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RECENT RESEARCH IN NETWORK PROBLEMS WITH APPLICATIONS

by

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Introduction

The decade of the 1970^fs witnessed a stunning improvement in the computability and applicability of network programming problems. Current applications involve problems having thousands or millions of variables and thousands of constraints. Network codes developed in the 1970's are 100 - 200 times as fast as their predecessors.

The main purpose of this talk is to survey the capabilities of network codes and their extensions to specially structured integer programming problems which can be solved by using the solutions of a series of ordinary network problems.

Most of the actual computational methods and results surveyed in this talk are taken from papers or working papers by the author and his students. This was done because of their easy availability. Many other authors have contributed important ideas to this area which we do not have time to discuss here. References to their work are given in the bibliography. It is not the purpose of this talk to give a historically accurate and complete account of the advances in network modelling and computing, so that the bibliography will have to suffice as a substitute for such a historical account.

Discussion of the Slides

SLIDE 1 gives the basic notation for a transportation problem having m factories and n markets. The factories have supplies a_i and the markets have demands b_j . Note that the sum of the supplies is assumed to be equal to the sum of the demands. When the demands are l^f s the problem is called a semi-assignment problem; and when the supplies are also l^f s, it is called an assignment problem.

SLIDE 2 gives the transportation problem constraints which the variables x_{ij} must satisfy. The first constraint says that the total amount shipped from warehouse i is equal to the amount it contains; the second constraint says that the sum of the amounts shipped to market j is equal to its demand; the last constraint is just nonnegativity. The (bipartite) graph at the bottom shows that the direction of shipping is from factories to markets.

SLIDE 3 gives the two kinds of objective functions we will consider. The sum objective adds together all the shipping costs from each warehouse to each market. It is appropriate for bulk shipments of nonperishable goods. The bottleneck objective is appropriate when c_{ij} is interpreted as the time to ship goods from factory i to market j , and the objective is to minimize the maximum time along any route which carries a positive shipment. The bottleneck objective is appropriate when considering problems such as: shipping perishable goods to markets in which we are concerned with the longest time for any shipment to get to its destination; or sending troops to staging areas in a case in which the unit is not ready to go until all sub-units have achieved their starting positions.

SLIDE 4 introduces the concept of a transshipment node, that is, one which is both a source and a sink. Network problems are transportation problems in which most nodes are transshipment nodes. Usually, not all of the possible arcs connecting pairs of nodes are assumed to exist in network problems, that is, the problems are sparse. Very large sparse problems have been formulated and solved relatively quickly.

The idea of computationally complexity is in vogue among computer science and OR practitioners. As noted on SLIDE 6, transportation and network problems

are among the easiest such problems since they are polynomially bounded; that is, in the worst case the maximum number of steps required to solve such a problem can be constrained by a bound which is a polynomial function of the amount of input data needed. Transportation problems are natural integer problems since they will have integer solutions when the a_i 's and b_j 's are integers. For both these reasons, these problems are important in applications.

SLIDE 7 shows a typical network application in the area of manpower planning. Here there are three ranks and a maximum of five years of organizational age. Separations from the organization are indicated by upward slanting arrows, promotions by downward slanting arrows, and continuations in rank by horizontal arrows. The full model also has upper bounds on flows in each of these arcs. Note that there is one source node and one sink node (retirement), and all other nodes are retirement nodes. This is fairly typical for a network application.

SLIDE 8 shows a fairly typical warehouse (or factory) to market application which is a straightforward transportation problem application. Unfortunately, many such applications also have other constraints which are not transportation type constraints. The single source constraint at the bottom of the slide is one such. It imposes the very commonly occurring requirement that all the demand at a given market be supplied from a single warehouse. We discuss methods for imposing such constraints next.

SLIDE 9 gives a computational flow diagram of the Regret Heuristic which (sometimes) finds good feasible solutions to single source problems. Note that it contains a random choice element so that each time it is run a (potentially) different solution is found. This heuristic is not guaranteed to get a feasible solution, but it usually does, and about half of the time the feasible solution is also optimal.

In contrast to a heuristic code which can only produce feasible solutions, an algorithm is a code which will, if it is run long enough, produce an optimal solution to a decision problem (when such an optimal solution exists).

SLIDES 10 and 11 discuss some of the concepts needed to implement a branch and bound algorithm for solving the single source problem. On SLIDE 10 note that the first thing we do is to relax the integer single source constraints to be just nonnegativity constraints. The relaxed problem is an ordinary transportation problem whose solution value gives a lower bound on the value of the unrelaxed problem. Usually the relaxed solution will not satisfy all the single source constraints; variables which violate these constraints are called fractional variables. We choose one such variable and branch, that is, we consider the two subproblem in which the fractional variable is set either to zero or to the total demand of the column it is in. We relax the remaining variables in these two subproblems and solve them as transportation problems to get their lower bounds. This process is continued until we get either a feasible solution which allows us to update the upper bound (UB) or else we obtain a lower bound greater than an already achieved upper bound and can terminate search on this branch of the search tree--the latter step is also called fathoming. A typical search tree is shown in SLIDE 11.

SLIDE 12 gives a LIFO (Last In First Out) or depth first branch and bound algorithm for solving the single source transportation problem. Note that it begins by finding a heuristic solution to give the initial value of UB the upper bound. Then the relaxed problem is solved. Then a column having a fractional variable is selected and one of the cells having largest flow is selected; it is fixed in, that is, made to supply the total demand. If the resulting solution is single source we update UB and backtrack, that is, go upward on the search tree. If it isn't a single source solution we test to

see if the value of the subproblem is \geq UB to see if we can fathom. If we can fathom we backtrack, otherwise we choose another fractional variable and search deeper in the tree. The computational procedure stops when we try to backtrack from the initial node of the search tree.

As noted at the top of SLIDE 13, the forward and backtrack movements in the search tree are actually performed by using cost operators which are computationally inexpensive. Also shown there are computation times obtained recently by Nagelhout and Thompson. Note that the heuristic frequently finds the optimum. Also note that solution times vary erratically depending on the size of the search tree. In one case computation was stopped because of excessive time. These are typical results for this kind of problem.

SLIDES 14 and 15 discuss the Travelling Salesman problem which can be solved by similar procedures. The rubber band heuristic for the travelling salesman proceeds as follows: choose any three cities and find their smallest subtour; now choose any city omitted and try inserting it in between pairs of cities on the subtour so far constructed; continue until a complete tour is obtained. The relaxed problem is an assignment problem which, if solved, will usually have loops or subtours. To develop a branch and bound code we solve the relaxed (assignment) problem, select a smallest subtour, choose an arc on that subtour and fix it out; now solve the new relaxed problem and iterate until a feasible tour is found; then backtrack etc. The rest of the code is similar to that for the single source problem.

SLIDE 15 gives some computation times for sum and bottleneck travelling salesman problems. Note that for the sum case, the total solution time goes up rapidly with the number of cities, but the average time to the first tour (which is usually within a few percent of the optimum) remains small. The first tour found by the algorithm can be used as an improved heuristic solution.

The most amazing results are for the bottleneck travelling salesman problem; Smith and Thompson have solved such problems up to 2,000 cities. The reason that this is possible is that the search trees remain surprisingly small as noted at the bottom of SLIDE 15.

The last major example to be discussed is the capacitated warehouse location problem stated in SLIDE 16. Note that the x_{ij} variables are as before, but the y_i variables take on only the integer values 1 if warehouse i is open, and 0 if it is closed. In the objective function a fixed charge F_i is added when warehouse i is opened. The relaxed problem here is obtained by just requiring $y_i \geq 0$, i.e., nonnegativity. We do not discuss further details of the branch and bound code.

SLIDE 17 gives computational results obtained by Nagelhout and Thompson on this problem. Note that the bottleneck problems are much easier than the sum problems, since the bottleneck code has no failures while the sum code failed to solve two sum objective problems. It is also true that the variance of times is much less for the bottleneck than for the sum objective problems.

The problems discussed so far are far from exhausting the applications of network and transportation problems.

SLIDE 18 lists 8 other application areas which will be briefly discussed. Also discussed are two other network models. The first is a network with gains in which the quantity of the good can increase or decrease as it flows along an arc. An example of an increase is: suppose the commodity is money and flowing on the arc means being on deposit in a savings account for a period of time; the money can then be augmented by an interest payment. An example of a decrease is: suppose the quantity is electrical power flowing in a wire; it can be decreased due to power losses. The final generalization is to multi-commodity flows in which we consider several commodities flowing on the same arc and competing for its capacity.

Many further results on these and other models are available in the literature cited in the following references.

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FACTORIES (SOURCES) $I = \{1, \dots, M\}$

SUPPLIES A_i FOR $i \in I$

MARKETS (SINKS) $J = \{1, \dots, N\}$

DEMANDS B_j FOR $j \in J$

ASSUME

$$\sum_{i \in I} A_i = \sum_{j \in J} B_j$$

SEMIASSIGNMENT PROBLEM

$$B_j = 1 \text{ FOR } j \in J$$

ASSIGNMENT PROBLEM

$$A_i = 1 \text{ FOR } i \in I, \text{ AND}$$

$$B_j = 1 \text{ FOR } j \in J$$

SLIDE 1

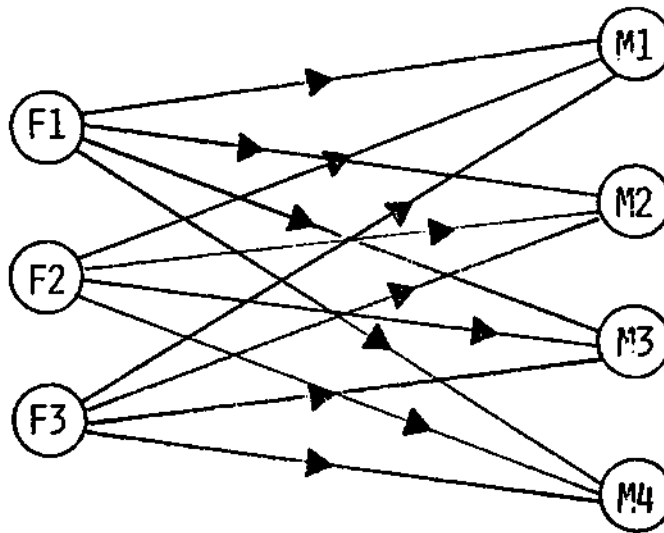
x_{ij} = AMOUNT SHIPPED FROM FACTORY i TO MARKET J ,

CONSTRAINTS

$$\sum_{J \in J} x_{iJ} = A_i \quad \text{FOR } i \in I$$

$$\sum_{i \in I} x_{iJ} = B_J \quad \wedge \quad J \in J^*$$

$$x_{ij} \geq 0 \quad \text{FOR } i \in I, J \in J.$$



SLIDE 2

OBJECTIVE FUNCTIONS

SUM OBJECTIVE

c_{ij} = COST OF SHIPPING ONE UNIT FROM I TO J.

MINIMIZE $\{ \sum_{i \in I} \sum_{j \in J} x_{ij} c_{ij} \}$

Z = TOTAL SHIPPING COST

EXAMPLE: GROCERY WAREHOUSES TO SUPERMARKETS

BOTTLENECK OBJECTIVE

c_{ij} = TIME TO SHIP ONE UNIT FROM I TO J

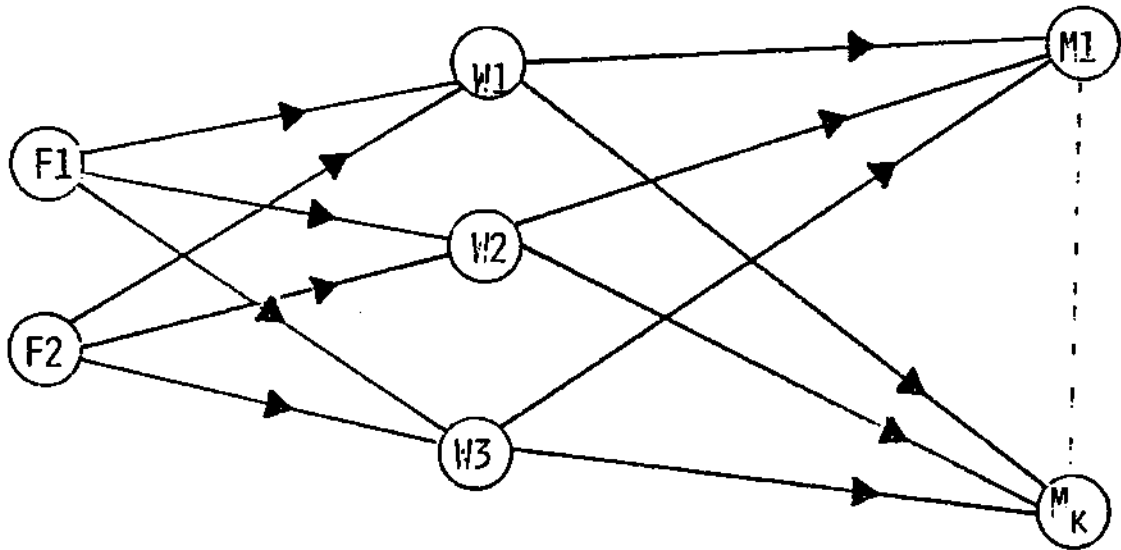
MINIMIZE $\{ Z = \text{MAXIMUM } c_{ij} \}$
 $x_{ij} \geq 0$

Z - MAXIMUM SHIPPING TIME

EXAMPLES: PERISHABLE GOODS, STAGING OF TROOPS

TRANSSHIPMENT NODE: ONE THAT APPEARS BOTH AS A SOURCE
AND AS A SINK.

EXAMPLE: FACTORY-HAREHOUSE-MARKET SYSTEM



NETWORK PROBLEMS: MOST NODES ARE TRANSSHIPMENT.

SPARSE: NOT ALL ARCS ARE USED. CAN RE TAKEN ADVANTAGE OF.

FAST PRIMAL METHODS FOR SOLVING BOTH SUM AND BOTTLENECK

PROBLEMS WERE DEVELOPED IN THE 1970's.

TYPICAL RESULTS: IP! x 100 DENSE PROBLEMS.

OBJECTIVE	COST CHOSEN IN INTERVAL			
	0-10	0-100	0-1000	0-10000
SUM	.433	2.090	2.351	2.419
BOTTLENECK	.490	1.229.....	1.123.....	1.023.....

NOTE THAT BOTTLENECK PROBLEMS ARE APPROXIMATELY ONE HALF AS
DIFFICULT AS SUM PROBLEMS.

NOTE MINIMUM COST EFFECT.

SPARSE PROBLEMS CAN BE SOLVED MUCH FASTER.

1000 x 1000 SPARSE PROBLEMS CAN BE SOLVED IN LESS THAN
2 MINUTES.

MUCH BIGGER PROBLEMS HAVE BEEN SOLVED

50,000 x 50,000

THERE ARE SEVERAL **POLYNOMIALLY** BOUNDED PRIMAL ALGORITHMS FOR
THE SUM PROBLEM, .

FORD-FULKERSON DUAL METHOD

BALINSKI-GOMORY PRIMAL METHOD

SRINIVASAN-THOMPSON COST OPERATOR METHOD

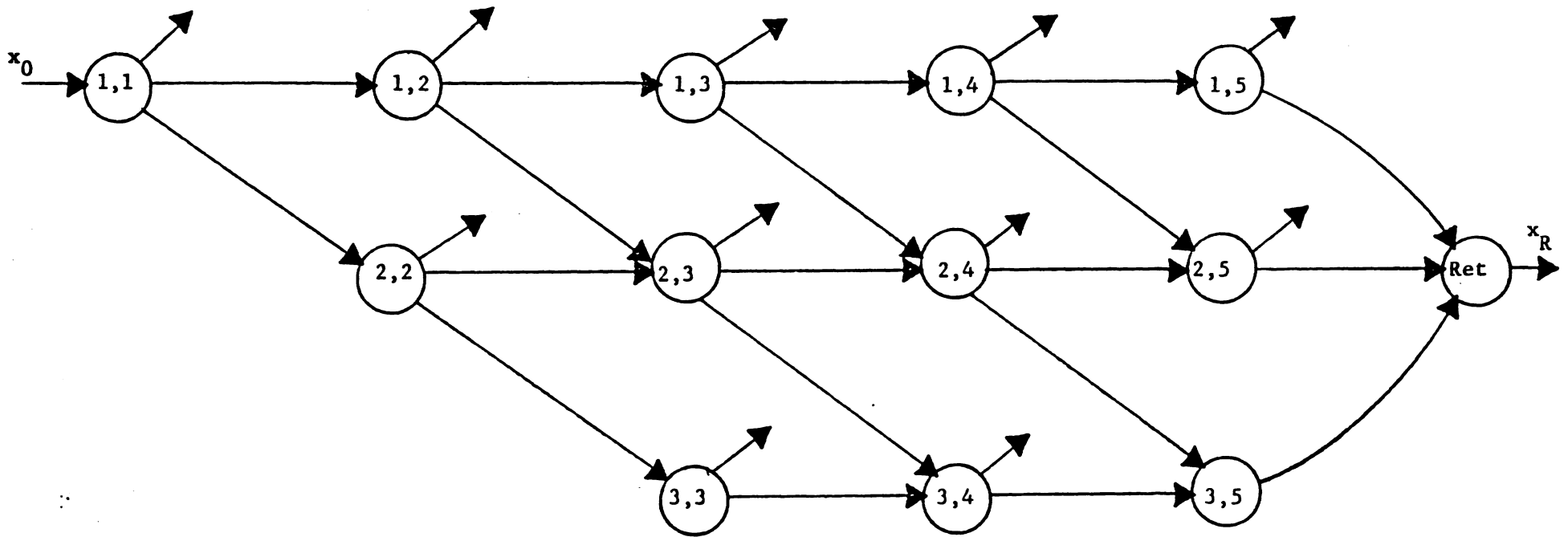
THE SRINIVASAN-THOMPSON-SZWARZ-HAMMER ALGORITHM CAN BE
SHOWN TO BE POLYNOMIALLY BOUNDED.

THOMPSON HAS A NEW RECURSIVE METHOD FOR BOTH SUM AND
BOTTLENECK PROBLEMS WHICH IS POLYNOMIALLY BOUNDED.

ALSO, IF THE A_i 'S AND B_i / S_i ARE INTEGRAL THEN A BASIC PRIMAL
FEASIBLE INTEGER SOLUTION WILL BE FOUND BY THESE
PRIMAL ALGORITHMS. THIS IS A NATURAL INTEGER PROBLEM.

THE NATURAL INTEGER PROPERTY MAKES THESE HOPELS USEFUL FOR
APPLICATIONS.

SLIDE 6



State version of the manpower model with $R = 3$ and $T = 5$. Upward slanting arrows denote separations, horizontal arrows denote continuation in rank, and downward slanting arrows denote promotions. The yearly number of new employees is x_0 , the yearly number of retirements is x_R , and the yearly separations (sum of flows on all upward slanting arrows) is x_S ; we require $x_R + x_S = x_0$.

EXAMPLE 2. WAREHOUSES TO MARKETS

	M1	M2	...	MN	
W1	c_{11}	c_{12}	...	c_{1N}	A_1
W2	c_{21}	c_{22}	...	c_{2N}	A_2
.		
WM	c_{M1}	c_{M2}	...	c_{MN}	A_M
	B_1	B_2	...	B_N	

IF WE ADD OTHER CONSTRAINTS WE USUALLY DESTROY THE NATURAL
INTEGER PROPERTY.

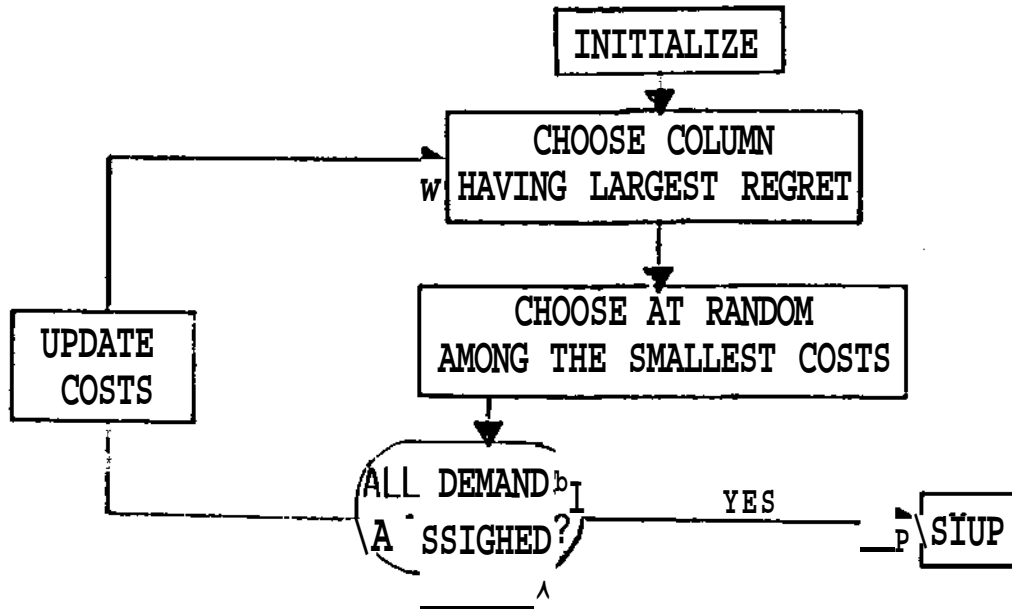
EXAMPLE: SINGLE (SOLE) SOURCE CONSTRAINT

$$x_{IJ} = \begin{cases} 0 \\ B_J \end{cases}$$

I.E., ALL THE DEMAND AT A MARKET MUST BE SUPPLIED FROM A
SINGLE WAREHOUSE.

SINGLE SOURCE REGRET HEURISTIC

$$\text{REGRET}_j = (\text{SECOND SMALLEST ENTRY} - \text{SMALLEST ENTRY})_j$$



RUN THIS PROGRAM SEVERAL (SAY 10) TIMES: SAVE BEST SOLUTION FOUND.

BRANCH AND BOUND ALGORITHM

RELAX THE SINGLE SOURCE CONSTRAINT

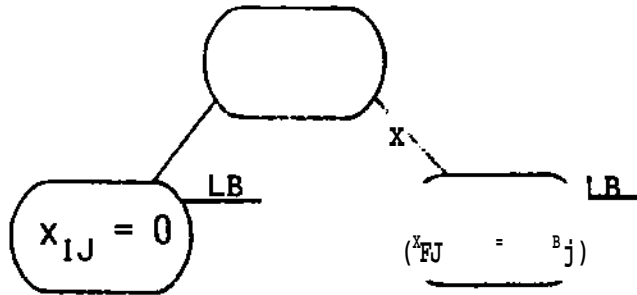
$$x_{ij} = \begin{cases} i & \text{TO} \\ 0 & x_{ij} \wedge 0 \end{cases}$$

SOLVE THE RESULTING TRANSPORTATION PROBLEM, ITS VALUE
GIVES A LOWER BOUND ON THE VALUE OF THE SINGLE
SOURCE PROBLEM

FIND A COLUMN WITH A FRACTIONAL VARIABLE, I.E.,

$$0 < x_{ij} < b_j$$

BRANCH: DEVELOP THE SEARCH TREE



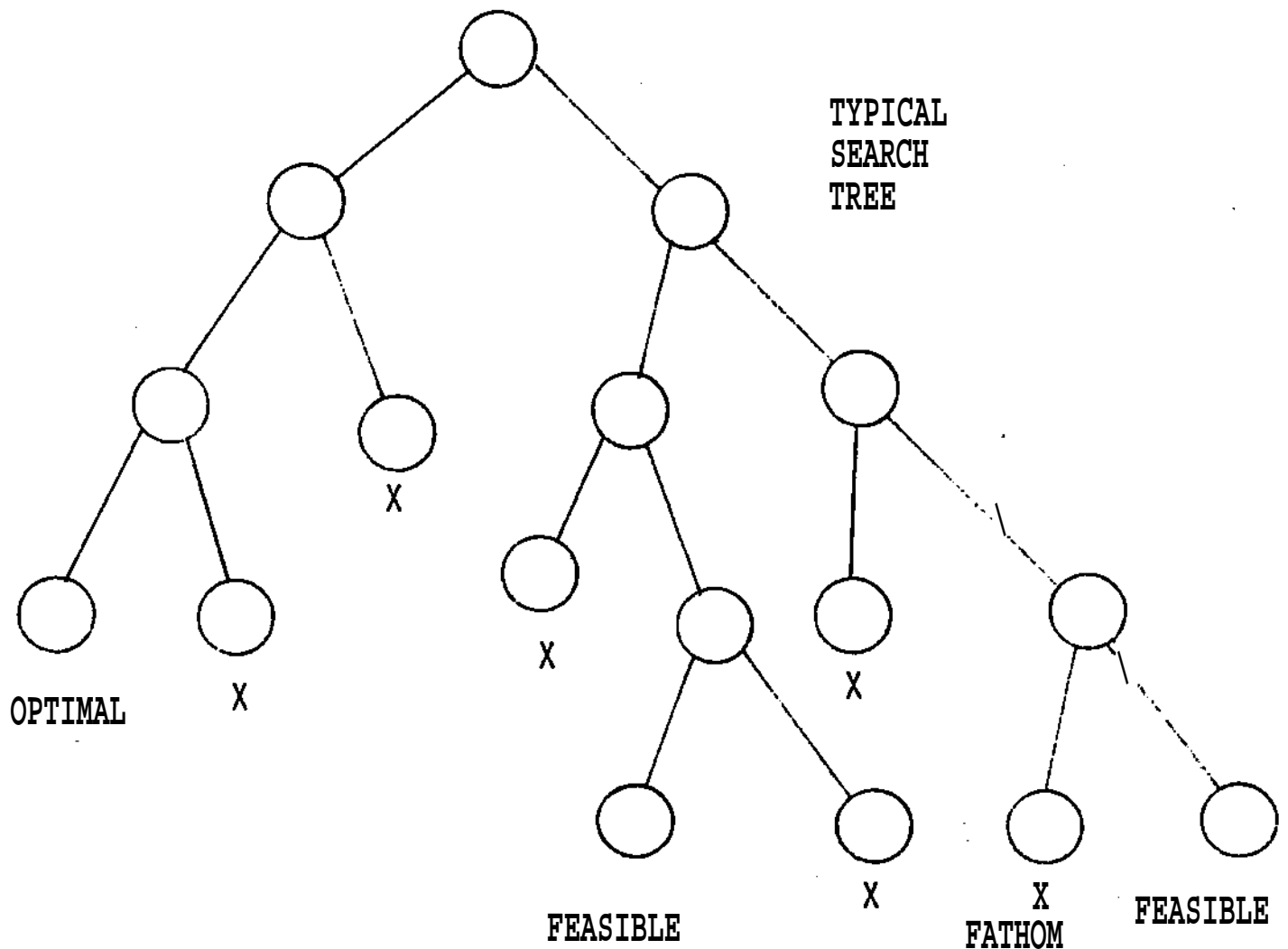
SOLVE EACH TRANSPORTATION PROBLEM TO GET LOVER BOUNDS (LB).

BRANCH AND BOUND (CONT)

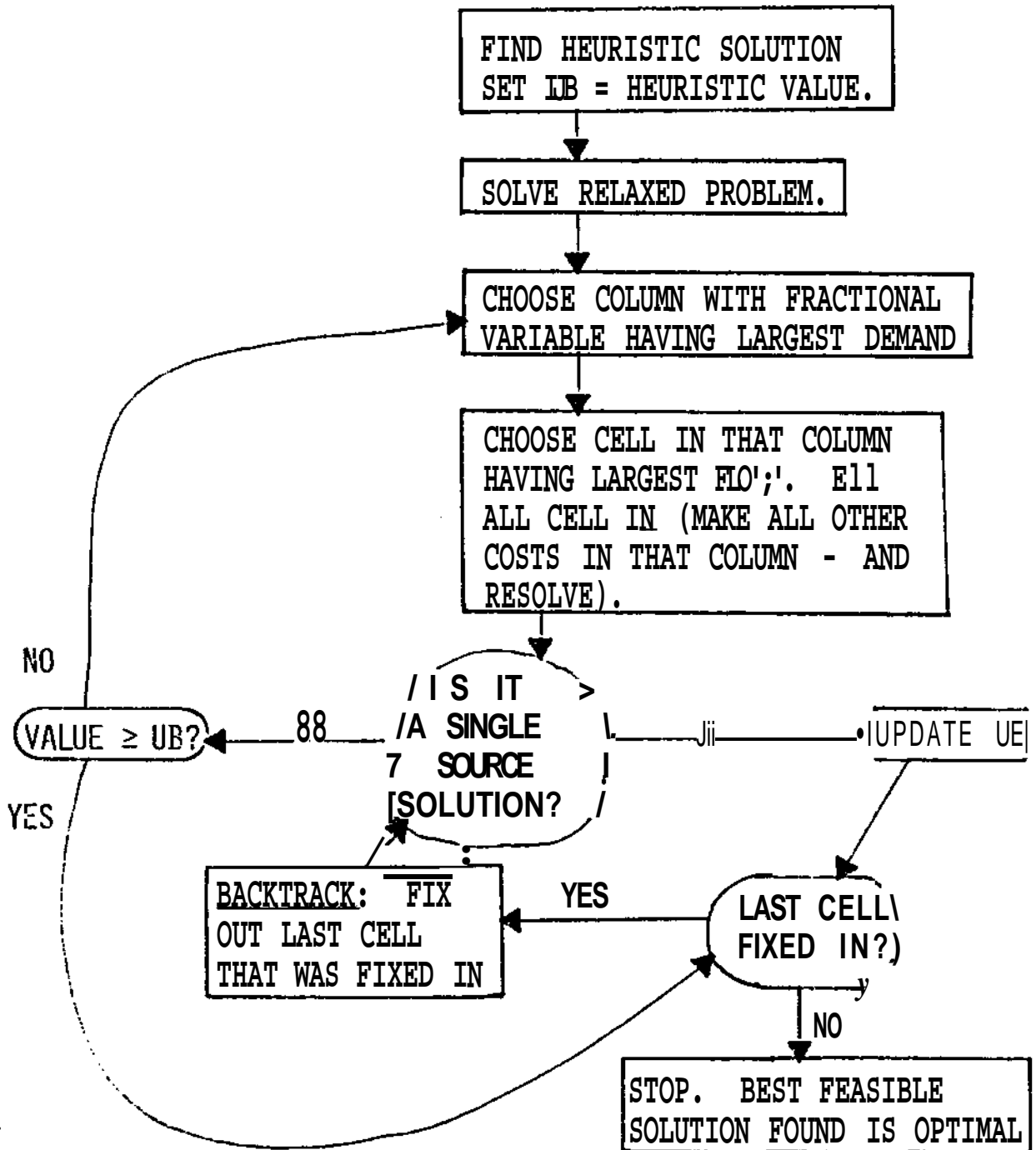
WHENEVER $LB \geq UB$ FATHOM, I.E., DON'T SEARCH
LOWER IN THE TREE.

WHENEVER A FEASIBLE SOLUTION IS FOUND (SATISFYING
SINGLE SOURCE CONDITIONS) WITH A BETTER BOUND
UPDATE UB

WHEN SEARCH IS COMPLETE HAVE OPTIMAL SOLUTION



LIFO BRANCH AND BOUND ALGORITHM
FOR THE SINGLE SOURCE PROBLEM



MOVING UP AND DOWN THE SEARCH TREE
IS DONE BY APPLYING COST OPERATORS,
A TYPE OF PARAMETRIC PROGRAMMING.

COMPUTATIONAL RESULTS FOR SINGLE-SOURCE SUM PROBLEMS

M X N	HEURISTIC		NUMBER OF SEARCH TREES NODES	TOTAL TIME (SECS)
	% ERROR	CPU TIME		
100 x 100	0	2.11	505	23.88
100 x 200	3.3	4.38	946	4P.07
100 x 300	0	7.71	20	18.75
100 x 350	-	9.94	6749	7600
100 x 400	0	10.78	3	23.39

COMPUTATIONAL TIMES FOR SINGLE-SOURCE BOTTLENECK PROBLEMS

M X N	HEURISTIC GET OPTIMAL?	NUMBER OF SEARCH TREE NODES	TOTAL TIFE (SECS)
100 x 100	No	46	19.1
100 x 150	No	9263	115.4
100 x 400	YES	-	12.48
100 x 400	No	133	106

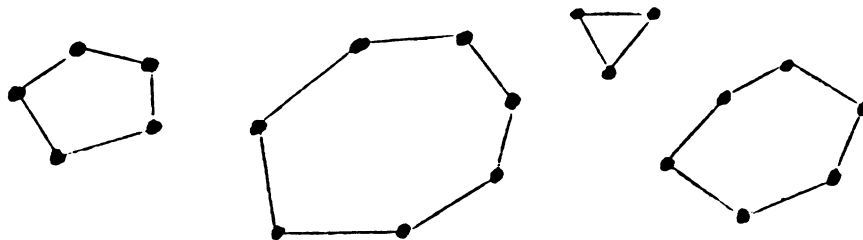
TRAVELLING SALESMAN PROBLEM

GIVEN n CITIES FIND A ROUTE THAT GOES THROUGH EACH CITY EXACTLY ONCE AND MINIMIZES THE TOTAL MILEAGE TRAVELLED (OR, MINIMIZES THE MAXIMUM INTERCITY DISTANCE.)

HEURISTIC SOLUTION: RUBBER BAND HEURISTIC.

PROBLEM RELAXATIONS: ASSIGNMENT PROBLEM OR BOTTLENECK ASSIGNMENT PROBLEM

RELAXED PROBLEM HAS SUBTOURS.



BRANCHING RULE. CHOOSE A SMALLEST SUBTOUR AND BRANCH ON SOME ARC IN IT.

RANDOMLY GENERATED ASYMMETRIC SUM PROBLEMS COSTS (0-100)
 (SMITH-SRIMIVASAN-THOMPSON, 1977)

NO. CITIES	50	100	150	180
AVE. TIME TO OPTIMALITY	1.72	52.98	65.28	617.12
AVE. TIME TO FIRST TOUR	.6	5.2	9.0	23.00

TIMES ARE MEASURES ON UMYAC-1108.

FIRST TOURS ARE ALWAYS WITHIN 5% OF OPTIMUM
 AND USUALLY MUCH CLOSER

BIVALENT (COSTS 0-1) PROBLEMS WITH 200 CITIES SOLVED
 IN LESS THAN 6 SECS.

RANDOMLY GENERATED BOTTLENECK PROBLEMS
 (SMITH-THOMPSON, 1975)

NO. CITIES	200	500	1000	1500	2000
AVE. TIME TO OPTIMALITY	2.75	20.08	33.72	206.43	313.87
AVE. NO. OF NODES IN SEARCH TREE	6.6	16.8	7	15	12

CAPACITATED WAREHOUSE LOCATION

VOTER REDISTRICTING

PROBLEM

$$\text{MINIMIZE } \left\{ \sum_{I \in I} \sum_{J \in J} x_{IJ} c_{IJ} + \sum_{I \in I} F_I y_I \right\}$$

$$\sum_{j \in J} x_{IJ} = A_I y_I$$

$$\sum_{I \in I} x_{IJ} = B_J$$

$$x_{IJ} \geq 0 \quad y_I = \begin{cases} 0 & \text{WAREHOUSE I CLOSED} \\ 1 & \text{WAREHOUSE I OPENED} \end{cases}$$

PROBLEM RELAXATION

TRANSPORTATION PROBLEM

	1		N	N+1	
1	c_{11}		c_{1N}	0	A_1
M	c_{1M}		c_{MN}	0	A_M
	B_1		B_N	B_{N+1}	

WAREHOUSE LOCATION PROBLEMS

SUM OBJECTIVE

<u>SIZE</u>	<u>SEARCH TREE NODES</u>	<u>TIME (SECS)</u>
15 x 50	35	3
25 x 50	500	30 (2 FAILURES)
15 x 45	1000	35

BOTTLENECK OBJECTIVE

<u>SIZE</u>	<u>SEARCH TREE NODES</u>	<u>TIME (SECS)</u>
15 x 50	102	2.4
30 x 90	214	5.3
50 x 150	457	17.0

NO FAILURES

OTHER APPLICATIONS

1. K-TOUR TRAVELLING SALESMAN
2. OPTIMAL GROWTH PATHS
3. CASH MANAGEMENT MODEL
4. ELECTRICAL POWER DISTRIBUTION
5. ELECTRICAL POWER CAPACITY **PLANNING**
6. TRANSPORTATION WITH STOCHASTIC DEMANDS
7. DECISION CPM
8. CLUSTER ANALYSIS

EXTENSIONS

1. NETWORKS WITH GAINS
2. MULTI-COMMODITY FLOW MODEL