An Integrated Approach for the Optimal Design of a Dynamic System and its Controller

by

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AN INTEGRATED APPROACH FOR THE OPTIMAL DESIGN OF A DYNAMIC SYSTEM AND ITS CONTROLLER

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Abstract

Optimal control system design traditionally involves the adjustment of state and input variables to optimize a specified performance index. However, in practice there may exist variables, such as plant parameters, that offer extra freedom in the design of systems that can achieve even better performance. In this paper, we describe an integrated approach for dynamic system/control system design which can satisfy a variety of constraints and allows for the introduction of three additional types of variables in the optimization process. The approach we outline is based on a direct search method which can be used in conjunction with standard optimal control algorithms such as the Riccati equation for unconstrained linear systems. The method is used to generate optimal state and input trajectories for a linear mass-spring-damper system.

1 Introduction

Traditionally, a mechanical system is designed to satisfy performance specifications, such as a stiffness requirement derived from peak loads expected during system operation. The design is "optimized" by adjusting system parameters, such as stiffness, mass, or dimension, for example, subject to mechanical and geometrical constraints such that the modifications satisfy the specified performance requirements. The design is then passed on to the controls engineer whose task is to design an optimal control scheme for the given system. Typically, the controls engineer has little input in the evolution of the mechanical design. This practice of separating the mechanical and control system design activities is promoted by the attitude that an "optimal control system" can be designed for any system. In fact, the solution to the optimal control synthesis problem can be quite different depending on the design of the mechanical system. The current design philosophy is that if each of the "subsystems" is optimized independently, then the total system will be at least nearly optimal. This may not be the case; ideally an integrated approach is necessary for real systems.

The substantial gap between the processes of dynamic system design and optimal control system design can, in part, be attributed to their different mathematical bases. Typically, a dynamic system design problem is formulated and solved by nonlinear programming techniques [1-3], while an optimal control system design is based on mathematical techniques such as variational methods and Pontryagin's maximum principle [4-6]. Nonlinear programming methods involve the adjustment of system parameters to optimize an objective function while satisfying system constraints, such as requirements resulting from stress-strain analyses, etc. On the other hand, optimal control system design is concerned with finding the optimal state variable and input variable histories that minimize a given performance index without violating any constraints on the state, input, and final time. The two design stages remain uncoupled principally due to the lack of a unifying approach which can account for the interaction between mechanical system variables and control system variables in a combined optimization process.

Recently, work in the development of integrated algorithms has been reported in [7-9]. In general, these efforts are directed toward the development of data-base software management packages that facilitate the transfer of information between application programs. These large-scale computer projects represent an important step toward integrated design, but do not include a unifying methodology linking the disparate stages of dynamic system design and control system design.

1.1 Scope

This paper explores the integration of dynamic system (plant) design and control system design in the interest of obtaining an optimal total system, rather than independently optimized subsystems. The goal is to present a systematic framework for unified dynamic system/control system design with application to optimal trajectory planning. We introduce an approach based on a direct search optimization method which is used in conjunction with standard optimal control algorithms. In addition to solving an illustrative example, we identify difficulties with the development of integrated design schemes and suggest possible future directions for research.

2 Background

In standard optimal control theory we seek an admissible control $\underline{u}^*$ that causes the dynamic system represented in state-space form

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t), \underline{u}(t), t), \qquad \underline{x}(0) = \underline{x}_0 \tag{1}$$

to follow an admissible trajectory $\underline{x}^*$ that minimizes the performance index J

$$J = h(\underline{x}(t_f), t_f) + \int_{t_0}^{t_f} g(\underline{x}(t), \underline{u}(t), t)\, dt \tag{2}$$

where $\underline{x}$ is the state vector, $\underline{u}$ is the vector of control inputs, h and g are real, scalar-valued functions of the indicated arguments and where the superscript * implies "optimal." Function h is the penalty or cost associated with the error in the terminal state at the final time t_f, whereas function g is the cost associated with the transient state errors and control effort. These functions are selected by the system designer to put more or less emphasis on terminal accuracy, transient behavior, and the expended control effort in the performance index J.

The calculus of variation approach [4-6] can be used to derive the necessary conditions for optimality when the admissible state and control regions are not bounded, *viz.*

$$\begin{aligned}
\dot{\underline{x}}^*(t) &= \frac{\partial \mathcal{H}}{\partial \underline{p}}(\underline{x}^*(t), \underline{u}^*(t), \underline{p}^*(t), t) \\
\dot{\underline{p}}^*(t) &= -\frac{\partial \mathcal{H}}{\partial \underline{x}}(\underline{x}^*(t), \underline{u}^*(t), \underline{p}^*(t), t) \\
0 &= \frac{\partial \mathcal{H}}{\partial \underline{u}}(\underline{x}^*(t), \underline{u}^*(t), \underline{p}^*(t), t)
\end{aligned} \tag{3}$$

for all $t \in [0, t_f]$ and boundary condition equation

$$\left[\frac{\partial h}{\partial \underline{x}}(\underline{x}^*(t_f), t_f) - \underline{p}^*(t_f)\right]^T \delta \underline{x}_f + \left[\mathcal{H}(\underline{x}^*(t_f), \underline{u}^*(t_f), \underline{p}^*(t_f), t_f) + \frac{\partial h}{\partial t}(\underline{x}^*(t_f), t_f)\right]\delta t_f = 0 \tag{4}$$

where superscript T represents transpose and the Hamiltonian $\mathcal{H}$ is the scalar function defined as

$$\mathcal{H}(\underline{x}(t), \underline{u}(t), \underline{p}(t), t) \triangleq g(\underline{x}(t), \underline{u}(t), t) + \underline{p}^T(t) [\underline{f}(\underline{x}(t), \underline{u}(t), t)] \tag{5}$$

where p(t) is the co-state vector of Lagrange multipliers. The co-state $\underline{p}(t)$ contains information about future effects of control perturbations and the Hamiltonian describes the way that this information can be used to describe the change in the value of the performance index. Thus, the structure of the optimal controller depends strongly on the co-state vector and on the Hamiltonian function (and its dependence on the control).

For a given problem, the boundary condition requirement reduces to a specific set of equations, as described in [5]. A problem may involve t_f fixed or free with $\underline{x}(t_f)$ specified, free, or constrained on a surface. For each case, a unique set of equations is obtained and different solution methods are usually required. The classic Riccati equation is an example of the necessary conditions for optimality for an unconstrained linear system with the final time fixed. Upon relaxing the final time constraint, a more complicated set of necessary conditions applies and a different approach must be employed. This distinct change required in the solution method due to a different constraint often hampers the design process.

3 Methodology

This paper introduces an integrated approach for the design of a plant and its control system. The approach can accommodate a variety of constraints and exploits freedom in both the dynamic system and the control system. In general, flexibility may exist in three types of variables: (1) system (or plant) parameters, (2) terminal conditions, and (3) weighting in the performance index. These variables are described below.

<u>System Parameters.</u> The selection of system parameters (as reflected by the function $\underline{f}$ in Eq(1)) usually occurs independently of the control system design. For instance, the mass and stiffness of a mass-spring-damper system are sometimes selected to meet certain natural frequency requirements, but the freedom in these system parameters is rarely exploited in control system design. There have been some attempts [1, 8, 10] to optimize the dynamic response of mechanical systems by including system parameters in optimization schemes. However, these approaches seem to tackle the problem as a totally new optimization problem without incorporating the results of standard optimal control theory which governs the original system with fixed parameters. As a result, they are often mathematically and numerically cumbersome. In the method proposed below, standard control tools can be included "naturally" in the optimization process.

<u>Terminal Conditions.</u> Assuming known system parameters, the necessary conditions for optimality have been developed and are described in standard textbooks on optimal control [4-6]. In practice, techniques for solving optimal control problems with free final state and fixed final time are well developed. In contrast, numerical difficulties are encountered when solving some optimal control problems with fixed final state and/or free final time. In this paper we include the terminal conditions as additional variables in such a way that the optimal control techniques associated with free final state and fixed final time can be employed.

<u>Performance Index Weighting.</u> The general optimal control problem involves the minimization of a performance index

$$\tilde{J} = \int_0^{t_f} g(\underline{x}(t), \underline{u}(t), t)\, dt \qquad (6)$$

subject to the inequality constraint $|*_k(t_f) - x_{Rf}|$ £ $<5_k$ for all k state variables where x_{kf} is the nominal terminal value of the k-th state variable and $\&_k$ is the tolerance on the k-th state variable at the terminal time. In order to avoid analytical and numerical difficulties associated with incorporating the inequality constraints on the terminal state, the alternative performance index J of Eq(2) is typically used. The first term in Eq(2) is a penalty (or cost) function of the final state and time, and it is usually written as

$$h(\underline{x}(t_f),\ t_f) = Vi\ \underline{x}^T(t_f)\ \underline{H}\ \underline{x}(t_f) \tag{7}$$

where the elements of weighting matrix $\underline{H}$ (usually diagonal) are selected large enough to drive the state to the region of acceptable tolerance. If the elements of $\underline{H}$ are too large, the state inequality constraint will be satisfied, but J will be larger than necessary. The selection of $\underline{H}$ involves trial-and-error or estimation methods [11]. It is clear that there exists an optimal $\underline{H}$ which satisfies the inequality constraint while minimizing J.

If all three types of variables are specified *(i.e.,* fixed), the optimal control problem can be solved for both linear and nonlinear systems. For unconstrained linear systems, the Riccati equation can be solved; for nonlinear systems and constrained linear systems, a variety of numerical techniques (including steepest descent and dynamic programming) are available for optimization.*

If flexibility exists for only one type of variable, specialized techniques can be used to solve the optimal control problem. However, the possibility of simultaneously exploiting freedom in system parameters, terminal conditions, and performance index weighting is not commonly discussed.

In this paper we assume freedom in all three types of variables. The proposed iterative method is outlined below.

1. Formulate the dynamic system in the state equation form:

$$\dot{\underline{x}} = \underline{f}(\ \underline{s},\ \underline{x},\ \underline{u},\ t\ ), \qquad \underline{x}(o) = \underline{x}_Q \tag{8}$$

subject to any constraints on (i) the state and input variables, (ii) the final state and final time, and (iii) the system parameters. In Eq. (8) $\underline{s}$ is the vector of free system parameters. The performance index, J, is

$$J = h(\underline{x}(t_f), \underline{w},\ t_f) \bullet \tilde{J} \tag{9}$$

where

*In a nonlinaar optimal control problem (undar torni smoothnass assumptions) a solution can always ba obtainad by discratizing tha problam with a dansa anough grid and than using discrata dynamic programming algorithms 1121.

$$\tilde{J} = \int_{o}^{t_f} g(\underline{s}, \underline{x}, \underline{u}, t) \, dt$$

and $\underline{w}$ is the vector of weighting coefficients on the terminal state.

Note that the optimality of the proposed performance index is a function of $\underline{s}$ and $\underline{w}$ as well as the state vector $\underline{x}$ and control (input) vector $\underline{u}$ of standard optimal control theory,.

2. Set initial guess values for $\underline{s}$, $\underline{x}(t_f)$, $\underline{w}$, and t_f.

3. Solve the standard optimal control problem. This can be accomplished because the problem is reduced to the standard problem of finding $\underline{x}^*$ and $\underline{u}^*$ for the system

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, t), \qquad \underline{x}(o) = \underline{x}_o \tag{10}$$

which will minimize the performance index

$$J = h(\underline{x}(t_f), t_f) + \int_{o}^{t_f} g(\underline{x}, \underline{u}, t) \, dt \tag{11}$$

with the final state and final time fixed and the system parameters and weighting coefficients given. In solving for $\underline{x}^*$ and $\underline{u}^*$ both equality and inequality constraints on the state and input variables must be satisfied.

4. Without violating any constraints on the system parameters, final state, and final time, modify $\underline{s}$, $\underline{x}(t_f)$, $\underline{w}$, and t_f according to an optimization algorithm, such as a direct search method, to minimize J.

5. If the termination criterion of the optimization algorithm is not satisfied, repeat Step 3 with updated $\underline{s}$, $\underline{x}(t_f)$, $\underline{w}$, and t_f. If the termination criterion is satisfied, then the method has identified the optimized "optimal" solution, *i.e.,* _s\ w\ t_f\ $\underline{x}^{\#}$\ and $\underline{u}^{**}$ where the single superscript « implies the optimal values and the double superscript *« implies the optimized *(i.e.,* minimum) optimal solution.

The scheme presented above represents an integrated design approach for solving a variety of optimal control problems involving freedom in the three types of variables, $\underline{s}$, $\underline{w}$, and t_f. The approach assumes that the optimal control solution can be obtained for the case when the three types of variables are fixed. A general flowchart of the proposed integrated design method is shown in Figure 1.

The optimization algorithm we have employed is based on a direct search method known as Simplex optimization [13,14]. The Simplex method is an effective and computationally compact scheme for minimizing the performance index J which is a function of $\underline{s}$, $\underline{x}(t_f)$, $\underline{w}$, and t_f, or n scalar variables. The method depends on the comparison of functional values of J (from Step 3) at (n+1) vertices of a general simplex, *i.e.,* a geometric figure that has one more vertex than the space in which it is defined has dimensions. To reach a minimum, the simplex is moved "downhill" by replacing the vertex with the highest value by another point. In comparison to other methods such as steepest descent and Newton-Raphson methods which depend on gradient values, the Simplex method requires only functional values of the objective function *(i.e.,* J) and thus can update the current guesses rapidly.

4 Example

We demonstrate the approach using the linear system shown in Figure 2. We consider a mass-spring-damper system with known mass m and known viscous damping constant c and variable *(i.e.,* "free") spring stiffness k. The mass is acted upon by force F(t). We seek the optimal stiffness and force in moving the mass to a new state, as described below.

Step 1. The system is described by state equation

$$\dot{\underline{x}}(t) = \underline{A}\ \underline{x}(t) + \underline{B}\ F(t) \tag{12}$$

where

$$\underline{A} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \quad \text{and} \quad \underline{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and the state is $\underline{x} = [\ x_i\ ,\ x_2\ ]^T * [\ x\ ,\ \dot{x}\ ]^T$. The initial state variables x_{10} * x^O) and $x_{2Q} = x_2(0)$ and the desired final state variables x_{1f} and x_{2f} are given. We seek the optimal control $F^{\#}$ that minimizes

$$J = {}^1/_2\ \underline{x}^T(t_f)\ \underline{H}\ \underline{x}(t_f) \bullet \int_0^{t_f} F^2(t)\ dt \tag{13}$$

where $\underline{H}$ is a diagonal weighting matrix whose diagonal elements are large enough to satisfy known terminal state constraints

$$\begin{aligned} |x_1(t_f) - x_{1f}| &\le \delta_1 \\ |x_2(t_f) - x_{2f}| &\le \delta_2 \end{aligned} \tag{14}$$

where the tolerances are δ_1 and *&2* and 0 < t_f £ T where T is the upperbound on acceptable t_f.

Step 2. Select initial guess values for stiffness k, final time t_f, and diagonal elements of weighting matrix $\underline{H}$.

Step 3. From standard linear optimal control theory, the optimal control law can be obtained by solving the Riccati equation

$$\dot{\underline{K}}(t) = -\ \underline{K}(t)\ \underline{A} - \underline{A}^T\ \underline{K}(t) * \underline{K}(t)\ \underline{B} \bullet \underline{K}(t)\ \underline{B}\ \underline{B}^T\ \underline{K}(t) \tag{15}$$

Eq (15) is solved by integration for $\underline{K}(t)$. The input is then calculated from

$$F(t) = - \underline{B}^T \underline{K}(t) \underline{x}(t) \tag{16}$$

which is substituted into the state equation (Eq(12)) and solved for the optimal state trajectory $\underline{x}^{\#}(t)$. The optimal input $F^{\#}(t)$ is known from Eq(16) with $\underline{x}(t) = \underline{x}^{*}(t)$.

Steps 4 and 5. Based on the results of Step 3, k, t_f, and the elements of $\underline{H}$ are updated using a Simplex direct search method to minimize

$$\bar{J} = \int_0^{t} F^2(t)\, dt \tag{17}$$

while not violating the terminal state and terminal time constraints. The updating continues until a termination criterion, such as

$$\sqrt{\left(1 - \frac{k}{\bar{k}}\right)^2 + \sum_m \left(1 - \frac{H_{mm}}{\bar{H}_{mm}}\right)^2 + \left(1 - \frac{t_f}{\bar{t}_f}\right)^2} < \epsilon$$

is satisfied where the numerator of the second term inside each pair of parentheses is the current iteration value, the superbar represents the mean of all previous iteration values, and * is the error bound.

The method has been implemented on a IBM PC. Simulation results have been obtained assuming the numerical data listed in Table 1.

Three cases have been considered, as shown in Table 2. Case 1 represents the standard optimal control problem where k, H_n, H_{22}, and t_f are fixed (having been selected arbitrarily). The index of control effort, $\bar{J}$, is 26.7 N^2. In Case 2, the final time t_f is fixed at 0.5 sec and the method described above is used to solve for the optimal k, H_n, and H_{22}. Here, the index $\bar{J}$ is reduced by 6.7 percent to 24.9 N^2. In Case 3, an inequality constraint is imposed on the final time. The optimal solution is reached at the upper bound of the final time constraint, *i.e.,* at 0.8 sec, where $\bar{J}$ = 5.23 N^2.

In comparing the results of Cases 1 and 2, we note that the terminal state constraints are satisfied at the extremes for Case 2 resulting in a decrease in $\bar{J}$. The decrease in weighting elements H_{11} and H_{22} on the terminal state implies that the weighting on the total control effort has been increased. In fact, the weighting on control effort cannot be increased further without violating the bound on the terminal state. Since the performance index J consists of the sum of the terminal penalty function and the trajectory function $\bar{J}$, we cannot claim to have found the minimum

$\tilde{J}$, *i.e.,* there may be an admissible trajectory which has a smaller $\tilde{J}$ but larger terminal penalty function with a larger total sum J. However, when the terminal state constraint is satisfied at the extreme values, as occurs in Case 2, and the terminal penalty function cannot be increased further without violation, then we are assured of achieving minimum $\tilde{J}$. Thus, Case 2 represents a truly minimum control effort solution.

In Case 3, the final time constraint is satisfied at its upper extreme, *i.e.,* t_f = 0.8 sec. Physically, this minimizes energy dissipation in the damper by reducing the velocity profile. Even though H_n and H_{22} are decreased, we cannot claim to have a truly minimum $\tilde{J}$ since one of the terminal state constraints is not satisfied at its extreme. Although the solution is optimal only with regard to J, the control effort index $\tilde{J}$ is reduced significantly and the performance is improved.

Graphs of displacement, velocity, and force as a function of time are shown in Figures 3, 4, and 5, respectively. Figure 3 shows that the displacement histories are sigmoidal ("S" shaped) in character corresponding to the fact that the mass is first accelerated and then decelerated. For Case #1 the displacement at the terminal time, t_f = 0.5 sec, is within the acceptable bounded region. There is potential for expending less control effort by reducing the weighting on the terminal constraint without violating the required specifications. This occurs in Case #2, where the terminal constraint is satisfied at the "lower" *{i.e.* closer) extreme (-0.05 m), thereby minimizing the energy dissipated in the damper. Case #3 represents the solution where the final time is free but limited to t_f £ 0.8 sec. The optimal solution occurs at t_f = 0.8 sec, at which time the displacement is close to zero, and not at an extreme. The solution obtained is a local minimum and the result suggests the potential for further improved performance.

Figure 4 shows the velocity profiles as a function of time for the three cases. These parabolic profiles again reflect an acceleration phase followed by a deceleration phase of the mass. For Cases #2 and #3 the terminal state constraint is satisfied at the "upper" extreme (0.05 m/sec). Physically, the mass is decelerating to just satisfy the constraint.

Figure 5 shows the minimum control effort solution for the three cases. In addition to minimizing control effort, the maximum magnitude of the input force is reduced for Case *02* relative to Case #1. The input force varies smoothly except at the boundaries *(i.e.,* near the initial and final times). The optimal control solution has finite initial force values, which may be physically impossible to achieve instantaneously.

5 Discussion

Due to the nature of the direct search method, the proposed design approach does not guarantee global minimization of the performance index; rather, only local minimization is achieved. This suggests the importance of selecting a reasonable initial guess for the optimization. In practice, the selection of an initial guess should be predicated, to the extent possible, on insight and experience based on the given physical problem. In many cases, it is advisable to repeat the simulation starting with different initial guesses, in order to search for the global minimum. Despite the possibility of missing the global minimum, the method still improves upon the standard optimal control solution by arriving at a smaller value of the performance index.

In the proposed method, an optimal control problem must be solved during each optimization iteration. For unconstrained linear systems, the optimal control solution is directly available by solving the Riccati equation. In contrast, for nonlinear and constrained linear systems, iterative methods, such as steepest-descent methods, are typically applied to solve the optimal control problem. In the latter cases, the double-nested iterative structure of the overall method makes the approach computationally-intensive and often-times unwieldy. Thus, due to its iterative nature, the method is best suited for off-line design.

Despite the above limitations related to finding the global minimum and on computational inefficiency, the method has several advantages, namely, (1) it can work in conjunction with existing linear and nonlinear optimal control theories and their numerical algorithms, and (2) it can handle effectively inequality constraints on the three types of free variables. The first advantage means that the method can draw upon the wealth of existing optimal control packages. The second advantage is due to the fact that the direct search method can easily convert a constrained problem to an equivalent unconstrained problem by suitable penalty function techniques. The advantage is significant since, in design, inequality constraints on system parameters are commonly encountered.

6 Conclusions

This paper describes a PC-based tool for integrated design of dynamic systems and control systems. The tool embeds an optimal control method inside an optimization algorithm and is capable of simultaneously adjusting state variables, input variables, system parameters, performance weighting, and final time to minimize an objective function while satisfying design constraints. By utilizing these degrees of freedom, it is possible to improve upon the standard optimal control solution.

The algorithm has been used to solve several integrated design problems and experience has shown that the method is best suited to unconstrained linear systems. While there is some art involved in application of the algorithm - to arrive at initial guess values - no difficulty has been encountered in obtaining optimal solutions.

We believe that this method represents a new trend in integrated plant/control system design and are currently exploring a suboptimal control approach that will overcome the double-iterative structure of the algorithm when tackling nonlinear and constrained linear problems.

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Table 1. Numerical Data for Example,

$m = 1.0$ kg

$c = 1.0$ N-sec/m

$x_{10} = -1.0$ m

$x_{20} = x_{1f} = x_{2f} = 0$

$\delta_1 = 0.05$ m

d_2 s 0.05 m/sec

T * 0.8 sec

ϵ * 10^{-6}

Table 2. A Comparison of the Results of Three Optimal Control Problems.

CASE	PROBLEM	k (N/m)	H_{11} (N/m)	H_{22} (N-seota)	*f (sec)	$x_1(t_f)$ (m)	$x_2(t_f)$ (m/sec)	$\bar{J} = \int u^2 dt$ (N^2)
1	Fixed k, ^{H}1T ^{H}22* *f ("Optimal Control")	20.0	500.	500.	0.50	0.0292	0.00919	26.7
2	Fixed t_f only; *free* k» "ii• H22* Constraint on $\lfloor x(t_f) \rvert$ 10.05	13.1	452.	233.	0.50	-0.050	0.050	24.9
3	Free t_f, k, ^{H}1T *^{H}22'* Constraint on $t_f \leq$ 0.8 sec	7.12	415.	96.5	0.80	0.00298	0.050	5.23

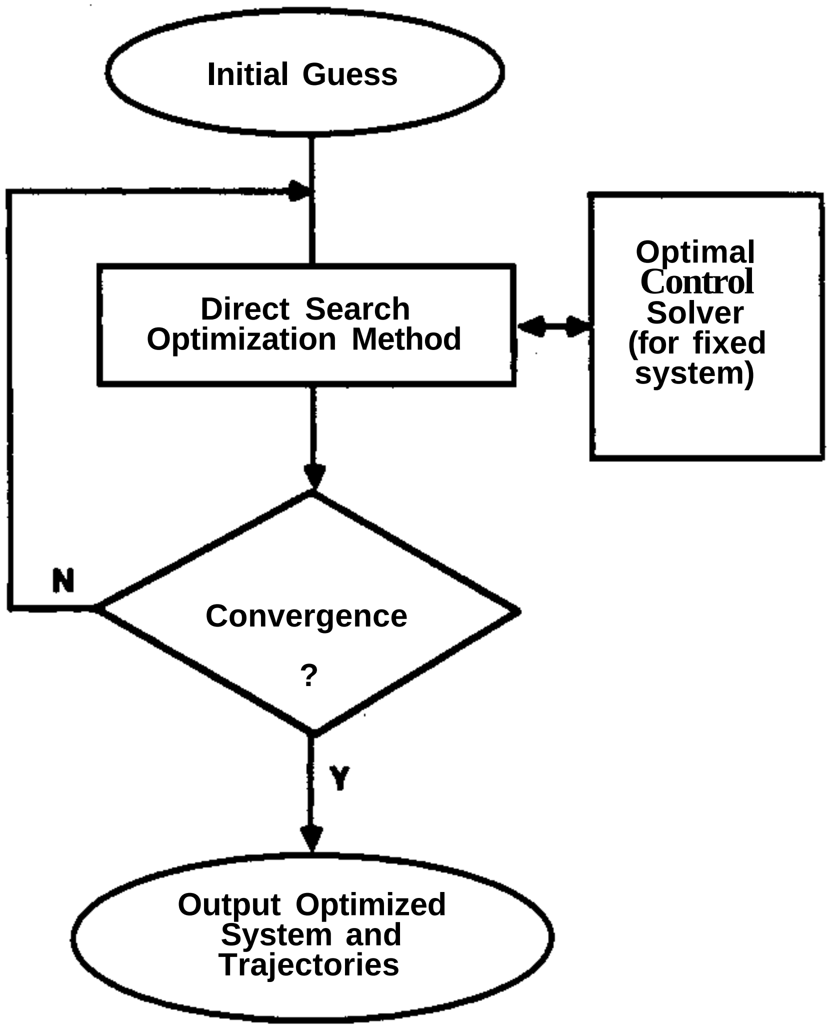

Figure 1. Simplified Flowchart of Integrated Design Algorithm.

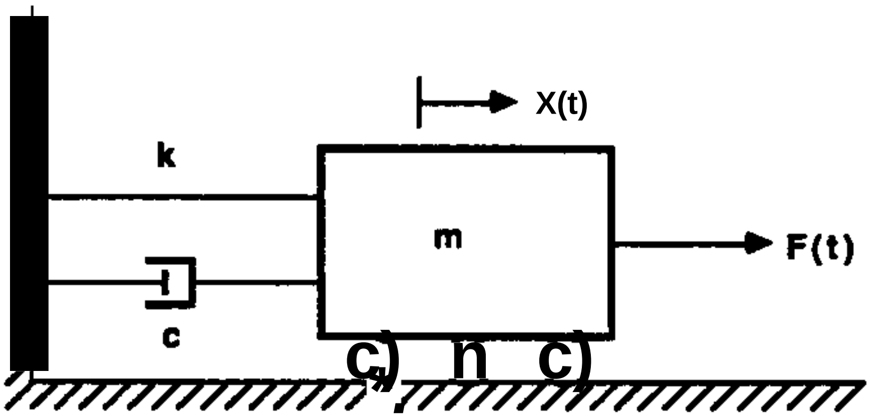

Figure 2. Mass-Spring-Damper System.

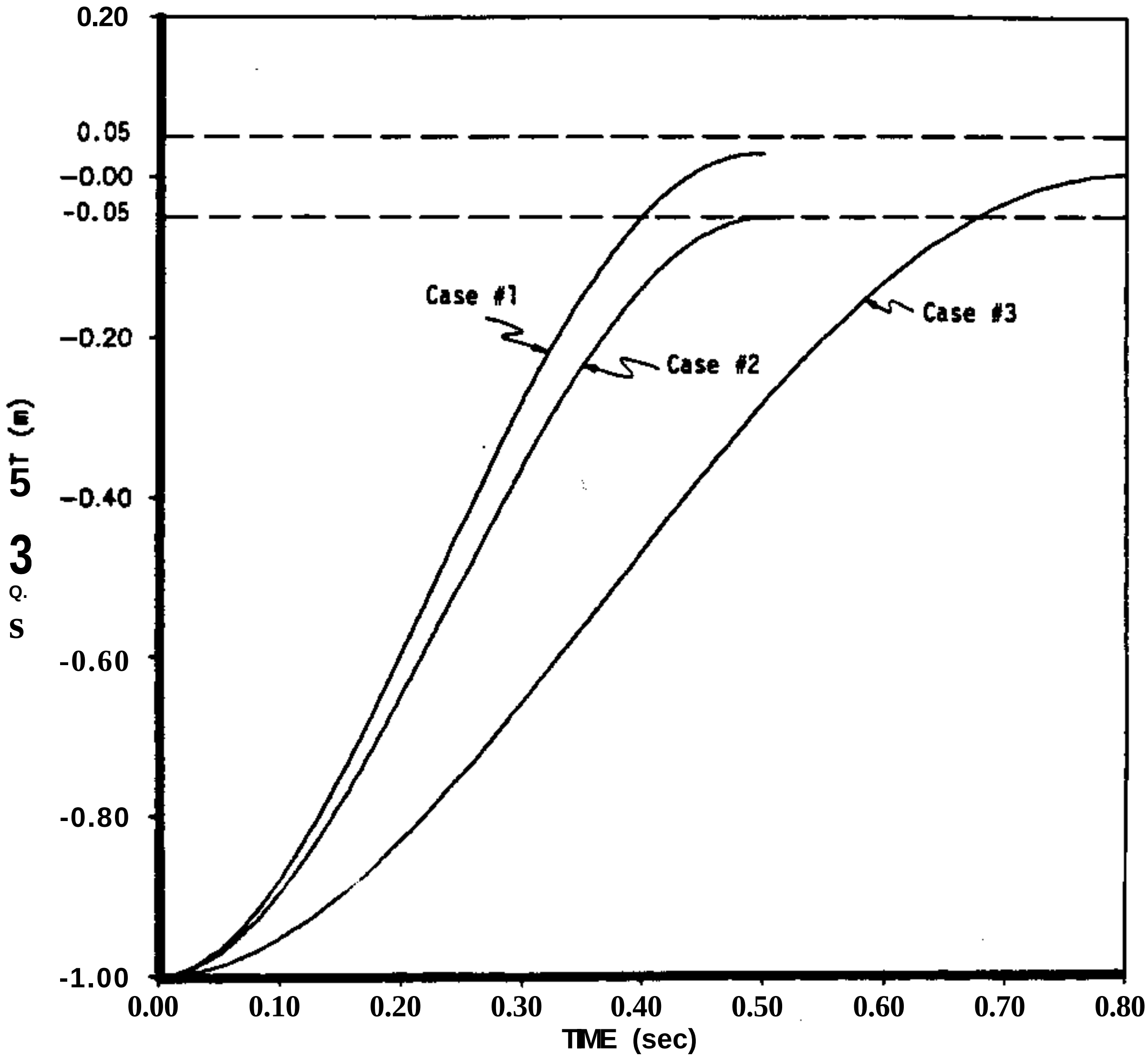

Figure 3. Displacement Histories for Three Cases of Example.

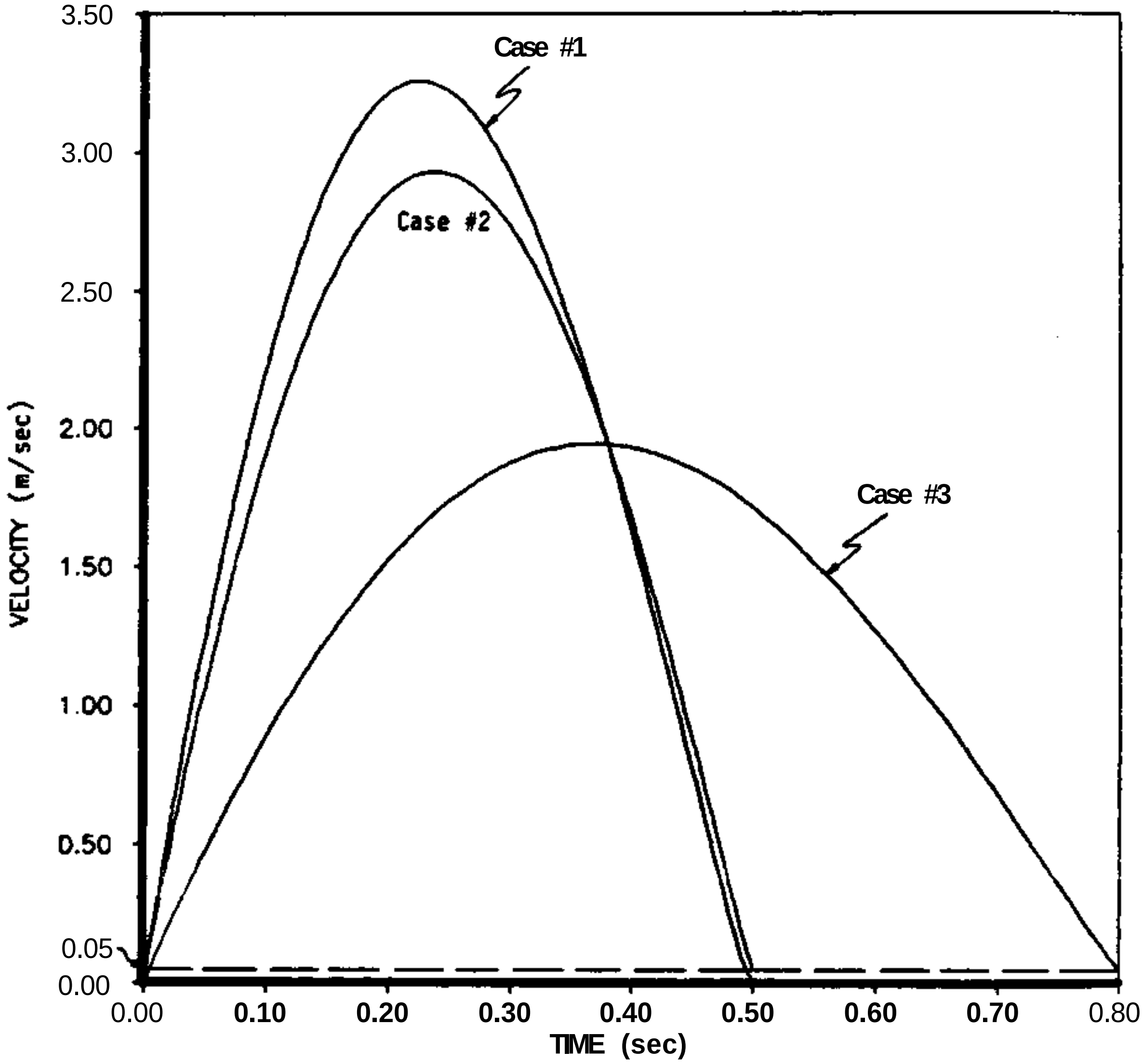

Figure 4. Velocity Histories for Three Cases of Example.

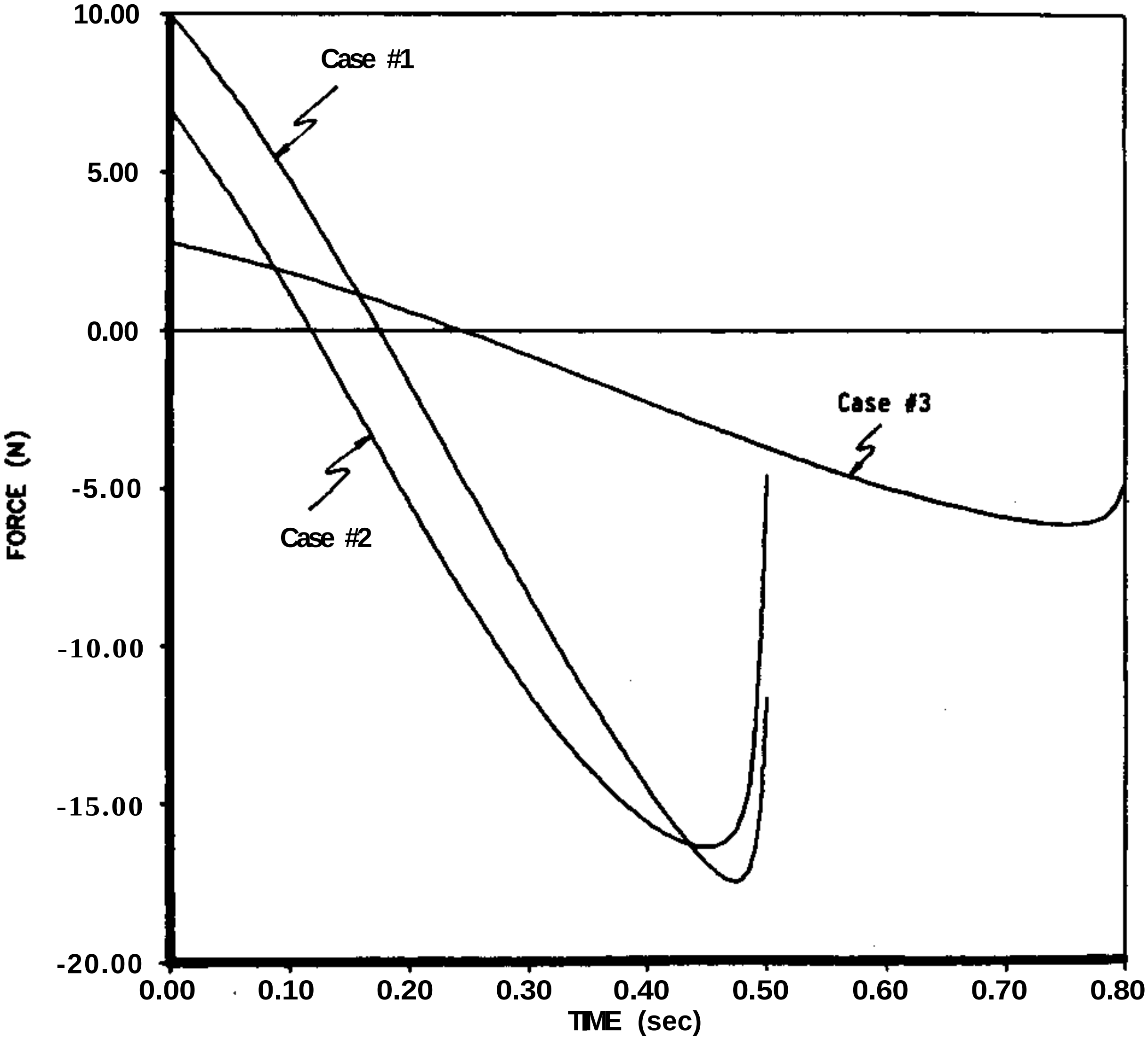

Figure 5. Force Histories for Three Cases of Example.