Acceleration Sets of Planar Manipulators
Part II: Applications and Experiments

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Abstract

The acceleration set theory developed in the companion report is applied to two important problems which arise in the design of manipulator systems for performance: manipulator type selection and actuator size determination. A systematic procedure is given for the comparison of the performance, based on acceleration, of a set of alternative manipulator types. This procedure is then used to compare the performance of three well-known manipulator designs which have been proposed for high performance. Simple algorithms, based on the acceleration set theory, are given for the determination of the minimum actuator sizes to obtain a specified isotropic acceleration. The ease of implementation of these algorithms is demonstrated by actual examples. The experimental determination of acceleration sets is also addressed and simple experimental results are presented and compared with those predicted by the theory.

1 Introduction*

In this paper, we apply the acceleration set theory developed in (Desa and Kim, 1989) to the following two important problems which arise in the design of manipulator systems:

1. The selection of manipulator type from a given set of feasible alternatives.
2. The determination of the actuator sizes for a given manipulator type.

One approach to solving the above two problems is to define suitable performance measures. These performance measures could then be used as a basis for comparing different manipulator types in order to select the "best" one. Furthermore, if the performance measures could be explicitly related to the input design variables of the problem, for example actuator size, then we could use these measures to obtain values ("sizes") of the design variables to meet a desired level of performance.

In this paper, we show how acceleration properties of the acceleration sets, when interpreted as performance measures can be used to provide solutions to the " manipulator type selection" problem and the " actuator sizing" problem state above.

Several performance measures for manipulators have been proposed in earlier studies (Asada, 1983; Yoshikawa, 1985; Khatib and Burdick, 1987; Graettinger and Krogh; 1988) and it is useful to briefly discuss these performance measures within the present context. (Asada, 1983) has defined a General Inertia Ellipsoid (GIE) to characterize manipulator dynamics: this measure does not have a clear physical meaning and is mostly useful in those cases where the nonlinearities in the joint velocities are zero. (Yoshikawa, 1985) defines a dynamic manipulability index which is essentially based on the linear mapping between the actuator torques and end-effector acceleration and therefore does not take into account the nonlinearities in joint velocities. (Khatib and Burdick, 1987) define a performance measure whose physical meaning is not clear and which, in addition, accounts for the nonlinearities in a somewhat ad-hoc fashion by evaluating the measure at one "high" and one "low" joint velocity vector. These drawbacks have been pointed out in (Graettinger and Krogh, 1988) who propose an acceleration radius, which in the terminology of (Khatib and Burdick, 1987) or (Desa and Kim, 1989) is the isotropic acceleration over an operating region and can be thought of as a "global isotropic acceleration". Since the isotropic acceleration does not always exist and is zero at a singular point, global isotropic acceleration (acceleration radius) will in general be zero

* Equation (1.x) refers to equation (1.x) in Part I (i.e. (Desa and Kim, 1989)). Every equation in the current paper (i.e. PartII) starts with "2.", for example (2.46).

unless the operating region is small enough, in which case it approaches the local isotropic acceleration, one of the measures proposed in the current paper. Furthermore, designing for global isotropic acceleration (acceleration radius) when possible will result in actuators which are grossly oversized.

In section 3, we propose a group of performance measures of increasing complexity, based on the theory developed in the companion paper, (Desa and Kim, 1989), which are attractive for the following reasons:

1. They have simple physical meanings.
2. They can be directly related to the manipulator parameters and input variable rates (actuator torques, joint variables) and therefore can be used for design and redesign.
3. The most " complex" performance measure, the local isotropic acceleration takes nonlinearities into account in an "exact" manner.

A direct consequence of (2) and (3) is that a typical design problem like the determination of actuator sizes to guarantee a specified isotropic acceleration can be solved in a relatively straightforward manner and without resort to complex nonlinear optimization as in (Graettinger and Krogh, 1988).

The paper is organized as follows: In section 2, we present a heuristic justification for using acceleration (and acceleration properties) as a measure of dynamic performance for manipulators. Several useful acceleration-based performance measures are then defined in section 3. These performance measures are then used to solve the "manipulator type selection" problem in section 4 and to solve the "actuator sizing" problem in section 5.

The experimental determination of acceleration sets is described in section 6. The simple experimental results presented in this section serve to validate the theory presented in the companion paper.

2 Dynamic performance

Dynamic systems are designed to perform a variety of tasks. Each task generally has an inherent measure of its performance which we will refer to as the task performance measure. For example, if the dynamic system is a manipulator and the task is for a reference point P to move from one point to another, then the manipulator: if one "improves" the acceleration capability of a manipulator, then the time required to perform the task is reduced. Therefore, the "acceleration capability" of a manipulator is a useful dynamic system performance measure.

Specifically, we use two properties of the acceleration sets (or acceleration capability) as dynamic system performance measures: the maximum acceleration and the isotropic acceleration. Furthermore, we are generally interested in these performance measures under three operating conditions, start-up, in-motion, and local, which are defined below. Figure 1 depicts the view of tasks and performance measures for manipulators presented in this chapter.

Comments:

1. The reason for defining three types of operating conditions is that the start-up condition is easier to design for than the in-motion condition which in turn is easier to design for than the local operating condition. Therefore, the start-up condition can be used to obtain very quick approximate results which can then be refined for other operating conditions (see section 6).

2. The isotropic acceleration is a measure of the ability of the manipulator to accelerate in all directions and can be thought of as a measure of the manuverability of the manipulator (Graettinger and Krogh, 1988) or its ability to avoid obstacles.

3 Performance measures

3.1 Start-up acceleration capability

Definition: The start-up acceleration capability of a manipulator, corresponding to a given configuration **q** in the workspace, is the set of all available acceleration vectors of a reference point P when the manipulator is at rest and input torques τ_1 and τ_2 are applied at the (driven) joints.

From the above definition, it is clear that the start-up acceleration capability as defined above is simply the acceleration set S_r, which is given by equations (1.34).

3.2 In-motion acceleration capability

Definition 1: The in-motion acceleration capability of a manipulator is the set of all available acceleration vectors of a reference point P when the point P is moving with a velocity $\dot{\mathbf{x}}^P$ at a given position $\mathbf{x}^P$ in the workspace.

When the point P is at a position $\mathbf{x}^P$ with a velocity $\dot{\mathbf{x}}^P$,

1. the corresponding configuration $\mathbf{q}$ of the manipulator can be obtained from $\mathbf{x}^P$ by solving the inverse kinematic problem (Desa and Roth, 1985), and
2. the corresponding joint variable rate vector $\dot{\mathbf{q}}$ can (except for a certain finite number of singular positions) be obtained from equation (1.22) as

$$\dot{\mathbf{q}} = \mathbf{J}^{-1}\dot{\mathbf{x}}^P. \tag{2.1}$$

We can therefore restate Definition 1 in the following equivalent form:

Definition 2: The in-motion acceleration capability of a manipulator is the set of all available acceleration vectors of a reference point P when the manipulator is in the dynamic state $\mathbf{u} = (\mathbf{q}, \dot{\mathbf{q}})$ and the actuator torques τ_1 and τ_2 are applied at the driven joints.

From the above definition, it is clear that the in-motion acceleration capability of the manipulator as defined above is simply the state acceleration set $S_{\mathbf{u}}$, which is given by equations (1.39).

3.3 Definition of performance measures

In order to be able to design a manipulator to have desirable acceleration capability, we need to be able to extract suitable performance measures. Six such measures are defined below: the first two characterize the acceleration capability at start-up, the next two characterize the acceleration capability when the manipulator is in motion, and the last two characterize the (local) acceleration capability at any configuration in the workspace. It should come as no surprise that the performance measures as defined below are the properties of the acceleration sets determined in section 5 of the companion paper (Desa and Kim, 1989).

1. Maximum start-up acceleration, $a_{max,su}$

Definition: The maximum start-up acceleration $a_{max,su}$ is the maximum available acceleration of a reference point P when the manipulator is at rest and (input) torques τ_1 and τ_2 are applied at the joints.

From the above definition, it is clear that the maximum start-up acceleration is given by

$$a_{max,su} = a_{max}(S_\tau) = \max[a(S_\tau)], \tag{2.2}$$

where $a_{max}(S_\tau)$ is given by equation (1.69).

2. Isotropic start-up acceleration, $a_{iso,su}$

Definition: The isotropic start-up acceleration $a_{iso,\,su}$ is the maximum available acceleration in all directions of a reference point P when the manipulator is at rest in a configuration $\mathbf{q}$ and (input) torques τ_1 and τ_2 are applied at the joints.

From the above definition, it is clear that the isotropic start-up acceleration is given by

$$a_{iso,su} = a_{iso}(S_\tau), \tag{2.3}$$

where $a_{iso}(S_\tau)$ is given by equation (1.70).

3. Maximum "in-motion" acceleration, $a_{max,im}$

Definition 1: The maximum "in-motion" acceleration of a manipulator is the maximum available acceleration when the reference point P moves with a velocity $\dot{\mathbf{x}}^P$ at a position $\mathbf{x}^P$ in the workspace.

An equivalent definition for $a_{max,im}$ is the following:

Definition 2: The maximum "in-motion" acceleration of a manipulator is the maximum available acceleration of a reference point P when the manipulator is in a dynamic state $\mathbf{u}$ and actuator torques τ_1 and τ_2 are applied at the joints.

From the above definition, it is clear that the maximum "in-motion" acceleration is given by

$$a_{\mathrm{max,im}} = a_{\mathrm{max}}(S_{\mathbf{u}}) = \max[a(S_{\mathbf{u}})], \tag{2.4}$$

where $a_{\mathrm{max}}(S_{\mathbf{u}})$ is given by equation (1.130).

4. Isotropic "in-motion"acceleration, $a_{\mathrm{iso,im}}$

Definition 1: The isotropic "in-motion" acceleration of a manipulator is the maximum available acceleration in all directions when the reference point P moves with a velocity $\dot{\mathbf{x}}^P$ at a position $\mathbf{x}^P$ and torques are applied at the driven joints.

An equivalent definition for $a_{\mathrm{iso,im}}$ is the following:

Definition 2: The isotropic "in-motion" acceleration of a manipulator is the maximum available acceleration of a reference point P in all directions when the manipulator is in a dynamic state $\mathbf{u}$ and torques τ_1 and τ_2 are applied at the driven joints.

From the definition above, it is clear that the isotropic in-motion acceleration is given by

$$a_{\mathrm{iso,im}} = a_{\mathrm{iso}}(S_{\mathbf{u}}), \tag{2.5}$$

where $a_{\mathrm{iso}}(S_{\mathbf{u}})$ is given by equation (1.133).

5. Maximum local acceleration, $a_{\mathrm{max,local}}$

Definition: The maximum local acceleration $a_{\mathrm{max,\ local}}$ of a manipulator is the maximum available acceleration of the reference point P at a configuration $\mathbf{q}$ of the manipulator.

The maximum local acceleration $a_{\mathrm{max,local}}$ is bounded by the upper bound $(a_{\mathrm{max,local}})_{\mathrm{ub}}$ given by (1.151) and the lower bound $(a_{\mathrm{max,local}})_{lb}$ given by (1.130) with the vector $\mathbf{k}$ evaluated at the joint variable vector $\dot{\mathbf{q}}$ which maximizes $l(\dot{q}_1,\ \dot{q}_2)$ in equation (1.89).

6. Isotropic local acceleration, $a_{\mathrm{iso,local}}$

Definition: The isotropic local acceleration $a_{iso,local}$ of a manipulator is the maximum available acceleration of the reference point P in all directions when the manipulator is at the (local) configuration q in the workspace.

The isotropic local acceleration $a_{iso,local}$ is given by equation (1.152).

3.4 Uses of the acceleration measures

The six acceleration measures can be used for the following purposes:

1. To compare different manipulator types in order to select a manipulator type with the "best" acceleration capabilities.

2. To design a manipulator to yield certain specified acceleration properties.

3. To redesign a given manipulator in order to improve its acceleration properties.

4. To yield estimates of the inertia forces which can then be used to size the links in very "high-performance" applications.

In the next two sections, we demonstrate the first two uses of the acceleration measures. In section 4, we also address simple redesign, i.e., performance improvement by changing actuator size.

Comment:

Since the isotropic acceleration is a measure which, by definition, is "direction-invariant", it is a more useful measure for the solution of problems 1 and 2.

4 Selection of manipulator type

After defining the manipulator type selection problem, we present a procedure for its solution (section 4.2). This procedure is applied in section 4.3 to three popular manipulator types which have been proposed for "high performance".

4.1 Definition of the problem

General problem statement

Given a set of alternate manipulator types, select the manipulator type which yields the best performance.

In section 5.1 and 5.2, we established the use of acceleration and acceleration properties as measures of performance. We can therefore restate the above general problem statement in a more precise manner for our purposes as follows:

Specific problem statement

Given a set of alternative manipulator types, select the manipulator type which yields the largest isotropic acceleration under various operating conditions (start-up, in-motion and local).

4.2 Procedure for type selection

1. Determine the geometric and inertia parameters for each manipulator type.
2. Determine the ranges for the inputs, $\dot{\mathbf{q}}$ and τ, of each manipulator type.
3. Determine the acceleration sets S_τ, $S_{\dot{\mathbf{q}}}$ and $S_{\mathbf{u}}$ for each manipulator type. (We did this in section 4 of Part I (Desa and Kim, 1989) for the planar two degree-of-freedom manipulator of Figure 4.)
4. Extract the isotropic acceleration for the sets S_τ, $S_{\dot{\mathbf{q}}}$ and $S_{\mathbf{u}}$ (using the theory developed in section 5 in Part I (Desa and Kim, 1989)).

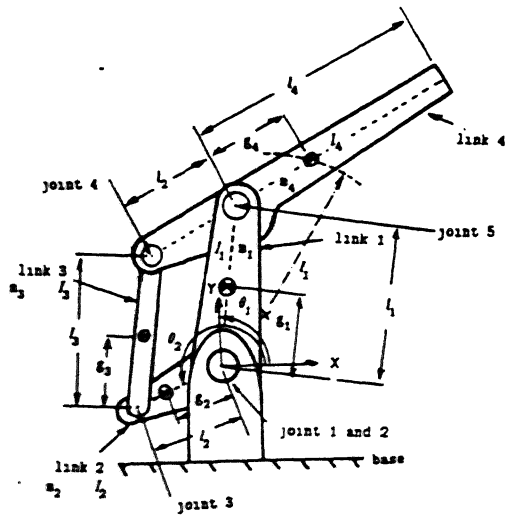

Figure 2: Manipulator type 2 (from Asada and Youcef-Toumi, 1985)

5. Obtain the isotropic acceleration for each manipulator type under various operating conditions (start-up, in-motion and local) using (1.70), (1.133) and (1.152).

6. The "best" manipulator type is the one which has the largest isotropic acceleration under the various operating conditions for the configuration (q) of interest.

7. Critically examine the possibility of redesigning each manipulator type and then repeat steps 1 through 6 for the redesigned manipulator.

8. Perform steps1 through 7 for various configurations (q) of interest.

4.3 Example

As an illustration of the above procedure, we compare the performance of the three manipulator types shown in Figure 8 (Asada and Kanade, 1983), Figure 2 (Asada and Youcef-Toumi, 1985) and Figure 3 (Newman, 1988), which will be referred to, respectively, as manipulator type 1, manipulator type 2 and manipulator type 3. Manipulator type 1 was the original direct drive manipulator. Manipulator type 2 in which both actuators are mounted at the base was proposed in order to improve the performance of

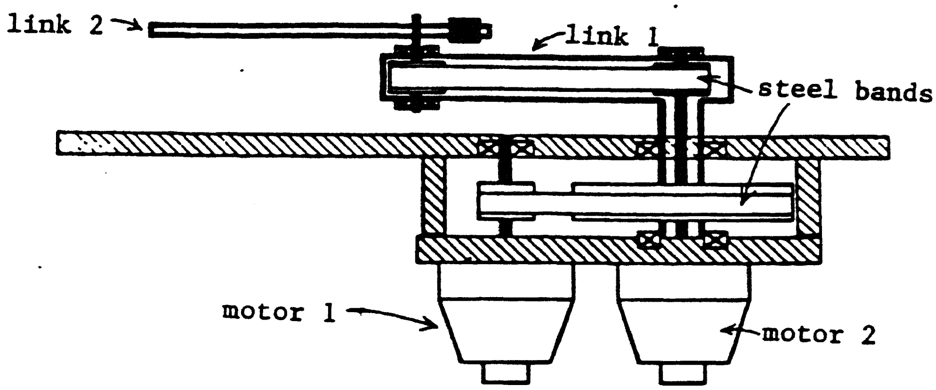

Figure 3: Manipulator type 3 (from Newman, 1987)

manipulator type 1; note that this manipulator type has a "closed kinematic chain". Later manipulator type 3 was proposed in order to improve the performance of manipulator type 2. The parameters and variables for manipulator types 1, 2 and 3 are given, respectively, in Figure 4, Figure 5 and Figure 6. (Note that the joint variable q_2 for manipulator type 1 is different from the joint variable q_2 for manipulator type 2).

The dynamic equations for each manipulator type are given in Appendix A and were used to determine and extract the properties of the acceleration sets S_T, $S_{\dot{\mathbf{q}}}$ and $S_{\mathbf{u}}$ using the theory developed in (Desa and Kim, 1989). The maximum and isotropic acceleration under the three operating conditions are then determined.

The numerical values of the link parameters for each manipulator type are given in Table 1. Two identical actuators, with maximum torques τ_{1o} and τ_{2o} of 30 Nm were used. The input torque set is given by

$$T = \{\tau| \ | \tau_i | \leq 30.0 \text{ Nm}, \ i = 1,2\} \tag{2.6}$$

and the set of joint variable rates given by

$$F = \{\dot{\mathbf{q}}| \ | \dot{q}_i | \leq 5.0 \text{ rad/s}, \ i = 1,2\}. \tag{2.7}$$

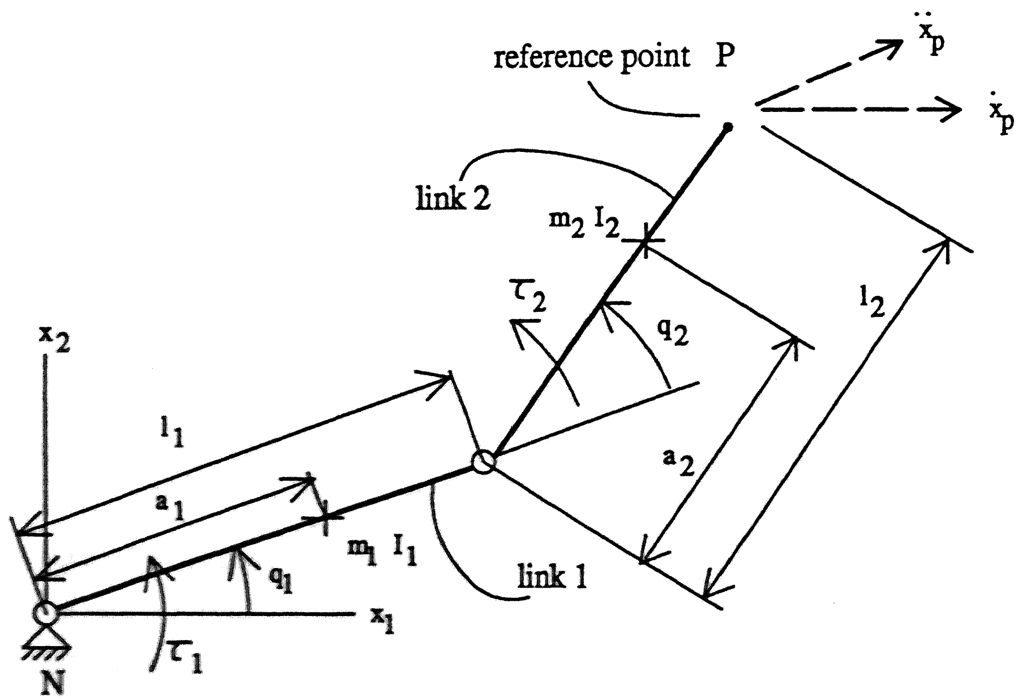

Figure 4: Parameters and variables of manipulator type 1

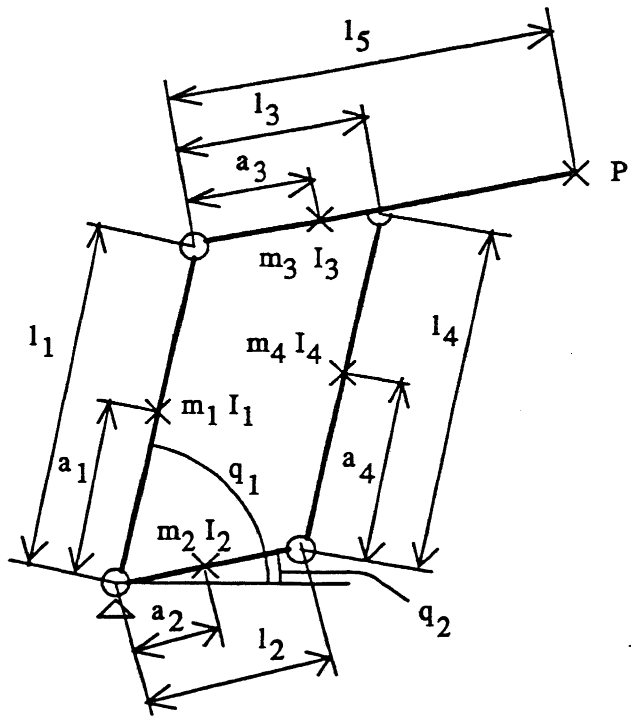

Figure 5: Parameters and variables for manipulator type 2

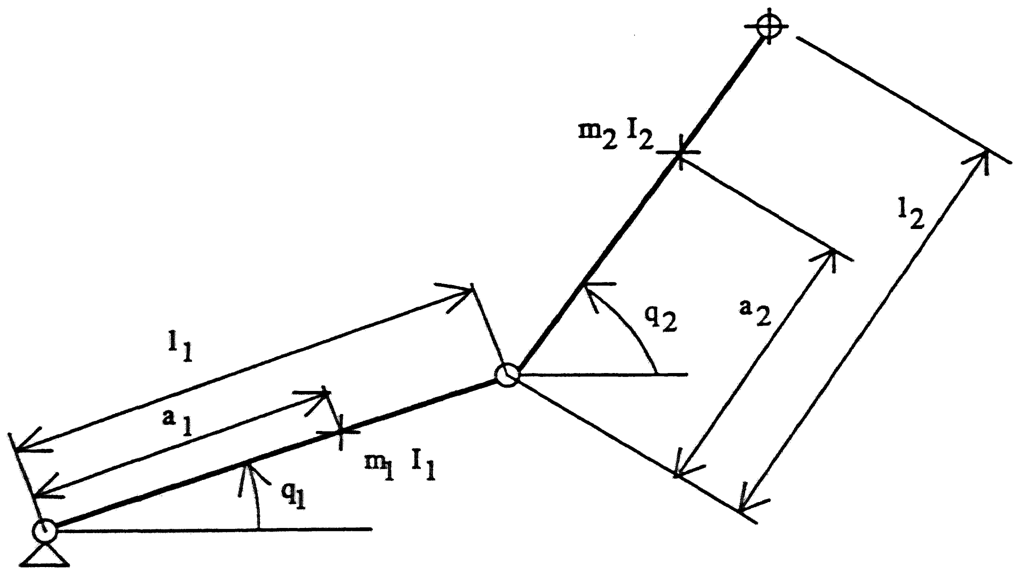

Figure 6: Parameters and variables for manipulator type 3

Manipulator 1:				
link 1:	$l_1 = 0.303$	$a_1 = 0.196$	$m_1 = 2.259$	$I_1 = 0.129$
link 2:	$l_2 = 0.303$	$a_2 = 0.088$	$m_2 = 1.126$	$I_2 = 0.103$
Manipulator 2:				
link 1:	$l_1 = 0.303$	$a_1 = 0.088$	$m_1 = 1.126$	$I_1 = 0.103$
link 2:	$l_2 = 0.303$	$a_2 = 0.088$	$m_2 = 1.126$	$I_2 = 0.103$
link 3:	$l_3 = 0.303$	$a_3 = 0.088$	$m_3 = 1.126$	$I_3 = 0.103$
link 4:	$l_4 = 0.303$	$a_4 = 0.088$	$m_4 = 1.126$	$I_4 = 0.103$
	$l_5 = 0.303$			
Manipulator 3:				
link 1:	$l_1 = 0.303$	$a_1 = 0.088$	$m_1 = 1.126$	$I_1 = 0.103$
link 2:	$l_2 = 0.303$	$a_2 = 0.088$	$m_2 = 1.126$	$I_2 = 0.103$

Table 1: Initial design parameters of manipulator types 1, 2 and 3

Operating condition	Configuration $q = (\dot{q}_1, \dot{q}_2)$	Isotropic acceleration, a_{iso} m/s^2		
		manipulator type 1	manipulator type 2	manipulator type 3
Start-up	(0°, 45°)	24.4	21.5	21.1
	(0°, 90°)	26.9	27.8	28.5
In-motion ($\dot{q}_1$ = 5 r/s, $\dot{q}_2$ = -5 r/s)	(0°, 45°)	20.8	5.14	8.83
Local	(0°, 45°)	5.74	5.14	8.83

Table 2: Isotropic acceleration of the initial design of three manipulator types

Each link of all the three manipulator types was chosen to be the same. The weight of the second actuator τ_2 (mounted on the second link) causes the values of a_1 and m_1 for link 1 of manipulator type 1 to be different from the corresponding values of link 1 for the other manipulator types.

The results obtained for isotropic acceleration for the initial design are given, respectively, in Table 2. (Note that the start-up accelerations were computed for two different configurations).

Let us now examine the possibility of performance improvement by increasing the actuator size of the two actuators. Increasing the size of actuator 2 (τ_2) for manipulator type 1 will have an adverse effect on its performance because the additional weight of the second actuator will be an additional inertial "load" on the first actuator. So, it is not advisable to increase the size of the second actuator. Furthermore, in the present example it is the size of actuator 1 which determines the isotropic acceleration and increasing the size of the first actuator alone will not change the isotropic acceleration (see section 5.5). Therefore, manipulator type 1 is not a good candidate for redesign. The actuator sizes of both actuators can be readily increased for manipulator types 2 and 3 since both actuators (for each of these types) are mounted at the base. We will therefore consider the effects of doubling the size of both actuators of manipulator types 2 and 3. The results obtained for the isotropic acceleration for the redesigned manipulator types 2 and 3 are given in Table 3.

From the results of Table 2 and Table 3, we can draw the following condition.

Operating condition	Configuration $q = (\dot{q}_1, \dot{q}_2)$	Isotropic acceleration, a_{iso} m/s^2	
		manipulator type 2	manipulator type 3
Start-up	(0°, 45°)	43.0	42.2
	(0°, 90°)	55.6	57.0
In-motion ($\dot{q}_1$ = 5 r/s, $\dot{q}_2$ = -5 r/s)	(0°, 45°)	10.2	17.6
Local	(0°, 45°)	10.2	17.6

Table 3: Isotropic acceleration of the redesigned manipulator types 2 and 3

1. Based on the local isotropic acceleration (which takes the nonlinearities into account) of the initial design (Table 2), the manipulator type 3 is "better" than the manipulator type 1 which slightly better than manipulator type 2.

2. When we take advantage of the fact that manipulator types 2 and 3 can be redesigned, we see that (based on local isotropic acceleration in Table 3) manipulator type 3 is better than manipultor type 2 which is better than (the initial) manipulator type 1.

These conclusions are borne out in practice: it is well-known that manipulator type 3 is "faster" than manipulator type 2 which in turn is much "faster" than manipulator type 1. (The reason manipulator types 2 and 3 are better than manipulator type 1 is because they both have all their actuators mounted at the base. The reason manipulator type 3 is better than manipulator type 2 is because the "steel-belt" used in manipulator type 3 to transmit the torque from the base actuator to the second link has negligible inertia compared to the linkages used in manipulator type 2 to transmit the torque from the base actuator to the second link.)

5 Determination of the actuator size for isotropic acceleration

In this section, we demonstrate how the theory developed in (Desa and Kim, 1989) can be used to solve the "actuator size determination" problem in a relatively straightforward fashion.

5.1 Introduction

Given a manipulator at a configuration **q** in the workspace with specified geometric (i.e., link lengths, etc.) and inertia parameters (i.e., masses, moment of inertias, etc.), and specified workspace and joint variable rate constraints, determine the actuator torques required to yield a specified (desired) acceleration property (for example, a specified local isotropic acceleration).

5.2 Definition of the problem

Definitions

$\eta_j \triangleq$ input parameter (or variable); the input parameters are the geometric and the inertia parameters.

$\eta \triangleq [l_1,\ l_2,\ a_1,\ a_2,\ m_1,\ m_2,\ I_1,\ I_2]^T$ input parameter vector with j^{th} component η_j.

W : workspace of the manipulator.

F : joint rate variable set.

T : torque set.

a : some specified acceleration property under a given operation condition (start-up, in-motion, local), for example, $a_{\text{iso,su}}$.

Problem statement

Given the input vector η of link parameters and the constraint sets

$$W = \{\mathbf{q} | q_{iL} \leq q_i \leq q_{iU},\ i = 1,2\}$$

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burgh,

and

$$F = \{\dot{q} \mid |\dot{q}_i| \leq \dot{q}_{io},\ i = 1,2\},$$

determine the torque set

$$T = \{\tau \mid |\tau_i| \leq \tau_{io}, i = 1,2\}$$

to yield the specified acceleration a. The required actuator sizes are of course τ_{1o} and τ_{2o}.

5.3 Solution procedure

We distinguish two cases, the first where the manipulator parameter vector η is independent of the weight of the actuators and therefore of τ_{1o} and τ_{2o} and the second where η depends on the actuator weights and therefore on τ_{1o} and τ_{2o}.

In each case we will obtain the actuator sizes to yield a desired isotropic acceleration under the three operating conditions.

Case 1: Manipulator parameter vector η is independent of the actuator sizes τ_{1o} and τ_{2o}

1 (a) Determination of actuator sizes for specified start-up isotropic acceleration

Given a specified manipulator parameter vector η, determine actuator sizes τ_{1o} and τ_{2o} to yield a specified start-up isotropic acceleration $a'_{\text{iso,su}}$ at a given configuration q in the workspace of the manipulator, i.e., determine τ_{1o} and τ_{2o} such that

$$a_{\text{iso,su}} \geq a'_{\text{iso,su}}. \tag{2.8}$$

The minimum actuator sizes $\tau_{1o,\min}$ and $\tau_{2o,\min}$ required to satisfy the requirements (2.8) are given by

$$\tau_{1o,\min} = \frac{a'_{\text{iso,su}}\sqrt{a_{12}^2 + a_{22}^2}}{|\det(A)|} \tag{2.9}$$

$$\tau_{2o,\min} = \frac{a'_{\text{iso,su}}\sqrt{a_{11}^2 + a_{21}^2}}{|\det(A)|}. \tag{2.10}$$

Proof: Equation (1.70) expresses the isotropic start-up acceleration in terms of the actuator torques τ_{1o} and τ_{2o}. Equation (1.70) is equivalent to the following two conditions

$$\frac{|\det(A)|\,\tau_{1o}}{\sqrt{a_{12}^2 + a_{22}^2}} \geq a_{iso,su} \tag{2.11}$$

$$\frac{|\det(A)|\,\tau_{2o}}{\sqrt{a_{11}^2 + a_{21}^2}} \geq a_{iso,su}. \tag{2.12}$$

Combining equations (2.8) (2.11) (2.12), we obtain

$$\frac{|\det(A)|\,\tau_{1o}}{\sqrt{a_{12}^2 + a_{22}^2}} \geq a'_{iso,su} \tag{2.13}$$

$$\frac{|\det(A)|\,\tau_{2o}}{\sqrt{a_{11}^2 + a_{21}^2}} \geq a'_{iso,su}. \tag{2.14}$$

For a given matrix A (i.e., for given a_{ij} and det(A), the actuator size τ_{1o} will be a minimum when equation (2.13) is an equality. Denoting by $\tau_{1o,min}$ the value of τ_{1o} when (2.13) is an equality and solving (2.13) for $\tau_{1o,min}$, we obtain the result (2.9). Starting with (2.14) and reasoning in a similar fashion, we obtain the result (2.10) for $\tau_{2o,min}$.

1 (b) Determination of actuator sizes for specified in-motion isotropic acceleration

Given a specified manipulator parameter vector η, determine actuator sizes τ_{1o} and τ_{2o} to yield a specified in-motion isotropic acceleration $a'_{iso,im}$ for a given manipulator state $\mathbf{u} = (q, \dot{q})$, i.e., determine τ_{1o} and τ_{2o} such that

$$a_{iso,im} \geq a'_{iso,im} \tag{2.15}$$

at $\mathbf{u} = (q, \dot{q})$.

The minimum actuator sizes $\tau_{1o,min}$ and $\tau_{2o,min}$ required to satisfy the requirement (2.15) are given by

$$\tau_{1o,min} = \frac{a_{iso,\,im}\sqrt{a_{12}^2 + a_{22}^2} + |\,a_{22}k_1 - a_{12}k_2\,|}{|\det(A)|} \tag{2.16}$$

$$\tau_{2o,min} = \frac{a_{iso,\,im}\sqrt{a_{11}^2 + a_{21}^2} + |\,a_{21}k_1 - a_{11}k_2\,|}{|\det(A)|}. \tag{2.17}$$

(Comment: k_1 and k_2 are the components of the vector $\mathbf{k}$ which is defined in section 4 of Part I (Desa and Kim, 1989).)

Proof: Equation (1.133) expresses the isotropic in-motion acceleration in terms of the actuator torques τ_{1o} and τ_{2o}. Equation (1.133) is equivalent to the following two conditions

$$\frac{|\det(A)|\tau_{1o} - |a_{22}k_1 - a_{12}k_2|}{\sqrt{a_{12}^2 + a_{22}^2}} \geq a_{\text{iso,im}} \tag{2.18}$$

$$\frac{|\det(A)|\tau_{2o} - |a_{21}k_1 - a_{11}k_2|}{\sqrt{a_{11}^2 + a_{21}^2}} \geq a_{\text{iso,im}}. \tag{2.19}$$

Combining (2.18) (2.19) and (2.15), we obtain

$$\frac{|\det(A)|\tau_{1o} - |a_{22}k_1 - a_{12}k_2|}{\sqrt{a_{12}^2 + a_{22}^2}} \geq a'_{\text{iso,im}} \tag{2.20}$$

$$\frac{|\det(A)|\tau_{2o} - |a_{21}k_1 - a_{11}k_2|}{\sqrt{a_{11}^2 + a_{21}^2}} \geq a'_{\text{iso,im}}. \tag{2.21}$$

For a given matrix **A** (i.e., for given a_{ij} and det(A) and coefficients k_1 and k_2, the actuator size τ_{1o} will be a minimum when equation (2.20) is an equality. Denoting by $\tau_{1o,\text{min}}$ the value of τ_{1o} when (2.20) is an equality and solving (2.20) for $\tau_{1o,\text{min}}$, we obtain the result (2.16). Starting with (2.21) and reasoning in a similar fashion, we obtain the result (2.17) for $\tau_{2o,\text{min}}$.

1 (c) Determination of actuator size for specified local isotropic acceleration

Given a specified manipulator parameter vector η, determine actuator sizes τ_{1o} and τ_{2o} to yield a specified local isotropic acceleration $a'_{\text{iso,im}}$ for a given configuration **q**, i.e., determine τ_{1o} and τ_{2o} such that

$$a_{\text{iso,local}} \geq a'_{\text{iso,local}}. \tag{2.22}$$

The minimum actuator sizes $\tau_{1o,\text{min}}$ and $\tau_{2o,\text{min}}$ required to satisfy the requirement (2.22) are given by

$$\tau_{1o,\text{min}} = \frac{\sqrt{a_{12}^2 + a_{22}^2}}{\det(A)}[a'_{\text{iso,local}} + \rho_{\max}(\ddot{x}(S_{\dot{q}}), l_1)] \tag{2.23}$$

$$\tau_{2o,\text{min}} = \frac{\sqrt{a_{11}^2 + a_{21}^2}}{\det(A)}[a'_{\text{iso,local}} + \rho_{\max}(\ddot{x}(S_{\dot{q}}), l_2)] \tag{2.24}$$

where $\rho_{\max}(\ddot{x}(S_{\dot{q}}), l_i)$, (i = 1, 2) are given by equation (1.92) in subsection 5.2 of Part I (Desa and Kim, 1989).

Proof: Equation (1.152) expresses the local isotropic acceleration in terms of the actuator torques τ_{1o} and τ_{2o}. Equation (1.152) is equivalent to the following two conditions

$$\frac{|\det(A)|\tau_{1o}}{\sqrt{a_{12}^2 + a_{22}^2}} - \rho_{\max}(\ddot{x}(S_{\dot{q}}), l_1) \geq a_{\text{iso,local}} \tag{2.25}$$

$$\frac{|\det(A)|\tau_{2o}}{\sqrt{a_{11}^2 + a_{21}^2}} - \rho_{\max}(\ddot{x}(S_{\dot{q}}), l_2) \geq a_{\text{iso,local}}. \tag{2.26}$$

Combining (2.25) (2.26) and (2.22), we obtain

$$\frac{|\det(A)|\tau_{1o}}{\sqrt{a_{12}^2 + a_{22}^2}} - \rho_{\max}(\ddot{x}(S_{\dot{q}}), l_1) \geq a'_{\text{iso,local}} \tag{2.27}$$

$$\frac{|\det(A)|\tau_{2o}}{\sqrt{a_{11}^2 + a_{21}^2}} - \rho_{\max}(\ddot{x}(S_{\dot{q}}), l_2) \geq a'_{\text{iso,local}}. \tag{2.28}$$

For a given matrix A (i.e., for given a_{ij} and det(A) and $\rho_{\max}(\ddot{x}(S_{\dot{q}}), l_1)$ (given by equation (1.92) in section 3 of Part I (Desa and Kim, 1989), the actuator size τ_{1o} will be a minimum when equation (2.27) is an equality. Denoting by $\tau_{1o,\min}$ the value of τ_{1o} when (2.27) is an equality and solving (2.27) for $\tau_{1o,\min}$, we obtain the result (2.23). Starting with (2.28) and reasoning in a similar fashion, we obtain the result (2.24) for $\tau_{2o,\min}$.

Case 2: Manipulator parameter vector η is dependent on the actuator sizes τ_{1o} and τ_{2o}

The algorithm for computing the actuator sizes is shown in Figure 7. Essentially, we should embed "Case 1" in a closed-loop which compensates for the fact that η does depend on τ_{1o} and τ_{2o}.

The algorithm (Figure 7) consists of the following steps:

1. Initialization. The initial parameter vector η is computed based on the actuator weights being set to zero. The values of the actuator sizes, denoted by τ_{1o}(old) and τ_{2o}(old), are set to zero.

2. Compute actuator sizes τ_{1o}(new) and τ_{2o}(new) based on a given parameter vector η as in Case 1 (Use Case 1(a) for start-up, Case 1(b) for in-motion and Case 1(c) for local).

3. Check whether τ_{1o} and τ_{2o} converge using the following convergence criteria

$$\frac{\tau_{1o}(\text{new}) - \tau_{1o}(\text{old})}{\tau_{1o}(\text{old})} \leq \epsilon_1 \tag{2.29}$$

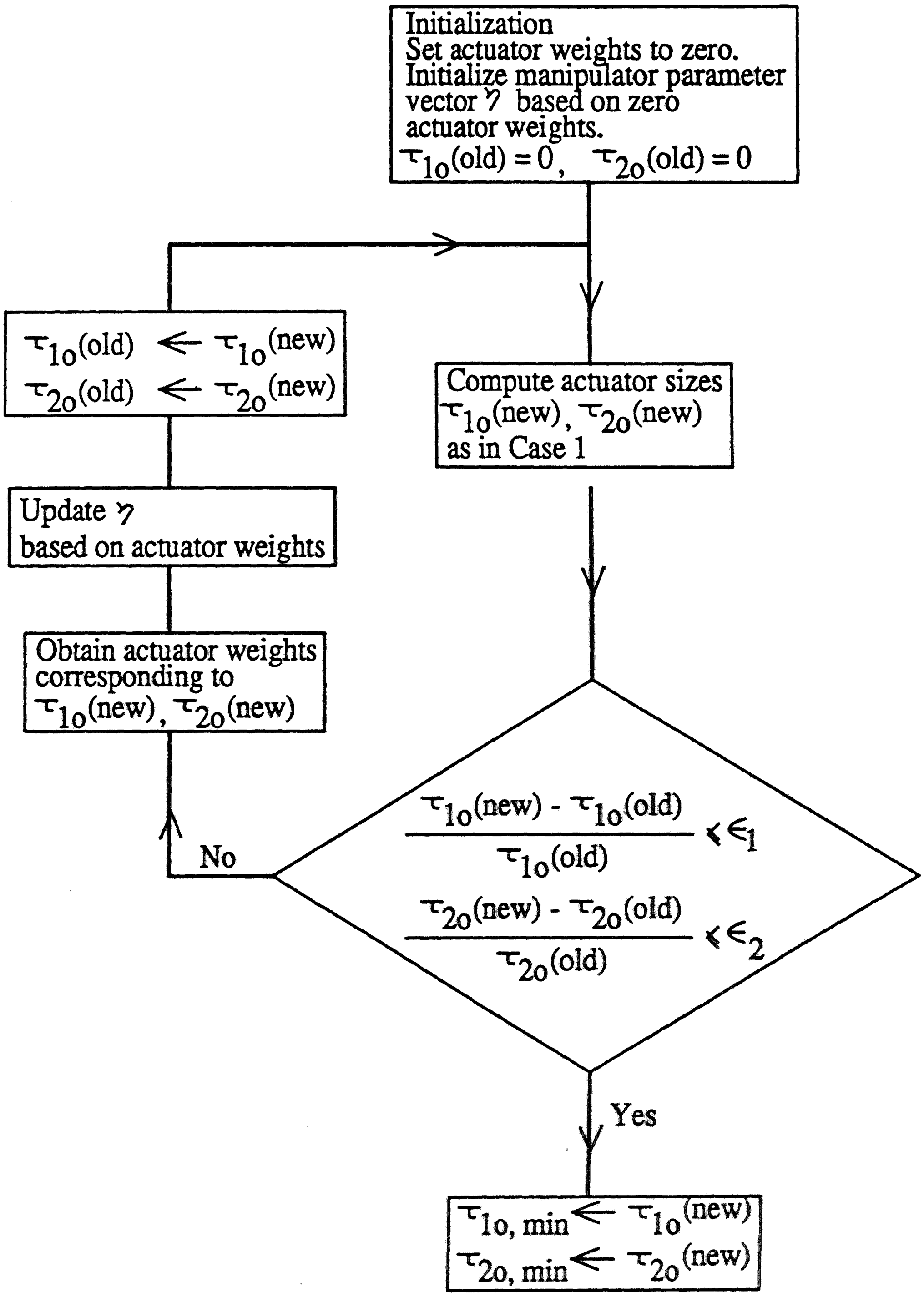

Figure 7: Algorithm for computation of actuator sizes to yield desired acceleration requirements for the case where the manipulator parameter vector η depends on the actuator sizes τ_{1o} and τ_{2o}

$$\frac{\tau_{2o}(\text{new}) - \tau_{2o}(\text{old})}{\tau_{2o}(\text{old})} \leq \epsilon_2 \tag{2.30}$$

where ϵ_1 and ϵ_2 are defined by the user. If (2.29) and (2.30) are satisfied, $\tau_{1o,\text{min}} = \tau_{1o}(\text{new})$ and $\tau_{2o,\text{min}} = \tau_{2o}(\text{new})$ and the design is complete.

4. If (2.29) and (2.30) are not satisfied, update the parameter vector η based on new actuator sizes $\tau_{1o}(\text{new})$ and $\tau_{2o}(\text{new})$, and go to step 2.

The closed loop shown in Figure 7 essentially performs iterations of step 2, 3, and 4 till the convergence criteria are satisfied.

Comment:

The "start-up" case can be used to get a quick design which can be successively refined by doing the "in-motion case" and "local case". This is demonstrated in the example below.

5.4 Example:

Determination of actuator sizes for acceleration properties for a two degree-of-freedom serial planar manipulator.

We illustrate how we determine the minimum actuator sizes of a planar two degree-of-freedom manipulator built in our laboratory for the following three cases

Case 1: (Start-up) $a_{\text{iso,su}} = 3\ \text{m/s}^2$ at $(q_1 = 0^o,\ q_2 = 90^o)$

Case 2: (In-motion) $a_{\text{iso,im}} = 3\ \text{m/s}^2$ at $(q_1 = 0^o,\ q_2 = 90^o,\ \dot{q}_1 = 1\text{rad/s}^2,\ \dot{q}_2 = 1\text{rad/s}^2)$

Case 3: (Local) $a_{\text{iso,local}} = 3\ \text{m/s}^2$ at $(q_1 = 0^o,\ q_2 = 90^o)$

Initialization

Initial link parameters for links 1 and 2 are as follows:

link 1: $l_1 = 0.303$ m, $a_1 = 0.088$ m, $m_1 = 1.126$ Kg, $I_1 = 0.103$ Kg m^2,
link 2: $l_2 = 0.254$ m, $a_2 = 0.094$ m, $m_2 = 1.120$ Kg, $I_2 = 0.003$ Kg m^2,

Since our manipulator belongs to manipulator type 1 in section 4, we use the loop-algorithm in Case 2.

Case 1: Design for start-up acceleration

To give the reader a feel for how to size actuators using the algorithm, we include the results of the three iterations which were needed to obtain the actuator sizes.

Iteration 1.

Using equations (2.9) and (2.10) with the initial link parameters, we come up with the following actuator sizes,

$$\begin{aligned} \tau_{1o,\mathrm{min}} &= 2.13Nm, \\ \tau_{2o,\mathrm{min}} &= 0.15Nm. \end{aligned} \tag{2.31}$$

Iteration 2.

Since we can vary the actuator torques between 0.2 - 5 Nm using the gear reduction, the weight of brushless motor is assumed to be around 1.1 Kg. Our manipulator is manipulator type 1 and we include the actual weight of actuator 2 to obtain a new set of parameters.

$$\begin{aligned} &\text{link } 1 : l_1 = 0.303, a_1 = 0.196, m_1 = 2.259, I_1 = 0.129, \\ &\text{link } 2 : l_2 = 0.254, a_2 = 0.094, m_2 = 1.129, I_2 = 0.003. \end{aligned} \tag{2.32}$$

If we use equations (2.9) and (2.10) with the new set of link parameters in (2.32), then we come up with the following actuator sizes,

$$\begin{aligned} \tau_{1o,\mathrm{min}} &= 3.17Nm, \\ \tau_{2o,\mathrm{min}} &= 0.15Nm. \end{aligned} \tag{2.33}$$

Iteration 3.

Since the weight of actuators is assumed to be around 1.1 Kg, we have the manipulator parameter set in (2.32). If we use equations (2.9) and (2.10) with the set of link parameters in (2.32), then we come up with the same actuator sizes as in calculation 2 as follows,

$$\begin{aligned} \tau_{1o,\mathrm{min}} &= 3.17Nm, \\ \tau_{2o,\mathrm{min}} &= 0.15Nm. \end{aligned} \tag{2.34}$$

The required actuator sizes, therefore, are the values in (2.33).

Case 2: Design for in-motion acceleration

Similarly, using (2.16) and (2.17) and employing the algorithm (Figure 7), we obtain the following minimum actuator sizes to satisfy the in-motion isotropic acceleration

$$\begin{aligned} \tau_{1o,\min} &= 3.42Nm, \\ \tau_{2o,\min} &= 0.17Nm. \end{aligned} \tag{2.35}$$

As expected, because of the non-linear effects when the manipulator is "in-motion", results (2.35) show that we should use bigger actuators in order to achieve the same level of acceleration properties as in manipulator start-up.

Case 3: Design for local acceleration

Using (2.23) and (2.24) and employing the algorithm, in Figure 7, we obtain the minimum actuator sizes to satisfy the local isotropic acceleration

$$\begin{aligned} \tau_{1o,\min} &= 4.12Nm, \\ \tau_{2o,\min} &= 0.17Nm. \end{aligned} \tag{2.36}$$

As expected, we come up with the bigger actuator sizes in (2.36) than those of the "in-motion" results in (2.35).

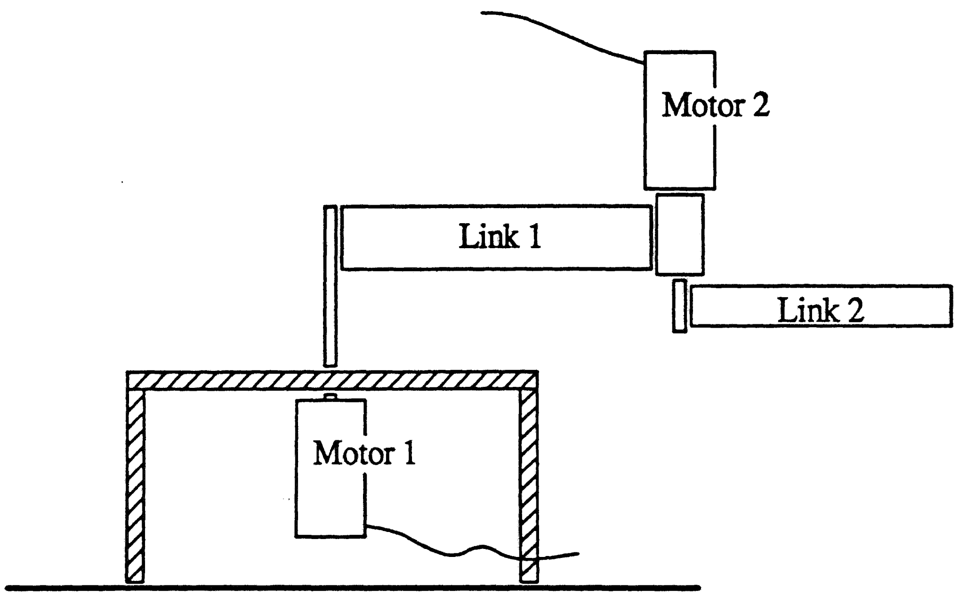

Figure 8: Mechanical components of a two degree-of-freedom manipulator

6 Experimental verification

In this section, we describe simple experiments which are used to determine the acceleration set S_τ (Start-up acceleration capability) and then compare the experimental results with those obtained using the analytical results of Part I.

6.1 Description of the two degree-of-freedom manipulator experimental set-up

The mechanical structure of the two degree-of-freedom manipulator is shown schematically in Figure 8. The design is modular so that the links can be easily changed, thus allowing one to study the effect of changing the link parameters. Each link is driven by a motor as shown in the Figure. A schematic of the control hardware which is used to drive each motor, and thereby control the torque applied to each link, is shown in Figure 9; the main points to note in the control hardware are the following:

1. A specified input torque commanded from a terminal (by the user) is transmitted to the pulse-width-modulation (PWM) generator by the MC68K microprocessor board.

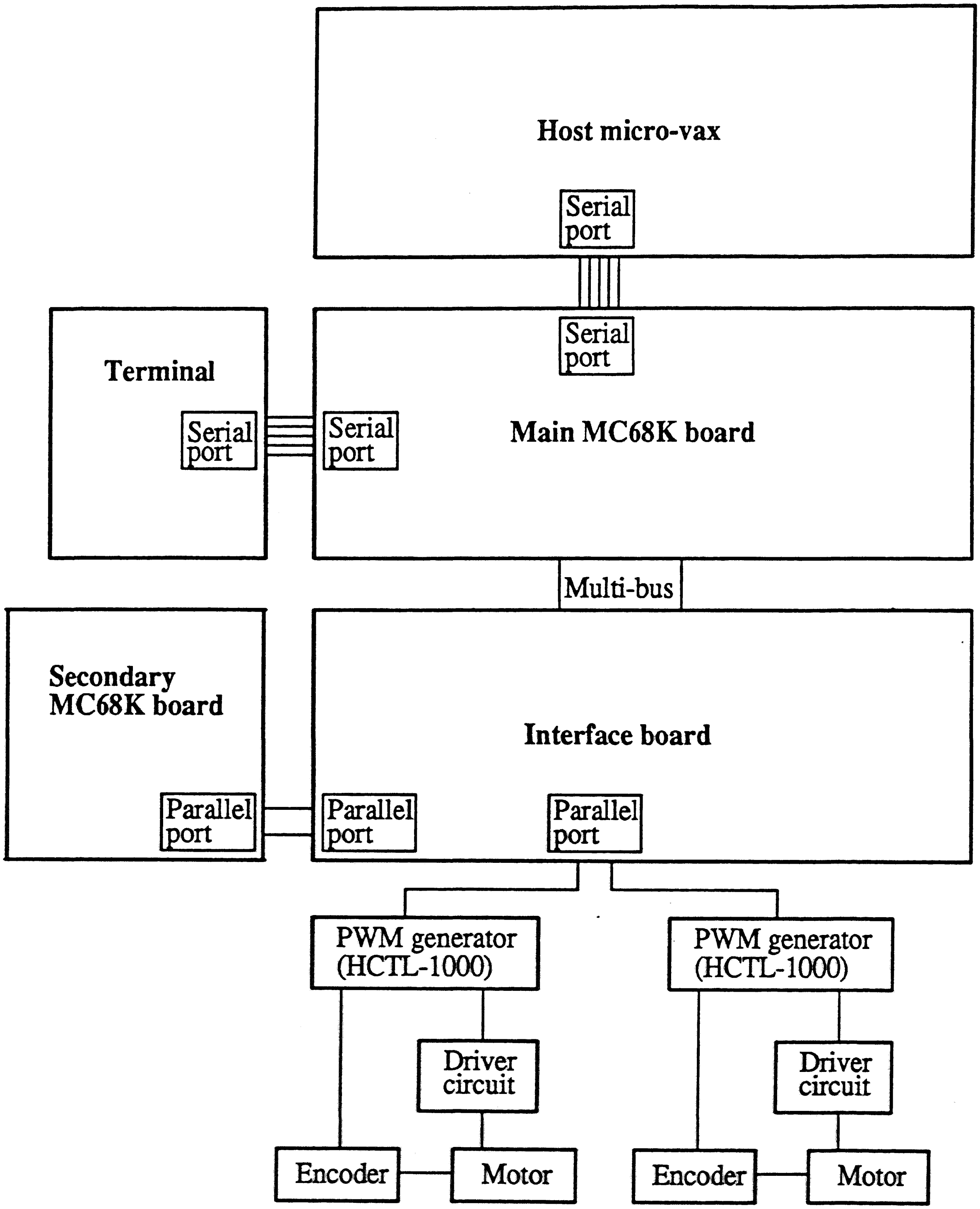

Figure 9: Control implementation of a two degree-of-freedom manipulator

2. The PWM generator converts the torque command into a pulse width modulated voltage signal to the motor resulting in the application of the torque to the link.

3. The motor position is measured by optical encoders and transmitted to the MC68K microprocessor board where it is stored until needed by the host computer (for various purposes).

6.2 Experimental procedure

We describe the procedure for experimentally determining S_τ. Because S_τ is a parallelogram in the $\ddot{\mathbf{x}}$ - plane (see Figure 5), it is sufficient to obtain the four vertices A', B', C', and D' of S_τ which correspond, respectively, to the vertices A, B, C, and D of the torque set T shown in Figure 3. Furthermore, because the origin of the acceleration plane is the centroid of the parallelogram $A'B'C'D'$, it is sufficient to determine the vertices A' and B' which correspond, respectively, to the vertices A and B of the torque set T.

If τ_{1o} and τ_{2o} denote, respectively, the magnitude of the maximum actuator torques at joints 1 and 2, then

1. in order to generate point A' of S_τ, we should apply actuator torques τ_{1o} and τ_{2o}, respectively, at joints 1 and 2, and

2. in order to generate point B' of S_τ, we should apply actuator torques τ_{1o} and $-\tau_{2o}$, respectively, at joints 1 and 2.

The procedure to obtain the image point in S_τ (for example, A') corresponding to a point (τ_1, τ_2) in T (for example, A) is as follows:

1. Apply the actuator torques τ_1 and τ_2 at, respectively, joints 1 and 2.

2. Measure the joint variables $q_1(t)$ and $q_2(t)$ at regular sampling instants. (The particular sampling time chosen was 0.01 second.)

3. Obtain the second rates-of-change of the joint variables $\ddot{q}_i(k)$, i=1,2, at the k^{th} sampling instant from the following finite-difference equations,

$$\ddot{q}_i(k) = \frac{q_i(k+2) + q_i(k) - 2q_i(k+1)}{\Delta t^2} \tag{2.37}$$

Experimental result	Calculated result	Error
$\ddot{\mathbf{x}}(A')_{exp} = (-4.08, 6.02)$	$\ddot{\mathbf{x}}(A')_{cal} = (-3.91, 6.65)$	
$\ddot{\mathbf{x}}(B')_{exp} = (2.96, 6.31)$	$\ddot{\mathbf{x}}(B')_{cal} = (3.91, 6.98)$	
$\mid \ddot{\mathbf{x}}(A')_{exp} \mid = 7.27$	$\mid \ddot{\mathbf{x}}(A')_{cal} \mid = 7.71$	6 %
$\mid \ddot{\mathbf{x}}(B')_{exp} \mid = 6.97$	$\mid \ddot{\mathbf{x}}(B')_{cal} \mid = 8.00$	13 %

Table 4: Comparison of experimental and calculated accelerations for two data points

where k, k+1 and k+2 denote, respectively, the k^{th}, $(k+1)^{th}$ and $(k+2)^{th}$ sampling instants, and Δt is the sampling time.

4. Determine the required acceleration of P, $\ddot{\mathbf{x}}^P$ from

$$\ddot{\mathbf{x}}^P = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \mathbf{J}\ddot{q} + \dot{\mathbf{J}}\dot{q}. \tag{2.38}$$

Since

$$\dot{q} \cong 0,$$

$$\ddot{\mathbf{x}}^P \cong \mathbf{J}\ddot{q}. \tag{2.39}$$

The coordinates $\ddot{x}_1$ and $\ddot{x}_2$ obtained from the equation (2.38) above is the required image point in $\bar{S}_\tau$. The following details apply to the particular experiments which we performed:

1. The experiments were performed for the configuration $q_1 = 0^o$ and $q_2 = 90^o$;

2. The parameters for the two links are given in section 5.

3. The maximum actuator torques applied were $\tau_{1o} = 8.12$ Nm and $\tau_{2o} = 0.17$ Nm, which were in the set of actuator constraints determined in the previous section.

6.3 Experimental results

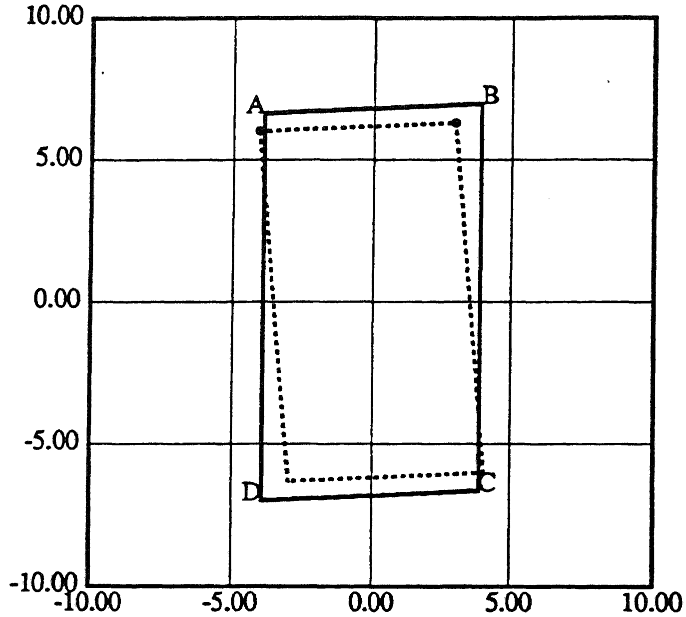

Figure 10: Comparison of experimental (dotted parallelogram) and calculated (solid parallelogram) start-up acceleration capability of the manipulator

The experimental results for the determination of the points A' and B' are given in Table 4 and graphically described in Figures 10. Also included are the theoretical results. From Table 4, we see that the experimental and theoretical results agree within experimental error (< 15 %) and are certainly good enough for our purposes.

In Table 5, we compare the values of the start-up acceleration properties $a_{max,su}$ and $a_{iso,su}$ obtained from experiment and theory; the theoretical and experimental results agree to within 10 %. The results of the experiment demonstrate the feasibility of using our theory to determine acceleration capabilities.

Acceleration properties (experiment)	Acceleration properties (theory)	Error
$a_{\mathrm{max,su}} = 7.27$	$a_{\mathrm{max,su}} = 8.0$	9 %
$a_{\mathrm{iso,su}} = 3.63$	$a_{\mathrm{iso,su}} = 3.91$	7 %

Table 5: Comparison of experimental and calculated acceleration properties (m/s^2), (Error $= | a_{exp} - a_{cal} | / a_{cal}$)

7 Summary and conclusions

Using the theory of acceleration sets, (Desa and Kim, 1989), we have defined (in section 3) six performance measures which can be used as a basis for designing manipulators for performance. We then illustrated the usefulness of these performance measures by applying them to the solution of the following two manipulator design problems.

1. Selection of the "best" manipulator type from a set of alternative manipulator types

2. Determination of minimum actuator sizes to achieve desired isotropic acceleration.

An explicit procedure was given in section 4 to solve the first problem, viz. "type selection". Algorithms for the determination of actuator sizes are given in section 5.

Finally in section 6, we addressed the experimental determination of the maximum and isotropic start-up acceleration and presented experimental results which verified the theory for start-up acceleration sets and start-up acceleration properties.

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Appendix. Equations of motion for planar manipulators

1. Jacobian matrix

The joint velocity is related to the velocity in Cartesian space by the Jacobian matrix,

$$\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}}.$$

The Jacobian matrix **J** for the three types of manipulators are as follows:

Manipulator type 1:

$$\mathbf{J} = \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos q_1 + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}$$

Manipulator type 2:

$$\mathbf{J} = \begin{bmatrix} -l_1 \sin q_1 & -l_5 \sin q_2 \\ l_1 \cos q_1 & l_5 \cos q_2 \end{bmatrix}$$

Manipulator type 3:

$$\mathbf{J} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix}.$$

When this relationship is differentiated with respect to the time, we obtain the following equation,

$$\ddot{\mathbf{x}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}} = \mathbf{J}\ddot{\mathbf{q}} - \mathbf{E}\{\dot{\mathbf{q}}\}^2 \tag{2.40}$$

where **E** is the matrix which has the following elements:

Manipulator 1:

$$\mathbf{E} = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \\ l_1 \sin q_1 + l_2 \sin(q_1 + q_2) & l_2 \sin(q_1 + q_2) \end{bmatrix}$$

Manipulator 2:

$$\mathbf{E} = \begin{bmatrix} l_1 \cos q_1 & l_5 \cos q_2 \\ l_1 \sin q_1 & l_5 \sin q_2 \end{bmatrix}$$

Manipulator 3:

$$\mathbf{E} = \begin{bmatrix} l_1 \cos q_1 & l_2 \cos q_2 \\ l_1 \sin q_1 & l_2 \sin q_2 \end{bmatrix}$$

2. Dynamic equations

The dynamics of a two-degree-of-freedom planar manipulator is described by the following equation:

$$\mathbf{D}\ddot{\mathbf{q}} + \mathbf{V}\{\dot{\mathbf{q}}\}^2 = \tau. \tag{2.41}$$

The components of matrices **D** and **V** are as follows:

Manipulator 1:

$$\mathbf{D} = \begin{bmatrix} I_1 + m_1 a_1^2 + I_2 + m_2(a_2^2 + 2a_2 l_1 \cos q_2 + l_1^2) & I_2 + m_2(a_2^2 + a_2 l_1 \cos q_2) \\ I_2 + m_2(a_2^2 + a_2 l_1 \cos q_2) & I_2 + m_2 a_2^2 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 0 & -m_2 a_2 l_1 \sin q_2 \\ m_2 a_2 l_1 \sin q_2 & 0 \end{bmatrix}$$

Manipulator 2:

$$\mathbf{D} = \begin{bmatrix} I_1 + m_1 a_1^2 + m_3 l_1^2 + I_4 + m_4 a_4^2 & (m_4 a_4 l_2 + m_3 a_3 l_1)\cos(q_1 - q_2) \\ (m_4 a_4 l_2 + m_3 a_3 l_1)\cos(q_1 - q_2) & I_2 + m_2 a_2^2 + m_4 l_2^2 + I_3 + m_3 a_3^2 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 0 & (m_4 a_4 l_2 + m_3 a_3 l_1)\sin(q_1 - q_2) \\ -(m_4 a_4 l_2 + m_3 a_3 l_1)\sin(q_1 - q_2) & 0 \end{bmatrix}$$

Manipulator 3:

$$\mathbf{D} = \begin{bmatrix} I_1 + m_1 a_1^2 + m_2 l_1^2 & m_2 a_2 l_1 \cos(q_1 - q_2) \\ m_2 a_2 l_1 \cos(q_1 - q_2) & I_2 + m_2 a_2^2 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 0 & m_2 a_2 l_1 \sin(q_1 - q_2) \\ -m_2 a_2 l_1 \sin(q_1 - q_2) & 0 \end{bmatrix}$$

The nonlinear vector $\{\dot{\mathbf{q}}\}^2$ is as follows:

Manipulator 1:

$$\{\dot{q}\}^2 = \begin{bmatrix} \dot{q}_1^2 \\ (\dot{q}_1 + \dot{q}_2)^2 - \dot{q}_1^2 \end{bmatrix}$$

Manipulator 2:

$$\{\dot{q}\}^2 = \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \end{bmatrix}$$

Manipulator 3:

$$\{\dot{q}\}^2 = \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \end{bmatrix}$$

3. Acceleration equation

The expression of the acceleration of the end-effector is as follows:

$$\ddot{\mathbf{x}} = \mathbf{A}\tau + \mathbf{B}\{\dot{\mathbf{q}}\}^2 \quad (2.42)$$

where

$$\mathbf{A} = \mathbf{J}\mathbf{D}^{-1} \quad (2.43)$$

$$\mathbf{B} = -\mathbf{A}\mathbf{V} - \mathbf{E} \quad (2.44)$$

where **J**, **D**, **V** and **E** are given above for each manipulator type.

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