THE STRATEGIC PETROLEUM RESERVE:
HOW LARGE SHOULD IT BE?

by

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Abstract

This paper is the final report on a study carried out for the U.S. Department of Energy, concerning the optimal size of the Strategic Petroleum Reserve (SPR). The purpose of the SPR is to diminish U.S. vulnerability to, as well as to offer protection against, possible future oil embargoes. In this study we formulate the problem of determining the optimal size of the SPR as a parametric bimatrix game between the U.S. and its potential opponent. The strategies of the opponent are embargoes of various intensities and lengths, including of course the no embargo option. The strategies of the U.S. are various ways of using the reserve. The size of the reserve itself is a parameter present in both payoff functions. Solving the game for the relevant reserve sizes yields interesting conclusions on the desirable size of the reserve, as well as on U.S. drawdown policies in case of an embargo. The crucial element in the game-theoretic approach is that, unlike the traditional cost-benefit analysis, it fully captures the embargo-deterrent effect of an appropriate SPR. We have solved the game for over 100 sets of assumptions organized into scenarios, and have derived qualitative and quantitative conclusions concerning the appropriate size of the SPR and related matters.

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Summary

In the wake of the 1973-1974 oil embargo, the U.S. Congress has legislated the creation of a Strategic Petroleum Reserve (SPR), meant to reduce the nation's vulnerability to similar supply interruptions. This study addresses the question about the appropriate overall size of the Reserve. On the one hand, it seeks answers to this and related questions under specific assumptions; on the other, it develops an analytical tool, implemented in a computer program, and available for further studies under different assumptions. The program runs on the DOE computer.

The purpose of the SPR is to reduce the vulnerability of the U.S. to petroleum supply interruptions. It can do this by absorbing some or all of the petroleum shortfall created by a supply interruption, but also by deterring an embargo through its mere existence. Earlier studies concerning the appropriate size of the SPR have focused on the loss-absorbent aspect of the reserve, and have not found adequate means of quantifying the embargo-deterrent aspect. This study uses a game theoretic model to analyze the optimal SPR size from the point of view of loss-absorption as well as embargo-deterrence.

Game theory is a mathematical discipline whose object is the analysis of competitive situations, conflicts, and related phenomena. Since embargoes obviously belong to this class, game theory is the most relevant analytical tool for our problem.

The problem of the optimal overall size of the SPR is formulated as a bimatrix game between a potential embargoer ("the Cartel") and the U.S., in which both sides pursue their objectives expressed in the form of two

payoff functions. We assume that periodically a situation arises, in which the Cartel wants to force upon the U.S. some political action that the U.S. is unwilling to take. We call such a situation a conflict. In pursuit of its political goal, the Cartel's objective in the game is to inflict maximum damage upon the U.S. quickly (damage caused later is of lesser "value"), with a minimum loss of its own. The U.S. objective is to minimize the total cost of the potential conflict to the U.S., consisting of the cost of the SPR, and the GNP loss caused by the Cartel's action. The game is nonzero-sum, i.e., one player's gain differs from the other's loss.

The parties to the game pursue their goals by a number of strategies available to them. The Cartel's strategies are embargoes of varying intensity and length; while the U.S. strategies are the various drawdown policies. The size of the SPR itself is a parameter in both payoff functions. Solving the game consists of finding an equilibrium point, i.e., a pair of strategies optimal for each player under the assumption that the other plays the game optimally for himself.

The way to use this model for finding the optimal size of the SPR is to solve the game for all relevant values of the SPR size, and find the value that minimizes the total cost of a conflict to the U.S. However, since formulating the player's payoff functions and strategies involves a number of specific assumptions, solving the game for one set of assumptions is interesting but not sufficient. In order to get more reliable answers, it is necessary to perform some sensitivity analysis, i.e., to solve the game (for all relevant SPR sizes) for different sets of assumptions that seen reasonable, and examine the effects on the outcome of changes in the assumptions.

We have solved the game for over 100 sets of assumptions organized into scenarios. Apart from the size of the SPR and its filling schedule, the main parameters defining a scenario are: the level of U.S. oil imports from the Cartel at the time of the potential conflict; the level of the Cartel's total oil exports at that time; the level of U.S. daily petroleum demand; the sensitivity of the U.S. GNP to a cutback in petroleum; the Cartel's sense of urgency in the pursuit of its goal; the Cartel's sensitivity to its own losses; the U.S. capacity of economic retaliation. Our main findings can be summarized as follows.

For small SPR sizes the optimal strategy for the Cartel in case of a conflict is to impose the most intensive type of embargo made possible by the level of U.S. imports from the Cartel, either for a full year or for 9 months, depending on certain assumptions about the Cartel's payoff function. In all the scenarios, there is a critical SPR size for which the "no embargo" strategy becomes optimal for the Cartel. This is the reserve size required to deter an embargo of the maximum intensity made possible by the U.S. import level. It is also the reserve size that minimizes the total cost of the conflict to the U.S., hence the optimal reserve size.

The reason for the above outcome is that, as the SPR size increases, the damage inflicted upon the U.S. economy by an embargo decreases; furthermore, it takes longer and longer for this damage to materialize, i.e., the larger the reserve, the later the U.S. "feels the pinch" of an embargo, if it does so at all. Since the utility to the Cartel of a damage inflicted upon the U.S. decreases with time, while the Cartel's sensitivity to its own loss increases, there is a point beyond which the embargo loses its justification.

The total cost to the U.S. of a conflict with the Cartel, viewed as a function of the SPR size, decreases monotonically as the reserve increases from 0, as a result of the loss-absorbing function of the SPR. Of the two components of this cost, the GNP loss is by far the more important one, exceeding several times the cost of the reserve for every SPR size up to the critical one. At the critical value of the SPR size, the total cost suddenly drops to the level of the cost of the reserve, as a result of the embargo-deterrent function of the SPR. Beyond the critical reserve size, the total cost to the U.S. increases slowly, as the reserve increases. The shape of the U.S. cost function is shown for 4 different scenarios in Figures 4, 5, 6 and 7, on pages 51 and 52.

The total benefit to the U.S. from the SPR, measured as the difference between the cost to the U.S. of a conflict with the Cartel without a reserve and with the SPR, increases with the size of the SPR up to the critical reserve size, then it decreases. The marginal benefit, i.e., the benefit obtained from increasing the SPR by 1 unit, is large for a small reserve, then decreases as the reserve increases, up to a point close to the critical value. Just before the SPR attains its critical size, the marginal benefit jumps to its highest value over the whole range of reserve sizes, corresponding to the switch in the Cartel's optimal strategy from a heavy embargo to "no embargo.[11] Beyond the critical value, the marginal benefit is a small negative constant. The U.S. marginal benefit corresponding to the most relevant SPR sizes for one of the more probable scenarios is presented in Table 25 and Figure 8 on pages 57 and 56 respectively.

The critical size of the SPR depends on several of the parameters defining the scenarios, first and foremost on the level of U.S. oil imports from the Cartel at the time of a potential conflict. Expressing the critical reserve size as a function of this import level, we conclude that within the range of realistic values for the other parameters, the critical SFR size is the equivalent of about 8-10 months[1] of U.S. imports from the Cartel at the time of the crisis. This suggests the following conclusion concerning the appropriate reserve size for various U.S. import levels:

U.S. Oil Imports from the Cartel (MMB/day)	Appropriate Size of SPR QMB)
3-3.5	750-1050
3.5-4	840-1200
4-4.5	960-1350
4.5-5	1080-1500

In addition to showing the optimal SPR size from the point of view of both the loss-absorbent and the embargo-deterrent functions of the SPR, the game theoretic model also provides a separate assessment of the value of these two functions, i.e., of the absorption value and the deterrent value of the SPR. The point of this distinction is the discovery that while the (loss-)absorption value of the reserve shows up as a direct benefit whenever there is an embargo, the deterrent value gets translated into benefits only if the SPR is at least equal to the critical size corresponding to the U.S. level of imports from the Cartel; otherwise it remains latent. This circumstance helps explain the surge in marginal benefits in the neighborhood of the critical SPR size: at that point the deterrent value, latent for smaller reserve sizes, suddenly gets translated into benefits.

1. Background

From mid-October 1973 till mid-March 1974, the U.S. was the object of an oil embargo on the part of the OAPEC (Organization of Arab Petroleum Exporting Countries, a grouping within OPEC). Right before the embargo, U.S. petroleum consumption was about 17 MMB/day (million barrels per day), of which about 6.2 MMB/day were being covered by imports. Roughly 227. of these imports, or about 1.4 **CB/day, were coining from the OAPEC countries. The embargo was accompanied by a substantial cutback of oil production in the OAPEC countries, and a quadrupling of world oil prices. All this had a most severe impact on the U.S. and world economy. In the U.S., the GNP loss directly ascribed to the embargo was officially estimated to have been between $10-20 billion [1, Appendix A V, p. 288], though other estimates run much higher.

The embargo has alerted U.S. policy makers and the public at large to the vulnerability that our high degree of dependence on oil imports entails. As a result, in December 1975, Congress passed the Energy Policy and Conservation Act (Public Law 94-163), which provided for the creation of a Strategic Petroleum Reserve, meant to reduce the vulnerability of the U.S. to petroleum supply interruptions and to alleviate the impact of such disruptions, should they occur. The law called for an Early Storage Reserve of at least 150 MMB to be put in place by December 1978, and for a total Strategic Petroleum Reserve of up to 1,000 MMB. It further specified that by December 1982 the reserve should reach a level corresponding to the volume of imports during the 3 consecutive highest import months of 1974-1975, subsequently determined to be about 500 *MB.

The Strategic Petroleum Reserve Plan [3] submitted to Congress in December 1976 recommended an overall size of 500 MMB for the reserve. One of the early decisions of the Carter Administration was to augment the size of the SPR to 1000 *MB. A recommendation to this effect was submitted by the Department of Energy [4] and approved by Congress in 1978. However, current budgetary allocations provide oaly for a reserve of 750 MMB, to be put in place by 1985.

At the end of 1978, five years after the oil embargo, U.S. dependence on petroleum imports has considerably increased: of the total petroleum consumption of 19 MMB/day, about 8 MMB/day were covered by imports. Furthermore, 3.2 MMB/day or roughly 40% of total imports were coming from the OAPEC countries, and another 0.9 MMB/day originated in Iran. On the other hand, the SFR had only 60 MMB of oil in place, instead of the 150 MMB required by the Energy Policy and Conservation Act.

U.S. dependence on imported oil, and in particular, Middle Eastern oil, is not a short term phenomenon. Conservation measures and various policies designed to replace oil by coal where possible, and to increase the efficiency of oil usage by motor cars, can certainly mitigate the growth of energy demand and, to a limited extent, even reduce petroleum consumption. Also, a successful policy of incentives can lead to an increase, within certain limits, of domestic oil production. However, the possibilities in either of these directions are too limited to be able to bring supply and demand into equilibrium without oil imports.

A rapid development of massive oil production in Mexico seems technically feasible and could possibly make North America self-sufficient in oil for several decades, but this is currently not the policy of the Mexican Government; and though a change of policy could possibly be induced by a sufficiently attractive and imaginative U.S. initiative, some sort of Marshall Plan for Mexico, nothing of the kind seems to be contemplated at the moment.

In the longer run, oil supplies will be supplemented by liquefied coal. A few years ago this seemed to be the music of a distant future, since at 30-35$/barrel the price of liquefied coal was several times higher than that of oil. But with the world price of oil at $18-20, the ratio is only 3:2, and the day when liquefied coal becomes commercially feasible seems much closer than it used to. However, lead times in this area are such that massive use of liquefied coal in the next 8-10 years is most unlikely.

Thus, massive oil imports from the Middle East are likely to continue over the next decade. The vulnerability of the U.S., far from having decreased since the last embargo, is considerably higher today than it was five years ago. The recent upheaval in Iran, which brought to power forces closely allied with the radical Arab regimes, as well as the success of the radicals in isolating Egypt in the wake of the conclusion of its peace treaty with Israel, show that another oil embargo against the U.S. is far from impossible; and that if it comes, Iran might join it.

Thus the need for an adequate Strategic Petroleum Reserve is more acute today than it ever was. While a petroleum stockpile cannot make the U.S. independent of oil imports, it can substantially reduce the vulnerability of the nation to sudden supply interruptions of the type experienced during the winter of 1973-1974.

There are two ways in which an adequate petroleum stockpile can reduce U.S. vulnerability. An oil embargo is not a common commercial practice; it is a tool of political blackmail, meant to force those at whom it is aimed, into some action they would otherwise not be willing to take. It is economic warfare, which also entails losses and carries risks for those who undertake it. Further, to be effective, it requires closely coordinated joint action on the part of many parties with diverging needs and often conflicting interests. For all these and other reasons, an oil embargo can only be imposed under circumstances of acute conflict, as a result of some international crisis, and only for a limited length of time. Should such an acute conflict arise, and should an embargo against the U.S. be contemplated, the existence of a sizeable petroleum stockpile in the U.S. would certainly diminish, if not abolish, the chances of success of a blackmailing attempt, and would therefore make the embargo a much less attractive policy option. This we will call the (embargo*)deterrent aspect of the SPR. Should an embargo nevertheless be imposed, the existence of a large enough petroleum stockpile would enable the U.S. to absorb the oil shortfall due to the embargo by drawing on the reserve for a considerable length of time. This we will call the (loss-)absorbent aspect of the SPR.

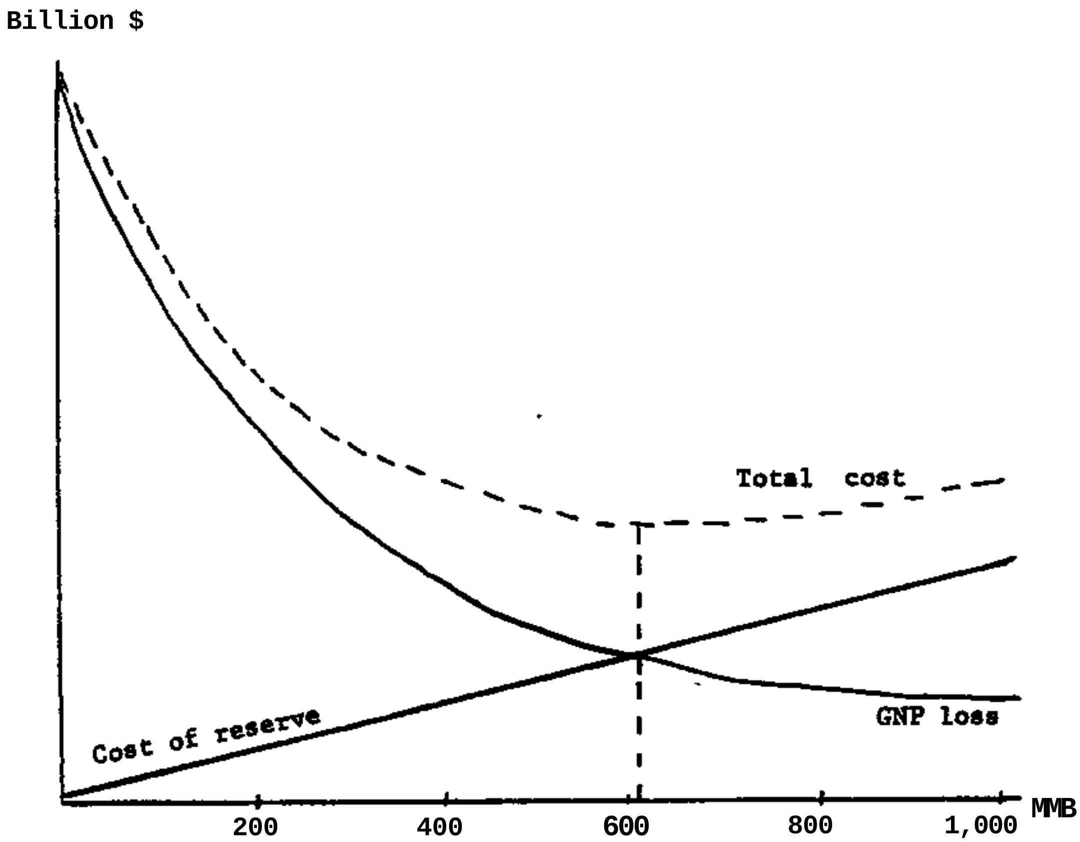
2. Criteria for Choosing the Overall Size of SPR

What is the most appropriate overall size for a U.S. Strategic Petroleum Reserve? One way of answering this question is to perform a cost-benefit analysis of various reserve sizes under assumptions organized into scenarios for possible future embargoes.

If we assume that an embargo will take place some time during the next 15 years, a good measure of the efficiency of the SPR is the total cost to the U.S. of the embargo, as a function of the reserve size. This total cost can be measured by the sum of (a) the cost of the reserve, and (b) the GNP loss caused by the petroleum shortfall that the reserve cannot replace. The cost of the reserve is an increasing linear function of the size of the reserve, whereas the GNP loss can be approximated by a quadratic function of the percentage petroleum shortfall, where the latter decreases linearly with the reserve size. As a result, the total cost is a convex function of the reserve size, which has a minimum. Figure 1 illustrates a case where the miirfimmi total cost occurs for a SPR of about 600 MMB.

This approach gives a perfectly correct assessment of the optimal reserve size if indeed an embargo of the assumed type occurs at the assumed time, and no other one occurs. These assumptions, however, are highly arbitrary. To mitigate the effects of their arbitrariness, one can replace

Fig. 1. Cost of hypothetical embargo as a function of SPR size.



them with a more complex set of assumptions on the probability of various types of embargoes. Constructing scenarios for all contingencies deemed possible and assigning a probability to each scenario, one can weight the optimal reserve size for each scenario with the presumed probability of its occurrence, and obtain this way an optimal reserve size corresponding to these expectations.

A cost-benefit analysis of this type is certainly relevant, and can yield valuable information. It has, however, two weaknesses.

The first one is that the outcome of the analysis depends heavily on the probabilities assigned to the various scenarios, and there is no objective basis for the assessment of these probabilities. Thus the results necessarily reflect to a very high degree the biases of the analyst. This inconvenience can partly be eliminated if one replaces the objective of finding an optimal reserve size, by that of determining the "break-even probability" of an embargo, corresponding to each reserve size; i.e., that probability of an embargo, for which the cost of a reserve of a given size equals the GNP loss from an embargo, times the probability of the embargo. The outcome of such an analysis is less dependent on assumptions, but also less conclusive with respect to what is a desirable reserve size.

The second, and in our view, main limitation of the cost-benefit analysis outlined above is the fact that it treats the probability of an embargo as independent of the size of the U.S. reserve. As mentioned before, an essential feature of the SFR is its embargo-deterrent aspect: the larger the U.S. reserve, the smaller the chances that the oil blackmail might work, hence the lower the probability of an embargo. This aspect is not captured by the above cost-benefit analysis, which addresses only the loss-absorbing aspect of the reserve. Thus, a reserve size that is shown optimal by the cost-benefit analysis, which ignores the effect of the reserve on the probability of an embargo, may be far from optimal from a point of view which takes into account this effect.

The purpose of this study is to develop a proper analytical tool for determining the optimal overall size of the SPR, viewed under both of its aspects, as a loss-absorbent and an embargo-deterrent. Since the loss-absorbent feature has been adequately studied earlier, this study makes full use of those earlier results; but adds to them a new dimension, by incorporating the deterrent aspect as a centerpiece of the analysis.

3. A Game-Theoretic Model

Since any embargo is the result of a conflict, and since conflict situations are best analyzed by the mathematical discipline called game theory, the proper analytical tool that suggests itself for determining the optimal size of the SPR, is a game theoretic model, namely a parametric bimatrix game. Such an approach, unlike the traditional cost-benefit analysis, addresses simultaneously both aspects of the SPR, i.e., its embargo-deterrent as well as its loss-absorbent aspects.. Rather than assuming the occurrence of an embargo to be an event independent of the existence and size of the SPR, this approach treats the embargoes of various lengths and intensities as possible outcomes of the game, whose probability of occurrence in a conflict is a function of, among other things, the size of the U.S. petroleum reserve.

In a bimatrix game there are two players, each of whom has several strategies at his disposal. The objectives pursued by the two players are expressed by their respective payoff functions, given by two m x n matrices,

$A = (a_{ij})$ and $B = (b_{ij})$. The interpretation of the latter is that if player 1 uses strategy i and player 2 uses strategy j, then player 1 gains a_{ij} units and player 2 gains b_{ij} units (here "gain" means loss whenever a_{ij} or b_{ij} is negative). When $A \neq -B$, like in our case, the game is nonzero-sum, i.e. one player's loss is not the other's gain. If each player uses a single strategy, we say that the players have used pure strategies. A mixed strategy x for player 1 is a nonnegative m-vector whose components sum to one, and whose interpretation is that player 1 uses strategy i with probability x_i. A mixed strategy y for player 2 is a nonnegative n-vector defined and interpreted in an analogous fashion. Pure strategies are special cases of mixed strategies. An equilibrium point is a pair of mixed strategies $(\bar{x},\bar{y})$, such that $\bar{x}A\bar{y} \geq xA\bar{y}$ for all strategies x available to player 1, and $\bar{x}B\bar{y} \geq \bar{x}By$ for all strategies y available to player 2. The essential characteristic of an equilibrium point is that it maximizes each player's gain (minimizes each player's loss), under the assumption that his opponent plays the game in a best (for himself) possible way. Every bimatrix game has a solution. For a good non-technical discussion of the basic ideas of game theory see [8].

To formulate the problem of the optimal size of the SPR as a parametric bimatrix game, we assume, based on the experience of the past 30 years, that a crisis in the Middle East arises periodically, say every 7-8 years. A crisis, or conflict, is defined as a situation in which the OAPEC, or some other group of countries, wish to pressure the U.S. into some action alien to its national goals. Such a situation may, but need not, lead to an embargo. Note that this is a considerably weaker assumption than the assigning of a probability to an embargo. For each conflict scenario, we formulate a bimatrix game in which the two players are the potential embargoer, briefly called the Cartel, and the U.S. The U.S. has a petroleum stockpile whose size is a parameter in the game, i.e., we wish to solve the game for all relevant stockpile sizes, and determine that range of stockpile sizes, for which the solutions are most convenient. The strategies available to the Cartel are the embargoes of varying length and intensity, including, of course, the "no embargo" option. The strategies available to the U.S. are the various possible drawdown policies (uniform, exponential with different rates, etc.).

The objective of the U.S. is to minimize the total cost, i.e., the sum of the GNP loss caused by the Cartel's action, and the cost of the stockpile over the whole period considered. Hence, the U.S. payoff function (to be maximized) is the negative of this sum. The objective of the Cartel in a crisis is to pressure the U.S. into some action. The pressure exerted increases with the damage caused to the U.S. On the other hand, actions which put pressure on the U.S. also carry some costs for the Cartel (loss of imports, risk of retaliation, etc.). The payoff function of the Cartel is therefore increasing in the damage caused to the U.S., and decreasing in the Cartel's own loss. The model that we are using is thus not a zero-sum game, in which one player's loss is the other player's gain; but a more general bimatrix game, in which the two players' payoff functions are interrelated in a more complex fashion.

A more detailed description of the model is given in the next three sections.

The goal of the game theoretic model is to analyze various conflict scenarios with a view of answering questions like these:

- How does the size of the SPR affect the optimal policies of the Cartel?

- Does an increase of the SPR create an incentive or a disincentive for a heavy, long embargo?

- Can a suitably large SPR deter an embargo, and how large need it be to achieve this?

- What is the optimal SPR size when the embargo-deterrent aspect is considered together with the loss absorbent aspect, and what are the factors affecting it?

Game theoretic models, like all analytical models assuming some kind of rational behavior, are often subjected to the criticism that in real life people may act irrationally. While this is certainly a limitation on the relevance of all such models, there is little evidence to suggest that the Cartel's actions are irrational. Shortsighted they may often be, but that is a different issue. In the light of the enormous profits realized by the Cartel as a result of the recent quadrupling of oil prices, brought about in no small part by the 1973-1974 embargo, one can hardly argue convincingly that the embargo was irrational.

4. Strategies Considered

The actions that the U.S. can take in connection with an embargo-prone conflict are of two kinds: actions to be taken in advance of the crisis, and those that can be taken during the crisis. These two kinds of actions are handled by the model in two different ways.

Among the actions of the first kind, the most important ones are those concerning the level of U.S. petroleum imports from the Cartel, and the size of the SPR. The outcome of these actions, in the form of a given level of imports and a given reserve size at the time of the crisis, are handled exogenously, as input data for various scenarios. In other words, in any given run of the game these data are fixed - reflecting the actual situation that they are the results of earlier decisions and do not represent options available to the U.S. at the time of a crisis.

The options available to the U.S., i.e., actions the U.S. can take during a crisis, have to do first of all with the way the reserve is used. The various drawdown policies that might be used during an embargo constitute the U.S. strategies in the game. We consider uniform drawdown policies, in which the reserve is used at a uniform rate, to replace the oil withheld by the Cartel either in its entirety, or diminished by a "conservation rate;" and exponential drawdown policies, in which the shortfall is replaced as above only until the reserve shrinks to the equivalent of 5 day's needs (where 6 is an input parameter), after which every day 1/5 of the remaining reserve is drawn down.

Other possible U.S. actions taken during an embargo, like various measures of economic retaliation, are considered via their effect on the Cartel's payoff function, and will be discussed in the next section.

The possibility of U.S. military action is not considered in this model.

The strategies available to the Cartel in a crisis are the oil embargoes of varying length and intensity, including the "no embargo" option. We consider embargoes directed solely against the U.S., as well as embargoes directed against the U.S. and the other members of the ISA. (Western Europe and Japan). Each embargo decision is accompanied by a corresponding production cutback decision. The embargoes considered by the model are specified by three input parameters: the percentage oil export cutback of the Cartel, the daily cutback in U.S. imports as a result of the embargo, and the length of the embargo. The model can also accommodate quarterly changes in the intensity of an embargo.

Oil export cutbacks of up to 50% on the part of the Cartel are considered. The daily cutback in U.S. imports is essentially limited by the level of U.S. imports from the Cartel at the time of a crisis; though in case of a total embargo against all the IEA countries one also has to (and the model is able to) take into account the possibility that the U.S. may lose some imports from non-Cartel countries in favor of its IEA partners who are importing a higher proportion of their petroleum. Naturally, when the U.S. alone is embargoed, the corresponding scenarios reflect the favorable effect of the IEA agreement, under which our partners are supposed to share with us a certain proportion of their imports.

As to the duration of the embargoes considered, the model can accommodate scenarios involving embargoes of any length up to 2 years. However, the scenarios for which the game was run, all involve embargoes of 6, 9 and 12 months' length.

5. The U.S. Payoff Function

The objective of the U.S. is to minimize the total cost K of a given conflict scenario. This total cost is of the form K = C + L, where C is the cost of the reserve, and L is the GNP loss.

The cost of the reserve, C, is the cost of creating the storage facilities, plus the opportunity cost of the capital invested in the reserves, estimated as follows. Each scenario contains a filling schedule for the reserve, specifying the amounts added each year. For every year of the time period considered, the prescribed increase in the reserve is multiplied by the unit cost of creating the storage facilities. In addition, the opportunity cost of the capital invested in the oil that is in place in the given year, is calculated at a discount rate specified by the scenario. The sum of these two kinds of costs is then present valued and added up for all the years of the time period considered. If *y* is the unit cost of creating the storage facilities and rr the price of oil (both in constant dollars), p the discount rate, R. the reserve in place in year t, and t. the length of the period considered (in years), the cost of the reserve (for every pair of U.S. and Cartel strategies) is

(1) $$C = \sum_{t=1}^{t} [Y(R_* - R_{t-1}) + PTTRJ(1 + p)^{-t}$$

In this calculation every barrel of oil in the SPR is accounted for via its annual opportunity cost of pn, present valued and added up for all the years from the time it is stored to the end of the period t^.

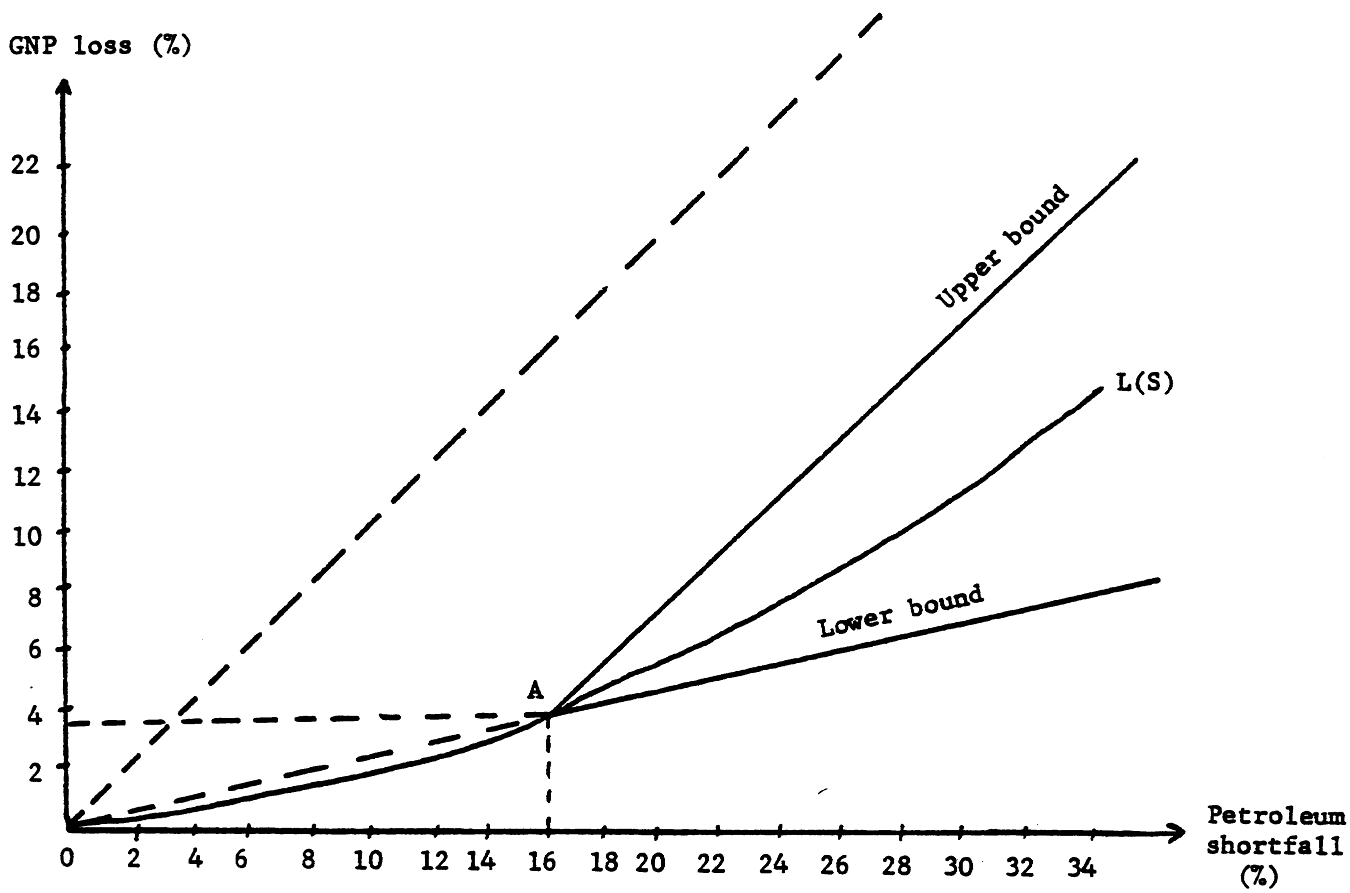
As for the GNP loss caused by an embargo, its estimation is a much less straightforward task. Several studies, using a variety of approaches, have proposed embargo loss estimators. The one used in our model (as well as in the Administration's SPR Plan) is due to R. G. Holcombe [5], whose loss estimate is conservative in comparison with most other studies. It is based on an analysis of the impact of the 1973-1974 embargo on the U.S. economy, via an 82 sector input-output model of that economy. It derives piecewise linear loss functions for each of the 82 sectors of the economy represented in the model, with lower and upper bounds on their slopes, then aggregates these into a GNP loss function that shows the percentage loss of GNP to be nearly quadratic in the percentage petroleum shortfall:

(2) $$100\,£ \cdot a\left(100\,§\right)^{1.91}.$$

Here D is the demand for petroleum and S the petroleum shortfall, expressed in the same units (say, MMB/day), and therefore 100 S/D is the petroleum shortfall in percent. Similarly, L is the GNP loss, expressed in the same units as GNP (say, $ billions), and therefore 100 L/GNP is the GNP loss in percent. Finally, the constant or = 0.01736 is a proportionality factor. The above functional relationship is shown in Fig. 2, where the point A corresponds to the 1973-1974 embargo. The lower bound obtained by passing a straight line from the origin through A corresponds to the assumption that the substitutions which were possible in 1973-1974 for a petroleum shortfall of 16%, could be increased proportionally for arbitrarily large shortfalls; whereas the upper bound, obtained by passing a 45^o line through A, represents the assumption that no substitution is possible at all, and therefore an incremental petroleum shortfall of 1% causes an incremental GNP loss of 1%.

Recently, the constant a in the GNP loss estimator has been reconsidered by DOE in the light of new information about the 1973-1974 embargo [6]. On the one hand, an analysis of the new data shows the actual petroleum shortfall during the embargo to have been considerably less than the 3 MMB/day assumed in [5]; on the other hand, a time lag analysis reveals a $1\frac{1}{2}$ month's lag between a change in petroleum supply and its impact on the economy. On the basis of these findings, it is recommended [6] to apply a correction factor to the constant *a* so as to increase its value by SOX. We represent this correction by replacing *a* in the above expression by a-j3, where 0 » 1 corresponds to the original Holcombe function, while the desired correction is obtained by setting p » 1.5. With this correction, Holcombe's loss estimator is still on the conservative side, in comparison with estimators arrived at by analyses using the Wharton and Data Research Institute econometric models.

Fig. 2. GNP loss as a function of petroleum shortfall



Unlike the cost of the reserve, which does not depend on the strategy choices of the players, the GNP loss suffered by the U.S. depends on the strategies selected by both the U.S. and the Cartel. If the Cartel's strategy i is an embargo involving a cutback in U.S. imports of c_i MMB of oil per day for k_j quarters, and if the U.S. strategy j involves a drawdown of d_{ij}^t MMB of oil on day t from the reserve, then the average daily petroleum shortfall in quarter k for the strategy pair (i,j) is

$$(3) \qquad S_{ij}^k = c_i - \frac{1}{90} \sum_{t=t_{k-1}+1}^{t_k} d_{ij}^t$$

where $t_k = 90k$, $k = 0,1,\dots,k_j$; and the (present valued) GNP loss in quarter k for the strategy pair (i,j) can be expressed, using (2) and (3), as

$$(4) \qquad L_{ij}^k = 10^{-2}\, a\theta\, (GNP/4)(100\, S_{ij}^k)^{\#}\, (1+p)^{-Y}$$

where p, as before, is the discount rate, and Y is the number of years after which the crisis occurs.

Thus the total coat to the U.S. of a given conflict scenario, if the two players choose the strategy pair (i,j), is

$$(5) \qquad K_{ij} = C + \sum_{k=1}^{k_j} L_{ij}^k$$

where k_j is the last quarter of the embargo specified by Cartel strategy j. The U.S. payoff matrix, that we denote by $P = (P_{ij})$, is the negative of the cost matrix whose elements are K_{ij}, i.e., $P_{ij} = -K_{ij}$ for all i, j.

6. The Cartel's Payoff Function

We denote the Cartel's payoff matrix by $F = (F_{ij})$ and we take it to be of the form $F = G - H$, where G is an increasing function of the GNP loss L inflicted upon the U.S., while H is an increasing function of the Cartel's own loss.

Since conflicts of the kind and gravity that might trigger an embargo usually involve fast-moving events, we conceive of an embargo as a blackmailing device whose objective is to obtain some action of considerable urgency. Therefore, while the function G used in our model increases with the GNP loss inflicted upon the U.S., it decreases with the length of the time over which the loss is spread. In other words, we assume that a GNP loss inflicted upon the U.S. in the first quarter of the embargo is of "full value" to the Cartel, a loss of the same magnitude inflicted in the second quarter is of somewhat lesser value, etc.; and losses suffered by the U.S. a year later are of little if any value.

On the other hand, the loss suffered by the Cartel as a consequence of a given petroleum export cutback (and of possible retaliatory action on the part of the U.S. and its allies) may be easy to bear at the beginning, in view of the sizeable surplus that some of the Cartel countries possess. As time goes by and the surplus is consumed, the loss becomes felt. It seems, therefore, reasonable to be conservative and assume that the function H starts at a level close to 0 and increases slowly with both the size of the percentage cutback in petroleum exports, and the length of time over which the cutback is imposed.

If L_{ij}^k denotes, as before, the U.S. GNP loss in the k^{th} quarter of the embargo when the strategy pair (i,j) is used, the function G corresponding to the strategy pair (i,j) is

(6) $$G_{ij} = \sum_{k=1}^{k^*} \lambda_k y_{ij}^k$$

where k^* is the last quarter of the embargo specified in the scenario, and the weights λ_k satisfy $\lambda_1 = 1$, $0 \le \lambda_k \le \lambda_{k-1}$, $k = 2,...,k^*$.

The function H is independent of the U.S. strategy. For the Cartel's i^{th} strategy, it is taken to be

(7) $$H_i = \frac{1}{8} Q \sum_{k=1}^{k^*} \mu_k \left(\frac{p_k}{50}\right)^2 (1+\rho)$$

where Q is the value of the Cartel's annual exports at the time of the crisis without an embargo, p_k is the Cartel's oil export cutback in percent (of the pre-embargo export level) during the k^{th} quarter of the embargo, ρ and γ are as in (4), and the weights μ_k satisfy $\mu_1 \ge \mu_k \ge 0$, $k = 2,...,k^*$. Thus for the largest oil export cutback considered (50%) and weights ($\mu_k = 1$, $k = 1,...,k^*$) one has $H_i = \frac{1}{2}Q$, i.e., the 50% export cutback translates into a 50% loss of export value. For smaller cutbacks the loss diminishes quadratically, while for smaller weighting factors it diminishes according to the choice of the latter.

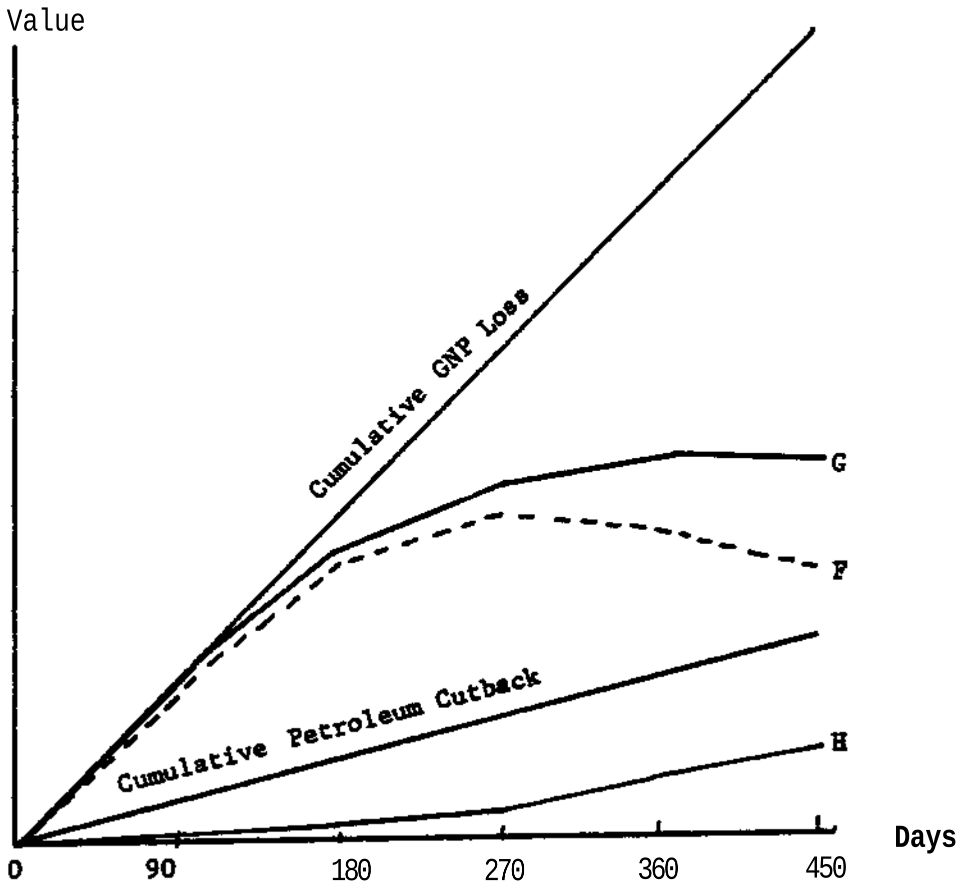
Thus the Cartel's payoff function for the strategy pair (i,j) is of the form

(8) $$F_{ij} = G_{ij} - H_i ,$$

where G_{ij} and H_i are given by (6) and (7) respectively.

The functions F, G and H are illustrated in Figure 3, which represents them as functions of time (number of days), for a given stockpile size and petroleum cutback (both the cutback in the Cartel's daily petroleum exports and the resulting daily GNP loss of the U.S. are assumed to be constant over time). The function G is equal to the cumulative GNP loss function in the first

Fig. 3. G. H and F as functions of time, for given SPR size and daily petroleum cutback.



quarter, then gradually approaches the horizontal line. The function H is almost horizontal in the first quarter, then gradually approaches a line parallel to the cumulative petroleum export cutback line. The resulting function F * G - H is first increasing, then decreasing; and its maximum corresponds to the optimal (for the Cartel) length of the embargo, given its assumed intensity and the size of the U.S. stockpile.

One can, of course, argue about the correct numerical values for the coefficients defining the functions 6, H, and hence F, but the general shape of these functions reflects the basic features of the type of conflict situation considered here.

By varying the weights X_k of the function G, one can represent situations corresponding to varying degrees of urgency in the crisis underlying the embargo. On the other hand, by varying the coefficients μ_k of the function H, one can represent various degrees of sensitivity or lack of sensitivity to economic losses on the part of the Cartel.

This latter point deserves some elaboration. It is often argued that some members of the Cartel have such enormous financial surpluses that they could easily withstand the loss of income from oil exports for a very long time. The truth of the matter is that while this applies to a few very oil-rich and low-population countries, like Saudi Arabia, Kuwait and Libya, other members of the Cartel whose participation is important for the embargo to have a chance of success, like Algeria, Iraq, Iran (a possible future, though not a past member), have large populations, no surpluses, and crucial import needs. Even if the surpluses of the sparsely populated oil-rich countries were pooled with the deficits of the others (not exactly a typical behavior pattern), such a joint monetary reserve would not last very long.

Besides, the sparsely populated oil-rich countries are themselves vulnerable to economic retaliation, in spite of their huge surpluses. The U.S. has a large number of on-going industrial projects in Saudi Arabia, whose interruption in retaliation for an embargo would cause substantial damage. The suspension of deliveries of food, arms, spare parts for military and other vehicles, the freezing or seizure of assets in the U.S. (currently more than 50 billion dollars), and other similar measures can certainly cause damage which is not negligible.

7. Conflict Scenarios

Having formulated a potential conflict between the Cartel and the U.S. as a bimatrix game, we then proceeded to solve the game for a number of relevant scenarios and values of the parameters defining the payoff functions.

Some of the data we used were the same for all the scenarios. Thus, the Cartel was taken to mean the group of OAPEC countries, plus Iran. Further, we assumed that conflicts of the kind that might lead to an embargo arise on the average once every 7 years. A sensitivity analysis to be discussed in the next section shows that replacing 7 years by 10 or even 20 years produces only a moderate shift in the outcomes. (The reasons for this are discussed in section 12.) We chose 1985 as the year of the potential conflict, not because an earlier conflict seems unlikely, but because that is the earliest date for which one can realistically assume that a reserve of the size to be analyzed is in place.

All our calculations were made in 1975 constant dollars, and everything was present valued to 1975 at a discount rate of 10%. The U.S. GOT in 1985 was taken to be 2,192.62 billion 1975 dollars, as forecast by the DRI econometric model (see [2], page B16). The cost of oil in 1985 was assumed to be $13 (in 1975 dollars), hence the opportunity cost of storing oil, calculated at the 107« discount rate, is $1.3/barrel/year. The cost of creating the storage facilities was taken to be $3/barrel. The U.S. demand for oil in 1985, at the above price, was assumed to be 20 ^WB/day.

For the rest, we used different input data for different scenarios. To avoid looking at an excessively large number of scenarios, we used only two sets of data for most parameters, but additional approximations can be obtained by interpolation or extrapolation, and of course additional runs can be made at any time on the DOE computer.

A crucial assumption in our model is the level of U.S. imports from the Cartel in 1985, since it sets a limit to the intensity of an embargo by the Cartel. We used two values for this parameter, namely 4.2 MlB/day and 5 MMB/day. In view of the steady increase of U.S. oil imports from the Middle East during the last few years, this is a rather conservative estimate. It should be remembered that the Cartel means the group of countries that might participate in an embargo against the U.S., and that under present circumstances that group includes Iran, besides the participants of the 1973 embargo.

The Cartel's total exports in 1985 were assumed to be 21 MKB/day in one set of scenarios, 24 MMB/day in another. For comparison, the 1977 level was 19 Mffi/day, This is a much less crucial assumption than the level of U.S. imports, and it only affects the size of the Cartel's own loss from an embargo.

The Cartel's strategies considered in each scenario were:

1. No embargo

2. An embargo directed only against the U.S., with a 25% cutback in exports, lasting 180 days.

3, 4. Same as 2, for 270 and 360 days respectively.

5. An embargo directed against the U.S. and its IEA partners (Western Europe and Japan), with a 50% export cutback, lasting 180 days.

6, 7. Same as 5, for 270 and 360 days respectively.

The export cutback is understood to mean a complete cessation of exports to the U.S., and a uniform cutback in exports to the IEA countries up to the level corresponding to the stated percentage cutback in total exports. Thus, a 25% export cutback corresponds to a total cutoff of exports to the U.S. and a very mild limitation of exports to the other IEA countries. Since under these circumstances the IEA countries would have to share some of their imports with the U.S., the actual U.S. shortfall was taken to be 2.6 MMB/day in case of the lower U.S. import level, and 2.8 MMB/day in case of the higher import level. On the other hand, a 50% export cutback on the part of the Cartel corresponds to a complete embargo of the U.S. and a very substantial cutback of exports to the IEA countries. Since most of the latter are importing a higher proportion of their needs than the U.S., we might have to share with them part of our remaining imports. Thus, for this situation the actual U.S. shortfall was taken to be 4.6 MMB/day in case of the lower U.S. import level, and 5.2 MMB/day in case of the higher import level.

The U.S. drawdown strategies are expressed in the model via the conservation rate and the drawdown parameter δ. The reserve is supposed to be drawn down at a uniform rate equal to the daily shortfall, less the amount to be conserved, until it reaches the level of δ days' requirements (at the above rate); after which every day $1/\delta$ of the remaining reserve is drawn down. Thus $\delta = 1$ corresponds to a uniform drawdown strategy, the other values of δ define a strategy consisting of uniform drawdown up to a certain point, and exponential drawdown afterwards. In all our scenarios the U.S. strategies were defined by drawdown parameters of 1, 45, 90, 135, and 180 (days). As to the conservation rate, in most scenarios it was taken to be 0, though we also looked at a few cases with conservation rates of 5% and 9%.

The U.S. payoff matrix P depends primarily on the GNP loss function. For the latter we used Holcombe's estimator as shown in (5), with the component 1.91 replaced by 2 in order to make the outcomes comparable to earlier calculations based on this approximation. Two values were considered for the correction factor β, namely 1.0 and 1.5. The Cartel's payoff matrix F depends on the weights λ_k and μ_k used with the functions G and H, as shown in (6) and (7). We used $\lambda = (1, .8, .5, .2)$ and $\lambda = (1, 1, .8, .5)$ as weighting factors in the expression for G, where the two cases differ in the rate at which the utility to the Cartel of the loss inflicted upon the U.S. declines with time (the 4 components of λ are multipliers of the quarterly GNP loss for successive quarters during which an embargo is imposed). As for the Cartel's loss function H, we used three different weighting factors, $v = (.25, .5, .75, 1)$, $v = (0, 0, .25, .5)$, and $v = (.5, .66, .83, 1)$. The first of these three vectors represents the assumption that the Cartel perceives its own loss of income from exports as only 1/4 of what it actually is in the first quarter, 1/2 in the second one, etc. The second vector expresses the assumption that the loss is completely ignored for two quarters, is perceived at 1/4 of its value in the third quarter, and at 1/2 of its value in the fourth quarter. Finally, the third vector corresponds to assuming a somewhat higher sensitivity of the Cartel to its own losses.

For each scenario, the game was solved for all relevant sizes of the SPR. Here relevant means from 0 to the reserve size for which the "no embargo" option becomes the optimal strategy for the Cartel. In each case, the reserve assumed to be in place in 1985 was supposed to have been built up according to a specific schedule. These filling schedules, shown in Table 1, were used in calculating the cost of the reserve. Apart from the values shown in the 1985 column of Table 1, the game had to be solved for intermediate values in the vicinity of points where a change in the reserve size implied a change in the optimal strategies.

8. Solving the Game

While in our choice of scenarios and parameter values we tried to cover those situations that seemed most relevant, one of the goals of our study was to create an analytical tool which can be used whenever the need arises to consider additional scenarios or parameter values. This tool now exists in the form of the computer program EMSA (Embargo Game) on the DOE computer. EMGA takes as input the data of a potential conflict between the Cartel and the U.S. and the values of the parameters defining the scenario, and provides as output

(a) the cost of the reserve C

(b) the U.S. GNP loss matrix L * (L_{ij})

(c) the Cartel's "gain" matrix G * (G_{ij})

(d) the Cartel's loss vector H » (H_i)

(e) the U.S. payoff matrix P = (P_{ij})

(f) the Cartel's payoff matrix F * (F_{ij})

(g) all equilibrium points in pure strategies.

The properties of the payoff matrices of this particular game happen to be such that for most values of the input parameters there is an equilibrium in pure strategies; i.e., there exists a pair of strategies

Table 1. SPR filling schedules used in the scenarios (MMB)

	1979	1980	1981	1982	1983	1984	1985
1	200	250	250	250	250	250	250
2	200	300	400	500	500	500	500
3						700	700
4						750	750
5						800	800
6	200	300	400	550	700		850
7							900
						850	
8							950
9							1000
10							1050
					700	900	
11							1100
12	200	300	400	550			1150
						950	
13					750		1200
14						1000	1250
15							1300
					850	1100	
16							1350
17							1400
						1150	
18	200	300	450	650			1450
19					900		1500
20						1200	1550
21							1600

that is optimal for both players in the sense that none of them can improve his position by a unilateral change of strategy. The relatively few cases where there exists no equilibrium in pure strategies, usually correspond to threshold values of some parameter; i.e., by slightly increasing the parameter one obtains an equilibrium in pure strategies, and by slightly decreasing it one obtains another equilibrium in pure strategies.

Nevertheless, the program is equipped to deal with situations where there is no equilibrium in pure strategies, in two different ways. First, it can be set to find all strategy pairs that would represent an equilibrium point if differences of magnitude between the elements of the payoff matrices, not exceeding a certain tolerance level, were ignored. The tolerance levels can be set separately for each of the two payoff matrices. This device of finding all pairs of strategies that are near-equilibrium points, or equilibrium points within a certain tolerance level, can also be used to test the sensitivity of the equilibria to changes in the data.

The second way the program deals with situations where there is no equilibrium in pure strategies, is to reformulate the game as a mixed integer program whose solutions yield the equilibrium points in mixed strategies. The mixed integer program itself has to be solved by an integer programming code (DOE's MFSX package has been used for this purpose). Solutions in mixed strategies can be interpreted as probabilities with which the various strategies would be used. While suck solutions often seem inconclusive when viewed by themselves, they usually make good sense when viewed as threshold situations between two solutions in pure strategies.

Next we show a typical scenario input and the corresponding outcome of the game.

Assume that a conflict arises in 1985, when the level of U.S. petroleum imports from the Cartel is 4.2 MMB/day, while the level of the

Cartel's total oil exports is 21 Mffl/day. Assume, further, that for the U.S. payoff function we use the correction factor P * 1.5, while for the Cartel's payoff function we use the parameters \ = (1, .8, .5, .2) and p, » (.25, .5, .75, 1). For all other parameters defining the scenario (discount rate, frequency of conflicts, U.S. GNP in 1985, U.S. demand for oil, price of oil, cost of storage facilities), assume the values specified in section 7.

The following strategies are assumed to be available to the Cartel:

1. No embargo

2. Embargo against the U.S., with a cutback of 25% on total petroleum exports, resulting in a shortfall in U.S. petroleum imports of 2.6 MMB/day, for 180 days.

3,4. Same as 2, for 270 and 360 days respectively.

5. Embargo against the U.S. (total) and the other IEA countries (partial), with a cutback of 50% on total petroleum exports, resulting in a shortfall in U.S. petroleum imports of 4.6 MMB/day, for 180 days.

6, 7. Same as 5, for 270 and 360 days respectively.

As to the U.S. strategies, we consider drawdown policies based on a conservation rate of 0 and drawdown parameters of 1, 45, 90, 135 and 180 (days). In other words, the following 5 strategies are considered:

1. Replace from the SPR the entire daily petroleum shortfall for as long as the reserve lasts.

2. Replace the daily shortfall up to the point where the reserve covers only 45 days[1] needs; after that, use every day 1/45 of the current reserve.

3. 4, 5. Same as 2, with drawdown parameters of 90, 135 and 180 (days) respectively.

Solving the game for a SFR of 400, 750 and 960 MMB yields the results shown in Tables 2, 3 and 4 respectively. These tables contain the payoff matrices with the equilibrium points corresponding to the optimal strategy pairs for each reserve size. A pair of indices (i,j), where i stands for a row and j for a column, represents an equilibrium pair of strategies, i.e., is an equilibrium point, if and only if the number in position (i,j) of the matrix F is a maximum in its column, and the number in the same position of the matrix -P is a minimum in its row. The additional output (which is not needed to interpret the results) is shown in Table 5 only for R = 960 *MSB.*

From Table 2 one sees that if the SPR has 400 MMB of oil, the optimal strategy for the Cartel, under the assumptions of this scenario, is 7, which is to impose the heaviest and longest embargo available as an option (50% total export cutback, producing a cutback of 4.6 MMB/day in U.S. petroleum imports for 360 days). The total cost to the U.S. of this scenario (in terms of GNP loss, plus the cost of the SPR) is 73.69 billion 1975 dollars (present valued for 1975).

In the presence of a SPR of 750 MMB (Table 3), the optimal strategy for the Cartel is 6, i.e., an embargo of the same heaviest kind, but for 270 days only. The cost to the U.S. is in this case $28.62 billion.

Finally, for a U*S. reserve of 960 MMB (Table 4), strategy 1 (no embargo) becomes optimal for the Cartel. In this case the U.S. loss reduces to the cost of the reserve, in this case 3.98 billion 1975 dollars (present valued to 1975). We chose the figure of 960 MMB since this is the smallest reserve size for which the "no embargo" policy is optimal for the Cartel.

In Table 3, if we ignore differences between entries of the same column of the matrix F not exceeding e_F = 0,08, and differences between entries of the same row

Table 2. Payoff Matrices for R -= 400 MMB

F jCCartel's payoff matrix)

U.S. strategies

Cartel's strategies	1	2	3	4	5
1	0	0	0	0	0
2	-0.82	0.13	1.10	1.70	2.58
3	3.30	3.04	3.28	3.53	4.23
4	3.88	3.57	3.63	3.72	4.30
5	19.10	14.02	14.27	17.34	20.68
6	29.62	23.99	22..54	24.56	27.49
7	30.33	24.68	22.92	^24Tsi]	27.28

C Negative of U.S. payoff matrix)

U.S. strategies

Cartel's strategies	1	2	3	4	5
1	2.41	2.41	2.41	2.41	2.41
2	3.18	3.81	4.95	5.69	6.57
3	12.29	11.39	11.17	11.16	11.73
4	21.39	20.28	19.16	18.33	18.31
5	30.95	24.32	23.85	26.78	30.20
6	59.46	51.72	47.87	48.68	51.30
7	87.97	80.08	74.69	73.69	75.19

MWB

Equilibrium point: (7,4)

Table 3. Payoff Matrices for R - 750 MMB

(F)

	1	2	3	4	5
1	0	0	0	0	0
2	-0.93	-0.93	-0.93	-0.92	-0.80
3	-1.87	-1.85	-1.60	-1.19	-0.78
4	-1.96	-2.04	-1.91	-1.51	-1.13
5	-2.93	-1.51	1.42	3.39	5.93
6	7.59	6.10	6.99	7.96	9.95
7	8.81	6.66	6.94	7.88	9.00

(-P)

	1	1	3	4	5
1	3.56	3.56	3.56	3.56	3.56
2	3.56	3.56	3.56	3.57	3.74
3	3.56	3.61	4.11	5.02	5.75
4	9.32	8.38	8.77	9.39	9.96
5	4.58	6.35	10.01	12.34	15.07
6	33.08	29.04	28.62	28.96	30.59
7	61.59	56.74	53.27	50.99	50.77

Equilibrium point; (6,3)

Near-Ecmilibrium point: (7,4) (e_F • 0,08, «$_p$ • 0.22)

Table 4. Payoff Matrices for R = 960 MMB

(F)

	1	2	3	4	5
1	[0]	[0]	[0]	0	0
2	-0.93	-0.93	-0.93	-0.93	-0.93
3	-1.87	-1.87	-1.87	-1.86	-1.74
4	-3.12	-3.09	-2.95	-2.75	-2.53
5	-3.74	-3.72	-2.96	-0.91	0.97
6	-0.86	-0.49	-0.10	1.71	3.45
7	-0.15	-0.22	-0.63	0.65	2.06

(-P)

	1	2	3	4	5
1	[3.98]	[3.98]	[3.98]	3.98	3.98
2	3.98	3.98	3.98	3.98	3.98
3	3.98	3.98	3.98	4.00	4.25
4	3.98	4.11	4.82	5.76	6.50
5	3.98	4.00	4.96	7.51	9.79
6	17.21	17.94	18.15	20.23	22.22
7	45.71	44.22	40.38	39.87	40.21

Equilibrium points: (1,1), (1,2), (1,3)

Table 5. Additional Output for R = 960

(L)

	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0.02	0.27
4	0	0.13	0.84	1.78	2.52
5	0	0.02	0.98	3.53	5.81
6	13.23	13.96	14.17	16.25	18.25
7	41.74	40.25	36.40	35.89	36.24

C = 3.98

(G)

	1	2	3	4	5
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0.01	0.14
4	0	0.03	0.17	0.36	0.58
5	0	0.02	0.78	2.83	4.71
6	6.61	6.99	7.38	9.19	10.92
7	12.32	12.24	11.83	13.11	14.52

(H)

1	0
2	0.93
3	1.87
4	3.12
5	3.74
6	7.48
7	12.46

of the matrix -P not exceeding $\epsilon_P = 0.22$, then the point (7,4) is also an equilibrium point. Such points will be called ε-equilibria or near-equilibria. In the case of a 750 MMB reserve, while strategy 6 is optimal for the Cartel, strategy 7 is near-optimal, within the above mentioned tolerance level.

As to the U.S. strategies, the optimum in each case is an exponential drawdown policy, with a drawdown parameter of 135 days (strategy 4) in the case of a SPR of 400 MMB, and of 90 days (strategy 3) in the case of a reserve of 750 MMB. The reason for the difference is that in the first case the Cartel's optimal strategy is a 360 days' embargo; whereas in the second case it is a 270 days' embargo. In the 750 MMB case, while the optimal U.S. strategy is 3, strategy 4 is near-optimal, with the tolerance level shown above.

9. The Outcomes and Their Interpretation

Before discussing and interpreting the results, we will present a synopsis of the outcomes for each scenario, as a function of the SPR size. Tables 6 and 7 contain the relevant outcome data for two typcial sets of scenarios, and they also explain how the subsequent set of more concise tables (8-23) was arrived at. In each table, I denotes the level of U.S. oil imports from the Cartel at the time of the conflict outbreak (1985) in MMB/day, E denotes the level of the Cartel's total oil exports in MMB/day, β the correction factor used in the U.S. GNP loss function (5), while $\lambda = (\lambda_k)$ and $\mu = (\mu_k)$ denote the weight-vectors used in the functions G and H respectively, as shown in (6) and (7). The parameters not listed on top of the tables are at their values shown in section 7.

Every column of Table 6 corresponds to the SPR size shown on top ot it. A column marked (i,j) at the bottom represents column j of the Cartel's payoff matrix F (top part) and of the negative of the U.S. payoff matrix,

-P (bottom part). In other words, having solved the game for each of the reserve sizes shown in Table 6, we then extracted from each pair of payoff matrices the pair of columns corresponding to the optimal U.S. strategy. In each column, the entry corresponding to the optimal Cartel strategy is enclosed in a box. Thus, for instance, the top part of the first column corresponding to the SPR size of 750 MMB is column 3 of the matrix F of Table 3, whereas the bottom part is column 3 of the matrix -P of the same table, since 3 is the optimal U.S. strategy. The optimal Cartel strategy is 6. In those cases where there is no equilibrium point in pure strategies, like for the "transitional" reserve sizes of 450 and 950 MMB, Table 6 contains the two pairs of columns corresponding to near-equilibium points. Similarly, in those cases where there is an equilibrium point, but with a relatively small tolerance there is also a near-equilibrium point, like for the reserve sizes of 750 and 930 MMB, the table contains both columns. The corresponding tolerances, which describe the "nearness" of the equilibrium, are shown at the bottom of the corresponding columns.

In reading Table 6 one must keep in mind that for most SPR sizes it contains only one column of the payoff matrices, and therefore comparing the entries of the column does not in itself always explain why the entry in the box is an equilibrium point. For instance, for R = 400, it seems that (6,4) should be an equilibrium point as well, since $F_{6,4} = F_{7,4} = 24.6$. However, a glance at Table 2 will show that $-P_{6,4} = 48.7$ is not a minimum in its row, hence (6,4) is not an equilibrium point.

Table 7 is analogous to 6, but describes the outcomes of a different set of scenarios.

Tables 8-23 are concise representations of the outcomes of 16 different sets of scenarios. They are extracted from more detailed tables, of which only

Table 6. Comparison of relevant columns of payoff matrices for various SPR sizes

I = 4.2 MMB/day, E - 21 Mffl/day, X - (1, .8, .5, .2), ji - (.25, .5, .75, 1), B = 1.5

R(MMB)	0	250	400	450		750		930	950		960	1000
1	0	0	0	0	0	0	0	0	0	[0]	[0]	[0]
2	15.5	4.1	1.7	1.0	0.3	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9
3	19.1	7.0	3.5	2.6	2.2	-1.6	-1.1	-1.9	-1.9	-1.9	-1.9	-1.9
4	19.7	7.4	3.7	2.7	2.5	-1.9	-1.5	-2.8	-2.9	-3.1	-2.9	-3.0
5	47.6	24.0	17.3	14.7	11.8	1.4	3.4	-2.6	-2.9	-3.7	-3.0	-3.3
6	58.1	33.1	24.6	21.5	[19.8]	[7.0]	8.0	[0.7]	[0.2]	-0.2	[-0.1]	-1.0
7	[58]	[33.6]	[24.6]	[21.4]	20.1	6.9	[illegible]	[0.2]	-0.4	0.1	-0.6	-1.7
1	0	1.6	2.4	2.6	2.6	3.6	3.6	4.0	4.0	[4.0]	[4.0]	[4.1]
2	18.2	7.7	5.7	5.0	4.2	3.6	3.6	4.0	4.0	4.0	4.0	4.1
3	27.3	15.3	11.2	10.1	9.8	4.1	5.0	4.0	4.0	4.0	4.0	4.1
4	36.5	23.8	18.3	17.0	17.5	8.8	9.4	5.3	4.9	4.2	4.8	4.4
5	57.0	33.6	26.8	24.0	21.3	10.0	12.3	5.3	5.1	4.0	5.6	4.6
6	85.5	59.2	48.7	45.2	[44.8]	[28.6]	29.0	[19.4]	[18.6]	18.5	[18.2]	16.6
7	[114.0]	[illegible]	[illegible]	[69.7]	71.4	53.3		[42.1]	40.9	44.9	40.3	33.2
	(7,3)	(7,3)	(7,4)	(7,4)* $\epsilon_F=0.08$ $\epsilon_p=0$	(6,3)* $e_p=0.35$ $\epsilon_p=0$	(6,3)	(7,4)* $\epsilon_F=0.58$ $e_p=0.22$	(6,3) (7,3)* $\epsilon_p=0.46$ $\epsilon_p=0.61$	(6,3)* $\epsilon_p=0$ $e_p=0.44$	(1,2)* y O.ll $e_p=0$	(1,3) (6,3)*	(1,3)

^Near-equilibrium points

Table 7. Comparison of relevant columns of payoff matrices for various SPR sizes

1 = 5 MMB/day, E = 24 MMB/day, X = (1, .8, .5, 2), p, - (.25, .5, .75, 1), p - 1.5

R(MMB)	0	250	500	750	1000		1100		1120	1250
1	0	0	0	0	0	0	0	[0]	0	0
2	17.9	5.4	1.0	-1.0	-1.0	-1.1	-1.0	-1.0	-1.1	-1.1
3	22.2	8.8	2.7	-0.8	-2.1	-2.1	-2.1	-2.1	-2.1	-2.1
4	22.9	9.3	2.9	-1.1	-3.0	-3.2	-3.5	-3.5	-3.6	-3.6
5	61.3	34.0	19.7	7.9	1.1	-1.9	-3.5	-3.5	-4.3	-4.3
6	75.3	46.9	29.0	15.2	5.4	[3.6]	0.5	0.5	-1.3	-6.6
7	[illegible]	[47.7]	[30.0]	[illegible]	[illegible]	3.8	[0.74]	0.4	-0.2	-5.0
1	0	1.6	2.8	3.6	4.0	4.0	4.2	[illegible]	0	[illegible]
2	21.1	9.4	5.3	3.7	4.0	4.0	4.2	4.2	4.3	4.6
3	31.7	18.2	11.0	6.1	4.1	4.0	4.2	4.2	4.3	4.6
4	42.3	28.1	18.9	11.8	6.6	5.6	4.5	4.5	4.3	4.6
5	72,9	45.5	30.7	18.4	10.7	7.0	5.3	5.3	4.3	4.6
6	109.8	78.7	57.9	41.4	28.9	[illegible]	21.7	21.7	17.7	8.5
7	[145.77]	[114.0]	[191.31]	[70.5]	[illegible]	56.3	[49.8]	49.8	54.1	44.0
	(7,3)	(7,3)	(7,4)	(7,4)	(7,4)*	(6,3)*	(7,3)*	(1,3)*	(1,1)	(1,1)
					$\epsilon_F=0$	$\epsilon_F=0.30$	$\epsilon_F=0.07$	$\epsilon_F=0.48$		
					$\epsilon_P=0.11$	$\epsilon_P=0$	$\epsilon_P=0.63$	$\epsilon_P=0$		

Tableft

I - 4.2, E = 21.0, P = 1•5, X - (1, .8, .5, .2), μ = (.25, .5, .75, 1)

SPR (MMB)	0	250	450	750	930	950	960	1000	1250
Equilibrium points	(7 : (7A)	(7,3)	(7,4)* (6,3)*	(6.3) (7,4)*	(6,3) (7,3)*	(1,2)* (6,3)*	(1,1) « ,3)	(1,D : (1 is)	(1,D (1,5)
Total cost to U.S. ($ billion 1975)	114.0	86.7	69.7* 44.8*	28.6* 51.0*	19.4 42.1*	4.0* 18.6*	4.0	4.1	4.6

*Near-equllibrlum points and associated costs

Table 9

I = 4.2, E « 21., p - 1.9 x - (1, .8, .5, .2), p, = (.25, .5, .75, 1)

SPR (MMB)	0	250	500	800	830	850	1000	1250
Equilibrium points	(6,1) • (6',5)	(6,3)	(6,3)	(6,3)	(1,2) (6.3)	(1,2) (1,3)	(1,1) (M)	(1,1) (1^5)
Total cost to U.S. ($ billion 1975)	57.0	40.0	28.9	18.5	3.8 17.4	3.8	4.1	4.6

Table 10

$I = 4.2,\ E = 21.,\ \rho = 1.5,\ \lambda = (1, .8, .5, .2),\ \mu = (0, 0, .25, .5)$

SFR (MMB)	0	250	500	750	1000	1250	1320	1350
Equilibrium points	(7,1) ⋮ (7,5)	(7,3)	(7,4)	(7,5)	(7,4)	(7,3)	(1,2) (2,2) ⋮ (5,2)	(1,1)(2,1)(5,1) ⋮ (1,3)(2,3)(5,3)
Total cost to U.S. ($ billion 1975)	114.0	86.7	66.1	50.8	37.8	25.0	4.9	5.0

Table 11

$I = 4.2,\ E = 21.,\ \rho = 1.0,\ \lambda = (1, .8, .5, .2),\ \mu = (0, 0, .25, .5)$

SPR (MMB)	0	250	500	750	1000	1150	1200	1250
Equilibrium points	(7,1) ⋮ (7,5)	(7,3)	(7,4)	(7,5)	(7,4)	(7,3)	(1,2) (2,2) ⋮ (5,2)	(1,1)(2,1)(5,1) ⋮ (1,3)(2,3)(5,3)
Total cost to U.S. ($ billion 1975)	76.0	58.3	45.0	35.0	26.5	21.6	4.5	4.6

Table 12

I - 4.2, E = 21.0, β = 1.5, X - (1, .8, .5, .2), \i - (.5, .66, .83, 1)

SPR (MMB)	0	250	500	750	870	880	1000	1250
Equilibrium points	(7,1) ⋮ (7,5)	(7,3)	(7,4)* (6,3)*	(6,3)	(6,3)	(1,2) (1,3)	(1,1) ⋮	(1,1) ⋮ (M)
Total cost to U.S. ($ billion 1975)	114.0	86.7	66.1* 41.9*	28.6	22.2	3.9	4.1	4.6

*Near-equilibrium points and associated costs

Table 13

I - 4.2, E « 21.0, 0 - 1.0, \ « (1, .8, .5, .2), p « (.5, .66, .83, 1)

SPR (MfB)	0	250	500	700	750	1000	1250
Equilibrium points	(6,1) (6J5)	(6,3)	(6,3)	(6,4)	(1,2) (1,3)	(1,1) ⋮ (0)	(1,1) ⋮ (1^5)
Total cost to U.S. ($ billion 1975)	57.0	40.0	28.9	22.0	3.6	4.1	4.6

Table 14

I = 4.2, E - 21.0, β = 1.5, X - (1, 1, .8, .5), p = (.25, .5, .75, 1)

SFR (MMB)	0	250	500	750	1000	1150	1200	1250
Equilibrium points	(7,1) • (7',5)	(7,3)	(7,4)	(7,5)	(7,4)	(7,3)	(7,3) (1,2)	(1,2) (1,3)
Total cost to U.S. ($ billion 1975)	114.0	86.7	66.1	50.8	37.8	30.3	27.5 4.5	4.6

Table 15

I = 4.2, E - 21.0, β - 1.0, A - (1, 1, .8, .5), \i = (.25, .5, .75, 1)

SPR (MMB)	0	250	500	750	1000	1070	1100	1250
Equilibrium points	(7,1) (7#,5)	(7,3)	(7,4)	(7,5)	(7,4)	(7,4) (1,1) • • (1,3)	(1,1) •	(1,1) a',5)
Total cost to U.S. ($ billion 1975)	76.0	58.3	45.0	35.0	26.5	24.2 4.2	4.2	4.6

Table it;

I - 4.2, B - 21.0, P - 1.5, X = (I, 1, .8, .5), p - (0, 0, .25, .5)

SPR (MMB)	0	500	750	1000	1250	1400	1450	1500
Equilibrium points	(7,1) : a!5)	(7,4)	(7,,5)	(7,,3)	(7,3)	(7,,1)	(1,1) (2,1) (5,1)	(1,1)(2,1)(5,,D (1,2)(2,2)(5,2)
Total cost to U.S. ($ billion 1975)	114.0	66.1	50.•8	37..8	24.8	16..1	5.3	5.4

Table 17

I = 4.2, E = 21.0, ^ - 1.0, \ - (1,1, .8, .5), p - (0, 0, .25, .5)

SPR (MMB)	0	500	750	1000	1250	1390	1400	1500
Equilibrium points	(7,D : (7,5)	(7,4)	(7,5)	(7,4)	(7,3)	(7,,D	(1,1) (2,1) (5,1)	(1,1)(2,1)(5,1) : : : a!3)(2,3)(5,3)
Total cost to U.S. ($ billion 1975)	76.0	45.0	35.0	26.5	18.1	12..9	5.2	5.4

Table 18

I = 4.2, E = 21.0, β = 1.5, λ = (1, 1, .8, .5), μ = (.5, .66, .83, 1)

SPR (MMB)	0	250	500	750	1000	1150	1170	1250
Equilibrium points	(7,1) ⋮ (7,5)	(7,3)	(7,4)	(7,5)	(7,4)	(7,3) (1,2)	(1,2)	(1,1) ⋮ (1,5)
Total cost to U.S. ($ billion 1975)	114.0	86.7	66.1	50.8	37.8	30.1 4.3	4.3	4.6

Table 19

I = 4.2, E = 21.0, β = 1.0, λ = (1, 1, .8, .5), μ = (.5, .66, .83, 1)

SPR (MMB)	0	250	500	750	950	970	1000	1250
Equilibrium points	(7,1) ⋮ (7,5)	(7,3)	(7,4)	(7,5)	(7,4)	(7,4) (1,3)	(1,1) ⋮ (1,4)	(1,1) ⋮ (1,5)
Total cost to U.S. ($ billion 1975)	76.0	58.3	45.0	35.0	28.3	27.6 4.0	4.1	4.6

Table 20

I = 5.0, E = 24.0, β = 1.5, λ = (1, .8, .5, .2), μ = (.25, .5, .75, 1)

SPR (MMB)	0	250	500	750	1000	1100	1120	1250
Equilibrium points)	(7,1) ⋮ (7,5)	(7,4)	(7,4)	(7,4)	(7,4)* (6,3)*	(7,3)* (1,3)*	(1,1) (1,3)	(1,1) ⋮ (1,4)
Total cost to U.S. ($ billion 1975)	145.7	114.0	89.9	70.5	55.2* 26.6*	49.8* 4.2*	4.3	4,6

*Near-equilibrium points and associated costs

Table 21

I = 5.0, E = 24.0, β = 1.0, λ = (1, .8, .5, .2), μ = (.25, .5, .75, 1)

SPR (MMB)	0	250	500	750	900	980	1000	1250
Equilibrium points	(6,1) ⋮ (6,5)	(6,3)	(6,3)	(6,4)	(6,3)	(6,3) (1,2)	(1,2) (1,3)	(1,1) ⋮ (1,5)
Total cost to U.S. ($ billion 1975)	72.9	53.0	39.1	28.8	23.0	19.8 4.0	4.1	4.6

Table 22

I = 5.0, E - 24.0, B = 1.5, \ - (1, .8, .5, .2), \i, = (0, 0, .25, .5)

SPR (MMB)	0	500	750	1000	1250	1500	1530	1550
Equilibrium points	(7,1) • 5)	(7,4)	(7,4)	(7,5)	(7,4)	(7,3) (1,2)*	(7,3) (1,D(2,1)(5,D (1,2)(2,2)(5,2)	(1,1)(2,1)(5,1) : : : (1;3)(2;3)(5;3)
Total cost to U.S. ($ billion 1975)	145.7	89.9	70.5	55.1	40.6	26.2 5.4*	24.7 5.4	5.5

*Near-equilibrlum point and associated cost

Table 23

I - 5.0, E = 24.0, B = 1.0, X - (1, .8, .5, .2), μ = (0, 0, .25, .5)

SPR (MMB)	0	500	750	1000	1250	1400	1430	1500
Equilibrium points	(7,1) : O\5)	(7,4)	(7.4)	(7,5)	(7,4)	(7,3) (1,2)(2,2)(5,2)	(1,2)(2,2)(5,2) (1,3)(2.3)(5,3)	(y)(2,D(5,i) (1,3)(2,3)(5,3)
Total cost to U.S. ($ billion 1975)	97.2	60.5	48.2	38.1	28.6	23.0 5.2	5.2	5.4

Fig. 4. Illustration of Table 8.

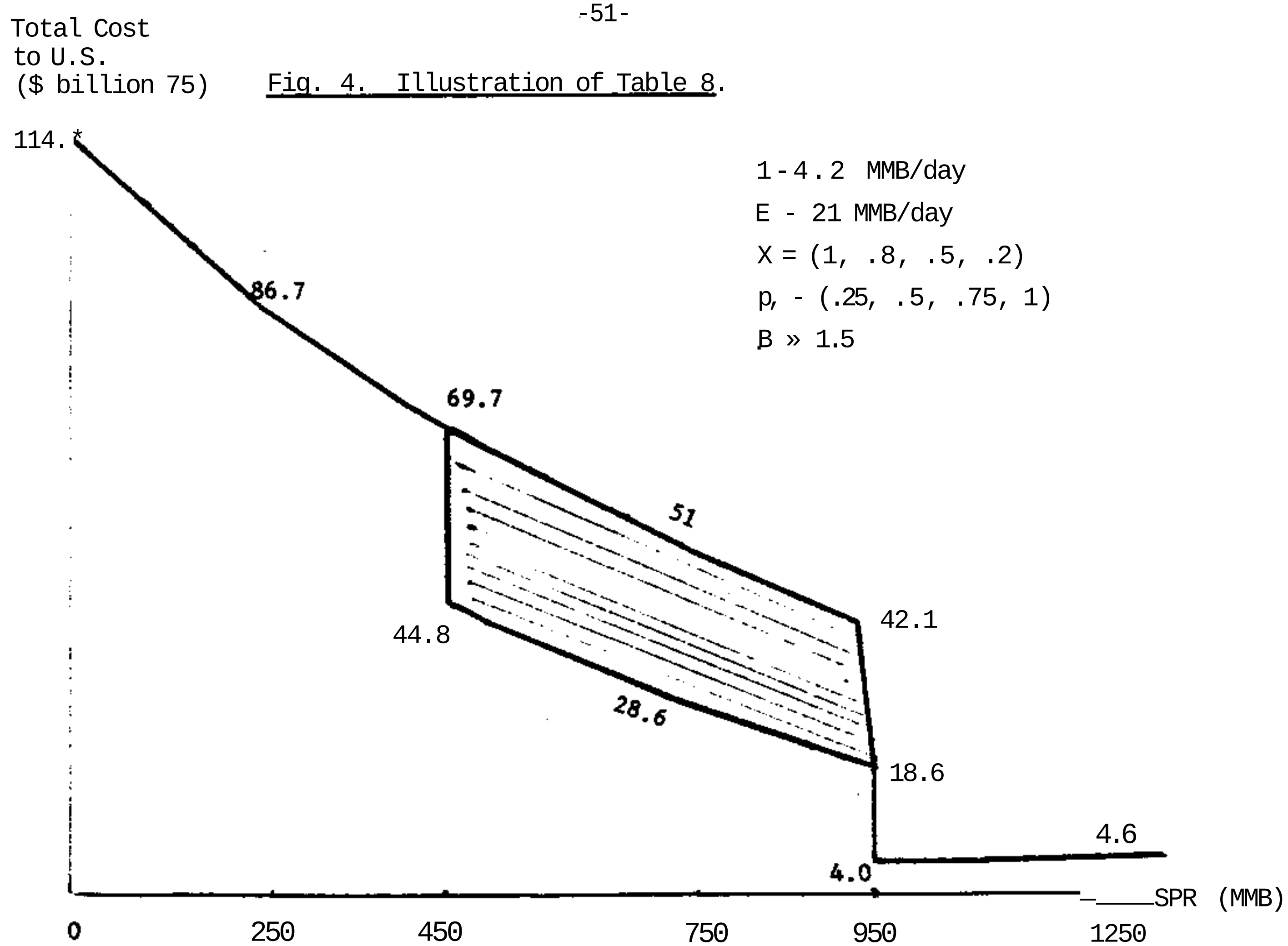


Fig. 5. Illustration of Table 9.

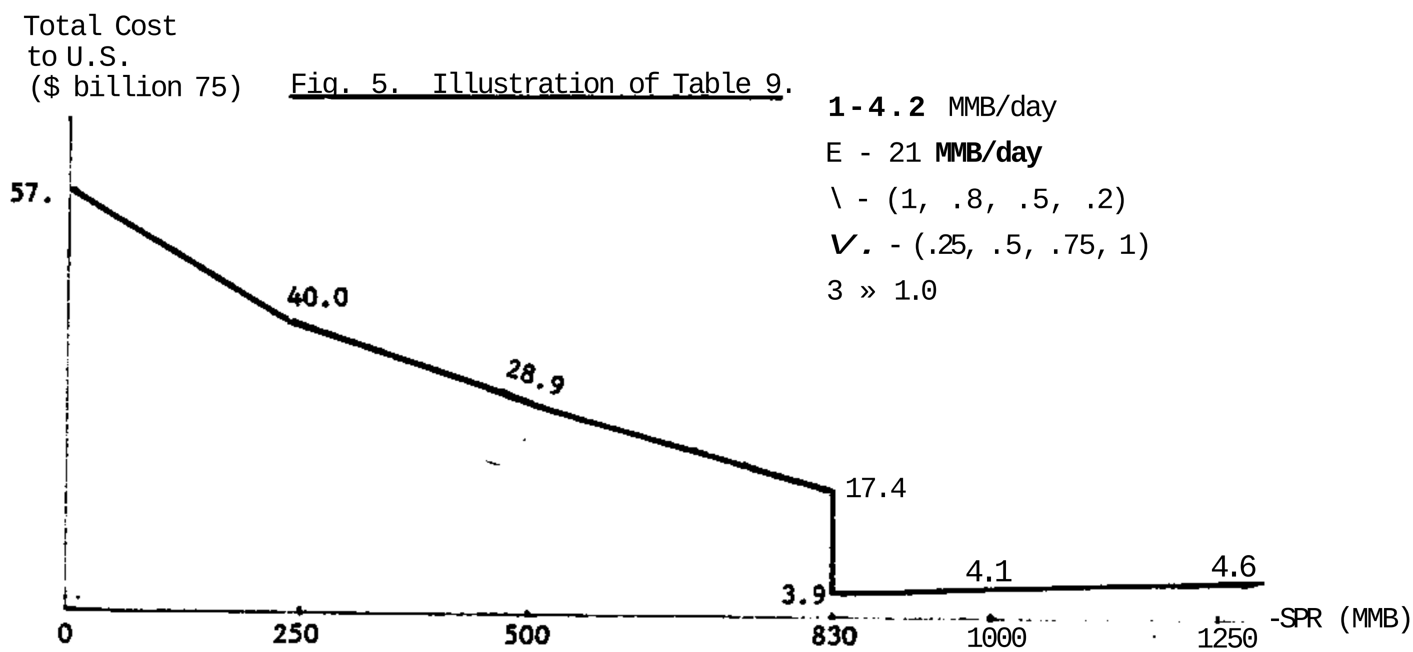


Fig. 6. Illustration of Table 20.

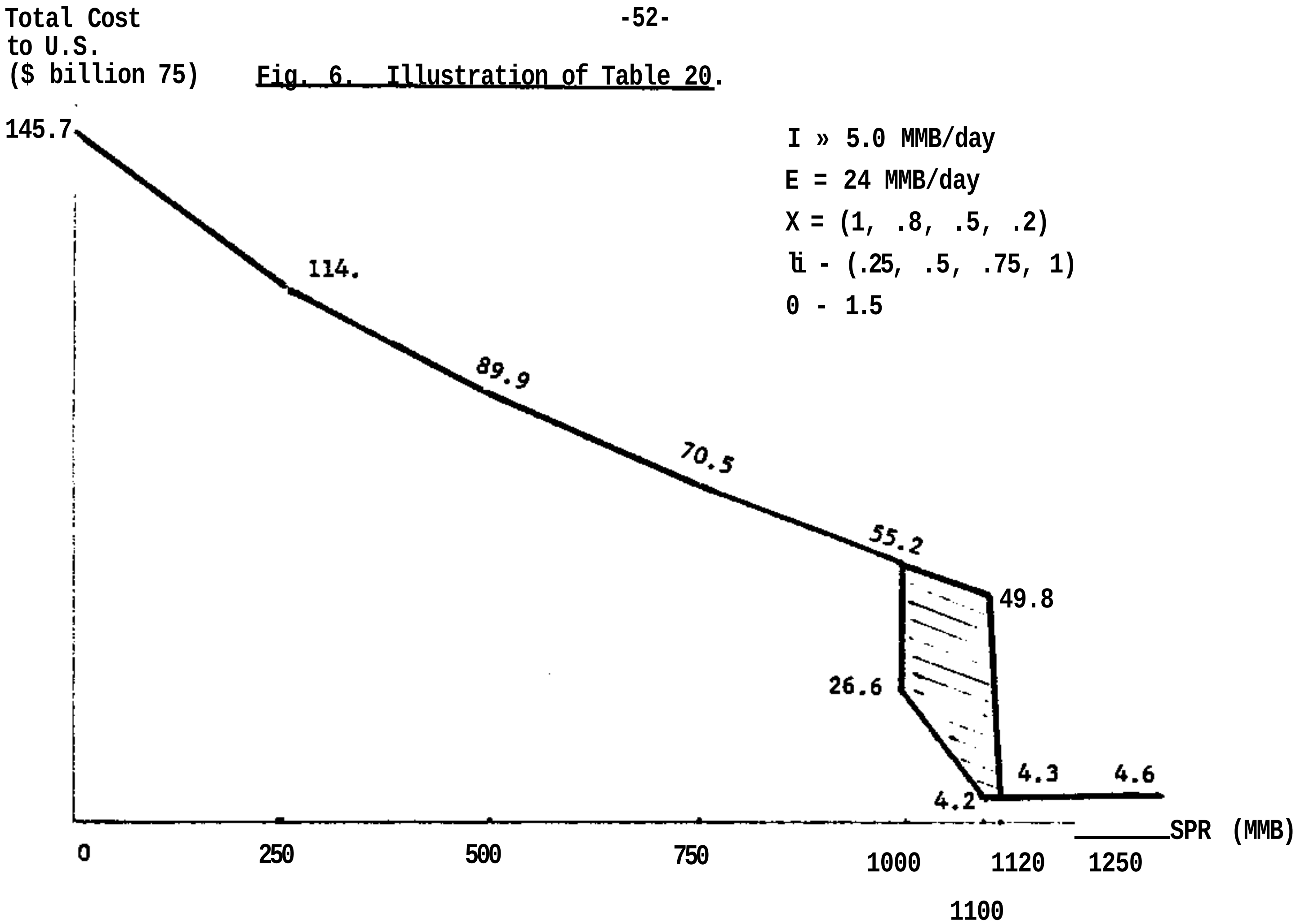
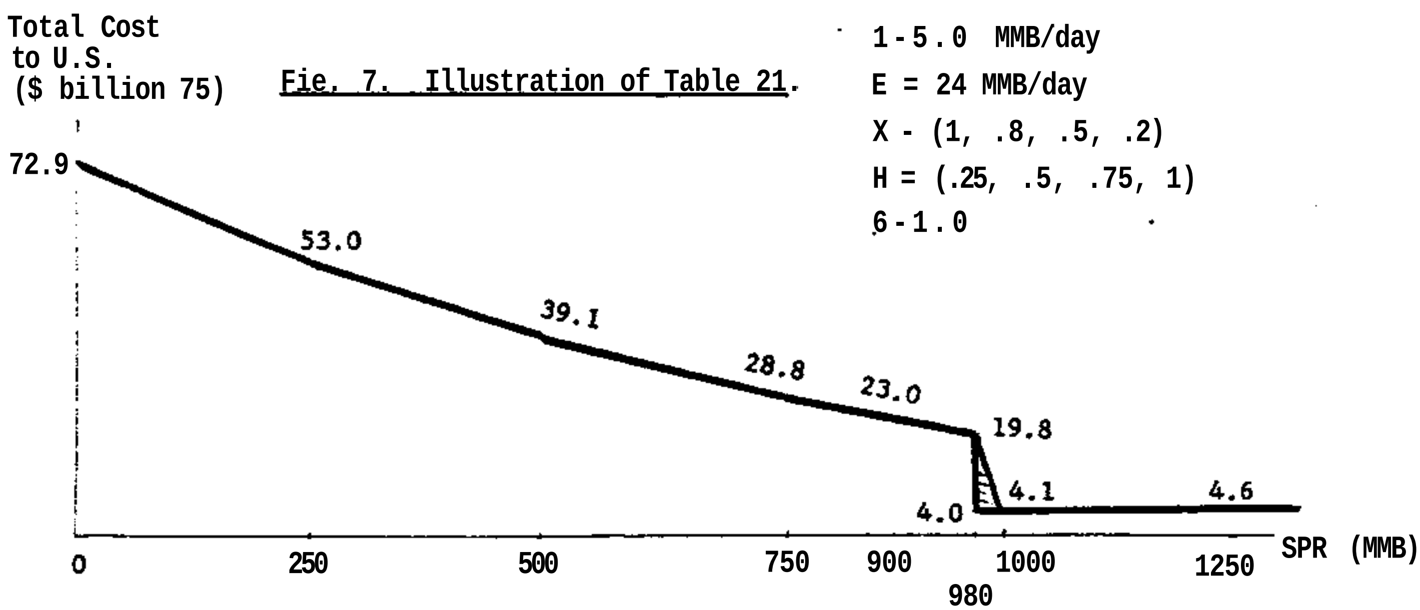


Fig. 7. Illustration of Table 21.



two, (tables 6 and 7) are shown here, to illustrate the procedure. Table 8 is extracted from Table 6. For every SPR size, it shows the corresponding equilibrium point and total cost to the U.S. in billions of 1975 dollars, present valued to 1975. These total costs are the boxed-in entries in the bottom part of Table 6. Similarly, Table 20 is extracted from Table 7 and bears the same relationship to the latter, as Table 8 to Table 6.

Figures 4, 5, 6 and 7 illustrate the outcomes for four typical scenarios, namely those corresponding to Tables 8, 9, 20 and 21.

Thus, figure 4 is a graphical representation of the last row of Table 8, i.e., of the total cost to the U.S. of the given scenario, as a function of the reserve size. Note that Table 8 and Figure 4 contain two values of the total U.S. cost for every reserve size between R = 450 and R = 930. The second set of values corresponds to near-equilibrium points with the tolerances shown in Table 6.

The interpretation of Table 8 (and Figure 4) is as follows. Given the scenario defined by the parameter values listed in section 7 and on top of Table 8, the optimal strategy for the Cartel in case of a conflict, in the absence of a Reserve, is 7, i.e., to impose the heaviest type of embargo for a full year. The corresponding cost to the U.S. is $114 billion (1975, present valued to 1975). This strategy remains optimal for the Cartel as the reserve size increases, up to a SPR of 450 M4B. At that point strategy 7 becomes tied for optimality with strategy 6, representing the same heaviest kind of embargo, for 270 days. The corresponding U.S. costs are $69.7 billion (for strategy 7) and $44.8 billion (for strategy 6). Strategies 7 and 6 remain optimal or near-optimal for the Cartel up to a reserve size of 930 MMB. From that point on, strategy 7 recedes, and for 950 *ffB strategy 6 shares near-optimality with strategy 1, the "no embargo" policy. This makes the U.S. costs decrease from $42.1 billion or $19.4 billion for a SPR of 930 MMB, to $18.6 billion or $4.0 billion, for a reserve size of 950 MMB. Finally, at 960 MMB strategy 1 becomes clearly optimal, by a margin that increases with the reserve size up to a point beyond which it remains constant. Accordingly, the total U.S. cost drops to the level of $4.0 billion, representing the cost of the SPR, and from there on increases slowly (linearly) with the size of the SPR.

Figure 6 bears the same relationship to Table 20, as Figure 4 to Table 8. The interpretation of the other tables in the series 8-23 is the same, i.e., they represent the total cost to the U.S. of a conflict with the Cartel as a function of the size of the SIR, for different sets of values of the parameters (i.e., different scenarios).

10. U.S. Costs, Benefits and Marginal Benefits

As with most mathematical models, the numerical results in themselves are less interesting than the way they change with the parameters defining the scenario. Since every single scenario contains some questionable assumptions, the most interesting features of the outcomes are those least affected by changes in the assumptions. In looking over the outcomes shown in Tables 8-23, as well as their illustrations in Figures 4-7, one finds the same general characteristcs of the U.S. cost function in all cases: it is monotone decreasing in the reserve size up to a critical value of the latter, at which the function drops to the level corresponding to the cost of the reserve. Beyond this critical SPR size, the U.S. cost function increases slightly, linearly with the reserve size. The critical SPR size is the one for which the "no embargo" policy becomes optimal for the Cartel. Since the U.S. cost function attains its unique minimum for this critical

SPR size, the latter is optimal for the U.S. Changes in the various parameters do affect the critical SPR size, but leave unchanged the general shape of the U.S. cost curve as a function of the SPR size.

The benefit from a SPR of size R can be defined as the savings in total U.S. costs brought about by a reserve of size R under a given conflict scenario; i.e., as the difference between the cost to the U.S. of the conflict without a reserve, and the cost with a SPR of size R. In case of the scenario represented in Fig. 4 and Table 6, for instance, the benefit from various reserve sizes is shown in Table 24. The entry under R » 250 is the difference between 114.0 and 86.7, the entry under R * 450 is the difference between 114.0 and 69.7, etc.

Table 24. Benefits from SPR of various sizes

I =4.2 *«B/day, E = 21 **©/day, \ = (1,.8,.5,.2), » = (.25,.5,.75,1), 0 » 1.5

R.QMB)	0	250	450	750	930	960	1000
Benefit ($ billion 75)	0	27.3	44.3	63.0	71.9	110.0	109.9

It is interesting to examine the marginal benefit obtained from every additional MMB of reserve as the SPR is increased from 0 to its critical size. Table 25 and Figure 8 show the average marginal benefit for every interval between two consecutive reserve sizes of Table 24. The average marginal benefit for such an interval is calculated by dividing the increase in benefit over the interval, by the increase in the reserve over the same interval, and multiplying by 1000 to obtain the results in $/barrel (constant 1975 dollars). Thus, ((27.3 - 0): 250) x 1000 = 109.2, ((44.3 - 27.3): 200) x 1000 * 85.0, etc.

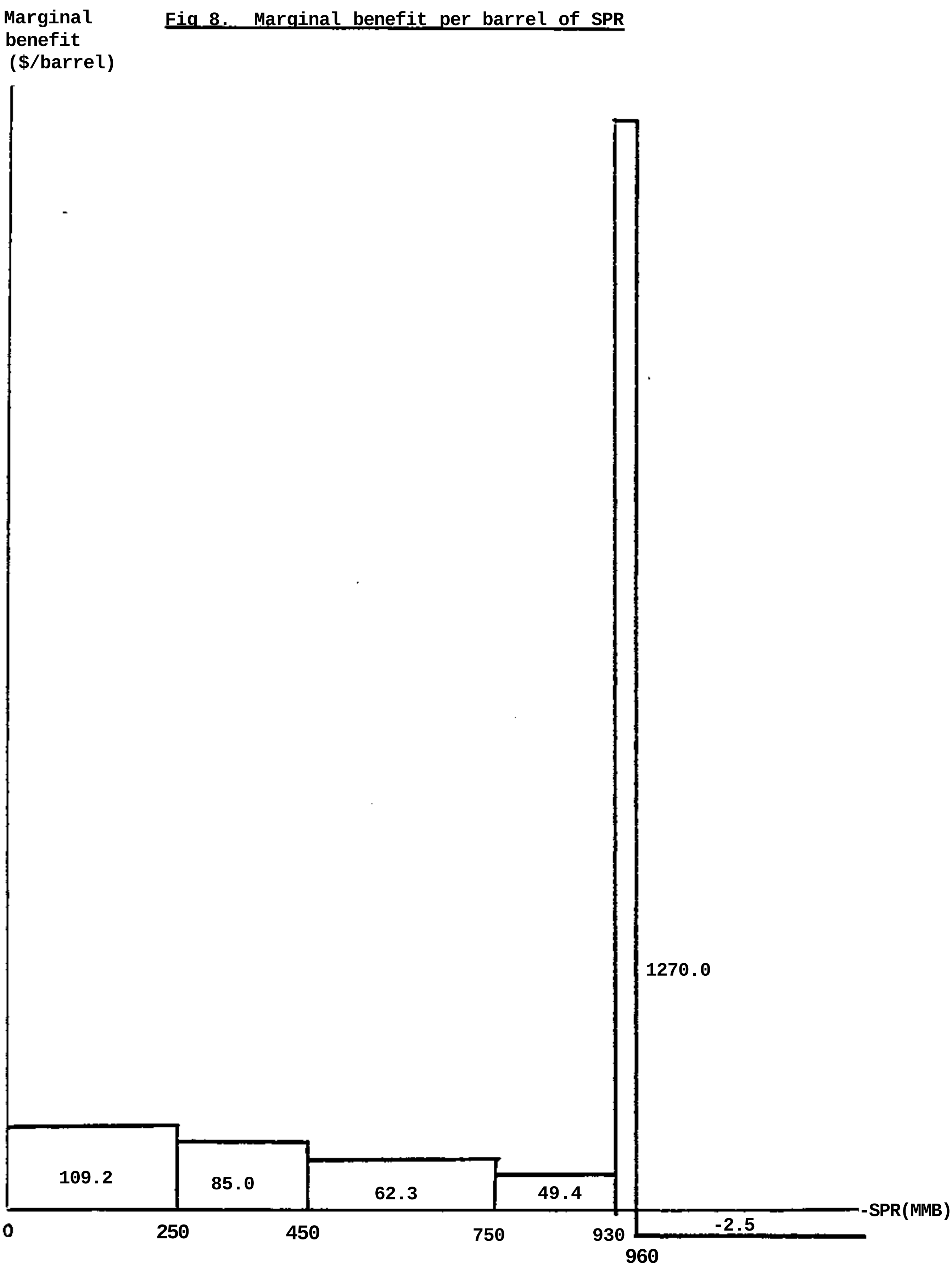

Marginal
benefit
($/barrel)
Fig 8. Marginal benefit per barrel of SPR
1270.0
109.2
85.0
62.3
49.4
-2.5
-SPR(MMB)
0
250
450
750
930
960

Table 25. Average marginal benefits for successive intervals of SPR size

I = 4.2 MMB/day, E - 21 MMB/day, \ - (1,.8,.5,.2), p, - (.25,.5,.75,1), 3 = 1.5

R(MMB)	0-250	250-450	450-750	750-930	930-960	> 960
Marginal benefit $/barrel	109.2	85.0	62.3	49.4	1270.0	-2.5

We see that as the reserve size increases from 0, the marginal benefit decreases until the reserve gets in the vicinity of the critical size, in this case 960 MMB; then it suddenly jumps, i.e., the last increment in R required to attain the critical size yields a much higher marginal benefit than any other increment. Finally, increasing the reserve beyond the critical size brings negative marginal benefits.

While the particular numbers are somewhat different for the different scenarios, the general pattern of decreasing marginal benefits up to a point close to the critical reserve size, then a sudden jump to the highest marginal benefit over the whole range of reserve sizes, and finally negative marginal benefits beyond the critical size, is the same for all scenarios. The jump in the vicinity of the critical reserve size reflects the fact that at that point a relatively small increment in the reserve size causes the no embargo strategy to become optimal for the Cartel.

11. The SPR as an Embargo-Deterrent

In our model the optimal size of the SPR is determined on the basis of its loss-absorbing as well as its embargo-deterrent function. It is sometimes useful, however, to evaluate these two functions separately.

Given an embargo E of a certain length and intensity, the existence of a reserve of size R reduces the petroleum shortfall created by the embargo, and thereby the GNP loss caused by the shortfall. If L(E,R) is the GNP loss from an embargo of type E in the presence of a reserve of size R, then the <u>absorption value</u> of a reserve of size R relative to an embargo of type E can be defined as the difference between the GNP loss caused by an embargo E without any reserve, and with a reserve of size R, i.e.:

$$V_r(E,R) = (L(E,0) - L(E,R).$$

On the other hand, for every reserve size R, there exists some number I(R) such that, if U.S. oil imports from the Cartel do not exceed I(R) MMB/day, then the reserve is capable of deterring an embargo. If we denote by E(I) the heaviest type of embargo that can be imposed when U.S. imports from the Cartel are at a level of I, the <u>deterrent value</u> of a reserve of size R can be defined as the GNP loss that would be caused, in the presence of a reserve of size R, by an embargo of type E(I(R)), i.e., by the heaviest type of embargo that the reserve R is capable of deterring; that is,

$$V_d(R) = L(E(I(R)), R).$$

For a given reserve size R, the deterrent value $V_d(R)$ can be readily found by looking at a table like 6, corresponding to a U.S. import level that makes R the critical SPR size. For example, Table 6 itself can be used to read off the deterrent value of a SPR of 960 MMB, as well as its absorption value corresponding to the given scenario. Indeed, the absorption value of a SPR of 960 MMB relative to the scenario underlying Table 6, is obtained by subtracting the last entry in the column under R = 960, which represents the total cost to the U.S. of an embargo corresponding to strategy 7 in the presence of a reserve of 960 MMB, from the last entry in the column under R = 0, which represents the cost to the U.S. of the same embargo in the absence of a reserve. This difference is 114.0 - 40.3 = 73.7 $billion. On the other hand, the deterrent value of a SPR of 960 MMB is obtained by subtracting the top entry in the second part of the column under R = 960, which represents the cost to the U.S. of the conflict when the Cartel uses the "no embargo" policy, i.e., the cost of the SPR, from the last entry in the same column, which shows what the cost to the U.S. would be if the SPR did not deter the embargo. This difference is 40.3 - 4.0 = 36.3 billion dollars.

The essential point in looking at the two functions of the SPR separately, is the observation that while the loss-absorbing function of the reserve is exercised in case of an embargo whether the reserve is small or large, the embargo-deterrent function gets exercised only if the reserve is sufficiently large to deter the heaviest type of embargo made possible by the level of U.S. imports from the Cartel. If the level of U.S. imports from the Cartel corresponds to a critical SPR size of R*, but the actual SPR size is smaller than R*, then a conflict may result in a heavy embargo and the deterrent effect of the reserve may be lost. To illustrate, suppose the U.S. imports 4.2 MMB/day from the Cartel, which requires an optimal reserve size of about 960 MMB, but the SPR is only 700 MMB. Though the 700 MMB is enough to deter an embargo if the U.S. imports from the Cartel no more than 2.5 MMB/day, and has a sizeable deterrent value for that case, with the actual import level of 4.2 MMB the deterrent value of the SPR gets lost.

This phenomenon helps to explain the behavior of the marginal benefit function in the neighborhood of the critical SPR size, discussed in the previous section. The deterrent value of a reserve of size R remains latent as long as the level of U.S. oil imports from the Cartel exceeds I(R); and gets translated into actual benefits only when the import level is less than or equal to I(R). Conversely, for a given level I of U.S. oil imports from the Cartel, if R(I) denotes the critical reserve size corresponding to I, then the deterrent value of all reserves smaller than R(I) remains latent; and it is only when the reserve attains its critical size R(I) that its deterrent value gets translated into actual benefits. This accounts for the sudden surge in marginal benefits in the neighborhood of the critical reserve size, shown in Table 25.

12. Changes in the U.S. Payoff Function

For each scenario represented in Tables 8-23, the game was solved for the values fJ » 1.5 and g • 1.0 of the correction factor P that appears in the U.S. GNP loss function (4). We have argued in section 5 why p • 1.5 seems a more realistic assumption than (3 = 1.0. Nevertheless, it is important to know the way in which the size of j3 affects the outcome. As can be seen by comparing every even numbered table (8, 10, etc.) with the table immediately following it, the change in the value of 3 from 1.5 to 1.0 reduces the size of total U.S. cost by about 1/3 to 1/2, depending on the particular scenario; but affects the critical value of the SPR size by only 10-15% in each case. Thus, important as it is for the assessment of the cost to the U.S. of a potential oil embargo, the correction factor P does not play a very important role in the determination of the optimal SPR size.

Another element of the U.S. payoff function whose influence on the outcome of the game we tested, is the length of the study period, or the assumed frequency of conflicts in the Middle East. Since this parameter determines in our model the length of the time for which the opportunity cost of the stored oil is added up in calculating the cost of the reserve, as shown in formula (1) of section 5, its value affects the U.S. cost. In all the runs represented in Tables 8-23, the length of this period was taken to be t^ = 7 years. To test the importance of this assumption, we ran the game for t^ = 10 years and t^ s 20 years, with everything else kept the same as in Table 8. The effect was surprisingly small: the cost of a 1000 MMB SPR, shown in Table 8 to be $4.0 billion, when calculated with t^ « 7, becomes $5.3 billion and $7.6 billion respectively, when calculated with t* = 10 and t^ * 20. Note that the calculation is based on the filling schedule shown in line 10 of Table 1, and that all costs are in constant 1975 dollars ($13/barrel), present valued to 1975 at a discount rate of 10%. The difference between the reserve costs of $4.0 billion on the one hand, and $5.3 billion or $7.6 billion on the other, is too small in comparison with the difference in GNP loss caused by the various embargoes to affect the outcome of the game in any significant way. In fact, it reduces the critical reserve size by less than 5% in either case.

13. Changes in the Cartel's Payoff Function

We examined two kinds of changes in the Cartel's payoff function: those in the vector parameter A. defining the function G, and those in the parameter JL defining the function H (see (6) and (7), in section 6). We recall that the Cartel's payoff function is of the form F • G - H.

As can be seen by comparing Table 8 with 14, Table 9 with 15, etc., replacing the vector X » (1, .8, .5, .2) with \ = (1, 1, .8, .5) causes an increase of between 9-33% in the critical size of the SPR. The vector \ expresses our assumptions about how urgent it is for the Cartel to obtain its political objective in a crisis. The first set of values of X_k assumes that a GNP loss inflicted upon the U.S. in the 2nd quarter of the embargo is only .8 times as useful to the Cartel as an equal loss inflicted in the 1st quarter, and the corresponding multipliers for the 3rd and 4th quarter are .5 and .2 respectively. The second set of values assumes that the loss inflicted in the 2nd quarter is of equal usefulness with that of the first quarter, and the multipliers for the 3rd and 4th quarter are .8 and .5 respectively. The conclusion from the above runs is that the weaker the sense of urgency in the Cartel's action, i.e., the less immediate the goals of a potential embargo, the larger the size of the SPR that the U.S. needs to protect itself. Since this "sense of urgency" is by and large a matter of judgment, rather hard to predict, there is nothing better that one can do than examine the outcomes for various hypotheses and be aware of the differences.

On the other hand, changes in the vector ų, defining the function H have a less subjective and more meaningful interpretation. Comparing the Tables 8 and 10, 9 and 11, 14 and 16, 15 and 17, 20 and 22, 21 and 23, one sees that replacing the vector *p* * (.25, .5, .75, 1) by ^ -- (0, 0, .25, .5) causes an increase in the critical SPR size of 15-40%. The vector H » (0, 0, .25, .5) represents the assumption that the Cartel is completely immune to its own losses from a potential embargo for as long as 6 months, and that for another quarter it perceives its losses as diminished to

•25 of what they actually are, while in the 4th quarter of the embargo the perceived loss is .5 of the actual one. This assumes a considerable ability of the Cartel to coordinate its actions, share its resources, use the reserves of its richer members to help the poorer ones, as well as an almost total inability (or unwillingness) of the U.S. to retaliate economically and in general to inflict pain upon its blackmailers. Far fetched as these assumptions seem, some people consider them realistic. The conclusion from our study is that if these assumptions prevail, a considerably larger reserve size is warranted, than otherwise. But there is another implication of these assumptions: if the Cartel is completely immune to its own losses for the first 6 months of an embargo, there is no incentive for it to use the "no embargo" policy, even when the U.S. reserve is sufficiently large to protect the U.S. from the consequences of an embargo. This phenomenon shows up in our Tables 10, 11, 16, 17, 22 and 23, in that the equilibrium points corresponding to the critical U.S. reserve size involve in every case, besides strategy 1 ("no embargo"), also strategies 2 and 5, corresponding to "short" embargoes (6 months). Under these circumstances, the deterrent effect of a sufficiently large SPR may consist in reducing the length of an embargo in case of a conflict, rather than completely eliminating it.

On the other hand, comparing Tables 8 and 12, 9 and 13, 14 and 18, 15 and 19, we find that replacing t* * (-25, .5, .75, 1) by ^ * (.5, .66, .83, 1) produces a decrease of 7-12% in the optimal size of the SPR. This change corresponds to assuming a somewhat heightened sensitivity of the Cartel to its own losses, due perhaps to the acute needs of its more populous members (Iraq, Algeria, possibly Iran), who are highly vulnerable to the loss of imports, or perhaps to U.S. economic retaliation against members of the Cartel that depend on the U.S. for imports, construction projects, military aid, or simply earnings through investments in the U.S. The conclusion that emerges is that if the U.S. is able to bring about such a heightened sensitivity of the Cartel to its own losses, if it is capable of serious economic retaliation, then a somewhat smaller reserve size will accomplish the same thing as the larger reserve needed in the absence of this capability.

14. U.S. Imports and the Optimal SPR Size

The single most important parameter on which the optimal SPR size depends is the level of U.S. petroleum imports from the Cartel at the time of the conflict. By comparing Tables 8, 9, 10 and 11 with Tables 20, 21, 22 and 23 respectively, one sees that an increase in the level of U.S. oil imports from the Cartel from 4.2 MMB to 5 MMB produces an almost proportional increase in the critical SPR size. Since this relationship is the most obvious and direct one, it is appropriate to express the optimal reserve size as a function of the import level rather than as an absolute magnitude. For a U.S. import level from the Cartel of 4.2 MMB/day, the optimal SPR size varies between 750 MMB and 1450 MMB, depending on the values of the other parameters. These two figures roughly correspond to 180 and 350 days[1] imports respectively. For a U.S. import level of 5 MMB/day, the optimal SPR size varies between 1000 MMB and 1550 MMB , which corresponds to 200 and 310 days respectively. If we differentiate between scenarios corresponding to different values of the correction factor P, the above two intervals become 210-345 days and 225-310 days, respectively, for

0 = 1.5, and 180-335 days and 200-300 days, respectively, for 0 * 1.0. As can be seen from these figures, a correction factor of 0 » 1.5 implies an optimal SPR size slightly larger than 0 = 1.0, but the difference is only about 10%. Since 0 * 1.5 is, according to our present knowledge, closer to the reality than 0 « 1.0, we conclude that the desirable size of the SPR is somewhere around 8-10 months' equivalent of U.S. oil imports from the Cartel at the time of a potential crisis. Translating this into specific reserve requirements for various possible future import levels, we obtain the following:

U.S. oil imports from the Cartel (MMB/day)	Appropriate Size of SPR OMB)
3-3.5	720-1050
3.5-4	840-1200
4-4.5	960-1350
4.5-5	1080-1500

These reserve sizes are close to, but slightly higher than, those recommended at the end of 1976 on the basis of a preliminary study of this model [6]. The increase is mainly due to the correction factor 0, whose role was explained in section 5. Besides, the current recommendation is based on solving the game for more than 100 scenarios and examining the sensitivity of the model to various changes in the assumptions.

15. Conclusions

The purpose of the U.S. Strategic Petroleum Reserve is to reduce the vulnerability of the nation to petroleum supply interruptions. It can do this by absorbing some or all of the petroleum shortfall created by a supply interruption, but also by deterring an embargo through its mere existence. Earlier studies concerning the appropriate size of the SPR have focused on the loss-absorbent aspect of the reserve, and have not found adequate means of quantifying the embargo-deterrent aspect. This study uses a game theoretic model to analyze the optimal SPR size from the point of view of loss-absorption as well as embargo-deterrence. The model views a potential conflict between the Cartel (a coalition of potential embargoers) and the U.S., as a bimatrix game in which both sides pursue their objectives expressed in the form of two payoff functions (matrices).

The Cartel's objective is to inflict maximum damage on the U.S. over a decreasingly weighted sequence of time periods, with a minimum loss of its own; whereas the U.S. objective is to minimize the total cost of the conflict, consisting of the cost of the reserve, plus the U.S. GNP loss. The Cartel's strategies are embargoes of varying intensity and length, the U.S. strategies are SPR drawdown policies, while the size of the SPR is a parameter in both payoff functions. Besides the SPR size, the payoff functions depend on many other parameters. Each set of values for these parameters defines a certain conflict scenario.

The game was solved for over 100 different scenarios, representing every combination of assumptions that seems reasonable, and some that look unlikely but are not impossible. Our conclusions are as follows.

1. For small values of the SPR size, the optimal strategy for the Cartel in a conflict with the U.S. is to impose the heaviest type of embargo made possible by the level of U.S. oil imports from the Cartel, either for a full year or for 9 months, depending on certain assumptions about

the Cartel's payoff function. Often both of the above strategies are near-optimal, and in certain scenarios the optimum shifts slowly from the 1-year to the 9 months' embargo as the SPR size increases. In all the scenarios, there is a critical reserve size for which the "no embargo" strategy becomes optimal for the Cartel. This is the reserve size required to deter an embargo of the maximum intensity made possible by the U.S. import level. While the "no embargo" strategy is optimal for the Cartel with a narrow margin at the critical reserve size, its margin of superiority increases with the reserve size up to a point beyond which it remains constant.

2. The optimal U.S. strategy in case of an embargo is a combination of uniform and exponential drawdown policies; namely, to replace the entire shortfall up to the point where the reserve becomes equal to 5 day's shortfall; after which to draw down daily 1/6 of the current reserve, with $90 \leq 6 \leq 135$.

3. The total cost to the U.S. of a conflict with the Cartel, viewed as a function of the SPR size, decreases monotonically as the reserve increases from 0, as a result of the loss-absorbing function of the SPR. At the critical reserve size, the U.S. total cost suddenly drops to the level of the cost of the reserve, as a result of the embargo-deterrent function of the SPR. Beyond the critical reserve size, the cost increases linearly with the reserve.

4. The total benefit from the SPR increases with the reserve size up to the critical size, then it decreases. The marginal benefit, i.e., the benefit obtained from increasing the reserve by 1 unit, is large for a small reserve, then decreases as the reserve increases, up to a point close to the critical value. Just before the reserve reaches its critical size, the marginal benefit jumps to its highest value over the whole range

of reserve sizes, corresponding to the switch in the Cartel's optimal strategy from a heavy embargo to "no embargo." Beyond the critical value, the marginal benefit is a small negative constant.

5. The game-theoretic approach makes it possible to evaluate separately the two functions of the SPR, i.e., to assess separately the (loss-)absorption value and the deterrent value of the SPR. The point of this exercise is the observation that, while the absorption value of the SPR gets translated into direct benefits whenever there is an embargo, the deterrent value gets translated into benefits only if the SPR is at least equal to the critical size corresponding to the level of U.S. imports from the Cartel; otherwise it remains latent. This explains the surge in marginal benefits in the neighborhood of the critical reserve size: this is the point where the deterrent value, latent for smaller reserve sizes, suddenly gets translated into benefits.

6. Changes in the parameters defining the U.S. payoff function affect heavily the total cost of a conflict to the U.S., but only slightly the critical size of the SPR. Thus, reducing β from 1.5 to 1.0 produces a decrease of at most 15% in the critical reserve size. Increasing the length of the study period, i.e., the assumed average time lapse between two conflicts, from 7 to 20 years, reduces the critical reserve size by less than 5%.

7. Changes in the parameters defining the Cartel's payoff function do affect substantially the critical reserve size. A not too excessive reduction of the rate at which the Cartel's perceived gain G from the loss inflicted upon the U.S. decreases with time, can increase the critical

reserve size by as much as 1/3. On the other hand, an increase in the Cartel's assumed capacity to absorb its own losses can raise the critical reserve size by as much as 40%.

Furthermore, if the Cartel is completely immune to its own economic losses, say, for the first 6 months of an embargo, then the deterrent effect of a sufficiently large SPR may consist in reducing the length of an embargo from 1 year to 6 months rather than completely eliminating it. If, on the other hand, the Cartel is sensitive to its own losses, either because of the needs of its poorer members, or because of U.S. economic retaliation, then a somewhat smaller SPR size suffices to deter an embargo.

8. The single most important parameter determining the optimal SPR size is the level of U.S. oil imports from the Cartel at the time of a potential conflict. Expressing the optimal SPR size as a function of this parameter leads to the conclusion that the appropriate overall size for the SPR is the equivalent of 8-10 months' imports from the Cartel at the time of a potential conflict.

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This paper is the final report on a study carried out for the U.S. Department of Energy, concerning the optimal size of the Strategic Petroleum Reserve (SPR). The purpose of the SPR is to diminish U.S. vulnerability to, as well as to offer protection against, possible future oil embargoes. In this study we formulate the problem of determining the optimal size of the SPR as a parametric bimatrix game between the U.S. and its potential opponent. The strategies of the opponent are embargoes of various intensities and lengths,

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) including of course the no embargo option. The strategies of the U.S. are various ways of using the reserve. The size of the reserve itself is a parameter present in both payoff functions. Solving the game for the relevant reserve sizes yields interesting conclusions on the desirable size of the reserve, as well as on U.S. drawdown policies in case of an embargo. The crucial element in the game*theoretic approach is that, unlike the traditional cost-benefit analysis, it fully captures* the embargo-deterrent effect of an appropriate SFR. We have solved the game for over 100 sets of assumptions organized into scenarios, aad- have derived qualitative and quantitative conclusions concerning the appropriate size of the SPR and related matters.