering Design Research Center at Carnegie Mellon University and the National Science Foundation under the Young Investigator's Award program (award #DDM-9258090) for providing funding for this work.

References:

Bends0e, M. and N. Kikuchi [1988], "Generating Optimal Topologies in Structural Design using Homogenization Method", *Computer Methods in Applied Mechanics and Engineering,* 71:197-224.

Cagan, J., and Kurfess, T.R. [1992], "Optimal Tolerance Allocation over Multiple Manufacturing Alternatives", proceedings of: *ASME Design Automation Conference,* Phoenix, AZ, September 13-16,2:165-172.

Cagan, J. and G. Reddy [1992], "An Improved Shape Annealing Algorithm for Optimally Directed Shape Generation", *Artificial Intelligence in Design '92* (Gero, J.S., ed.) Kluwer Academic Publishers, Dordecht, pp. 307-324.

Cagan, J. and W. J. Mitchell [1993], "Optimally Directed Shape Generation by Shape Annealing", in press: *Environment and Planning B.*

Cohn, J. M., D. J. Garrod, R. A. Rutenbar, and L. R. Carley, "KOAN/ANAGRAMII: New Tools for Device-Level Analog Placement and Routing", *IEEE Journal of Solid-State Circuits,* 26(3):330-342.

Hemming, U. [1987], "More than the Sum of Parts: The Grammar of Queen Anne Houses", *Environment and Planning B,* 14:323-350.

Huang, M. D., R Romeo and A. Sangiovanni-Vincentilli [1986], "An Efficient Cooling Schedule for Simulated Annealing", *Proc. 1986 IEEE Int. Conf CAD,* Nov. 1986, pp. 381-384.

Jain, P., and A. M. Agogino [1990], "Theory of Design: An Optimization Perspective, *Mechanism and Machine Theory,* 25(3):287-303.

Jain, P., P. Fenyes, and R. Richter [1990], "Optimal Blank Nesting Using Simulated Annealing", *Transactions of ASME: Journal of Mechanical Design,* 114:160-165.

Koning, H., and J. Eizenberg [1981], "The Language of The Prairie: Frank Lloyd Wright's Prairie Houses", *Environment and Planning B,* 8:295-323.

Kirkpatrick, S., C. D. Gelatt, Jr., and M. P. Vecchi [1983], "Optimization by Simulated Annealing", *Science,* 220(4598):671-679.

van Laarhoven, P. J. M., and E. H. L. Aarts [1987], *Simulated Annealing: Theory and Applications,* D. Reidel Publishing Co., Boston.

Lakmazaheri, S. and W. Rasdorf [1990], "The Analysis and Partial Synthesis of Truss Structures via Theorem Proving", *Engineering with Computers,* 6:31-45.

Lundy, M., and A. Mees [1986] "Convergence of an Annealing Algorithm", *Mathematical Programming,* 34:111-124.

Metropolis, N., A. Rosenbluth, M. Rosenbluth, A. Teller, and E. Teller [1953], *Journal of Chemical Physics,* 21:1087-1091.

Papalambros, P., and M. Chirehdast [1990], "An Integrated Environment for Structural Configuration Design", *Journal of Engineering Design,* 1:73-96.

Rogers, J., S. Feycock, J. Sobieszczanski-Sobieski [1988], "STRUTEX- A Prototype Knowledge Based System for Initially Configuring a Structure to Support Point Loads in Two Dimensions", in *Artificial Intelligence in Engineering: Design* (Gero, J.S., ed.), Elseiver publications, Amsterdam, pp. 315-335.

Shah, J. J. [1988], "Synthesis of Initial Form for Structural Shape Optimization", *Journal of Vibration, Acoustics, Stress, and Reliability in Design,* 110:564-570.

Spillers, W. [1985], "Shape Optimization of Structures", in: *Design Optimization* (Gero, J.S., ed.), Academic Press Inc., Orlando, pp. 41-70.

Stiny, G. [1980], "Introduction to Shape and Shape Grammars", *Environment and Planning B,* 7:343-351.

Stiny, G. [1991], "The Algebras of Design", *Research in Engineering Design,* 2:171-181.

Stiny, G., and W. J. Mitchell [1978], "The Palladian Grammar", *Environment and Planning B,* 5:5-18.

Stiny, G., and W. J. Mitchell [1980], "The Grammar of Paradise: on the Generation of Mughul Gardens", *Environment and Planning B,* 7:209-226.

Topping, B. H. V. [1983], "Shape Optimization of Skeletal Structures: A Review", *Journal of Structural Engineering,* 109:1933-1947.

VMA [1990], *DOT Users Manual,* Vanderplaats, Miura & Associates, Inc., Goleta, CA.

Figure 1: Example shape rule.

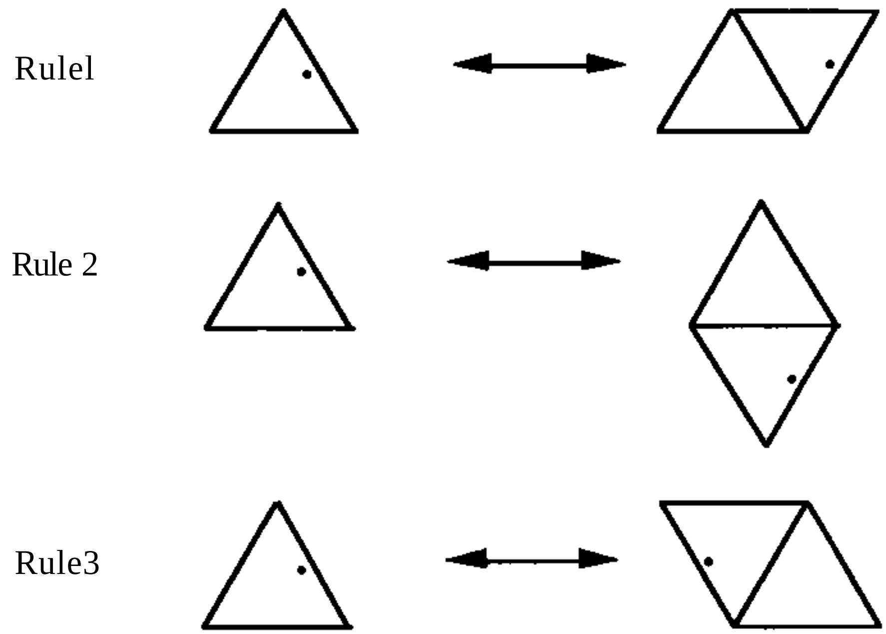


Figure 2: Shape grammar for truss topology generation

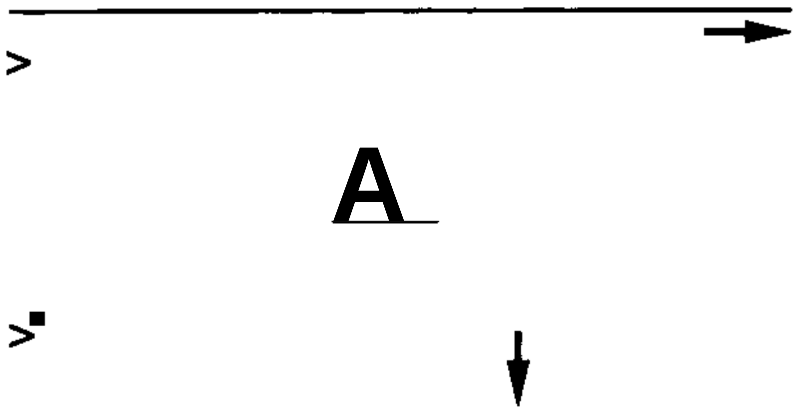


Figure 3a

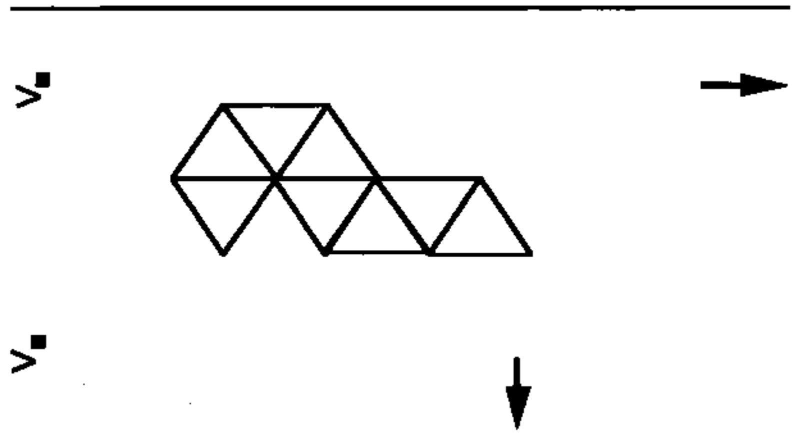


Figure 3b

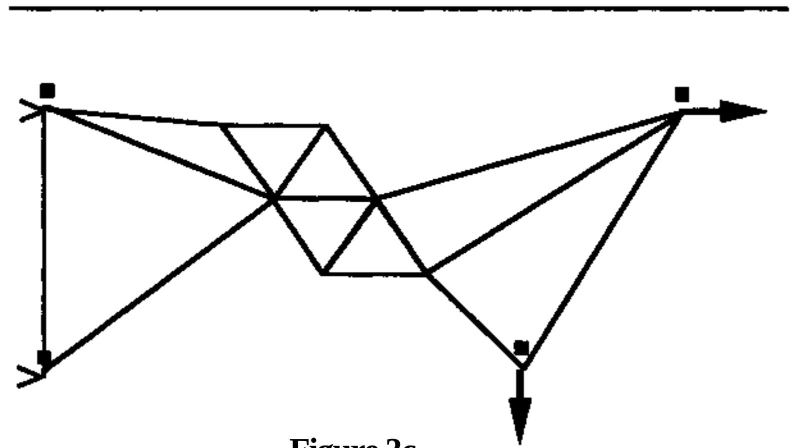


Figure 3c

Figure 3: Shape Space steps

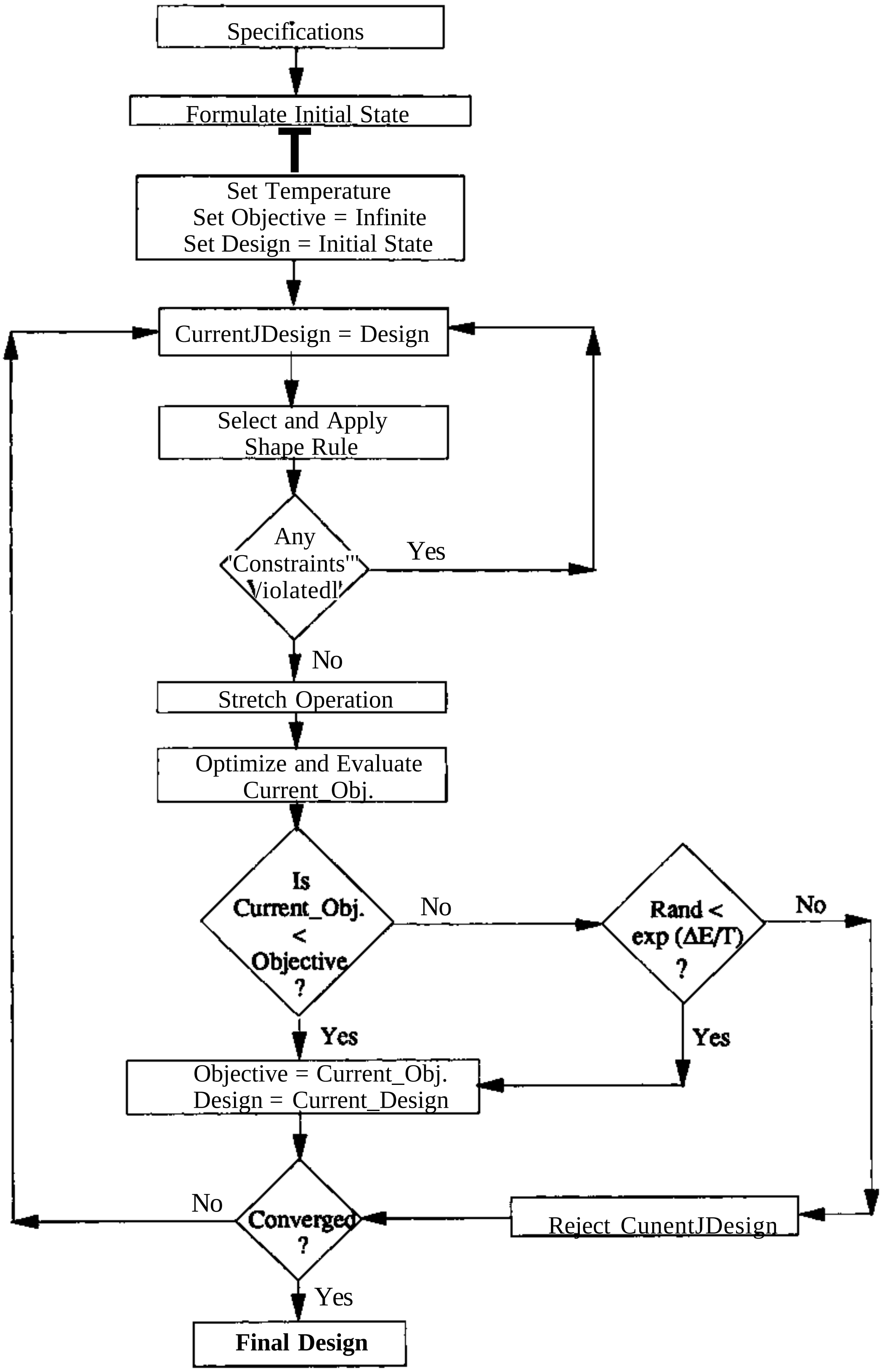


Figure 4: Truss Generation algorithm

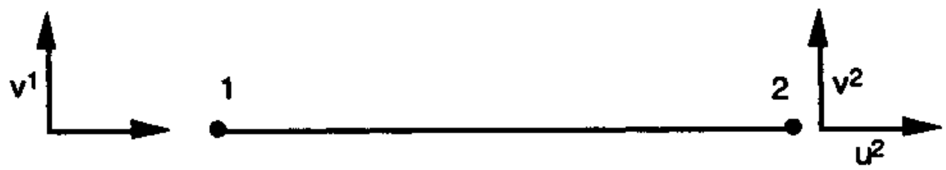


Figure 5: Displacements at the nodes of a truss element

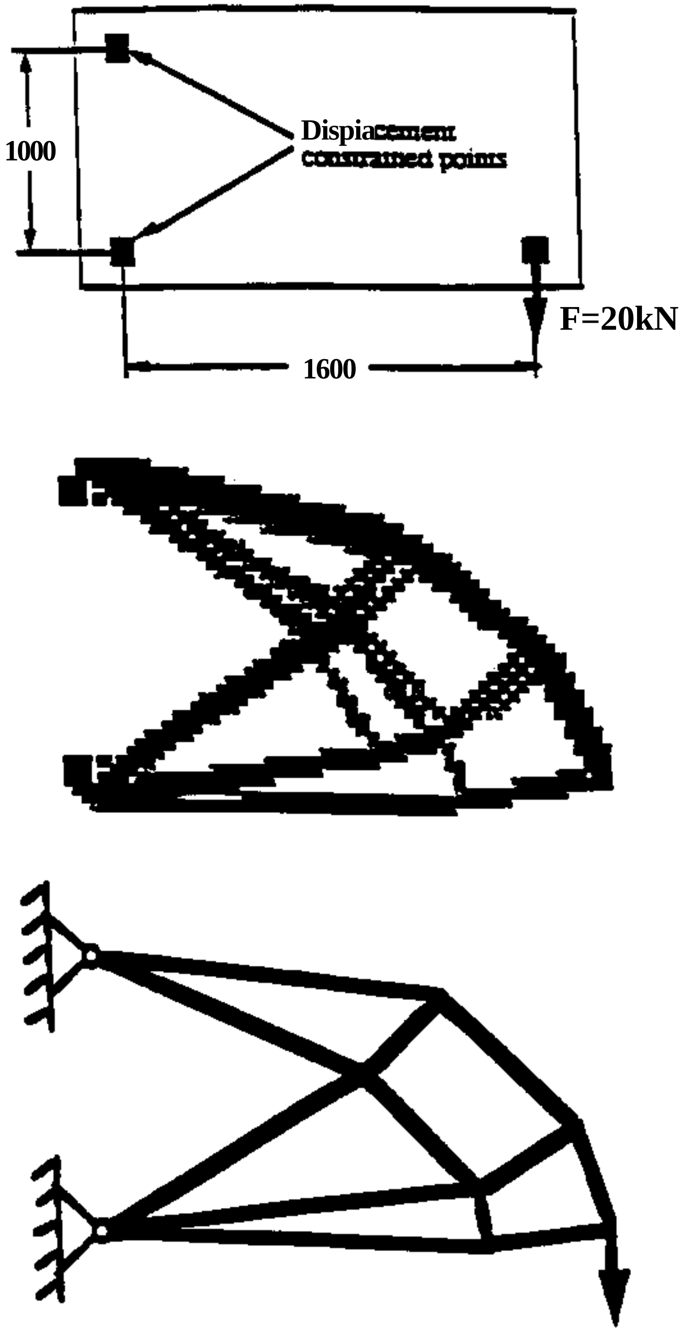


Figure 6: Homogenization method (From Papalambros, *et* a/., [1990]): Input (6a), homogenization result (6b), final design after refinement (6c)

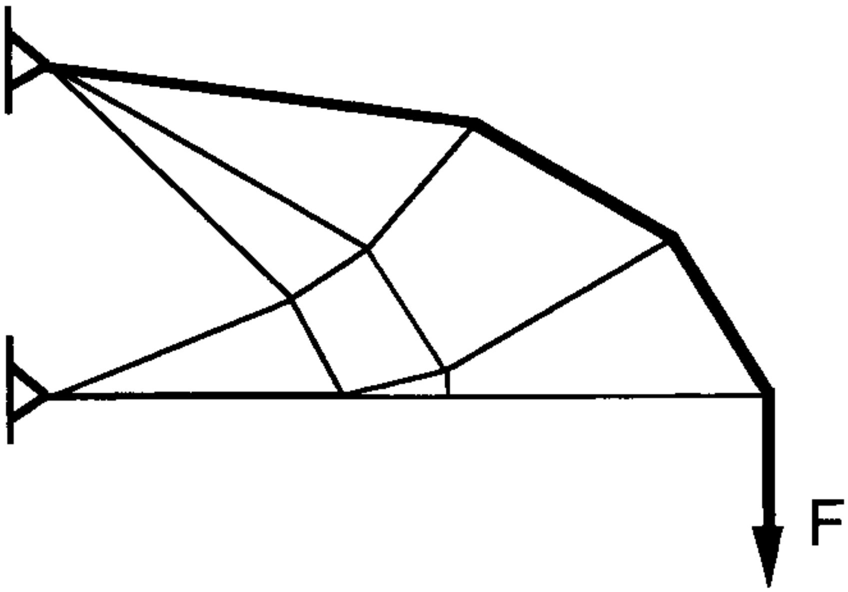


Figure 7a

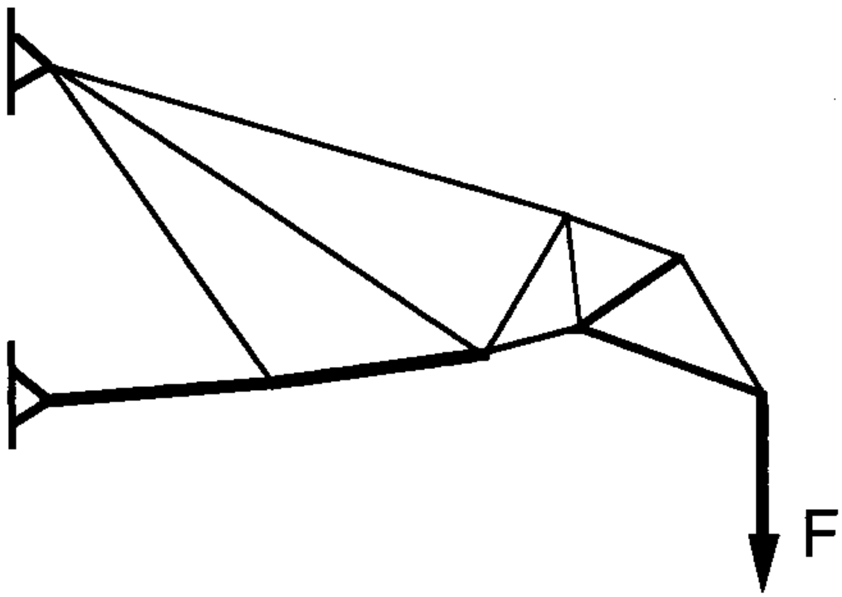


Figure 7b

Figure 7: Shape annealing topology, without buckling constraints (7a);
Shape annealing topology, with buckling constraints (7b)

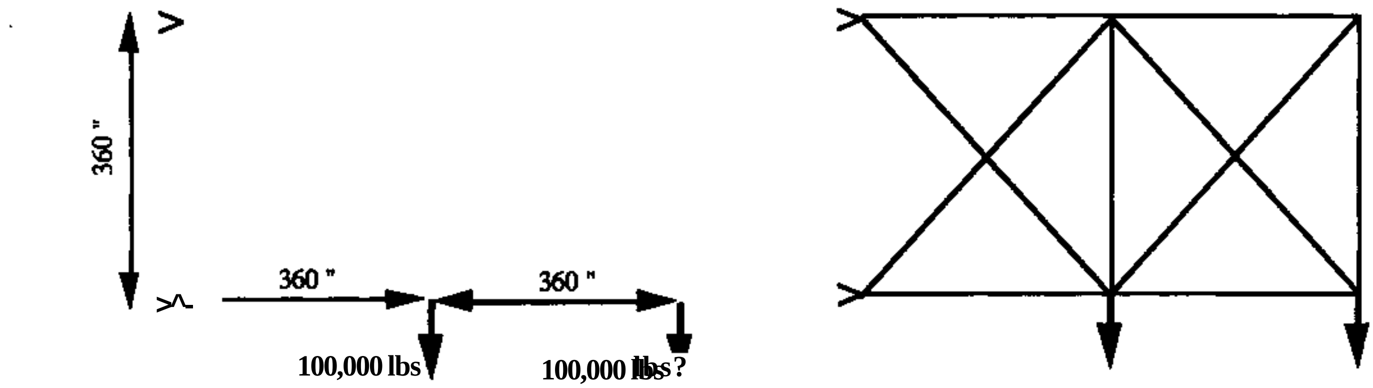
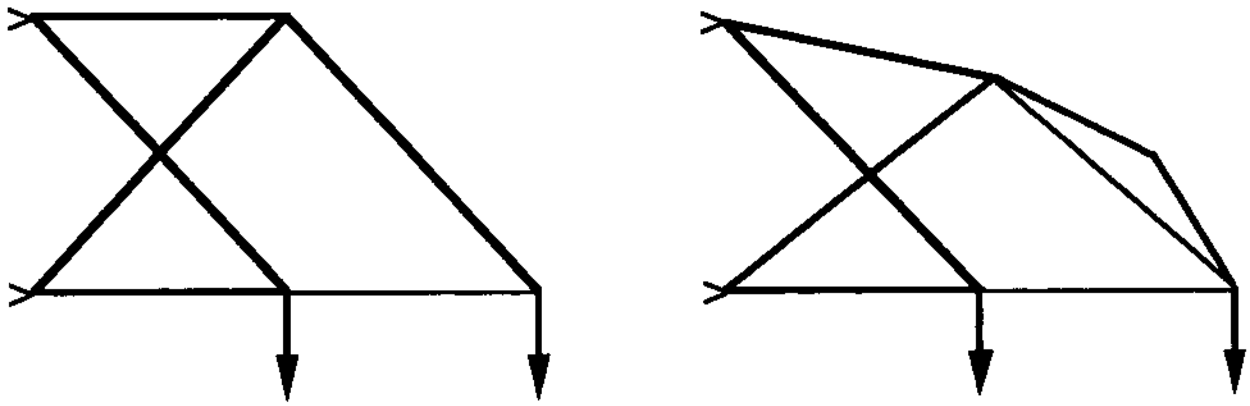


Figure 8a



Area optimized

Shape optimized

Figure 8b

Figure 8: Specifications of a 10-bar truss problem (8a). Area optimized 10-bar truss and shape optimized 10-bar truss, without buckling constraints (8b).

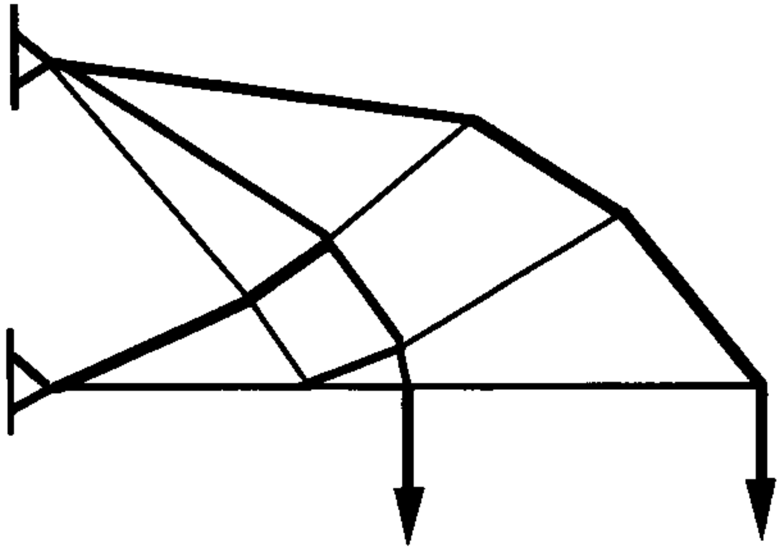


Figure 9a

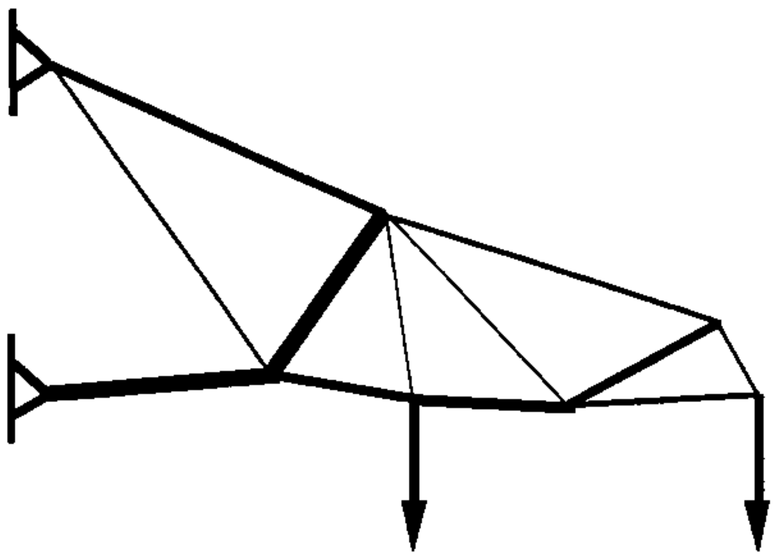


Figure 9b

Figure 9: Shape annealing topology, without buckling constraints (9a); Shape annealing topology, with buckling constraints (9b)

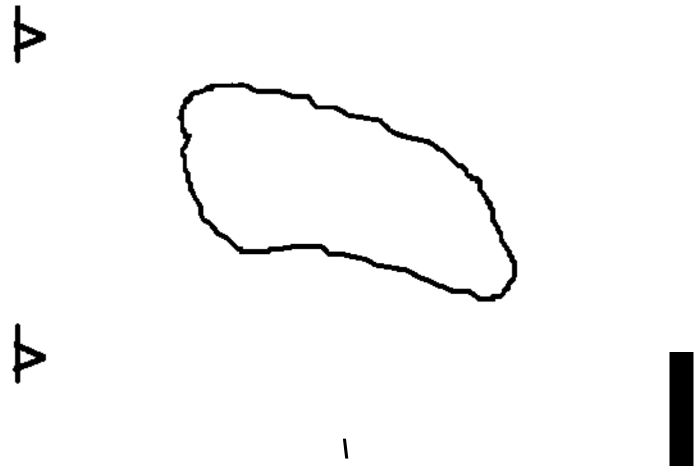


Figure 10: Interference between load and anchor points.