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**A Screening and Optimization Approach
for the Retrofit of Heat Exchanger Networks**

by

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FOR THE RETROFIT OF HEAT EXCHANGER NETWORKS**

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ABSTRACT

This paper presents a systematic procedure for the retrofit design of heat exchanger networks. The approach is divided into a prescreening stage and an optimization stage. In the prescreening stage, the economic feasibility of the project is analyzed with lower bounds on cost for utility, additional area, and structural modifications. The bounds are used to construct a prescreening cost plot which shows the best possible savings that can be derived from a retrofit project. In the optimization stage, information from the prescreening stage is used to construct a novel superstructure which has embedded alternative retrofit designs for reassigning existing exchangers to different streams, introducing new exchangers to the network, and adding piping and area modifications. The superstructure is optimized with a mixed integer nonlinear programming (MINLP) formulation to determine the retrofit network requiring least total annual cost. Heat loads, minimum approach temperature (EMAT), and hot stream/cold stream matches are not fixed but are optimized in order to accurately account for the tradeoffs between capital and energy cost. Several examples are presented to illustrate this method.

INTRODUCTION

In recent years, the retrofit of existing heat exchanger networks (HEN) has become more important than the design of new networks. Often times, proper redesign of an existing network can reduce significantly the operating costs in a process. In addition, a network may require redesign when process modifications in the plant alter the conditions of the process streams. Previous research work, however, has mainly been directed to developing methods for the design of grassroots HEN's (e.g. Cerda and Westerberg (1983), Linnhoff and Hindmarsh (1983), Floudas et al. (1986)). For an extensive review, also see Gundersen and Naess (1988). Grassroots methods are of limited use in retrofit applications since they cannot account for the existing equipment and piping layout. In general, the existing equipment and the layout must be considered in order to properly account for the tradeoffs between energy and capital cost.

Systematic identification of optimal retrofit designs is considerably more difficult than for grassroots networks. Consider as an example the existing network shown in Figure 1a. The small network consists of five exchangers involving one hot and two cold process streams. Steam and cooling water are used as utility streams. The optimal retrofit design with a project payback time of about 1 year is shown in Figure 1b, and which in fact was obtained with the method suggested in this paper. The design actually calls for the total use of steam to heat up stream C1, while the network is repiped so that both exchangers 1 and 2 are now being used to heat up stream C2. This simple repiping along with some additional area in exchanger 1 reduces the utility cost by \$60,000/yr with an investment of \$62,000. Thus, with this example, it should be clear that there is an economic incentive to develop systematic procedures for retrofit design.

Only a few papers have been published on the retrofit of HEN's. Tjoe and Linnhoff (1986) proposed a procedure that is based on the pinch design method. A level of energy recovery is first determined by a targeting procedure accounting for desired payback time. The existing exchangers that exchange heat across the pinch are shifted or rematched strictly above or below the pinch or eliminated. Additional exchangers are also placed above and below the pinch if necessary. Finally the network is evolved manually using heat loops and paths to a retrofit network that is as closely compatible to the existing one as possible.

Jones et al. (1986) proposed an evolutionary technique which takes advantage of the capabilities of the computer package HEXTRAN. The method generates alternative grassroots designs and then

selects one subject to desired payback time. Modified versions of the selected design are simulated to evolve a final retrofit design that is compatible with the existing network. Saboo et al. (1986) use an evolutionary approach based on RESHEX's capabilities for nonlinear optimization, constrained MILP synthesis and feasibility evaluation. The program is used to generate a number of alternative retrofit designs of varying complexity. All the alternatives are then analyzed and the best retrofit design is selected.

Yee and Grossmann (1987) developed the MILP assignment-transportation model to predict the fewest structural modifications needed in order for an existing network to achieve a certain level of energy recovery (HRAT). The motivation for the model is to determine network configurations which closely resemble the existing one from a structural standpoint. The objective function considered in this model is to maximize the use of the existing exchangers, while minimizing first addition of new units and second the reassignment of existing units to different streams. Ciric and Floudas (1988) extended the work by Yee and Grossmann (1987) by further categorizing the levels of structural modifications in the MILP model. They also attempt to estimate cost for additional area requirements although the model will generally overestimate this requirement. The optimal solution determined in the MILP model is then used to postulate a superstructure with fixed heat loads and matches and solved by the NLP model by Floudas et al. (1986) to attain the final retrofit network configuration.

It should be noted that none of the methods presented above can accurately account for the tradeoff between energy and capital costs. Some of the methods are evolutionary in nature and mainly utilize grassroots techniques, though with some extension or modification. Also, all the methods appear to follow the grassroots techniques of optimizing for targets progressively; ie. optimize first utility requirement, then number of units, and finally area. For grassroots design, this may not be a very significant limitation. However, for the retrofit case, since capital cost arises from modifications on the existing network, a progressive optimization method cannot ensure near minimum annual cost. This is due to the fact that different levels of energy recovery may require significantly different modifications and that the capital expenses must be restricted to reasonable payout times. Thus, simultaneous consideration for energy and modification cost appears necessary.

In this paper, a two stage prescreening and optimization approach will be presented which will properly account for the tradeoffs between energy and capital cost. The procedure starts with a prescreening stage in which the economic feasibility of a retrofit project for the existing network at hand is

evaluated. This involves the calculation of lower bounds on cost for utility, additional area, and structural modifications subject to various levels of heat recovery (HRAT). The bounds are used to construct a prescreening cost plot which shows the best savings that can be achieved by the retrofit project. If the savings appear promising, then the optimization stage is carried out to determine a retrofit network requiring least annual cost. In the optimization stage, information on structural modifications from the prescreening stage is used to construct a novel superstructure which for a fixed number of units has embedded all alternative retrofit designs of interest. In the superstructure, energy recovery, heat loads, minimum approach temperature (EMAT), and stream matches are not fixed and are left to be optimized in order to properly account for tradeoffs between capital and energy cost. The optimal retrofit design is determined by the solution of a MINLP model with an objective of minimizing annual cost arising from utility, additional area, additional units, and repiping requirements. It should be noted that the model can easily handle constraints on piping structures, matches and heat loads.

PROBLEM STATEMENT

For the retrofit design problem considered in this paper, it will be assumed that the following information is available from the existing network:

- The hot and cold streams for heat integration and their respective flowrates and heat capacities
- The available utilities for heating and cooling and their cost
- The inlet and required outlet temperatures for the streams and utilities
- The individual heat transfer coefficients for the streams or overall heat transfer coefficients for all potential matches
- The stream matches involved in each of the existing exchangers
- The heat transfer areas of all the existing exchangers
- The existing piping structure of the network.
- The utility usage by the existing network
- The cost data for additional area, additional exchangers, and piping modifications
- The constraints on the possible reassignments of the existing exchangers other matches.

With the given information, the objective of the proposed method is to determine a retrofit design exhibiting least annual cost and possibly subject to a constraint for payout time. The solution defines the retrofit network by providing the required operating conditions and the necessary modifications to transform the existing network to the retrofit design. Specifically, the solution provides the following:

1. Utilities required
2. Stream matches involved at each exchanger
3. Heat loads and operating temperatures of each exchanger
4. Piping modifications required and flows of all branches of piping
5. Additional exchangers and additional area required

The proposed methodology for determining the retrofit design with least annual cost is illustrated in Figure 2. As mentioned previously, the general strategy for determining the optimal retrofit design is a two stage approach involving a prescreening stage and an optimization stage. In the next two sections, the methodology is presented in detail starting with the prescreening stage.

PRESCREENING STAGE

The primary motivation behind the prescreening step is to determine the economic feasibility of the retrofit project. Lower bounds for total annual cost comprising of cost for utility, cost for additional area requirement and fixed cost for structural modifications (ie. repiping and installation of new exchanger units), are estimated for projects with various levels of energy recovery (HRAT). These lower bounds can then be compared to the existing annual cost to evaluate the incentive for performing retrofit. In order to determine lower bounds for annual total cost lower bounds for each of the contributing costs are determined. Some grassroots ideas are used for this purpose.

Minimum utility requirement for a particular value of HRAT can be predicted using the IP transshipment model of Papoulias and Grossmann (1983) which can easily accommodate constraints on stream matches. Since for the prescreening an entire range of HRAT must be considered, a proposed method is outlined in Appendix A which generates the minimum utility cost plot by parametrically solving the transshipment model for a range of HRAT. Solution of the model provides the slope(s) of the plot and the HRAT's for which the slope changes; eg. where the pinch location changes. This information can then be used to construct the minimum utility cost vs. HRAT plot.

Minimum additional heat transfer area requirement can be estimated using the area targeting method proposed by Townsend and Linnhoff (1984). This method determines an area target, which provides a good estimation of the area for the grassroots network. For prescreening in the retrofit case, this target is used to estimate the additional area required by assuming that all of the existing area can be fully utilized in the retrofit network. This assumption, though, may not hold true for the general case.

However, since the objective is to identify a lower bound for the additional area required, the assumption is valid even though it may be conservative. The minimum additional area required is thus estimated by the following equation:

$$\text{Additional area required} \ll \max(0, \text{area target} - \text{existing area}) \quad (1)$$

Besides the cost for additional area, structural modifications performed on the network can constitute another major capital expenditure. Minimum structural modifications required can be predicted by the MILP assignment-transshipment model proposed by Yee and Grossmann (1987). Optimization of the model determines, for a particular value of HRAT, a set of matches that allows for maximum use of existing exchanger units, which means a minimum number of reassignments of existing exchangers to different streams and more importantly, minimum number of new exchangers required. Lower bounds on structural modification cost, therefore, can be estimated by assigning fixed charges to the minimum structural modifications identified by the model. Since for prescreening, a range of HRAT values must be considered, a parametric solution of the assignment-transshipment model is proposed in Appendix B. The idea is to divide the range of HRAT into small intervals (e.g. 5K) and then solve an assignment-transshipment model subject to any HRAT within that interval. The fixed cost associated to the structural modifications predicted by the model will then be taken as the lower bound for all the HRAT's in the small interval.

To clarify, assume that the actual structural modifications needed are represented with fixed charges on the plot in Figure 3(a). By dividing the range of HRAT into small intervals and solving an assignment-transshipment model for each interval, the fixed charges estimated are represented in Figure 3(b). It is clear that the estimates in Figure 3(b) are lower bounds for the actual fixed charges indicated in Figure 3(a). It is interesting to note, as indicated by Figure 3(a), that structural modifications do not necessarily increase with higher levels of energy recovery. Change of pinch point location, exact matchings of heat loads, and the changes in stream population in the subnetworks all affect the number of structural changes required.

Having estimated the lower bounds, cost information can be summarized by the construction of a prescreening cost plot of Annual Cost vs. HRAT. A typical plot is illustrated in Figure 4. Minimum annual costs for utility, additional area and structural modifications are summed up to determine the total annual cost curve. Inspection of the prescreening cost plot can identify regions of HRAT for which retrofit

projects are economically unattractive. More importantly, it provides an estimate on the best potential savings for the retrofit project. This information can be used to determine whether the project can satisfy minimum payback time requirements prior to the design stage.

If the prescreening step indicates the desirability of the retrofit project, structural modification information (ie. the number of new units that may be required by the retrofit design) may be carried into the optimization stage for the construction of a retrofit superstructure. It should be noted that the design decisions on the heat recovery are postponed for the optimization stage, and therefore, no value of HRAT gets fixed in the prescreening stage. The next section will describe in detail the optimization stage, including the construction of the retrofit superstructure and the formulation and solution of its MINLP representation.

OPTIMIZATION STAGE

Construction of Superstructure

The first step in the optimization stage is the construction of a retrofit superstructure. The superstructure, with a given number of potential exchangers, is a two dimensional network layout which has embedded within all the alternative retrofit designs of interest. The major novelty in this superstructure is the fact that exchanger units are not committed to any particular stream matches. That is, in principle, each exchanger can be assigned to any pair of hot and cold streams. Also, the two dimensional layout of the piping is explicitly considered and the possibilities of mixing different process streams can also be examined. A three exchanger superstructure involving one hot stream and two cold streams is shown in Figure 5. The components of the superstructure are the following:

- Initial Stream Splitters
- Exchanger Inlet Mixers
- Heat Exchanger Units
- Exchanger Outlet Splitters
- Final Stream Mixers
- Piping Segments

The number of exchangers to be included in the superstructure can be determined in the prescreening stage from the information on minimum structural modifications necessary along with the prescreening cost plot. Specifically, embedded in the superstructure should be the existing units along with the

maximum number of potential new units that may be involved in an attractive retrofit design. It should be noted, though, that the inclusion of a new unit in the superstructure does not mandate its use in the final retrofit network. Besides heat exchangers, mixers and splitters are explicitly shown in the superstructure to represent mixing and splitting points in the HEN. Segments of piping are also shown for connecting the different components in all possible ways.

The flow scheme through the superstructure can be explained starting from the Initial Stream Splitters which represent the entry points to the HEN (see Figure 5). Inlet flow of a stream is distributed from the Initial Splitter to the Inlet Mixers of each exchanger. Each Inlet Mixer combines flows from the Initial Splitters and flows from the outlet of other exchangers (via Outlet Splitters) and sends the resulting flow to the Heat Exchanger Unit. The stream exchanges heat in the Heat Exchanger Unit and then enters an Outlet Splitter where it is either sent to another exchanger via an Inlet Mixer or exits the HEN at a Final Stream Mixer.

The configuration of a particular network embedded in the superstructure is designated by a particular selection of the Piping Segments and Heat Exchanger Units. For the superstructure in Figure 5, two embedded alternative retrofit designs are shown in Figure 6. In Figure 6(a), a simple series configuration is shown to be embedded in the superstructure. In Figure 6(b), a more complicated series and parallel configuration is shown where the cold streams are now assigned to different exchangers as compared to Figure 6(a). It should be obvious that many other retrofit configurations involving combinations of series, parallel, and bypass structures are present within the superstructure.

The attractive features of this superstructure and its applicability to the retrofit problem can be summarized by the following:

- Reassignment of existing exchangers to different streams can be considered in order to maximize the use of existing area and exchangers
- Possibility of mixing different process streams can be considered
- Complex piping structures can be explored
- Each segment of repiping can be accounted for explicitly
- Addition of new heat exchanger units can be considered
- Heat loads (ie. HRAT), minimum approach temperatures (EMAT), and stream matches need not be fixed
- Constraints on heat loads, piping structures and stream matches can be easily incorporated

MINLP Formulation

In order to determine the best retrofit design embedded in the superstructure, a mixed integer nonlinear programming (MINLP) model is formulated in this section. Binary variables are associated with each segment of piping and the selection of streams for each exchanger. The general model involves mass balances at mixers and splitters and energy balances at mixers and exchangers. In addition, constraints can be included to prevent mixing of different process streams. Design equations determine the additional heat transfer area required at each existing exchanger as well as new exchangers. The MINLP model is solved to select from amongst all the alternatives embedded, the one that exhibits least annual cost.

In developing the MINLP model of the superstructure proposed in this paper, it should be noted that a complication that arises in the modelling is the fact that the identity of the streams and of the exchangers is not predetermined. For example, for any hot side piping segment between a given pair of exchangers, it is not known a priori which hot process stream will be assigned to it. Also, in each exchanger, it is not known a priori which pair of streams will be assigned to it. This is in great contrast with the superstructure developed by Floudas et al. (1986) where the modelling is greatly simplified by the fact that it is known in advance what particular process streams correspond to each piping segment in the superstructure, and what pair of streams are involved in each exchanger.

In order to formulate the proposed MINLP model, the following definitions are necessary:

(i) Indices

i * hot process or utility stream

j « cold process or utility stream

k - index for heat exchanger

l - index for heat exchanger

(ii) Subscripts and superscripts

c - cold side

A - hot side

s m hot or cold process or utility stream

$exit$ » network exit point

IN m inlet condition to superstructure

OUT « outlet condition from superstructure

in m inlet of an exchanger

out m outlet of an exchanger

(iii) Sets

HP « $\{i|j\}$ is a hot process stream)

HU - $\{i|j\}$ is a hot utility stream)

CP « $\{j\}$ is a cold process stream

CU « $\{j\}$ is a cold utility stream

$CmCPUCU$

$H-HPUHU$

$HCmHPUCP$

$HCTmHUC$

$MA(k)$ « $\{(ij)\}$ is a possible match for exchanger k

E_m « $\{k\}$ is an exchanger in the superstructure

$EE - \{k\}$ represents an existing exchanger in the superstructure

NE « $\{k\}$ represents a new exchanger in the superstructure

$NP - \{(kj)\}$ a piping segment connecting exchangers k and l does not exist in the existing network

$NPE - \{(j|t)\}$ stream s does not enter the existing HEN at exchanger k

(iv) Parameters

co « cost parameter

DT - temperature change

EA_k - existing area for exchanger k

F_m heat capacity flowrate

T » temperature

K_m total number of exchangers in superstructure

U^* an upper bound

ϵ_{ij} - overall heat transfer coefficient for match (ij)

(v) Variables

A_{iA} - Additional area required by heat exchanger k

AEA_k^* Existing area assigned to the location of exchanger k

TD_k » temperature difference or driving force for heat transfer

dt_k » variable temperature change

f » variable heat capacity flowrate

Q_{ijk} » heat exchanged between hot i and cold j at exchanger k

t - variable temperature

v_u » assignment variable to assign existing area of exchanger l to the location of exchanger k

w_{ij}^* « binary variable to denote service of exchanger k for stream s

x_{ij}^* » binary variable to denote that exchanger k and l service the same process stream s

y » binary variable to denote that the inlet of stream s is assigned to exchanger k

z_u - binary variable to denote existence of a piping segment connecting exchangers k and l (hot or cold side)

$z_{k,exit}$ binary variable to denote existence of piping segment from exchanger k to exit of HEN

With the definitions above, the constraints are as follows:

Mass balances for Splitters and Mixers

Mass balance constraints at each splitter and mixer determine flows of each Piping Segment. For simplicity, no sets of mixers or splitters are defined but rather it is assumed that the constraints are written for each applicable splitter or mixer in the superstructure:

Mass balance at each Initial Stream Splitter

$$\sum_{k \in E} f_k^* - f_l - \theta_{seHC} = 0 \quad (2)$$

Mass balance at each Exchanger Inlet Mixer

$$\sum_{i \in HP} t_i + \sum_{\substack{l \in E \\ l^*k}} A_{l^*k} + \sum_{ti \in HU} F_{i \rightarrow l} - f_k^{in} = 0 \quad k=1,2,\dots,K$$

$$\sum_j f_j^* + \sum_{fi \in I} f_{i \rightarrow k} - \sum_{FJ \in W} F_{J \rightarrow k} - f_k^{in} = 0 \quad k=1,2,\dots,K \quad (3)$$

Mass balance at each Exchanger Outlet Splitter

$$f_k^{in} - \sum_{E} f_{kl} - f_{k,exit} = 0 \quad k=1,2,\dots,K$$

$$f_k^{in} - \sum_{E} F_{k \rightarrow l} = 0 \quad k=1,2,\dots,K \quad (4)$$

To avoid mixing different process streams in equation (3), the following logical constraints can be included:

Logical constraints for Initial Piping Segments

$$y_j - U \cdot y_l \leq 0 \quad k^* \setminus X, X \quad s \in HC \quad (5)$$

Selection of entry point for process streams

$$\sum_{k \in E} y_k^* \leq 1 \quad seHC$$

$$Y_{j^*} \cdot y_l \leq 1 \quad 4=U..JT \quad (6)$$

sthC

Note that these constraints can be relaxed if mixing of different process streams is allowed.

Logical Assignment Constraints

Pure integer constraints are needed for proper assignment of streams to exchangers. For existing exchangers, only one hot and one cold stream can be assigned. For new exchangers, at most one hot and one cold stream can be assigned depending on whether or not the new exchanger gets selected for the retrofit network:

Selection Of exchanger service

$$\begin{aligned}
 \sum_{i \in H} w_i^k &= 1 & k \in EE \\
 \sum_{j \in C} w_j^k &= 1 & k \in EE \\
 \sum_{i \in H} w_i^k &\leq 1 & k \in NE \\
 \sum_{j \in C} w_j^k &\leq 1 & k \in NE
 \end{aligned} \tag{7}$$

Also, to avoid mixing of heat loads of different process streams, the following logical constraints are needed:

Logical constraints for service selection at each exchanger

$$Q_{ijk} - w_s^k \leq 0 \quad \forall s \in HCT \quad k=1,2..j \quad (i,j) \in MA(k) \tag{8}$$

Logical Interconnection Constraints

Consistency constraints are necessary to allow for a connection between two exchangers only if the two exchangers service the same process streams. By defining the variable x_{ij} to denote that stream s is assigned to both exchangers k and l , the variables z_{ij}^k, z_{ij}^l which denote the existence of piping sections can be related to the assignment exchanger variables w_i^k, w_j^l as follows:

Constraints for Heat Exchanger Units

The Heat Exchanger Units are modelled by heat balances around each exchanger to relate inlet and outlet temperatures with heat loads and design equations to determine the additional area needed. Also, minimum approach temperature constraints are needed to ensure thermodynamic feasibility for heat transfer.

Heat balance around exchanger

For determining temperature change around an exchanger, the heat balance equation is relaxed as an inequality. This relaxed form aside from being rigorous (from the Kuhn-Tucker conditions) is also needed since the utility streams are not preassigned to exchangers in the superstructure. As a result, the equality form of the heat balance equation would not be appropriate for utility streams with a specified temperature change.

$$\begin{aligned}
 \sum_{(i,j) \in MA(k)} Q_{out} - f_i^* \Delta t^h & \leq 0 & * = 1, 2, \dots, JT \\
 \sum_{(i,j) \in MA(k)} Q_{ijk} - f_k^* \Delta t^c & \leq 0 & b^* |X \times X \\
 dt_k^h & = t_k^{h,in} - t_k & k = |X, JC \\
 dt_k^c & = t_k^{c,out} - t_k^{c,in} & k = 1, 2, \dots, K
 \end{aligned} \tag{12}$$

Furthermore, for determining temperature changes in cases of utility streams, the following bounding equations are needed:

Bounds on change of temperature around exchangers

$$\begin{aligned}
 t_k^{h,in} - t_k^{h,out} & \geq \sum_{(ij) \in MA(k)} (Q_{ijk} / F_i^{IN}) + \sum_{i \in HU} (COT, w_i^h) & k = 1, 2, \dots, K \\
 t_k^{c,out} - t_k^{c,in} & \geq \sum_{\substack{(V) \in MA(k) \\ > \epsilon CP}} (Q_{ijk} / F_j^{IN}) + \sum_{j \in CU} (DT_j, w_j^c) & k = 1, 2, \dots, K
 \end{aligned} \tag{13}$$

Exchanger design equation

A relaxed form of the exchanger design equation can be written as follows to determine additional area requirements:

$$\sum_{i \in H} \sum_{j \in C} (Q_{ijk} / U_{ij}) - (EA_k + AEA_k) TD_k \leq 0 \quad k=1,2..K \quad (14)$$

where TD_k can be the arithmetic mean temperature difference or the log mean temperature difference:

$$TD_k = (t_k^{h,in} - t_k^{c,out} + t_k^{h,out} - t_k^{c,in}) / 2 \text{ or}$$

$$TD_k = ((t_k^{h,in} - t_k^{c,out}) - (t_k^{h,out} - t_k^{c,in})) / \text{LOG}((t_k^{h,in} - t_k^{c,out}) / (t_k^{h,out} - t_k^{c,in}))$$

It should be noted that in cases where exchangers can be physically moved to a different location in the network, e.g. exchanger k can trade location with exchanger l , additional assignment variables and constraints can account for these possibilities. Given assignment variables v_{kl} , existing area of exchanger l , EA_l , can be assigned to exchanger k by the following equations:

$$\begin{aligned} \sum_{l=1}^K v_{kl} &= 1 & k=1,2..K \\ \sum_{k=1}^K v_{kl} &= 1 & l=1,2..K \\ AEA_k &= \sum_{l=1}^K (EA_l v_{kl}) \end{aligned} \quad (14a)$$

With the introduction of these constraints, the parameter EA_k in the exchanger design equation (14) is then replaced by the assigned area variable AEA_k .

Approach temperature constraints

Even though minimum approach temperatures are not fixed in the model, constraints are needed to ensure thermodynamic feasibility.

$$\begin{aligned} t_k^{h,out} - t_k^{c,in} &\geq 0 & k=1,2..K \\ t_k^{h,in} - t_k^{c,out} &\geq 0 & k=1,2..K \end{aligned} \quad (15)$$

Overall heat balance

An overall heat balance for each process stream ensures sufficient heating or cooling of the process streams.

$$\sum_{k \in E} Y_k - \sum_{j \in C} Y_j = F \quad (i,j) \in MA(k)$$

$$\sum_{k \in E} X_k - \sum_{j \in C} X_j = \text{fly}^* - \text{rf}^* \quad j \in CP \quad (i,j) \in MA(k) \quad (16)$$

Objective function

Finally, the objective function determines total annual cost for the retrofit networks by summing cost for additional area, fixed charges for new units and new segments of piping, and cost for hot and cold utilities required. The function is minimized to determine the retrofit network which exhibits minimum total annual cost.

$$\begin{aligned} \min \text{ Annual Cost} &= c_{OM} \sum_{k \in E} AA_k + c_{NB} \sum_{k \in NE} (w_i^* / 2 \cdot w_j^* / 2) \\ &+ c_{NP} \left(\sum_{(i,j) \in NP} (z_{ij}^h + z_{ij}^c) + \sum_{(i,out) \in NP} y_{ij}^h \right) \\ &+ c_{HV} \left(\sum_{(iV) \in MA(A)} X_{iV} \cdot c^A \right) \cdot \sum_{(ij) \in MA(k)} X_{ij} \cdot fl_{ij} \end{aligned} \quad (17)$$

Constraints (2) to (16) along with the objective function in equation (17) defines the generalized MINLP model for the superstructure. It should be noted that heat and mass balance constraints have not been included for the Final Stream Mixer. This is due to the fact that these constraints are dependent in that by satisfying the other constraints in the model, these constraints are automatically satisfied. However, if the stream outlets are specified as inequalities, these constraints can be included to account for possible piping modifications at the HEN outlet.

Also, it should be noted that the 0-1 variables for process streams se_{HC} , x_g^* and w_g^* can actually be treated as continuous variables that lie between 0 and 1. This is due to the fact that when binary variables, w_g^k for utility streams $g \in HUUCU$, y_{ij}^k , z_u and z^{\wedge} define the piping structure of the network by taking on 0-1 values, x_g^* and $se_{1 \in HC}$ automatically take on 0-1 values. Hence, these variables along with the remaining variables can be treated as continuous.

It should be noted that restrictions on the network structure can be easily included in the model by writing additional integer constraints. Certain assumptions, such as the no stream split requirement, can simplify the model significantly; ie. for the no stream split requirement, the heat balance constraints for the exchangers units, equations (12) can be eliminated since equations (13) will determine the change of temperature around the exchangers precisely.

REMARKS

In the solution of the proposed MINLP model, two important challenges arise. First, due to the many retrofit alternatives considered in the superstructure, the MINLP model will in general be of large scale requiring significant computational time to solve. Furthermore, the solution of the relaxed NLP (where binary variables can take on noninteger values) will usually have a significantly lower objective function value than the optimal integer solution for the MINLP model. This large gap is an indication of the difficulty for solving this problem. Its major implication is that the MILP master problem of any of the current MINLP methods will be expensive to solve. A second challenge arises from the fact that the model is nonconvex due to the presence of bilinear terms in the energy balance and exchanger design equations. For the general MINLP solution methods, the effects of these nonconvexities in the model may prevent the determination of a globally optimal solution, ie. only local optimality can be guaranteed.

As a result of these challenges, it is desirable to include reasonable simplifications and assumptions in the formulation which will ease the solution process. One simplification that can be made is the relaxation of nonlinear equations to inequalities (see equations (12), (13) and (14)). These relaxations can easily be shown to be rigorous, and they follow from a direct application of the Kuhn-Tucker conditions. Other assumptions can often be derived from the characteristics of the existing network such as the no stream split restriction discussed previously. However, one useful assumption that can be introduced is that no bypasses are allowed. A discussion is presented in Appendix C. This restriction helps to limit the piping complexity of the retrofit design, and also, bypasses are usually not needed if the exchanger approach temperature (EMAT) can be small (e.g. see Gundersen and Grossmann (1988)), which is allowed in the formulation. Based on this restriction, the nonconvex equations in (11) can be simplified by using a linear representation of the bilinear heat mixing constraints. This linear representation, therefore, eliminates the use of the many bilinear terms in the heat mixing equations which are nonconvex.

Another simplification that can be incorporated into the model is the use of arithmetic mean

temperature difference (AMTD) in the exchanger design equations. The use of AMID allows for a much simpler expression and, more importantly, better handling of nonconvexities in the design equations. In **general, the AMTD** provides reasonable approximations to log mean temperature difference (LMTD). **However, in cases** where the temperature approach of one side of the exchanger is significantly different than the other side, the approximation tends to overestimate the driving force, which results in an underestimation of the required heat exchange area. The effects of this underestimation, though, can be handled in the solution procedures introduced in the next section by a combined use of AMTD and LMTD (where AMTD is used in the master problem to predict rigorous lower bounds, while LMTD is used in the NLP subproblem to predict upper bounds).

MINLP SOLUTION METHODS

In view of the model difficulties discussed in the previous section, several methods have been tested for solving the proposed MINLP model. Specifically, the following methods were examined:

- Outer Approximation/Equality Relaxation (OA/ER) (Kocis and Grossmann (1987)) with piecewise approximations
- Generalized Benders Decomposition (GBD) (Geoffrion (1972)) with valid outer approximations
- Augmented Penalty version of the OA/ER (AP/OA/ER) (Viswanathan and Grossmann (1988))

All of these methods have the common characteristics of decomposing the model into an NLP subproblem and an MILP master problem. The NLP subproblem optimizes particular network structures, ie. binary variables are fixed, and yields an upper bound to the cost. The MILP master problem is optimized with an approximated feasible region, to select new network structures and to predict lower bounds on the cost. The procedure involves alternately solving a sequence of the subproblems and master problems until the lower bound is equal or exceeds the best upper bound for the cost. The main difference between the methods lies in the formulation of the MILP master problem.

OA/ER with Piecewise Approximations

The general steps of the OA/ER algorithm by Kocis and Grossmann (1987) are shown in Figure 7. The NLP subproblem optimizes particular network structures to identify an upper bound, $NLP(UB)$, for the overall MINLP minimization problem. The MILP master problem determines the potentially best network structures and predicts successively tighter lower bounds, $MILP(LB)$, for the MINLP model. The master problem is formulated by progressively adding linear outer approximations of the feasible regions. These outer approximations are derived by linearization of nonlinear terms in the MINLP formulation at points determined by the subproblem optimizations. They have the effect of overestimating the feasible region while underestimating the objective function value. The sequence of NLP subproblems and MILP master problems is solved until the $MILP(LB)$ predicted is greater or equal to the lowest $NLP(UB)$ identified. The network structure associated with this lowest $NLP(UB)$ represents the optimal solution for the MINLP model.

The straightforward application of the OA/ER method for the proposed MINLP formulation, however, cannot guarantee global optimality due to the existence of nonconvex bilinear terms in the heat balance and exchanger design equations. The linearization of these nonconvex terms in the master problem may actually cut off part of the feasible region. As a result, the solution of the master problem cannot be guaranteed to predict a rigorous lower bound and the OA/ER algorithm can therefore converge to a local optimum that is different from the global optimum.

In order to remedy this problem, the nonconvex bilinear terms can be eliminated through a transformation based on separable programming and by using piecewise linear approximations. The procedure is outlined in Appendix D. The basic idea is to transform the bilinear terms into separable form and then use piecewise linear approximation to estimate the nonconvex terms from the transformation. This scheme eliminates all the nonconvexities in the model and thus guarantees rigorous lower bounds for the OA/ER method. The expense, however, is that extra binary and continuous variables and constraints are added to the master problem, although the structure of the added constraints can be exploited, e.g. SOS2 constraints, as discussed in Appendix D.

In general, the major bottleneck in the proposed OA/ER method is that the master problem can be very large and computationally expensive. Even though the method can ensure global optimality from a structural standpoint, for problems involving a large superstructure, the method is computationally very expensive for solving the master problem. The advantages of the method though include the fact that

problems can generally be solved in a very few iterations. In addition, the NLP subproblems are small and easy to solve since they only involve the model for the particular network structure being analyzed. As a result additional details of retrofit can be included in the NLP subproblems to make the model more rigorous.

GBD with Valid Outer Approximations

An approach based on Generalized Benders Decomposition method (Geoffrion (1972)) can also be used to solve the proposed MINLP model. The main difference between the OA/ER and the GBO method is the formulation of the MILP master problem. For GBD, only dual constraints in terms of integer variables are generated by the solution of each NLP subproblem and included in the master problem along with pure integer constraints. A dual integer constraint is derived by evaluating the parametrized lagrangian in the integer variables at each NLP subproblem optimization. Typically, the master formulation for GBD is significantly smaller as compared to the one for OA/ER. In addition, the likelihood of obtaining a global optimal solution when nonconvexities are present increases since the bounding of the feasible region is very much less restrictive. However, the major drawback is that since the feasible region is loosely bounded in the master problem, the lower bounds are often weak, and hence, many iterations are usually required to reach convergence. In fact, many structures identified by the master problem may be infeasible. To circumvent this problem, it is desirable to include valid outer approximations in the master problem to strengthen the lower bound and avoid obtaining too many infeasible structures.

The basic idea behind the proposed outer approximations for the GBD method is to predict lower bounds on cost for retrofit structures so that designs that are infeasible or that have potentially high cost can be screened out. The outer approximations involve logical constraints which ensure proper assignment of service selection through the binary variables z_j . In addition, transshipment constraints (with HRAT-0) are included to estimate lower bounds for utility cost while linear exchanger design equations based on maximum temperature differences are used to estimate the additional area cost. These costs are combined with the structural modification cost which is predicted by assigning fixed charges to the binary variables in order to form a constraint on total cost. This estimated total cost is then bounded by the value of the best solution from the NLP subproblems so that no structure will be selected which has an estimated cost higher than the best NLP solution. Although making the master problem

larger, the incorporation of these outer approximations can screen out many undesirable or infeasible structures. However, it should be noted that for large problems, convergence of the method may still require a relatively large number of iterations.

Augmented Penalty OA/ER

A third method that was tested is a recently modified version of the OA/ER algorithm by Viswanathan and Grossmann (1988). The modification appears in the master formulation where a penalty function is included in the objective function to minimize the sum of slacks of the relaxed linearization constraints that cut into the feasible region. In addition, bounding of the feasible region in the master problem is also performed with a relaxed NLP solution of the MINLP formulation. As a result of these modifications, the AP/OA/ER method can often handle the effect of nonconvexities in the model. The effectiveness of this algorithm has been shown with the solution of a number of nonconvex problems where the AP/OA/ER algorithm was always able to obtain the global optimum solution.

In the next few examples, the three methods mentioned in this section were applied. Although some trends have emerged in terms of computational times, none of the three methods can yet efficiently solve very large superstructure problems. Development of improved algorithms to solve the problem in this paper is a subject of current research.

EXAMPLES

Example 1

In Example 1, the proposed method is applied to integrate the heat of a compressor aftercooler with a distillation reboiler. The existing structure and stream data are shown in Figure 8. The existing network consists of the compressor aftercooler and the column reboiler. The annual utility cost for the network is \$74,060. A retrofit project is proposed to decrease the utility consumption of the network by recovering heat from the hot stream (H1) of the compressor aftercooler to the cold stream (C1) of the column reboiler. The existing matches of the network are S1-C1 and H1-W1, and a constraint for the retrofit problem is that the outlet temperature of the cooling utility (W1) must be between 100°F and 135°F.

Since the problem is rather small, from inspection, a retrofit design can be determined as shown in Figure 9. In this intuitive solution, the total column reboiler is reassigned to exchange heat between H1

and C1. As a result, no heating utility is needed. With some minor repiping and 702 ft² of additional heat exchange area, the annual cost accounting for capital investment can be reduced to about \$35,000/yr, which represents a payback time of 1.32 year.

In order to compare this intuitive solution, the proposed method was used to determine the optimal retrofit design. Following the proposed procedure, the prescreening cost plot as shown in Figure 10 was constructed by targeting for minimum utility, structural modifications, and additional heat exchange area. As indicated by the plot, structural modifications needed for heat integration is either the addition of a new exchanger unit or at threshold condition, i.e. where only cold utility is required, the reassignment of an exchanger. Fixed charges for these modifications are \$10,000 and \$5,000 respectively. From inspection of the total cost plot in Figure 10, it is determined that a maximum savings of about \$50,000/yr is possible from a retrofit project.

With this incentive, the optimization stage of the proposed method was carried out. A superstructure embedding the two existing exchangers along with a potentially new exchanger unit which was predicted from the prescreening was constructed. Its corresponding MINLP model was formulated to minimize annual cost arising from utility, additional area, and fixed charges for new branches of piping and the installation of the new exchanger unit. The solution methods discussed in the previous section were applied to determine the retrofit design embedded within the superstructure which exhibits least annual cost. All three methods determined as the optimal retrofit design the network shown in Figure 11. The solution actually calls for the reassignment of the column reboiler to be used as a cooler exchanging heat between H1 and W1 while the aftercooler becomes the column reboiler as it gets reassigned to the match H1-C1. Also, note that stream H1 is being split. With an investment cost of \$74,350 for the additional piping and 405 ft² of additional area, the annual cost for the network reduces to \$29,390/yr or a payback time of 1.07 year. This represents an annual savings of about \$44,670, which is close to the savings estimated from the prescreening stage of \$50,000.

In comparison with the intuitive solution of Figure 9, the optimal retrofit network obtained from the proposed method is a significantly better design in that it requires \$17,520 less in capital investment with a reduction in annual cost of \$5550. In the optimal retrofit network, a somewhat more complicated piping structure is used as a tradeoff for requiring less additional area. From the comparison of the two structures, it clearly shows that the proposed method not only can take into account tradeoffs between

utility and capital costs, but also between different types of capital costs.

The computational results of the three solution methods used are listed in Table 1. The solution times for the problem ranged from 3 CPU minutes (IBM 3083 using MINOS and MPSX via GAMS (Kendrick and Meeraus (1985)) for the AP/OA/ER method to about 7 CPU minutes for QBD with outer approximations. Only 2 iterations were required for both the OA/ER with piecewise approximation and the AP/OA/ER algorithms, while 17 iterations were required for GBD. It should be noted, however, that although GBD required significantly more iterations, the master problems were much smaller and thus easier to solve.

Example 2

Example 2 involves an existing network integrating two hot and two cold process streams. The existing structure along with the required data for retrofit are shown in Figure 12. The annual utility cost for the existing network is \$158,000. The proposed retrofit method is applied to redesign the network in order to decrease this utility cost. To favor designs with simple piping structures, no stream splitting will be allowed in the retrofit network. In addition, the heater and cooler in the network will remain as utility exchangers since realignments of the steam or cooling water side of these exchangers are not appropriate.

Following the proposed procedure, a prescreening cost plot was generated to evaluate the incentive of the project. Fixed charges for minimum structural modifications along with minimum utility cost and additional area cost are used to construct the prescreening cost plot shown in Figure 13 which predicted that a potential savings of about \$80,000/yr (at HRAT \ll 10K) is possible for the retrofit project. In determining minimum structural modifications required, an EMAT of 5K was selected. Solution of the assignment-transportation models determined that besides the five existing exchangers, an additional unit may be needed in the retrofit network.

With the incentive established for performing the redesign, a superstructure is next constructed. Since the utility exchangers cannot be reassigned to exchange heat between two process streams, a simplified superstructure can be constructed which positions the utility exchangers at the outlet of the superstructure. As a result, the superstructure, for the most part, considers only four exchangers - three existing ones and one new unit. This simplification reduces the number of alternatives embedded within the superstructure, and as a result, a smaller MINLP formulation can be developed for this problem. In

addition, the assumption of no stream splitting in the retrofit network allows the use of the linear representation of the heat mixing equations to predict exactly the mixer temperatures.

To solve the MINLP model, both the O_A/E_R with piecewise approximation and AP/O_A/E_R algorithms were used. Both methods obtained the same optimal solution and the results are shown in Table 2. It is first noted that 6 iterations were required for O_A/E_R with piecewise approximation while only 2 iterations were required for AP/O_A/E_R. In addition, the master problems of the former required more time to solve. Note that the computational time is rather significant for this problem.

The optimal retrofit network derived from the two solution methods is shown in Figure 14. The network requires the addition of a new exchanger unit and additional area in exchanger 1. Some minor piping modifications are also needed. These modifications require an investment cost of \$206,500. The utility cost for this network is \$24,670/yr. Accounting for capital cost, the total annual cost is \$93,490 which represents a payback time of about 1.55 year. The annual savings from the retrofit project, therefore, is \$64,510/year, which compares well with the maximum savings of \$80,000/year predicted in the prescreening stage.

Example 3

Example 3 was briefly mentioned in the beginning of the paper as a motivating example. The existing network exchanging heat between one hot and two cold process streams is shown in Figure 15 with an annual utility cost of \$180,000. Due to changes in process conditions in the plant, excess capacity is currently available in the heaters. Also, there is an incentive to check whether the changes in operating conditions can be better accommodated by further heat integration. Due to space limitations, no new exchanger can be added to the network, however, repiping can be performed.

For this example, the prescreening stage of the proposed method was not considered and the superstructure as shown in Figure 16 was constructed. Note that in this superstructure, only exchangers 1 and 2 are present. This is due to the additional constraint that utility exchangers cannot be reassigned to exchange heat between two process streams. Note also that in the superstructure, the cold streams C1 and C2 are allowed to exit the superstructure and go directly to a utility exchanger.

The MINLP model for the superstructure was formulated and solved using the GBD with valid outer approximation and AP/O_A/E_R algorithms. The results are shown in Table 3. Once again, AP/O_A/E_R was

able to solve the model in 2 iterations while GBD required 5 iterations. In addition, AP/OA/ER required less than half of the CPU time as compared to GBD.

The optimal retrofit design obtained is shown in Figure 17. This design exploits the difference in the overall heat transfer coefficient between match (H1-C1) and (H1-C2). Since the coefficient is significantly smaller for match (H1-C1), cold stream C1 in the retrofit network is completely heated by hot utility in heaters where excess capacity was available. Cold stream C2 is reassigned to exchanger 1 where with a higher heat transfer coefficient, the area can be better utilized. From the repiping and an additional 9 m² of new area in exchanger 1, the utility cost can be reduced to \$120,000/yr. Accounting for the investment cost of \$64,800, the annual cost for the retrofit network is \$141,600, which corresponds to a payback time of about 1.08 year.

This example further illustrates the fact that restrictions and characteristics in the existing heat exchanger network can be incorporated in the solution method to eliminate alternatives in the superstructure thus making the solution process easier.

Example 4

Example 4 is a small example which will illustrate the capabilities of the superstructure for allowing the mixing of different process streams. Consider an existing network structure as shown in Figure 18 where two exchangers are available for cooling stream H1 to a desired level. It is proposed that streams C1 and C2, streams which will eventually need to be mixed downstream be incorporated into the network.

A superstructure for this problem is constructed embedding the two existing exchangers and the hot and cold streams. This superstructure is similar to the one shown in Figure 16 for Example 3. The alternatives in the general superstructure include the assignment of the two exchangers to the two process streams and several options for the mixing of the two cold streams. This involves a small modification of the general MINLP formulation outlined previously by proper adjustment of upper bounds U, relaxing constraints to allow for more than one process stream to enter the Exchanger Inlet Mixer, and eliminating the logical interconnection constraints.

The MINLP model for the superstructure was solved using the AP/OA/ER algorithm requiring 3 iterations and a total solution time of 40 seconds. The retrofit structure is shown in Figure 19(a). The structure indeed calls for the mixing of the cold streams within the network. The location of the mixing

point accounts for the existing area of the exchanger. As a result the desired level of energy recovery can be accomplished by the minor repiping as shown in Figure 19(a) and the addition of 23.6 m² of area in exchanger 2. This corresponds to an investment cost of \$27,800.

It should be noted that if no stream mixing is allowed, the optimal retrofit network of Figure 19(b) is obtained. The design requires an investment cost of \$35,000, which is about 26% higher than the investment required for the optimal retrofit network of Figure 19(a), which allows for stream mixing.

CONCLUSION

In this paper, a systematic method has been proposed for the retrofit of heat exchanger networks. The method is a two stage approach involving a prescreening stage and an optimization stage. In the prescreening stage, the economic feasibility of a retrofit project is evaluated through the estimation of costs for utility, additional area, and structural modifications. These estimates are used to construct a prescreening cost plot which shows the total annual cost of a retrofit project for various levels of energy recovery (HRAT). Analysis of the plot determines the maximum savings that can be achieved from the project and thus determines prior to any design steps whether network redesign should be performed. The plot also provides information on the maximum number of new potential units that may be required.

In the optimization stage, an optimal retrofit design is obtained through the construction of a superstructure and the optimization of its MINLP model. The superstructure has the unique feature that it has embedded alternative retrofit designs for reassigning existing exchangers to different streams, introducing new exchangers to the network, adding piping and area modifications, and has the possibility of mixing different process streams. Energy recovery (HRAT), heat loads, minimum approach temperature (EMAT), and stream matches are not fixed in order to allow for tradeoffs between capital and energy cost. In addition, the superstructure takes each potential piping segment for the layout into consideration, and as a result it can also account for tradeoffs between different types of capital costs (ie. fixed charges for piping and new units versus cost of additional area). The MINLP model for the superstructure is optimized to determine the retrofit design requiring minimum total annual cost. Several methods have been discussed for solving the MINLP model. While for small examples the computational cost is modest, it is quite significant for large problems.

The application and usefulness of the proposed method has been shown with four examples. The results indicate that the method can properly account for the tradeoffs between energy and capital costs.

In addition, the method can determine optimal retrofit designs that may not be straightforward to identify. Finally, an example illustrated the method's ability to account for cases where the mixing of process streams is allowed.

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APPENDIX A: CONSTRUCTION OF UTILITY COST PLOT

In the prescreening procedure, it is necessary to construct a utility cost vs. HRAT plot for the estimation of the overall cost. The following outlines the procedure for constructing this plot. A basic understanding of the transshipment model by Papoulias and Grossmann (1983) is assumed.

When constructing the Utility Cost vs. HRAT plot (see Figure 20(a)), the information needed are slopes (change of cost wrt HRAT) of the various segments of the plot and the points (HRAT) at which changes of slope occur. To determine this information, one can use modified LP transshipment models parametrically while noting the following:

- Since the objective function in the LP can be selected to be the utility cost, the lagrange multiplier corresponding to (changes of) HRAT is the same as the slope of a segment of the plot.
- A change of slope implies a change of pinch point as defined by the composite curve.

Two versions of the LP transshipment model are used, specifically, version (V1) to determine the utility cost slopes and version (V2) to determine the points of slope change. In both, the transshipment formulations are written so that HRAT is expressed as a variable. The first formulation (V1) identifies the slope of the utility cost plot at a particular HRAT point.

$$\min UC \ll \sum_{m \in S} \alpha_m QH_{mJ}^S + \sum_{n \in W} \beta_n QC_{nJ}^W$$

SJ.

(VI)

$$(2) \quad \begin{aligned} & \text{me } S, \quad \sum_{n \in W} QC_{nJ}^W \\ & = \sum_{i \in TW} F_i \Delta T_i^i - \sum_{j \in C} F_j \Delta T_j^j \quad i=1,2,\dots,J_T \\ & \text{HRAT} \gg 11 \end{aligned}$$

$$(3) \quad \begin{aligned} & *; * 0 \quad QH^{\wedge} \quad 2 \quad 0 \quad QC_{nJ}^W \geq 0 \\ & i=1,2,\dots,J_T, \quad m \in S \end{aligned}$$

where $A7J$ and $A7^{\wedge}$ in (1) are functions of HRAT and

With:

- H, m {j|hot stream i is present in interval 1}
- $C_i - I/cokj$ streamy is present in interval 1}
- S, m {m|hot utility m is present in interval 1}

W_i - cost of hot utility i is present in interval i
 I_T - total number of transshipment intervals
 a_m - cost of hot utility m
 P_i - cost of cold utility i

ΔT_i - change of stream temperature in interval i
 R_i - Residual heat load of interval i
 F - heat capacity flow rate
 T_i - fixed value of HRAT at which slope is to be determined

$Q_{H_{m,i}}^S$ - heat load of hot utility in interval i

$Q_{C_{j,i}}^W$ - heat load of cold utility in interval i

Constraints (1) and (3) are the transshipment constraints. HRAT is written as a variable in constraint (1) by defining ΔT_i by the representation of the transshipment intervals as shown in Figure 20(b).

Solution of (V1) which predicts the minimum utility cost, UC, yields the Lagrange multiplier, X , corresponding to the constraint HRAT = T_i . X then is the slope of the utility cost plot at HRAT = T_i . Also, the pinch point is located by identifying the interval i at which the sum of the residuals (R_i) are zero.

The second formulation (V2) determines the lowest value of HRAT that is closest to T_i , and for which the utility cost plot slope changes. It incorporates information from (V1), specifically the location of the pinch and the slope X , into the transshipment formulation.

$$\begin{aligned}
 \min \quad & \text{HRAT} \\
 \text{s.t.} \quad & \text{transshipment constraints (1) and (3)} \\
 & \sum R_p \leq 0 \quad \text{at pinch location (p)} \\
 & UC = \sum_{m \in S} a_m Q_{H_{m,i}}^S + \sum_{j \in J} P_j Q_{C_{j,i}}^W \\
 & UC \geq UC(\text{HRAT} = T_i) - (T_i - \text{HRAT}) X
 \end{aligned} \tag{V2}$$

where $UC(\text{HRAT} = T_i)$ is the utility cost at $\text{HRAT} = T_i$ as predicted by (V1)

Solution of (V2) then yields the HRAT at which the next change of pinch point occurs.

It is clear that in order to determine the plot of cost versus HRAT, problems (V1) and (V2) need to be solved sequentially. In addition, due to the fact that HRAT is a variable in (V2), the model has to be reformulated whenever the transshipment intervals or warehouses are disturbed due to changes in

HRAT, eg. addition of interval(s) or extension of hot or cold streams into other intervals. An approach is presented where first a range of HRAT that is of interest is selected. This range, for the retrofit case, may be defined from the HRAT corresponding to the existing network to $HRAT=EMAT$.

After selecting, the range of HRAT, it is divided into intervals of HRAT for which disturbances of transshipment warehouses do not occur within the interval, ie. for all HRAT's within the interval, the same set of warehouses are used by each stream.

With the HRAT intervals determined, the following steps will generate the utility plot by sequentially solving formulations (V1) and (V2) for each interval.

1. Select the largest HRAT of each interval as the starting point.
2. Use (V1), setting $HRAT =$ largest HRAT, to solve for the slope of plot and location of pinch.
3. For (V2), bound HRAT to be values within the interval and constrain residuals to be zero at the location of the pinch. Solve to determine an HRAT (if any) within the interval for which the slope of the cost plot changes. Note that if no change of slope occur within the interval, the formulation will yield the lowest HRAT within the interval as the solution. If this occurs, the cost plot is simply one straight line having slope λ . Therefore, terminate examination of the interval by proceeding to step (5).
4. Constrain $HRAT = (HRAT \text{ from (3)} - \epsilon)$ in (V1) and solve to determine the new slope and new location of pinch. ϵ is an arbitrarily small number subtracted from HRAT from (3) to ensure calculation of the new slope. Go back to step (3) with information of new slope and new location of pinch.
5. Investigate the next interval by going back to step (1) or if entire range has been investigated, terminate.

Completion of the steps above would provide all the necessary information to construct the utility cost plot. The plot can then be incorporated in the prescreening to develop the prescreening cost plot.

APPENDIX B: Parametric Analysis of Minimum Structural Modifications

A primary prescreening step is the determination of a lower bound on the structural modification cost. To do so, one needs to identify the minimum structural modifications that are required for the potential retrofit network. The assignment-transshipment model of Yee and Grossmann (1987) was proposed for this purpose. Solution of this model identifies the minimum structural modifications required for *one* particular value of HRAT. In the prescreening stage, however, since various degrees of heat recovery are of interest, an entire range of HRAT needs to be investigated. In order to do this with a good degree of accuracy and efficiency, it is proposed that the assignment-transshipment model be extended to find the minimum structural modifications over different intervals of HRAT. The following steps outline

the procedure:

1. Divide the range of HRAT to be investigated into small divisions, e.g. 5K.
2. Develop a modified assignment-transshipment model for each small division of HRAT.
3. Solve the models to determine the minimum structural modifications required within each small division of HRAT.

In step 1, the range of HRAT is subdivided. In general, the size of the divisions reflect a tradeoff between the accuracy of the prescreening and the number of assignment-transshipment model to be solved. When the divisions are small, the analysis involves more effort but the prescreening becomes more accurate with respect to each HRAT in the interval.

In step 2, the assignment-transshipment model is formulated for predicting minimum structural modifications for the small divisions of HRAT. In order for HRAT to take on any value within the division of HRAT, the transshipment constraints in the assignment-transshipment model is modified so that HRAT becomes a variable. This is done in the same manner as presented in Appendix A.

Also, similar to the utility cost case in Appendix A, changes of the transshipment interval conditions must be accounted. Since HRAT is a variable in the model, within a particular HRAT division, the transshipment intervals may change, e.g. streams extending into additional warehouses. To avoid this problem, it is suggested that the HRAT divisions be selected so that points of change for the transshipment intervals correspond to certain HRAT division limits. To determine at what HRAT's the changes for the intervals occur, see Appendix A.

It should also be noted that construction of the transshipment intervals can be based on an EMAT, for $EMAT < HRAT$. The idea is to have utility requirements and pinch locations determined by HRAT and flexibility of matching, ie. transshipment intervals, based on EMAT. A smaller value for partitioning the intervals will allow greater flexibility of matching and thus may decrease the structural modifications required.

Parametric solution of the modified assignment-transshipment model from step 3 predicts the minimum structural modifications (number of new units and reassignments) necessary for each HRAT interval. Fixed costs can then be assigned to each of the modifications predicted to determine an estimated cost for structural changes. The estimated cost for each HRAT interval is then incorporated to develop the prescreening cost plot.

APPENDIX C: Linear Heat Mixing Constraints

In order to develop a linear representation of the heat mixing equations for the master problem in the MINLP for the retrofit superstructure, the following procedure that is illustrated by example is proposed. Consider the heat mixing constraint for the hot side mixer in Figure 21:

In order to represent equation (C1) linearly, the assumption that no bypasses be allowed is made. For the mixer in Figure 21, this explicitly constrains that only one of the two streams, y_i or z_i^h , entering the mixer can be nonzero. Typically, this assumption is not a serious one in that bypasses are usually not needed if the exchanger approach temperature (EMAT) can be small, which is allowed in the formulation. Since only one stream is nonzero, the mixer outlet temperature, T^* is either T_i or $T_{i,mt}^h$. This can be formulated easily by taking advantage of the fact that in the MINLP model, binary variables are used to designate the existence of branches of flow. For the hot stream as represented by the mixer in Figure 21, the linear heat mixing constraints are written as follows:

$$T_k^{in} \leq T_c + U(1 - y_i^k)$$

$$T_k^{in} \leq T_h^{out} + U(1 - z_i^h)$$

where U is an upper bound.

$$y_i^k + z_i^h \leq 1$$

where y_i^k and z_i^h are binary variables denoting existence of the branches of flow.

Note that if in case 1, $y_i^k = 1$, $z_i^h = 0$, this leads to:

$$T_k^{in} \leq T_c \quad \text{and} \quad T^* \leq T_h^{out} + U$$

If in case 2, $z_i^h = 1$, and $y_i^k = 0$, this leads to:

$$t_k^{in} \leq T_i + U \quad \text{and} \quad t_k^{in} \leq t_h^{out}$$

By this scheme, the first inequality becomes active for the first case while the second inequality becomes active for the second case. The correct temperature is thus selected for ϵ .

In the MINLP formulation, these linear heat mixing constraints can be incorporated by substituting the nonlinear heat mixing constraints of equation (11) by the following equations:

No bypass constraints

First, constraints to enforce the no bypass assumption is included:

$$\begin{aligned} \sum_{l \in E} x_{*l} - U (1 - \sum_{i \in P} y_i) &\leq 0 \quad k=1,2,\dots,K \\ \sum_{l \in E} z_{li} - U (0 - \sum_{j \in CP} x_j) &\leq 0 \quad k=1,2,\dots,K \end{aligned} \quad (C2)$$

Linear heat balance at each Exchanger Inlet Mixer

Linear heat mixing constraints are included which accounts for utility stream temperatures as well.

$$\begin{aligned} t_k^{in} &\leq \sum_{i \in HP} (T_i^{IN} - U) \sum_{l \in E} z_{lk}^h + \sum_{i \in HU} (T_i^{IN} - w_i^h) \quad k=1,2,\dots,K \\ t_k^{in} &\geq \sum_{i \in CP} (T_i^{IN} - y_i^h) - U \sum_{l \in E} z_{lk}^c + \sum_{j \in CU} (T_j^{IN} - w_j^h) \quad k=1,2,\dots,JT \end{aligned} \quad (C3)$$

It should be noted that these constraints are in general only an approximation on the actual heat mixing constraints. However, for an arbitrary two exchanger superstructure or any superstructure where no stream splits are allowed, the addition of the following constraints can predict exactly the temperature at the mixer outlet

Additional constraints for two exchanger superstructure or no stream splitting restriction

$$\begin{aligned} t_k^{in} &\leq t_i^{out} \cdot U (1 - \epsilon) \quad kmiz.jl \quad tmIX.X \quad \text{where } l \neq k \\ t_k^{in} &\geq t_i^{out} - U (1 - z_{jj}) \quad lo^*IX-X \quad MZ.JC \quad \text{where } l \neq * \end{aligned} \quad (C4)$$

Through the use of these linear representations, the nonconvexities from the nonlinear heat mixing equations can be eliminated.

APPENDIX D: Piecewise Approximation for Nonconvex Equations

Bilinear terms appear in equations involving heat balance and area calculations of exchangers. These bilinear terms introduce nonconvexities in the MINLP formulation. In the solution of the MINLP, nonconvexities can cause part of the feasible region to be extended in the master problem of the outer-approximation/equality relaxation scheme (Kocis and Grossmann (1987)). As a result, a global optimal solution cannot be guaranteed. To remedy this problem, the bilinear equations are first relaxed based on the Kuhn-Tucker conditions and then transformed in the master problem. The transformation is based on separable programming by using piecewise approximation. This process allows explicit treatment of nonconvexities at the expense, however, of increased number of binary and continuous variables and constraints. The transformation is summarized below for the relaxed heat balance equation:

$$\text{Heat balance equation: } \quad \overline{\varepsilon} Q - I dt z = 0 \quad \text{where } \begin{cases} t'' - t^* & \text{for hot streams} \\ dt m < t^* - t^* & \text{for cold streams} \end{cases}$$

$$\text{Define new variable } u: \quad u \gg I + dt$$

$$\text{Algebraic manipulation: } \quad u^2 = I^2 + dt^2 + 2I dt$$

$$J \quad 2 \quad 2 \quad \overline{2}$$

$$\text{Substituting: } \quad 2 \wedge Q + I + dt^2 - M^2 \leq 0$$

In the above equation, the only nonconvex term is $-u^2$. To provide a valid linear underestimation for this term, piecewise linear approximation can be used. An outline of the steps involved for the piecewise approximation is presented below (see Garfinkel and Nemhauser (1972)). Refer to Figure 22 for clarification.

1. Identify the range for u .
2. Select discrete points in the range for the approximation (in the figure, points XH1, XH2, XH3, and XH4).
3. Evaluate functional values $(-XH1^2, -XH2^2, -XH3^2, -XH4^2)$ for the points selected.
4. Introduce continuous variables WH_1, WH_2, WH_3, WH_4 , and approximate u and u^2 by the following equations:

$$i? \gg WH1 XH1^2 + WH2 XH2^2 + WH3 XH3^2 + WH4 XH4^2 \quad (D1)$$

$$ii * WH1 XH1 + WH2 XH2 + WH3 XH3 + WH4 XH4 \quad (D2)$$

5. Substitute equations (D1) and (D2) for u and u^2 in the master problem of the MINLP.

6. Introduce binary variables YH and add the following constraints to ensure proper selection of WH (only two adjacent WH can be nonzero):

$$WH_1 + WH_2 + WH_3 + WH_4 \leq 1$$

$$\begin{aligned} WH_1 & \leq Z_{YH_1} \\ WH_2 & \leq Z_{YH_2} + YH_1 \\ WH_3 & \leq Z_{YH_3} + YH_2 \\ WH_4 & \leq 17/3 \end{aligned}$$

$$\begin{aligned} 17/1 & \leq YH_2 + YH_3 \leq 1 \\ YH_1, YH_2, YH_3 & \in \{0, 1\} \end{aligned}$$

The approximation of the function is represented by the dashed lines in the Figure 22. The accuracy is dependent on the number of discrete points used.

It should be noted that the constraints given in step 6 above corresponds to the so-called Special Order Set of Type 2 (SOS2)(Schrage (1984)). A number of MILP codes allow the declaration of such a structure without explicitly writing out the constraints of step 6 in the formulation. Such an MILP code also accounts for the SOS2 structure when constructing the nodes in the branch and bound tree used to solve the problem, which makes the solution process more efficient.

The approximation described above provides the desired result that the master problem of the outer approximation scheme will overestimate the feasible region. As a result, the master problem is a rigorous model to identify the global optimum.

An analogous procedure is used to approximate the relaxed exchanger design equations. In the area calculations, arithmetic mean temperature difference (AMTD) is used to give the following equation:

$$\sum (Q_i U_i) - EA \text{ AMTD} - AA \text{ AMTD} \leq 0$$

To eliminate the nonconvexities arising from the bilinear term $(AA \text{ AMTD})$, transformation and piecewise approximation are carried out as outlined above.

Method	Iterations Required	Approximate size of Master problem	Approximate Solution time of Master Prob.	Total Solution Time **
OA/ER* with piece-wise approx.	2	166 constraints 123 variables 47 binaries	2 min.	4 min.
GBD with valid outer approx.	17	113 constraints 53 variables 30 binaries	25 sec.	7 min.
AP/OA/ER	2	144 constraints 104 variables 30 binaries	1.3 min	3 min.

* solved using a modified superstructure representation

** solution times are for the use of MINOS and MPSX via GAMS on the IBM 3083

table 1 Solution Results for Example 1

Method	Iterations Required	Approximate size of Master problem	Approximate Solution time of Master Prob.
OA/ER [*] with piece-wise approx.	6	299 constraints 169 variables 65 binaries	35 min.
AP/OA/ER	2	252 constraints 145 variables 48 binaries	30 min.

* solved using a modified superstructure representation

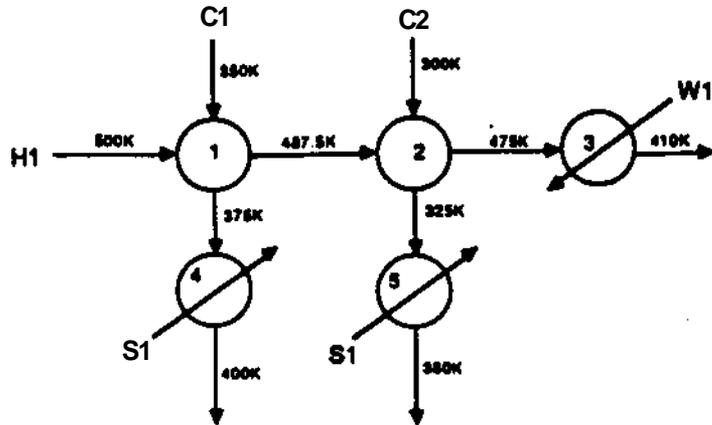
** solution times are for MINOS and MPSX via GAMS on the IBM 3083

Table 2 Solution Results for Example 2

Method	Iterations Required	Approximate size of Master problem	Approximate Solution time of Master Prob.	Total Solution Time "
GBD with valid outer approx.	5	67 constraints 36 variables 12 binaries	10 sec.	90 sec.
AP/OA/ER	2	102 constraints 65 variables 12 binaries	13 sec	34 sec

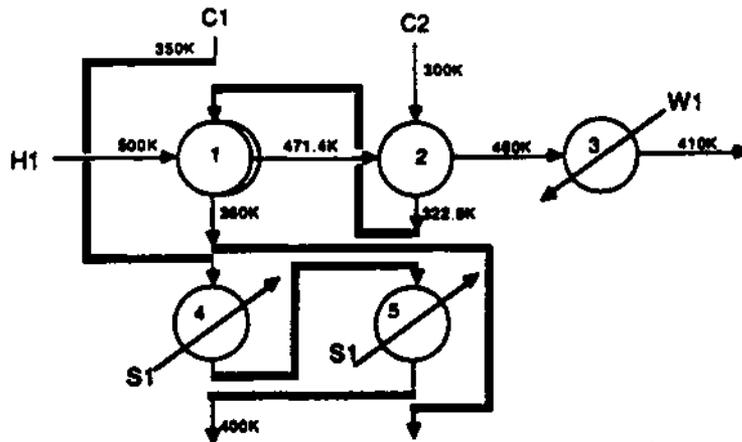
** solution times are for the use of MINOS and MPSX via GAMS on the IBM 3083

table 3 Solution Results for Example 3



Utility Cost « \$180,000/yr

Figure 1a Existing Network for Motivating Example



Utility Cost - \$120,000/yr

Annual Cost « \$141,600/yr

Payback time « 1 year

•» Bold Un« indicates n«w piping

Figure 1b Retrofit Network for Motivating Example

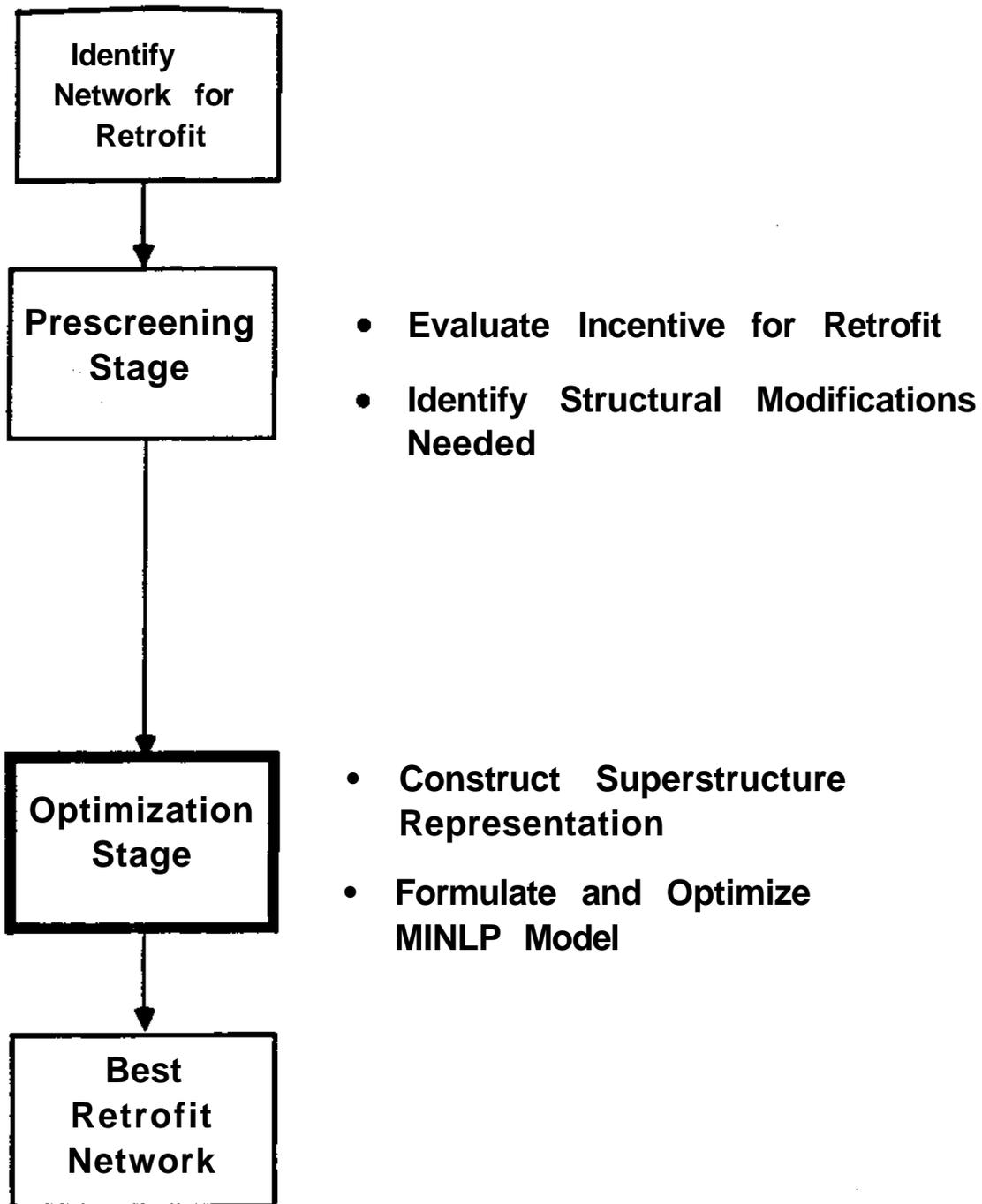


Figure 2 General Strategy for Proposed Methodology

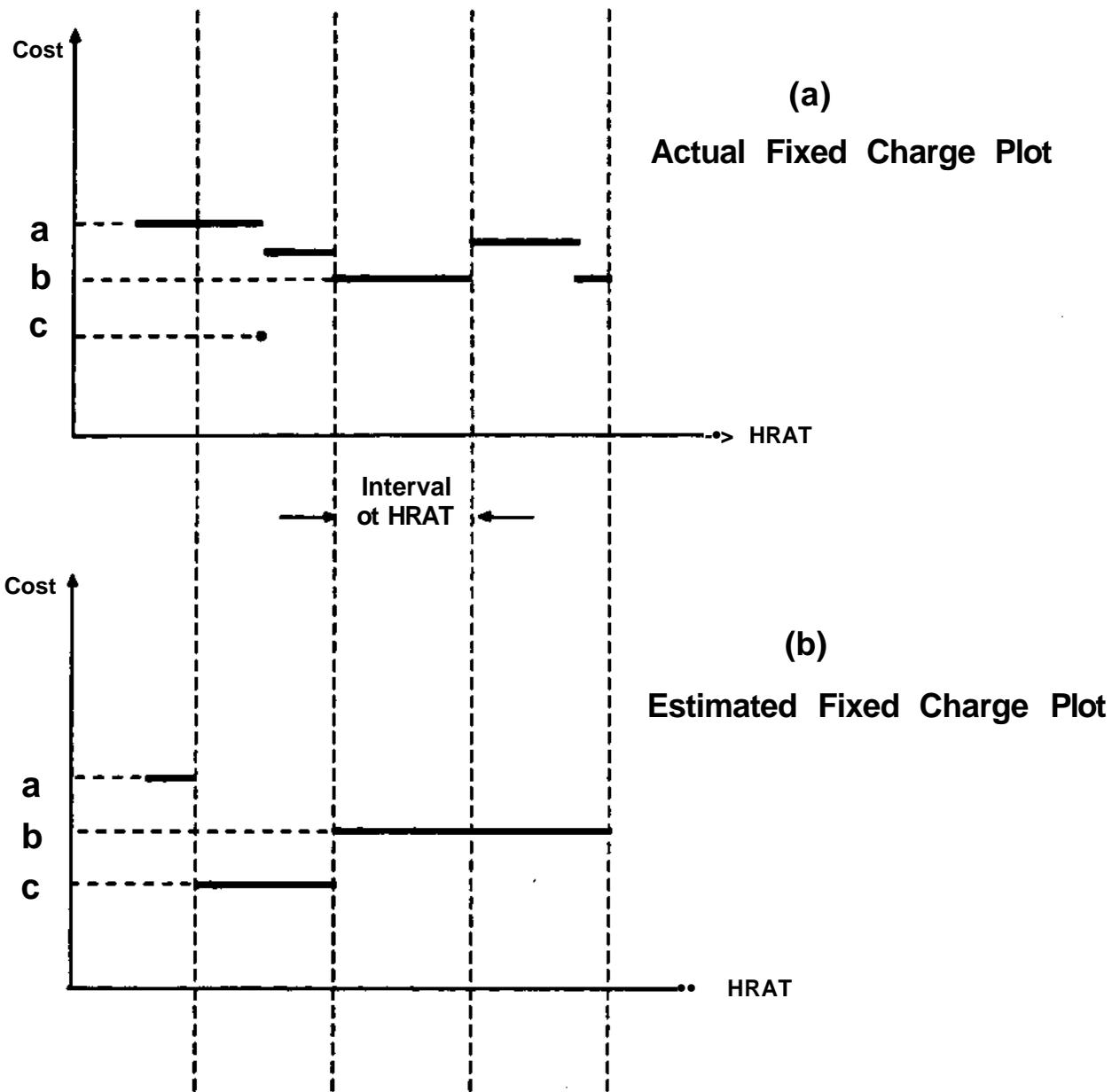


Figure 3 Fixed Charge Plots

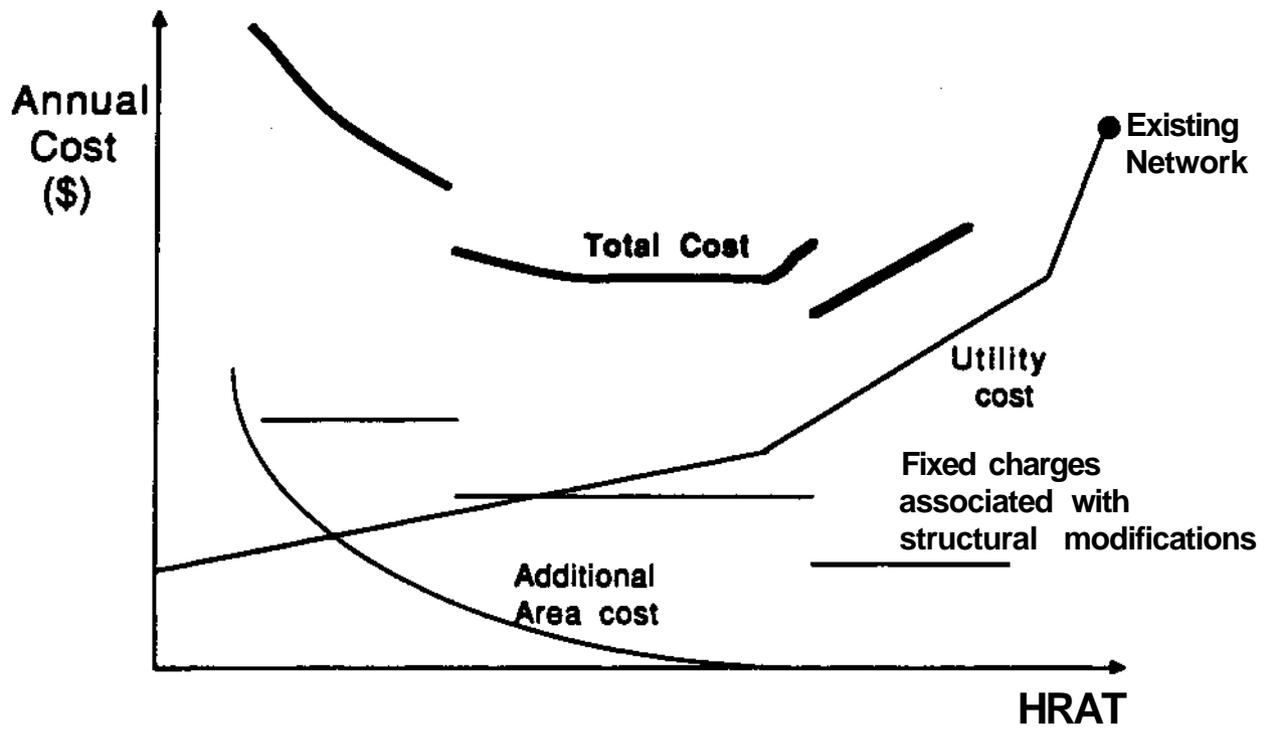


Figure 4 Typical Prescreening Cost Plot

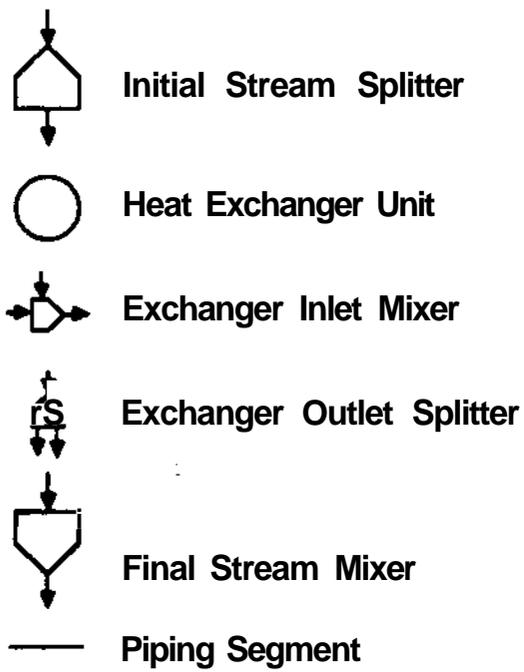
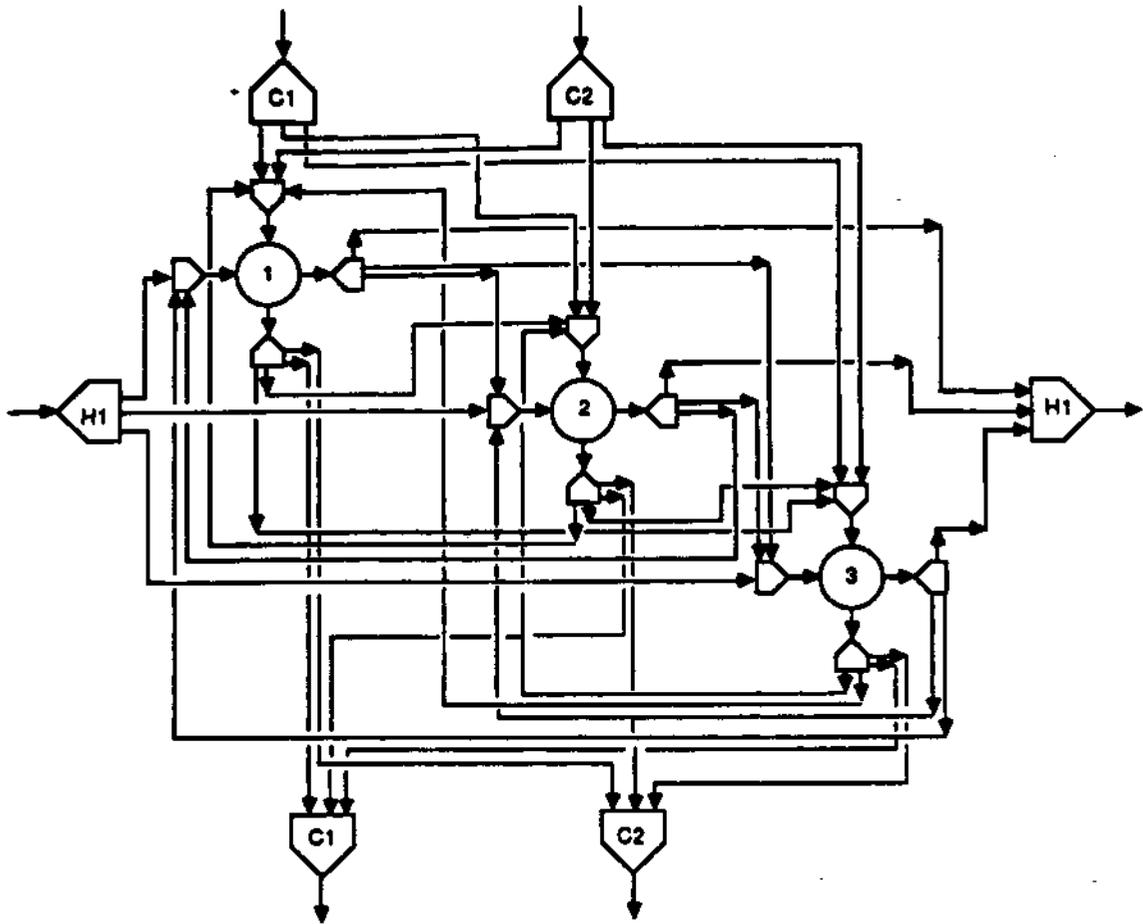


Figure 5 A Three Exchanger Retrofit Superstructure

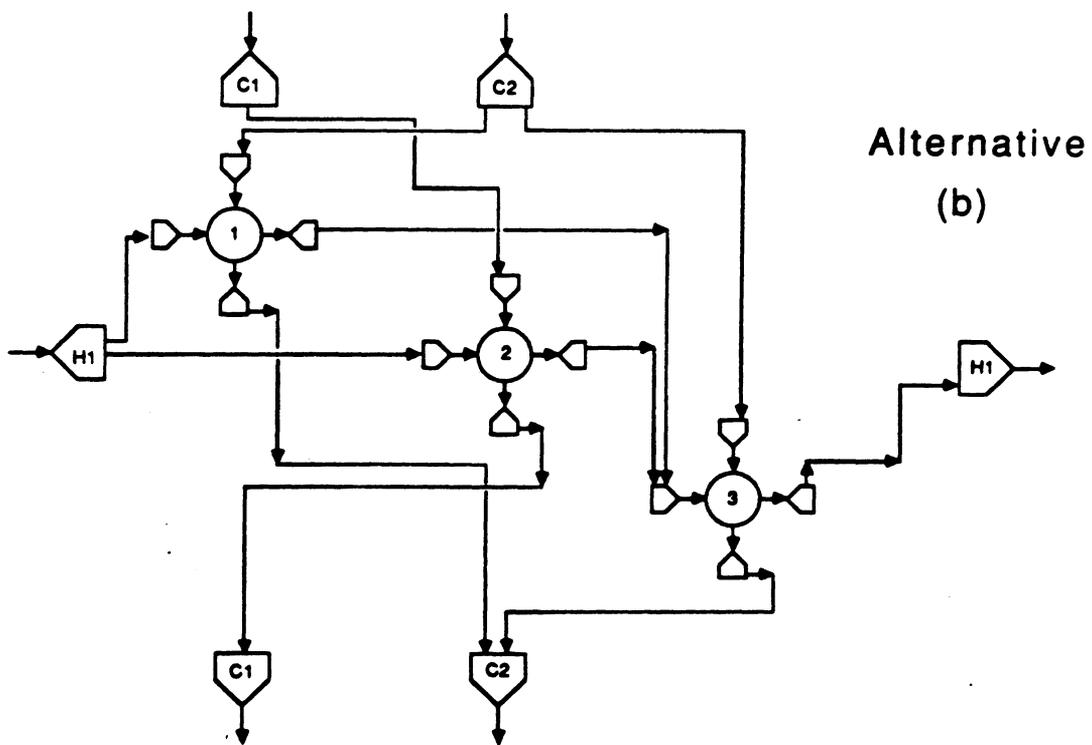
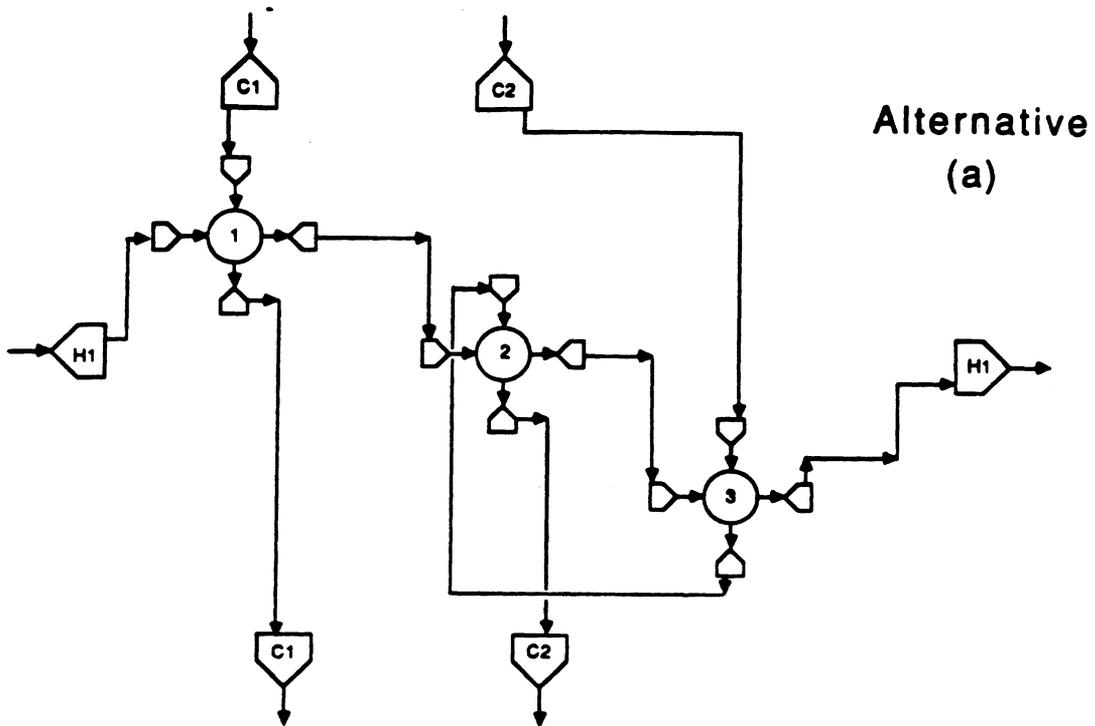


Figure 6 Alternative Retrofit Designs Embedded within Superstructure

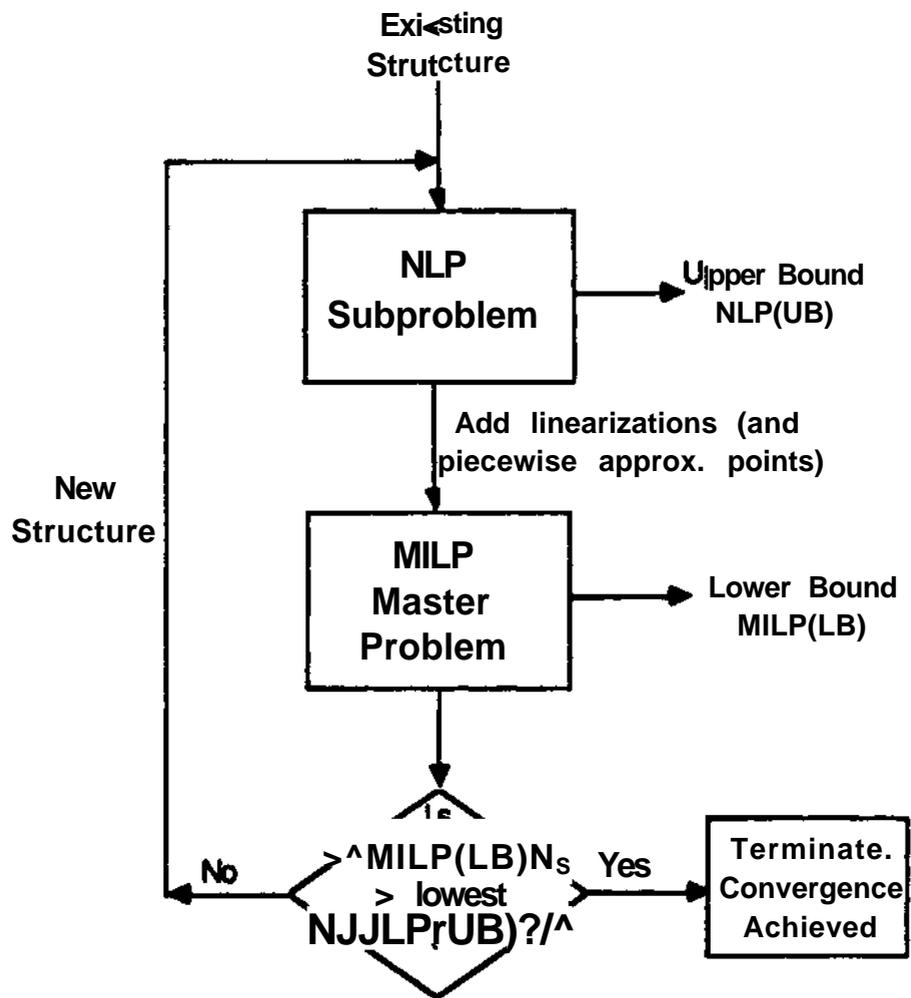
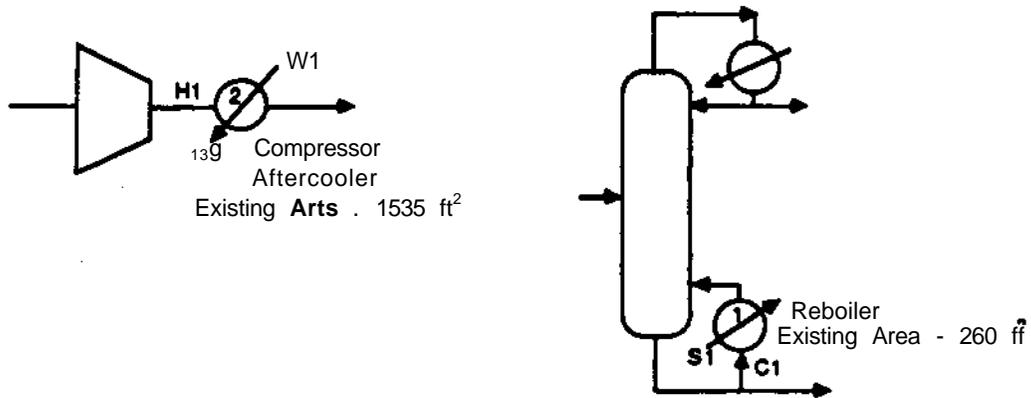


Figure 7 Outer Approximation/ Equality Relaxation Method

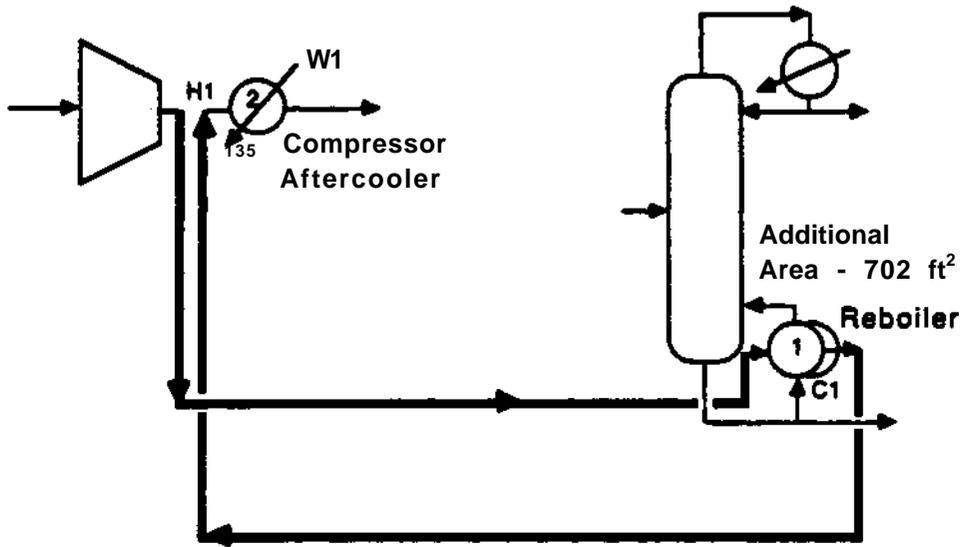


Annual Utility Cost « \$74,060

	$T_{in} (^{\circ}F)$	$T_{out} (^{\circ}F)$	F_{cp} (Btu/hr $\cdot^{\circ}F$)	Cost
H1	285	140	57190	
C1	92.3	135	103900	
W1	80	«*100 C.135	—	(0.0616/ F_{cp})
S1	210	210	—	$\frac{\$0.0146}{(\text{Btu/hr})}$

Match	Overall Heat Transfer Coefficient (Btu/(hr-ft ² - Δ))
H1 - W1	55
S1 - C1	180
H1 - C1	35

Figure 8 Existing Network for Example 1



•• Bold lines indicate new piping

Investment Cost - \$91,870

Utility Cost - \$4,310/ yr

Annual Cost - \$34,940/ yr

Annual Savings * \$39,120/yr

Payback Time • 1.32 yr

Figure 9 Intuitive Solution for Example 1

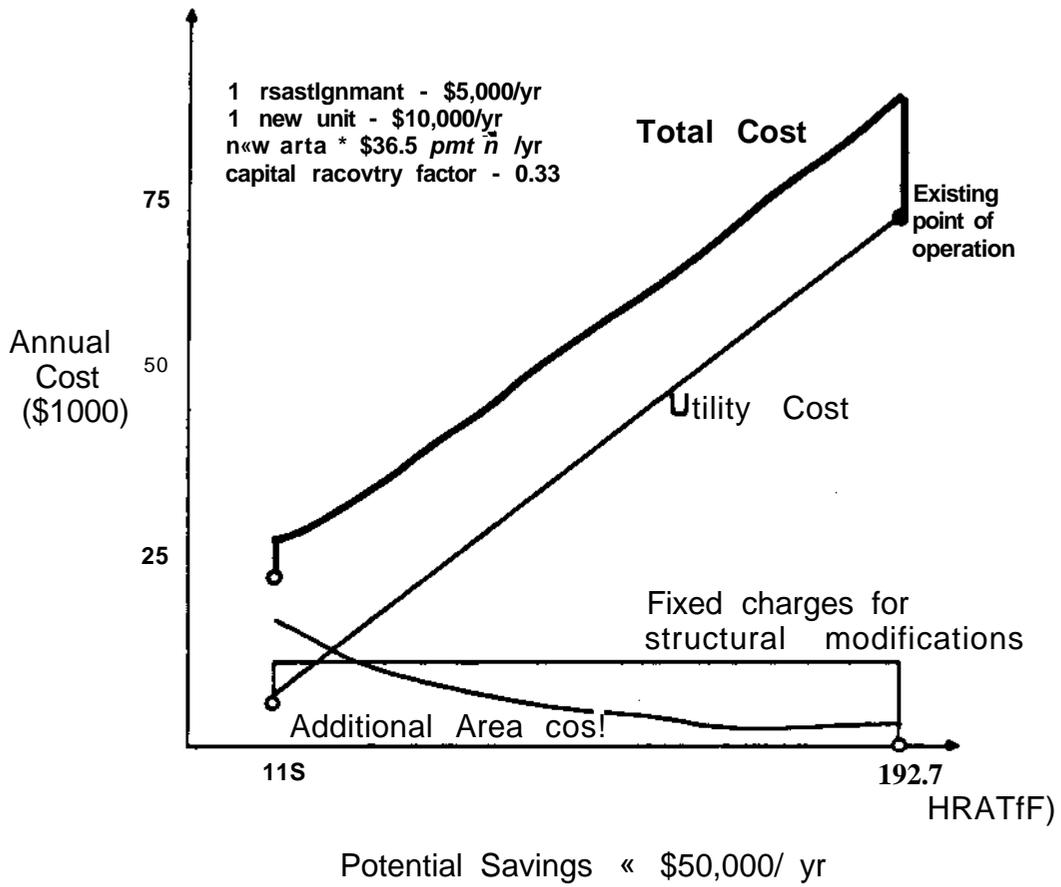
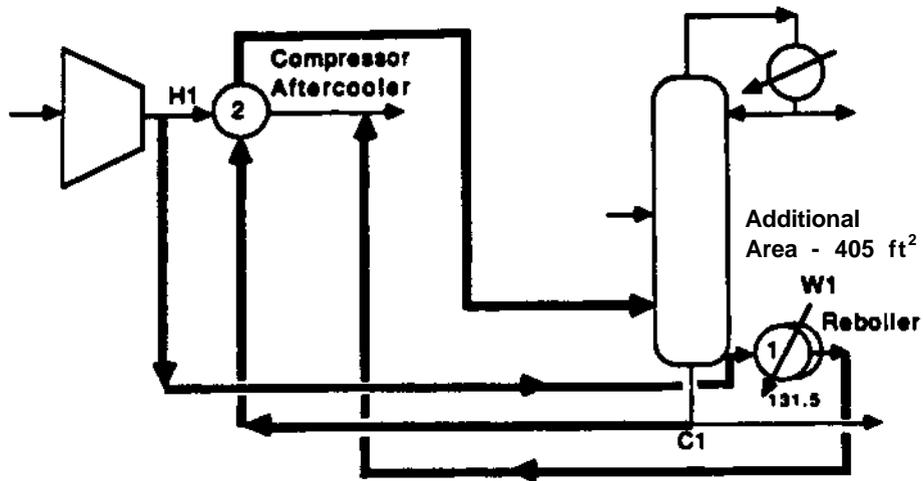


Figure 10 Prescreening Cost Plot for Example 1



— Bold lines indicate new piping

Investment Cost - \$74,350

Utility Cost - \$4,610/ yr

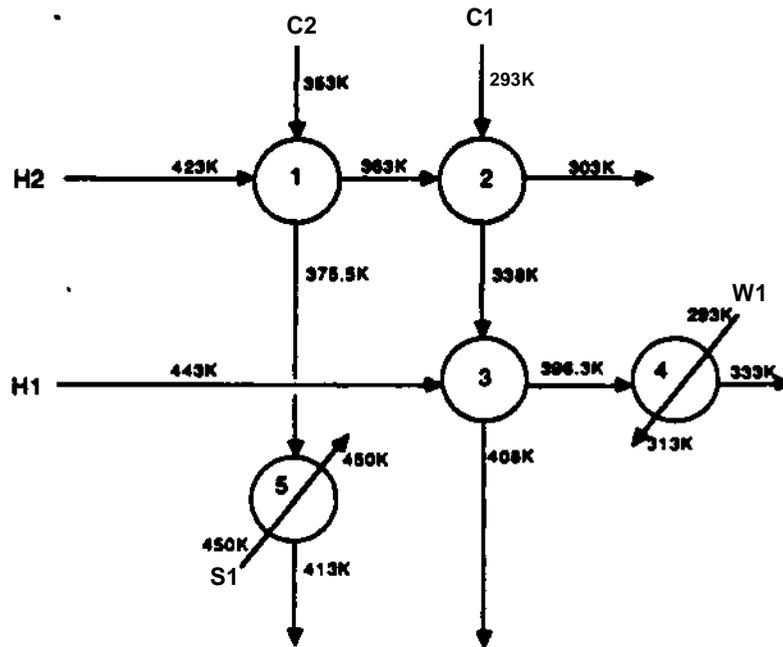
Annual Cost - \$29,390/ yr

Annual Savings - \$44,670/yr

Payback Time • 1.07 yr

Improvement over Intuitive Solution is
\$17,520 reduction in initial investment

Figure 11 Retrofit Network for Example 1



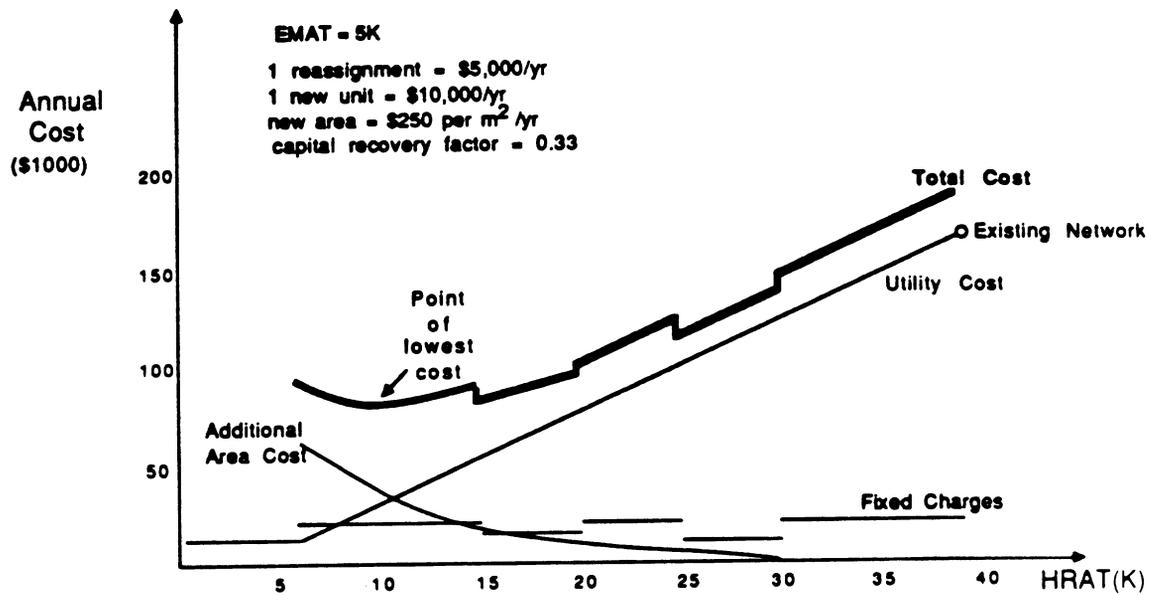
Utility Cost - \$158,000/ yr

$U \ll 0.8 \text{ kW}/(\text{m}^2 \text{ K})$ for all exchangers except ones involving S1
 $U * 1.2 \text{ kW}/(\text{m}^2 \text{ K})$ for exchanger involving S1

Exchanger	Area (m ²)	HNtLotd (KW)
1	46.74	900
2	68.72	900
3	38.31	1400
4	40.23	1900
5	23.33	1500

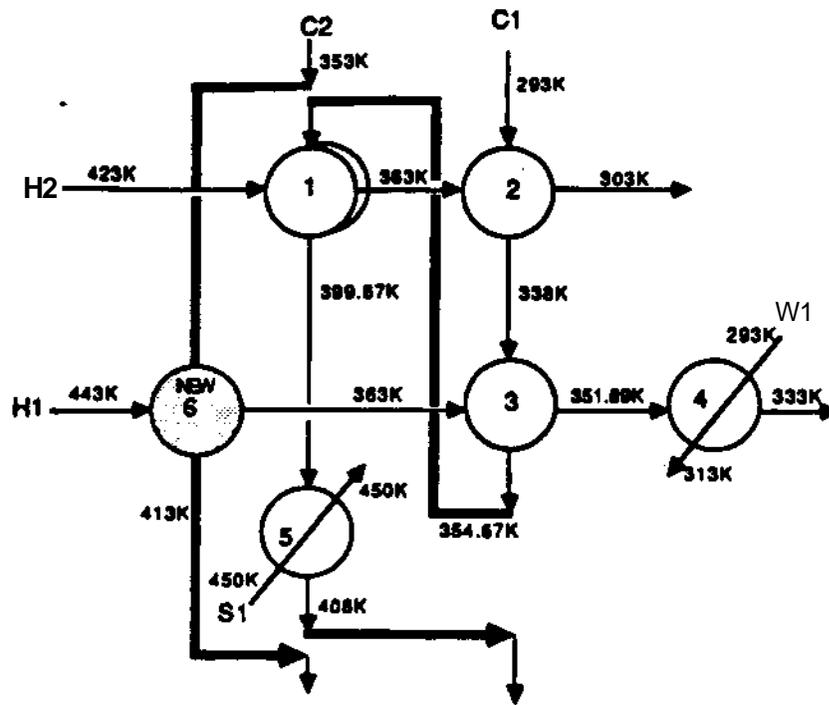
Stream	F _{cp} (kW/K)	T _{JnW}	T _{out<K>}	Cost t/kW-y
H1	30	443	333	--
H2	15	423	303	--
S1	••	450	450	80
C1	20	293	408	--
C2	40	353	413	••
W1	••	293	313	20

Figure 12 Existing Network for Example 2



Potential Savings \cong \$80,000

Figure 13 Prescreening Plot for Example 2



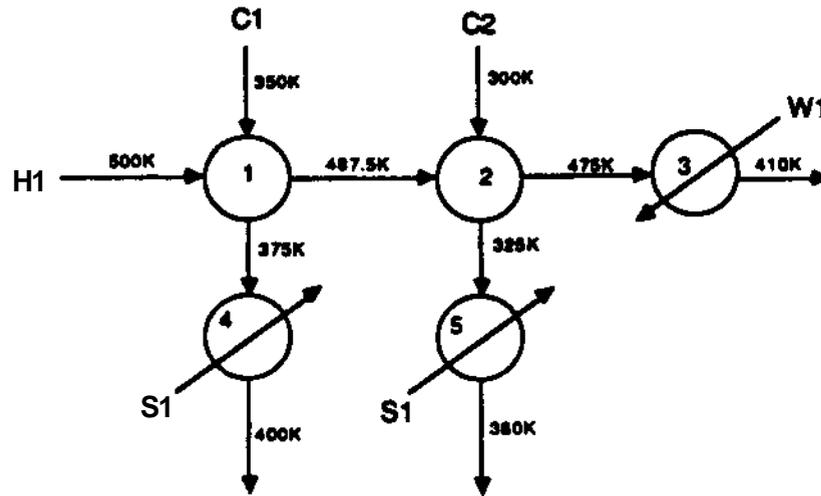
— Bold lines indicate new piping

Investment Cost • \$206,500
 Utility Cost « \$24,670/ yr

Total Annual Cost « \$93,490
 Payback time « 1.55 year

Exchanger	Area (m ²)	Retrofit Area (m ²)	ATM Change (m)	Heat Load (kW)
1	46.74	77.22	+30.48	900
2	68.72	68.72	0	900
3	38.31	38.31	0	333.33
4	40.23	17.96	-22.27	566.67
5	23.33	3.02	•20.32	166.67
6	0	164.79	—	2400

Figure 14 Optimal Retrofit Network for Example 2



Utility Cost * \$180,000/yr

Stream	$T_{in}(K)$	$T_{out}(K)$	$F_{cp} (kW/K)$
H1	500	410	40
C1	350	400	20
C2	300	380	20

Match	Overall Heat Transfer Coefficient ($kW/(m^2K)$)
H1 - C1	0.05
H1 - C2	0.10

Existing Area for exchanger 1 • 76.25 m²
 Existing Area for exchanger 2 - 29.64 m²

Figure 15 Existing Network for Example 3

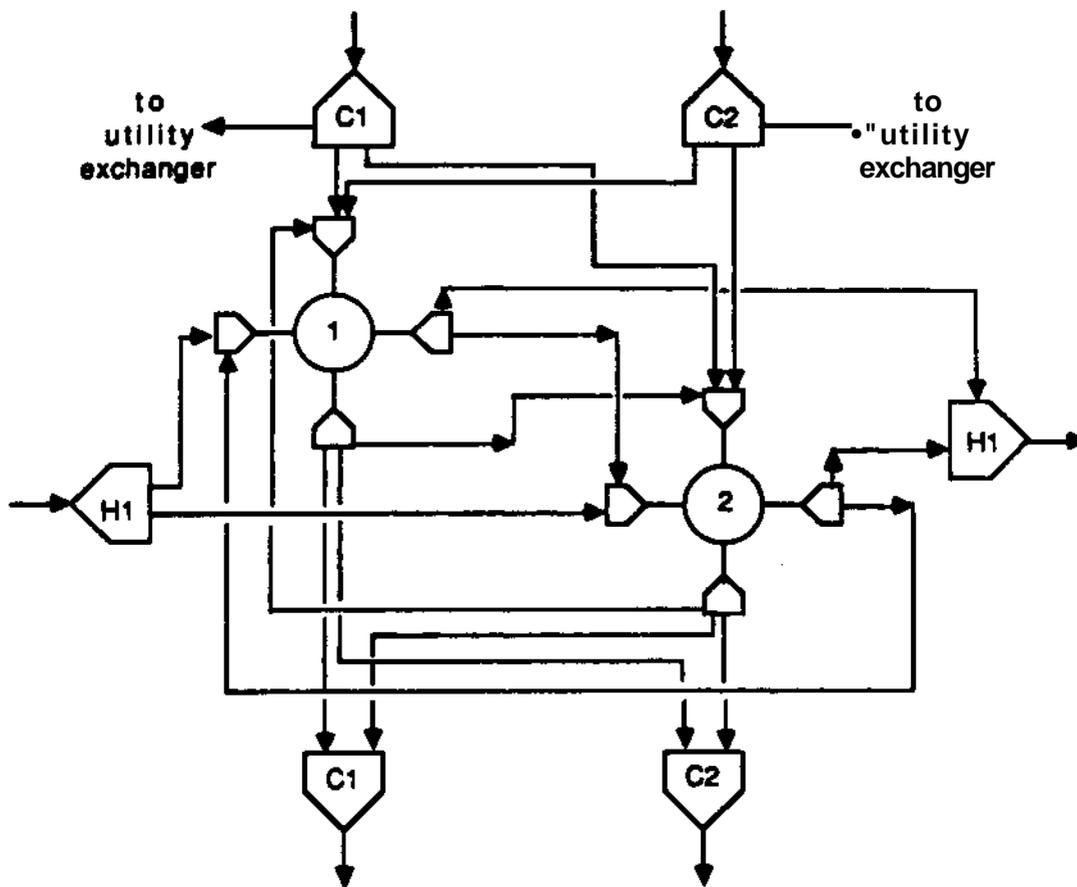
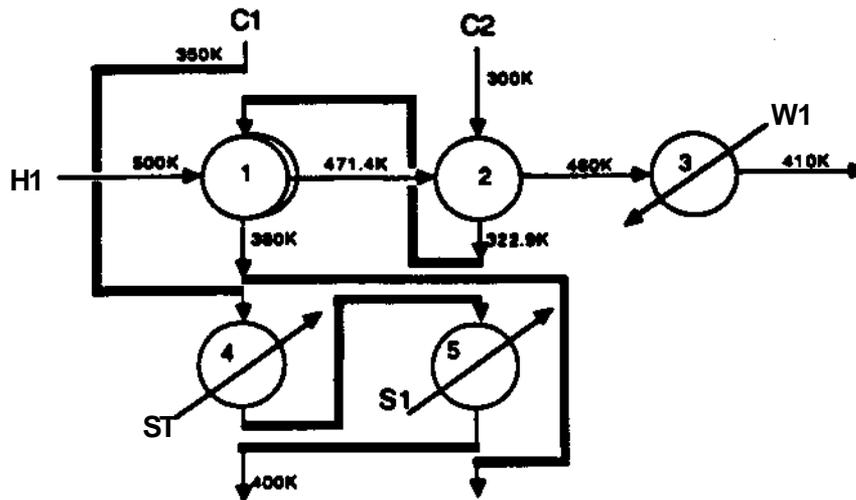


Figure 16 Superstructure for Example 3

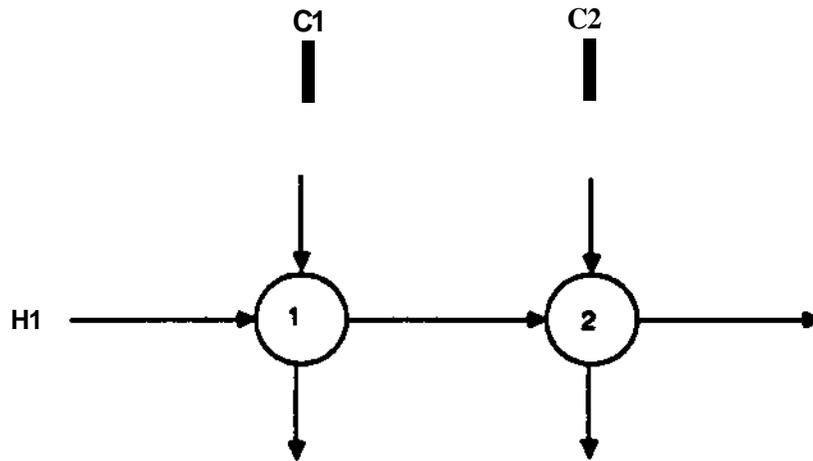


—•Bold line indicates new piping

Investment Cost • \$64,800
 Utility Cost * \$120,000/yr

Annual Cost = \$141,600/yr
 Payback time * 1.08 year

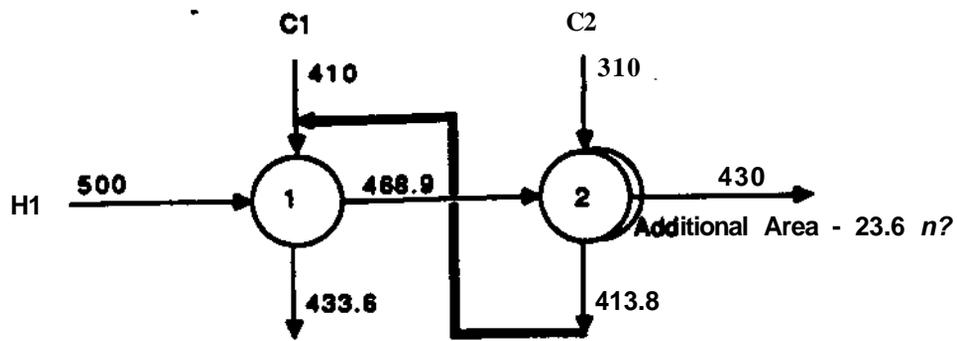
Figure 17 Retrofit Network for Example 3



Stream	T_{in} (K)	WK	Fcp (kW/K)
H1	500	430	40
C1	410	433.6	40
C2	310		15

Existing Area for exchanger 1 « 250 m²
 Existing Area for exchanger 2 « 210 m²
 U - 0.08 (kW/Cm² K) for all exchanges

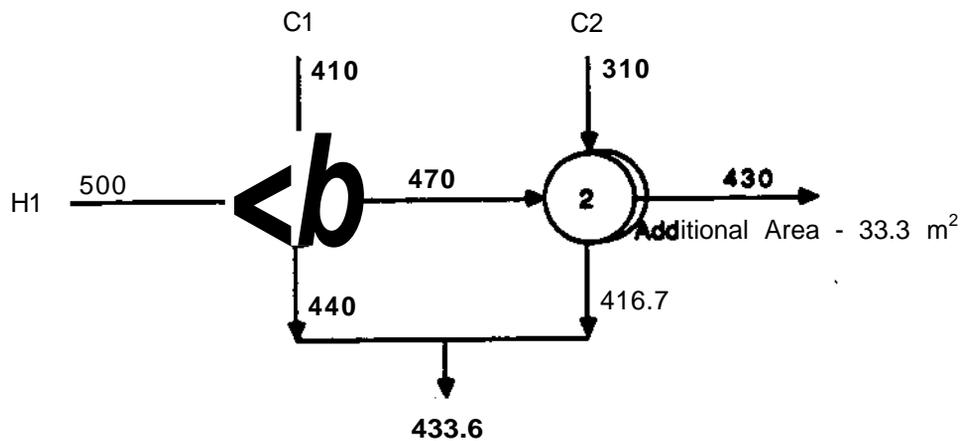
Figure 18 Existing Network for Example 4



— Bold line indicate new piping

Investment Cost • \$27,800

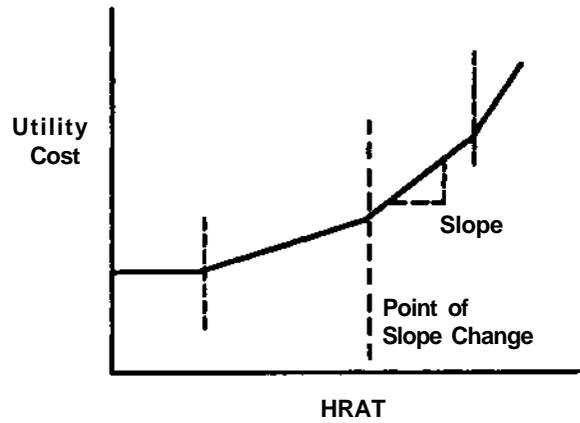
(a) Retrofit Network with Mixing of Streams



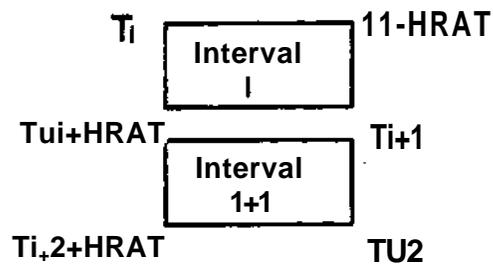
Investment Cost • \$35,000

(b) Retrofit Network without Mixing of Streams

Figure 19 Retrofit Networks for Example 4



(a) Utility Cost vs. HRAT Plot



(b) Transshipment Intervals with HRAT as a Variable

Figure 20 Construction of Utility Cost Plot

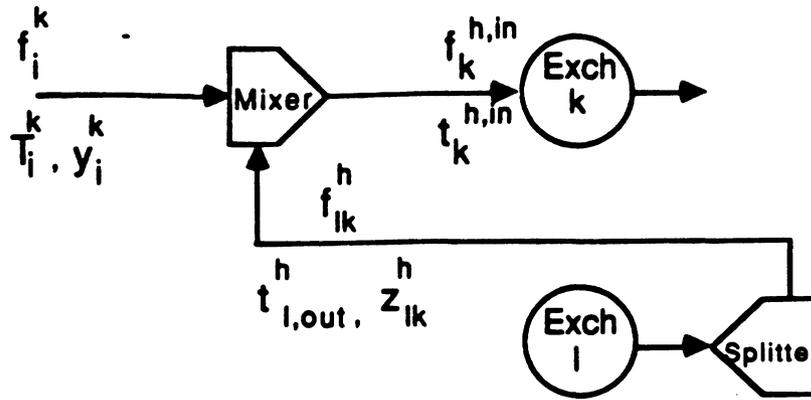


Figure 21 Mixing Point for Exchanger k

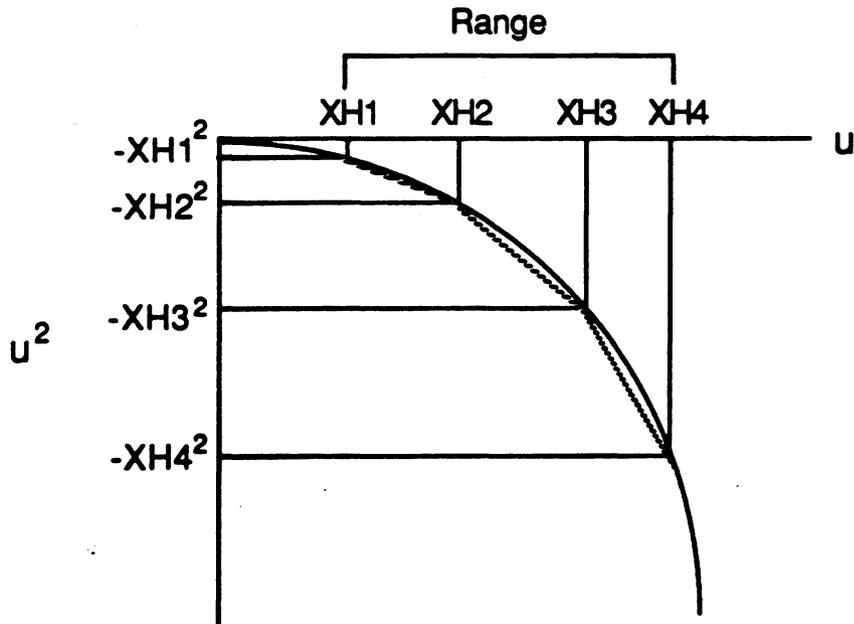


Figure 22 Piecewise Linear Approximation