Decision Making in Design for Quality
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0. Abstract

We consider the design issues of characterizing and appropriately choosing the quality of a product for maximum customer satisfaction and maximum corporate profit. We assume that the basic design configuration of a product is determined and that the designer must select the product quality by imposing statistical tolerances (manufacturing accuracy specifications) and choosing appropriate manufacturing processes. We claim that these design considerations, although often neglected, impact the behavior, cost, and even marketability of the product. A design concept without these specifications is incomplete because i) its feasibility is questionable ii) its cost of production is undetermined iii) its performance is unknown. The design process is seen here as an exploratory activity constrained by the manufacturing systems available. In particular, we provide a decision-analytic methodology to reason about the quality of product and the influences on profit resulting in a specification of manufacturing process and machine accuracy to match customer demands. The method has been implemented and applied to the design of a simple three component assembly to illustrate its capabilities.

1. Introduction

Designia^t product for mass production requires numerous complex decisions pertaining to manufacturinpaptkms. These decisions include quantitative factors such as performance, cost and accuracy, but also vaguely defined qualitative characteristics such as quality and customer satisfaction. Further, when employing mass production, economic factors become primary concerns since slight variations in cost can have a substantial impact on corporate profits. This implies that the manufacturing process becomes an important concern of the design process and should be optimized for minimum cost. Unfortunately, defining the objective function is often a major problem because of vaguely defined components (e.g., quality or customer satisfaction). In this paper, we take a unique approach to decision making for the competitive design problem. We introduce a model to assist an engineer in the selection of product designs that are best suited to respond to a given customer demand based on performance requirements. Our method allows a designer to quantify the abstract notion of design quality before the actual manufacture of the product We call our model a *decision-analytic* method because it enables a designer to make decisions with respect to poorly defined, contradictory objectives by quantifying such concepts as quality or customer satisfaction through the use of probability theory (Bradley and Agogino, 1991).

2. Conventional Tolerances: A First Step Toward Design Quality

Since the beginning of mass production, manufacturers have been struggling with the variability inherent in any manufacturing operation. For a given design, the performance or characteristics of the assemblies produced may vary and deteriorate because it is impossible to manufacture parts with perfect nominal specification. This degradation can be reduced if the variations of the part characteristics are limited. It is therefore common to permit constrained variations in part characteristics by allocating tolerances and removing out-of-tolerance parts. A widely used fount for these specifications is

$$\text{Nominal Value} \pm \text{Tolerance.}$$

Thus, the design of a peg to be inserted in a hole (Example 1) requires the designer to provide the diameters of the hole and peg, but also to allocate tolerances on these two dimensions (see Figure 1).
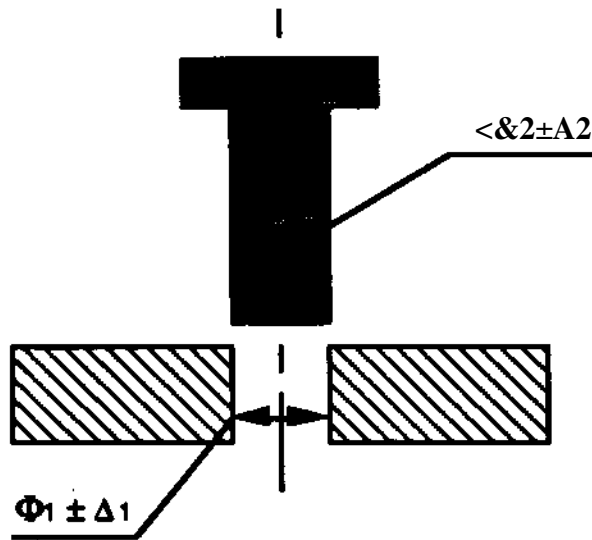
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Figure 1: Tolerances on the peg and hole diameters guarantee the proper functioning

The first concern in allocating tolerances is to guarantee the proper functioning of the product, and therefore to satisfy technical constraints. In the previous example, a constraint imposes the diameter of the peg to be smaller than that of the hole. Tolerances are to be allocated on the diameters in such a way that $F(\phi_1, \phi_2)$ is always positive, where ϕ_1 and $\$2$ are the actual diameters of the manufactured parts (as opposed to their nominal design values O_i and O_2), and $F(c_1, \phi_2)$ the difference $\$1 - \$2$. F is known as the design function (Martino and Gabriele, 1989), an i is a function of the part characteristics whose variations induce changes in the functioning or pe ormance of the assembly:

Performances F(partcharacteristics).

For a known requirement on the performance (usually defined in terms of tolerance), the problem of tolerance synthesis consists of choosing tolerances on the part characteristics, in such a way that the performance F, satisfies the requirement(s) when the part characteristics vary within permitted tolerances. For example 1, the tolerance synthesis problem consists of choosing A_i and A_2 in such a way that for any value of $\$1$ in $O_i \pm A_i$ and any value of ϕ_2 in $O_2 \pm A_2$, F should be positive. This simple requirement immediately yields the constraint

$$O_i - A_i - (O_2 + A_2) \geq 0 .$$

Tolerance allocation is therefore a way to sort a process output by quality level into two different categories: acceptable and defective.

3. Design Quality as a Continuum: Statistical Tolerances

' As seen in the previous section, the first definition of quality is conformance to specifications where manufacturers measure the quality of their production by the percentage of defective items produced. This approach is reasonable when defective items have a significant probability of being shipped to customers. When the rate of defects is reduced significantly, or when the final products are inspected, quality cannot be assessed solely by the percentage of defective items produced; this measurement does not differentiate between items that possess characteristics near to the design nominal values and those that, despite being within specifications, deviate significantly from the target values. In many cases, the nominal value is that which provides the desired performance and, thus, the best product (Taguchi, 1986). Tolerances exist only to limit the degradation of the performance. An item off target is more likely to break down than a product that has parameters closer to the target values. It is important to determine if a product is within specifications, and if so, how far it is from the target value. Thus, the quality of a product is actually a continuous measure, and conformance to conventional tolerances may not be the best means to measure quality.

In the case of mass production, product characteristics are more often than not statistically distributed in a continuous manner, and the attributes of the statistical distribution (e.g., mean, standard deviation) are the real quality indicators. In the industrial case, part tolerances have a small impact on the average product performance, compared to part distributions. Therefore, it seems better for a designer to use statistical tolerances that specify a standard deviation for each part dimension rather than a conventional tolerance. The standard deviation characterizes the distribution of the part characteristics better than the tolerance which is only the limit placed on the distribution. This approach does not make tolerances obsolete; they still can be of great use, especially when inspection has to be performed during the production stage. Our thesis is that tolerances alone do not contain enough information for the efficient manufacture of a design concept. For that matter, our method makes use of statistical tolerances which we define as the suitable probability distributions of manufactured part characteristics. Because of the large quantities manufactured in mass production, we assume that part characteristics are normally distributed about their nominal value. Therefore, selecting the standard deviations completely characterizes the distributions.

The design function, which is used for conventional tolerances as discussed in section 2, is also used for the determination of the statistical distribution of the assembly characteristics during the production stage: Assuming a normal distribution for the hole diameter $N(O_i, o_i)$ and for the peg diameter $N(E_2, O_2)$, the assembly characteristic F is the difference of two normal random variables, therefore also normally distributed, with mean $O_i - O_2$ and standard deviation

$V\,ai^1 + 02^2$. We can see at this point that two problems arise: i) the designer has to know which distribution of the assemblies is good enough for his purpose, ii) as for conventional tolerances, there are many possible solutions to the tolerance synthesis problem. Namely, in our example, the two problems are i) What is the right value for $y\sqrt{ai^2 + O2^2}$? and ii) what is the best combination $(01,02)$ among all the possible combinations that yield this value? In the following sections, we introduce a decision-analytic methodology to address these questions, based on machining cost, quality and design for manufacturing considerations.

4. Redefining the Cost of Machining

Several authors have investigated the economic aspect of design accuracy specifications. Various representations of the cost of an individual item have been used. Generally, the cost is decomposed into the sum of a constant part (cost of raw material, FO and a variable part (cost of holding the tolerance A|). The functions most commonly used are:

$$C_i(\Delta_i) = F_i + \frac{\wedge}{A_i?} \quad \text{or} \quad C_i(\Delta_i) = + \frac{j}{(Pi - Aj} - \wedge, \quad \text{or} \quad C_i(\Delta_i) = F_i + \lambda_i\, e^{-(\Delta_i/\tau_i)},$$

where ai, pi, λ_i and T_i are problem dependent parameters. From these relationships and from industrial data, we see that cost increases with accuracy. The assembly cost may be computed as the sum of the costs of the individual components. Tolerances are usually allocated by minimizing this cost under various design constraints. Several authors have presented a solution based on linear programming (Patel, 1980; Bjorke, 1989), on Lagrange's multipliers method (Bennett and Gupta 1969; Speckhart, 1972; Spotts, 1973; Wilde, 1978; Chase and Greenwood, 1988; Chase, *et. aU* 1990), on non-linear programming (Michael and Sidall, 1981-82; Parkinson, 1985; Lee and Woo, 1989; Lee and Woo, 1990), on geometric programing (Wilde and Prentice, 1975), on a graphical method (Peters, 1970), or on a simulated annealing technique (Cagan and Kurfess, 1991). It is also possible to choose the manufacturing processes concurrently with the tolerances (Lee and Woo, 1989; Cagan and Kurfess, 1991). None of these models provide a designer with the cost information necessary to allocate statistical tolerances on a design. Indeed, the attempts to allocate statistical tolerances based on cost considerations are still influenced by conventional tolerance allocation. Typical models were based on an implicit arbitrary relationship between the tolerance and the yield of the process, the famous relationship $A = 3a$, where A is the tolerance and a the standard deviation of the process output. This relationship is recommended by the theory of statistical process control (SPC) but there is no reason why there should be a constant linear relationship between A and a. Moreover, it is very likely that the coefficient 3 was originally

chosen only because it is simple and "probably" not too far from the optimal. For a model using this a priori relationship, statistical tolerance allocation is not fundamentally different from conventional tolerance allocation since they are based on the same machining costs viewed as functions of conventional tolerances. We believe that this cost representation based on tolerances rather than process output distribution may camouflage part of the design decision problem because the tolerance on a part is often an arbitrary number, chosen by the designer that has nothing to do with the process itself.

(a)$A = 3a$ (b)$A = 6\sigma$

Figure 2: SPC solution (a) and higher quality solution (b)

A better model would view machining cost as a function of the process output spread, rather than the tolerance on the part Since we are addressing industrial mass production, processes produce part populations that are approximately spread about a nominal target value: unavoidable sporadic trends are eventually detected, corrected and averaged to the nominal value. Because of the high quantities produced it is also reasonable to assume that the parts are normally distributed. The standard deviation of the part population can then be easily estimated, and constitutes an intrinsic representation of the process quality. Therefore, our model treats the cost of machining a ptrt with a given process as a decreasing function of the *standard deviation* of the part population produced by the process. Where a designer would traditionally impose a tolerance A, our approach characterizes the appropriate distribution, by specifying its standard deviation. The correct distribution may be the one recommended by SPC theory, or a different one (sec Figure 2). A is a limit beyond which the product is not acceptable, a determines how the pan characteristics are distributed and therefore what is the quality level of the parts. This view point is consistent with the current aggressive competition between manufacturers, which leads to

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constantly increasing performances. An extreme example is computer chip manufacturing, where the products of die same manufacturing process may perform at different speeds and be sold as different product grades at various prices depending on their characteristics. In that case, there is a clear direct relationship between the distribution of the product characteristics and the profit of the manufacturer. This relationship will be established for the general case in the next section.

5. Quantifying Design Quality

We have seen that in many cases, quality can be related to the machining inaccuracies that occur during the production stage. It is the designer's responsibility to determine what level of inaccuracy is acceptable. Here we analyze the particular case of an assembly whose performance is measurable. We assume that there is a nominal design value for the performance. This target value is accompanied with a tolerance corresponding to a customer requirement (bearing sizes) or to a technical constraint (voltage of a generator to be hooked on appliances). This tolerance on the overall assembly performance is assumed to be given, as opposed to the distribution of the manufactured assembly performances.

We have established in section 3 that the closer a part characteristic is to its target value, the higher the quality of the product. The overall tolerance exists only to limit the degradation of the assembly performance. An off target product may involve later warranty costs because is it more likely to break down than a product that has a performance closer to the target value. A product that does not perform exactiy as it is supposed to may also generate a loss in the future sales of the manufacturer. Therefore, any deviation from the nominal performance is a potential loss for the manufacturer. In order to be used in our analysis, this loss needs to be quantified monetarily. This is the purpose of the quality loss function, L(F) where F is the variable to be optimized (such as the performance or geometry of the assembly) (Taguchi, 1986). Consider the situation where F is not at the nominal value D (despite being possibly within the given tolerance A). Let us compute an approximatioii of the function L. A second order Taylor expansion of L about D provides the following approximation:

$$L(F) = L(D) + L'(DXF - D) + {}^{\wedge}h^* - Df.$$

Two relationships allow for the determination of the derivatives of L: because L is minimum for F = D, $L^f(D) = 0$. For F = D + A, the product is defective therefore the manufacturer looses the product production cost denoted by C:

$$C - {}^{LH(D)}A^2$$

An approximation of the quality loss function is, therefore,

$$L(F) = \frac{C}{A^2} \ - D)^2.$$

For the industrial production of a product, the expected value of the loss function (as defined for instance in Raiffa, 1968) becomes a function of σ_F, the standard deviation of the manufactured assembly performances:

$$L = \frac{C}{A^2} a p^2. \tag{1}$$

This formula is important because it relates C, the production cost of a product, with L, the quality loss, which is the only means to estimate customer satisfaction before the manufacture of the product C and a_F are functions of the design accuracies on the parts imposed by the designer. Therefore, an optimal design decision consists of allocating the statistical tolerances on the parts to minimize the sum of the cost of production and the quality loss denoted by Total Cost

$$Min\left[C + \frac{C}{\Delta^2} \sigma_F^2 \right].$$

6. Statistical Tolerance Specification Algorithm

Let us assume that a product has performance F, depending on the part characteristics xi, x2, ...xn. The designer must choose the σ_F s, statistical tolerances on the xi's (i.e., the standard deviations of the manufactured part characteristics). We believe that an efficient design accuracy specification must be based on manufacturing cost information. Namely, for each part, it is necessary to know what are the processes available for the manufacture, what is their accuracy, and what is their cost Some processes have a single possible machining accuracy, others can be continuously adjusted over a certain range and their cost is a function of the required accuracy. Because the use of statistical tolerances is recent, very little information has been published in the literature. Presently, for application of our method, the collection of information has to be done by each company on its own machines. One can expect that eventually statistical tolerance cost information will be made publicly available; such information exists for conventional tolerances (Peat, 1968), and the growing interest for statistical tolerances will lead to the development of similar standard tables.

The information that has to be provided to the algorithm consists of the characteristics of the processes **available** for the manufacture. For each process, the range of possible accuracies has to be specified $[Q^\wedge;$ $a^\wedge]$ (with possibly $o_{ndn}^\wedge$ $a^\wedge$), as well as the cost functions over the range of accuracies, $C_i(G_i)$. One must also indicate to the algorithm the processes that are potentially suitable for each part The algorithm proceeds then as shown in see Figure 3. For every possible combination of processes, the production cost functions of all the components are summed to yield the production cost function of the assembly, and the quality loss function is computed with equation (1). The objective function, consisting of the sum of the assembly cost function and the quality loss function is then minimized. The accuracies that minimize the objective are the optimal statistical tolerances under the particular process selection. The processes that provide the lowest value for the objective function are selected and their suitable accuracies provide the statistical tolerances. We are currently using the stochastic optimization technique of simulated annealing for selection of both the continuous and discrete variables. The algorithm has been implemented in C on a Mac II; results are presented in the next section.

```
Jtegjn StatisticalJTolerance.Allocation
        Lowest_Total_Cost = Max_Float;                    /• maximum real number •/
        Eor all potential combinations of processes  Bfifi££iH
                Select Qirrent_Process_.Combination from potential combinations;
                Minimize Total_Cost( Current_Process_Combination);
                Save Minimizing_Accuracies for Cunent.Process.Combination;
                If (TotaLCost < LowestJTotaLCost) Then fifian
                        LowcstJTotaLCost    = TotaLCost;
                        StatisticaLTolerances = Minimizing_Accuracies;
                        .Best_Processes          = Cuirent_Process_Combination;
                End:
        End:
        Toknmce.Solution     = StatisticaLTolerances;
        Process_Solution     = BestJProcesses;
    End:
```

Figure 4: Statistical tolerance allocation algorithm

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7. Example Application

For illustration of our theory, we examine the particular example of an assembly of three friction wheels with different diameters and, therefore, different production costs. Each wheel, from largest to smallest, has a given nominal diameter of: 4, 3, and 1 inches. The customer specifications require that the total length of the assembly must be the sum of the three nominal wheel diameters, with a given overall tolerance A (inches) as shown in Figure 4.
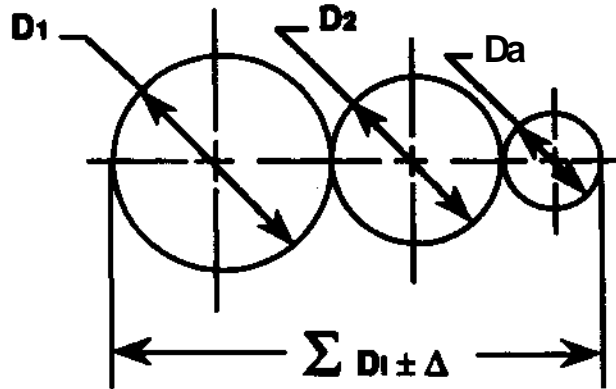

Figure 4: System of friction wheels.

The wheels can be manufactured with three different processes: sawing cutting bar stock, turning on a lathe, or grinding. For each of those processes, there is a best possible machining accuracy involving the highest cost Decreasing the processing times decreases the accuracy as well as the cost per part Each machine is best suited for a given range of accuracy over which the manufacturing cost per part can be represented as a hyperbolic function of accuracy (process output standard deviation) as shown in Figure 5 (Cagan and Kurfess, 1991). Each wheel has its own cost function due to the different sizes of the wheels: the larger the diameter, the more expensive the part We determine the manufacturing cost, C, of an assembly by summing the costs of machining the individual wheels, $Q(G_i)$, with a process of accuracy Q_j. The quality loss function is denoted by L. The populations of parts are assumed to be normally distributed about their nominal or target diameters. Note that we do not specify conventional tolerances on individual components. The only tolerance is on the overall length of the assembly and is customer defined. Given this tolerance A, we are going to determine the optimal distribution of the assembly lengths, as well as that of the component diameters.

Applying (1) yields the quality loss:

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$$L = \frac{C}{\Delta^2} \text{OF}, \text{ with } C = \sum_{i=1}^{3} C_i(\sigma_i), \text{ and OF} = 2 * \sum_{i=1}^{3} a_i^2$$

The function to minimize is therefore:

$$\text{TotalCost} = \left(\sum_{i=1}^{3} C_i(\sigma_i) \right)\left(1 + \frac{1}{\Delta^2} \sum_{i=1}^{3} \sigma_i^2 \right).$$
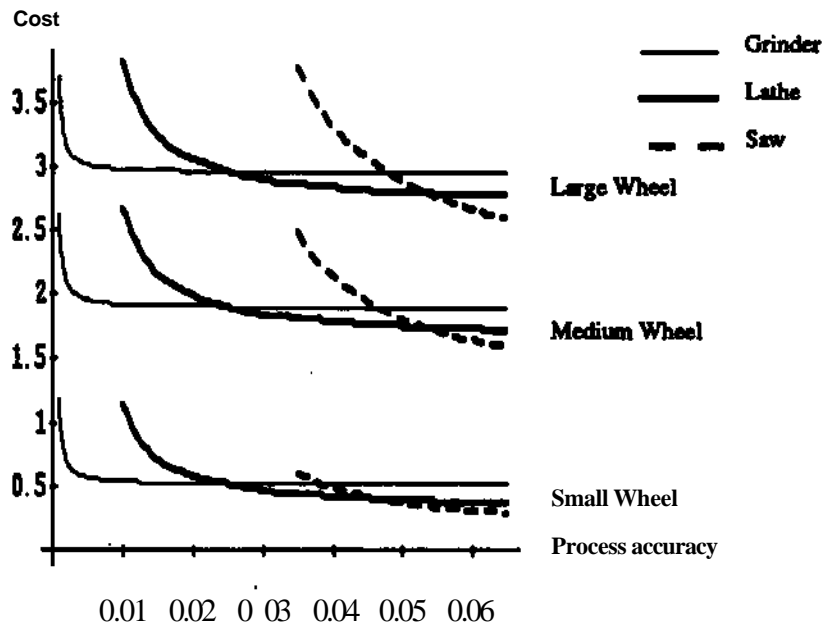

Figure 5: Cost Functions of the Components for Different Processes

The results of the minimization are presented in Figure 6. We have studied the friction wheel design for several values of the overall tolerance. The components with the highest manufacturing costs (the largest wheels) are allocated the loosest statistical tolerances (the cheapest processes), and the components with the lowest manufacturing costs receive the tightest statistical tolerances (requiring expensive processes). This is due to the fact that it is cheaper to hold tighter tolerances on **the** smaller components; Large components usually require a longer processing time since more **material** has to be removed. Note that the slope discontinuities in Figure 6 relate to production process changes (from grinding to turning to saw cutting with increasing overall tolerance).

Figure 7 displays the costs incurred by manufacturing and quality loss for various overall requirer us. The high quality loss for the large overall tolerances is due to the fact that large deviatk •> from nominal are permitted by the larger overall tolerances, while the manufacturing costs at ^se loose tolerances are relatively low. However, as the tolerances are tightened, the

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quality loss is reduced significantly, while the manufacturing costs are increased. As the requirements **a*fc tightened** even further, the quality loss again increases due to the fact that the processes can **n$ longer** satisfy these quality requirements. This indicates that at the tighter tolerances, die **avaflaWc** processes may not have sufficient capabilities.
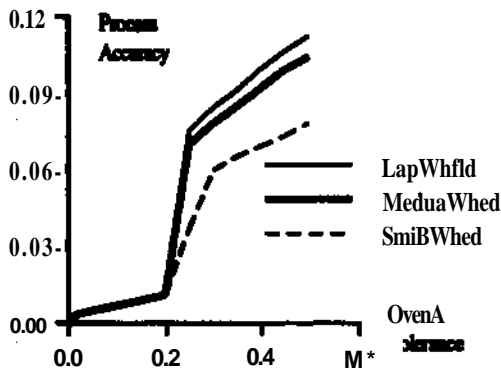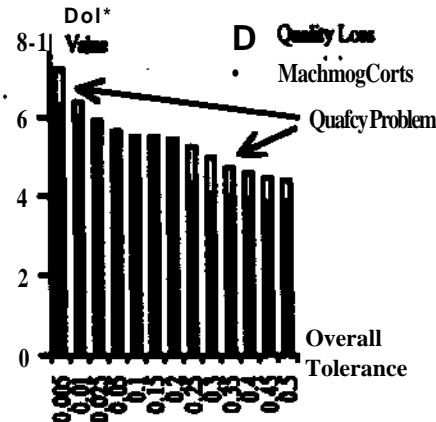

Figure 6: Optimal Process Accuracies   Figure 7: Decomposition of the Total Cost

An interesting point should be mentioned about the cost variations associated with the requirements $A = 0.25$ and $A = 0.20$. The overall costs of the part (the sum of production and quality loss costs) are quite similar, yet the looser tolerance has a substantially larger quality loss cost component. This occurs because of a change in production process that increases production cost (shown in black in Figure 7) and decreases quality loss (shown in white). In connection with this observation, section 8 investigates the meaning of the quality loss and its variations.

8. Selecting Appropriate Products for a Given Set of Machines

In die first sections we developed a model to optimize the quality of a design. Quality was measured with **flGIpect** to the design target performance of the product as well as to the tolerance set on this **perfornnóce.** The optimal statistical tolerances are those that minimize the aggregate quantity Total Cost, defined as

$$\text{Total Cost} = \text{Cost} + \text{Quality Loss},$$

where "Cost" is the cost of production, and "Quality Loss" estimates a future loss incurred by the manufacturer due to the fact that he sells products that are not perfect. Let us consider a product for which the market situation is such that prices are approximately markedHip production costs.

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We consider also a medium sized firm in the price taking situation (as opposed to a large firm in the price setting situation). For a given customer requirement A, the method described above computes a production cost C(A) and a quality loss L(A). If the given selling price of the product P_A is

$$P_A = MC(A),$$

where M is the manufacturer's mark-up, the profit n achieved by selling the product is

$$n = MC(A) - C(A) - ML(A).$$

It should be noted that the quality loss is marked-up by M since the value of the perfect product changes once out of the factory. L(A) is the loss corresponding to the case where the manufacturer is his own customer (e.g. for intermediate products). This is statistically consistent with the scenario in which an unsatisfied customer obtains his money back, thus generating a negative profit for the manufacturer. Two remarks are in order: i) the quality loss is a direct cut in the manufacturer's profit; therefore product designs for which the quality loss is important should be avoided; ii) if product designs with important quality losses are to be produced, it may be possible to increase profit by replacing the ordered product design with a higher grade design. Indeed, there exists a tradeoff between the conflicting goals of manufacturer's cost and profit Thus, the concept of Pareto optimality becomes relevant A Pareto optimal solution is one where both goals reach an equilibrium of satisfaction; a positive change in one goal's direction affects a negative change in the other goal's direction and vice-versa (Jain and Agogino, 1990). In many cases, the solution to this model is not a Pareto equilibrium; it is possible to make both the manufacturer and the customer "better off" without making either "worse off." If the selling price remains the same but the manufacturer reduces the quality loss by increasing the manufacturing cost, the customer is certainly "better off." In some cases, they can be both better off if there exists a A^1 that imposes higher quality standards (such that A' < A), and

$$C(A) + L(A) M > C(A^f) + L(A') M.$$

Therefore, a method to address a large quality loss is to maximize the profit under a constraint imposing the selling price P_A:

$$Max [P_A - C(A') - L(A') M],$$

such that A^1 < A. Thus, a new overall tolerance of the assembly, A^1, is determined yielding a performance superior to that requested by the customer; simultaneously, profits are increased. This yields a Pareto equilibrium where the customer is happy (happier that expected) and the producer is better off as well Note that the customer received better product but does not pay a

higher price; rather, the company just increases its profit margin. Thus, the company could potentially decrease profits to remain competitive while delivering superior product.

The results of the Pareto optimization for the friction wheel example are displayed in Figure 8. The optimization was performed assuming a manufacturer's mark up of 1.75. The quality loss and profit *before* Pareto optimization is an optimization with the user-defined tolerance; the quality loss and profit *after* Pareto optimization is found as discussed above. The Pareto principle is illustrated by the fact that the new profit curve is always above the previous one (the manufacturer is never "worse off), and the value of the quality loss is always lower than previous ones (the customer is never "worse off* either). Figure 8 shows that the quality loss is significant over two different ranges of requirements: above 0.2S and under 0.025 inches. The benefits of this optimization are clear over the first range whereas they are more questionable over the second. Obviously the second problem is a technical problem that cannot be solved within the predefined possible machining accuracies: the three machines available simply cannot efficiently meet the tighter requirements imposed.
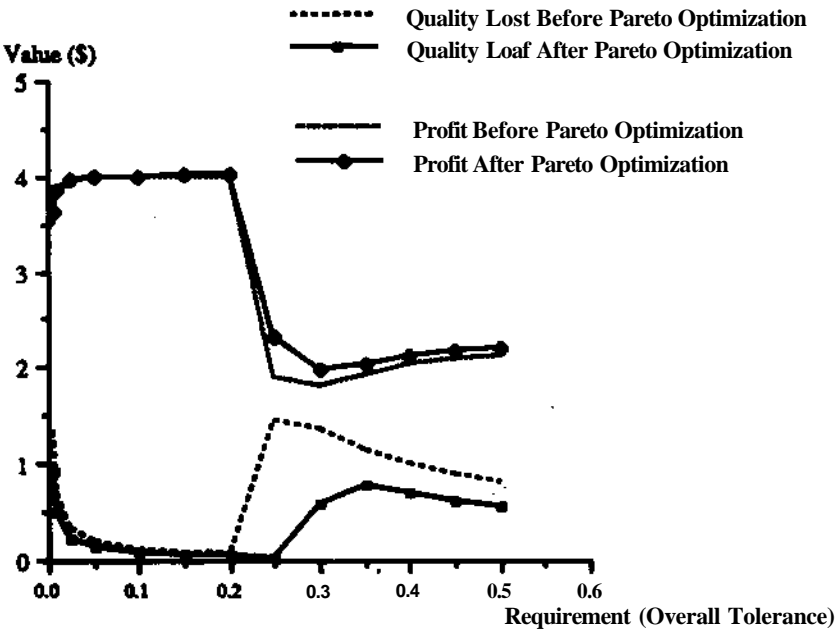

Figure 8: Results of the Pareto optimization.

Figure 9 establishes the relationship between the performance required by a customer (based on the specified output tolerance) and the optimal performance that should be delivered by a manufacturer to maximize the profit Over the range of performances above 0.2 inches, the quality

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loss is so important that the manufacturer must deliver a higher quality product than that required by the customer to maximize profit This analysis makes an important, but counter-intuitive statement that the manufacturer may have to deliver *better* product than requested by the customer to maximize profit! Notice that the design with requirements in the range 0.35 to 0.5 inches are not even worth manufacturing, no matter what the customer demand is; at any of these tolerances the manufacturer is required to hold at least 0.35 inch tolerance in order to maximize profit. In particular, follow the arrows in Figure 8 where the customer required 0.5 inch tolerance, yet Pareto optimality demonstrates the customer and manufacturer will be most happy at a tolerance of 0.35 inches. Note that this analysis is based on the assumption that only a given set of machines is available. The results may be significantly different if the machining equipment is changed.
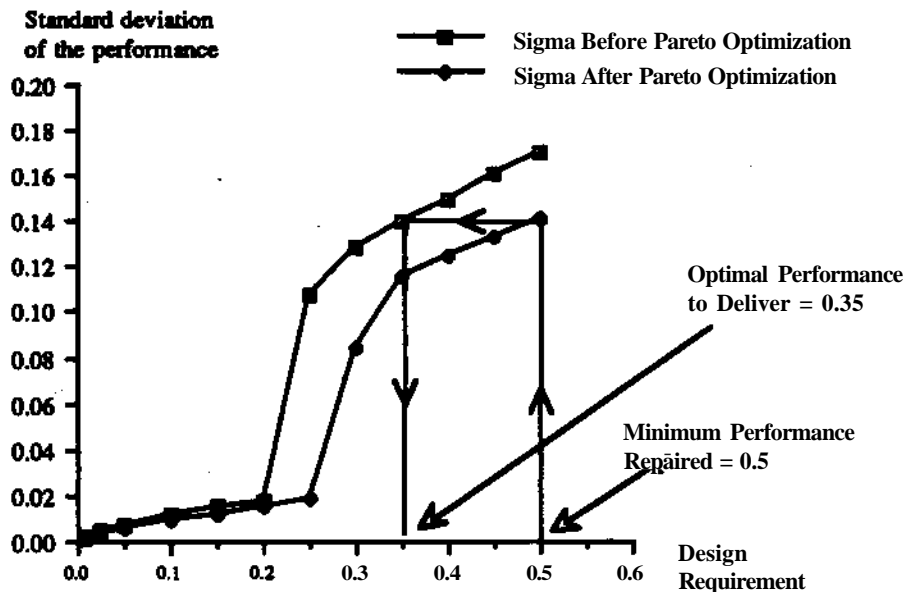

Figure 9: Computation of the optimal performance to deliver for a given requirement

9. Discussion

This research has established a relationship between the abstract concept of design quality and statistical tolerances, and developed a method for maximizing profit while also maximizing customer satisfaction. The solutions demonstrated in the friction wheel example indicate that companies should avoid designs involving a significant quality loss. When designs generating substantial quality losses have to be used, it may be profitable to deliver a product beyond the

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expectation **of the customer to reduce** the **quality** loss. This emphasizes the concept of *Design for Quality* in **the marketplace.** Contrary to industrial frustration with quality limitations, this work indicates **that improved quality can** significantly improve immediate and long-term profit.

The **method has been** applied to a simple example. The extension to a more complex product is possible provided that **a** design function can be defined. This not always easy, and some research is being done on automatic design function generation (Martino and Gabriele, 1989). The computation of the assembly performance distribution can also be more complex than in our example but can always be estimated by Monte Carlo methods.

This work can be extended to include other design variables such as choosing the physical dimensions of the parts. Often, the specific dimension of a part given by the designer is somewhat arbitrary. It could be that a slight modification in dimension could lead to a significant change in profit. By including the physical dimensions as design variables in this analysis, implications of such design decisions could be formally analyzed. Other design criteria besides accuracy could also be considered. Examples are surface finish, material hardness, surface cleanliness.

As a design reasoning tool, a significant amount of knowledge must be assembled to **make** the decisions illustrated in this paper. For example, to determine the hyperbolic cost functions from Cagan and Kurfess (1991) used in our example, the feeds, speeds, and costs of the material, the cost of machine capital, and the cost of labor all had to be considered to determine the manufacturing time and cost of the product. The current analysis is performed on a spread sheet, but more complicated problems (such as the design of a gear train or VLSI chip) could impose a large amount of knowledge better suited in a rule-based environment

Although we have emphasized the use of statistical tolerances, conventional tolerances may be beneficial to the manufacturer as well. Conventional tolerances can be determined with our method by including additional variables corresponding to inspection procedures via the truncation of the Gaussian distributions.

10. Conclusions

A decision analytic method has been presented to reason about product quality, manufacturing processes, statistical tolerances and corporate profit. Quality is quantified with the quality loss function, and manufacturing processes are quantified as functions of cost versus machine accuracy. A Pareto optimization analysis is then performed to determine the design that *both* maximizes customer satisfaction and corporate profits. Results are important as they allow

for an **a** *priori* estimation of a product quality, based only on its design specifications. This technique **can readily** incorporate additional design variables for a more complete design tool.

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