A MIXED INTEGER PROGRAMMING MODEL FOR TRANSMISSION SYSTEM PIANNING IN TELEOTMMDNICATIONS NETWORKS WITH GENERAL CIRCUIT REQUIREMENTS+

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DRC-21-02-80

May 1980

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+Presented at the (1) Eleventh Annual Conference on Modeling and Simulation, University of Pittsburgh, Pittsburgh, BA, May 1-2, 1980, and (2) ORSA/TDB Joint National Meeting, Washington, DC, May 4-7, 1980.

*This research was partially supported by the National Science Foundation, Grant NSF ENG78-25488,

ABSTRACT

It is known in the telecommunications network facility design area that a transmission system cost function may be decomposed into a fixed charge part and a linear cost part. The fixed charge part represents the initial investment cost of installing a transmission system on a link and the linear cost part represents the cost of installing circuits of the system. Using the cost function of this type, a mixed integer linear programming model is developed to minimize the present value of transmission systems installation costs subject to satisfying circuit requirements for the telecommunications network in each period of a fixed planning horizon.

To make the problem computationally tractable, simplifications are made by using a fixed network hierarchy, i.e., high usage links vs. final links, for routing the required circuits. We present numerical examples solved by using (1) a branch-and-bound procedure, and (2) a heuristic methodology.

1. INTRODUCTION

This paper deals with the facilities design problem in a telecommunications network over a fixed planning horizon. A telecommunications network is a collection of points (sources and/or destinations) some or all of which are joined by direct communication links. A link of a telecommunications network is a collection of facilities known as "transmission equipment[11] and various equipments are employed for various transmission systems. These equipments wiil be referred to as "circuits.[11] In a typical telephone network a call originating at point A and destined for point B can be transmitted to point B through a sequence of other links. Furthermore, if the number of point A to point B calls exceed the capacity of the direct A to point B link, some of the calls can be transmitted through an alternate route.

The problem that we approach here is one of determining a minimum cost (present value) facility installation scheme for the telecommunications network while satisfying point-to-point circuit requirements for each period of a fixed planning horizon. We shall assume that alternate transmission systems are available at the beginning of the planning horizon, and additional new systems may be made available during certain periods of the planning horizon.

The quantity to be determined is the number of circuits of a specific transmission system to be installed on each link of the network during each period of the planning horizon. It is widely known in the telecommunications networks facility design area that cost functions associated with installing transmission systems on the links of the network are concave, reflecting "economies of scale" and that these functions may be decomposed — approximately — into a fixed charge part and a linear variable part. The fixed charge part represents the initial investment cost of installing a specific transmission system on the link for the first time, and the variable part represents the cost of installing each circuit of that system on that link.

A static (one period) version of the above problem may be stated as a fixed-charge multi-commodity flow synthesis problem (see, for instance, [10]). The dynamic (multi-period) version that we consider here is more complicated because the economies of scale involved must be utilized over the planning horizon,

Yaged [14] has developed a methodology for solving the dynamic problem. His method is one of "heuristic" and consists of three iteratively related tasks. Task I solves the minimum cost static routing problem to satisfy point-to-point circuit requirements for each time instant, given linearized costs. The output of Task I is annual demand on each link. Task II, then, determines the minimum cost-approximate-facility installation scheme for each link. This is attained by a dynamic programming procedure in which the only state variable is time. Finally, Task III modifies the cost coefficients (that were used in the static routing problem of Task *I)* to take into account the economies of scale.

In contrast with the other Yaged model, Smith [13] assumes that any economy of scales effects are dominated by savings resulting from expenditure deferral and presents a heuristic algorithm for the deferral of expenditures associated with capacity expansion for a dynamic communication network. He emphasizes savings due to facility deferral as opposed to economies of scale. His model deals with minimizing the present worth of expenditures for expanding the capacity of a communications network in the face of increasing demand for service, while assuming that any economy of scales effects are dominated by savings resulting from expenditure deferral.

Kochman and McCallum [8], pn the other hand, present optimum seeking models for planning for the economic growth of a communications network given a projection of future circuit requirements for two alternate transmission systems. Their objective is to find an optimal placement of cables (type, location and time) and the routing of individual circuits between demand points such that the total discounted cost over a T-period horizon is minimized. They present two mathematical models differing in their provision for network reliability.

Finally, Perry [12] presents a mathematical programming algorithm for constructing or updating a minimum cost communication facility layout given point-to-point demand requirements subject to capacity constraints, for a two-level communication network. Such a network deals with a "network of mastergroup sections," the underlying capacity for the construction of a "network of supergroup sections" and a "network of channel group sections."

2. A NETWORK HIERARCHY FOR ROUTING POINT-TO-POINT CIRCUIT REQUIREMENTS

Given circuit requirements on the links of the telecommunications network for each period of the planning horizon, we simplify the multi-period facility design problem described above in the manner Kortanek, Lee and Polak [9] simplified the problem of determining point-to-point circuit requirements in the network. A network hierarchy is established by classifying the links into two: (i) final links, and (ii) high-usage links.

The network hierarchy can be described in graph theoretic terms[1] as follows: Given a graph $G = (P,L)$ with point set P and link set L, find a spanning tree[2] (i.e., a connected subgraph containing all the points of G but not containing any cycles) of G. Let $T = (P,F_T)$ be the spanning tree. . The links in T are referred to as the <u>final links</u> of the graph. On the other hand, the chords of T, i.e., those links of G which are not in T, are referred to as the <u>high-usage links</u>. The rationale behind this is the following: A link designated as a high-usage link carries only the traffic originating at one of its endpoints and destined to the other one. If customer demand over such a link m exceeds its circuit capacity, then it is possible to switch some of these calls to the link adjacent to m lying on a path joining

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[1] Readers not familiar with graph theoretic terms can refer to [7].

[2] Determination of such a spanning tree is discussed in [2].

the end points of m. Thus, calls are transmitted through an alternate route. The spanning tree contains exactly one such path for each chord. The final links, on the other hand, can not switch their traffic to any other route. They carry as much as they can, otherwise calls are lost. The designation of the links of the network in such a manner that the final links induce a spanning subgraph is what is termed as the network hierarchy, and it leads to a simplification of the more general, but almost impossible to solve (in terms of computational complexity), problem of the facilities design with no restriction on to which calls can be switched or not. In our case the final links induce a spanning tree, and hence a unique alternate route is prescribed to carry the (excessive) traffic of each high-usage link. Clearly, with no such restriction, an exponential number of alternate routes must be considered in a general case.

An example of a network hierarchy is depicted in Figure 1. The dashed lines represent high-usage links and the solid lines represent final links. The arrows specify the links to which the excess high-usage link customer demand can be switched.

The Kortanek, Lee and Polak model for determining point-to-point circuit requirements considers a unique alternate route for each high-usage link as prescribed by the network hierarchy and assumes that cusomter demand on any high-usage link can be switched to the unique alternate route joining

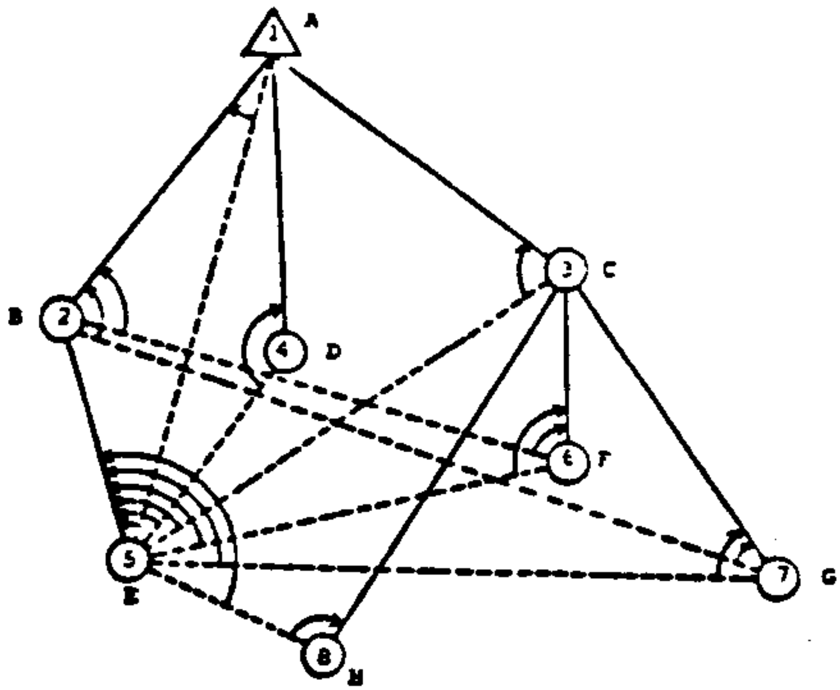

FIGURE 1. A Netvork Hierarchy with High-Usage (——) and Final (━━) Links Where Overflow Froc a High-Usage Link Onto a Final Link is Indicated by an Arrow.

(Baybars, Kortanek and Mizuno)

the endpoints of that link. Furthermore, a high-usage link carries only its own traffic. On the other side, no switching is possible on final links. In their model, customer demand is stochastic and they consider various times-of-day. They reduce the optimization problem to a linear programming problem and present numerical results as well as comparisons with earlier studies.

The output of the Kortanek, Lee and Polak model is the number of trunks required on each link. We shall refer to those trunks as circuits and they will be input to our model. We, thus, assume that point-to-point circuit requirements on each link of the network are known with certainty.[3] We shall consider a finite planning horizon and a finite number of alternate transmission systems.

[3] Same assumption is also made by Yaged [14].

3. A MIXED-INTEGER PROGRAMMING MODEL

Let P and L denote the point and link sets, respectively, of the telecommunications network. Let p and q denote the cardinalities of the sets P and L, respectively. We shall denote by F the set of the final links, and by H the set of the high-usage links. By definitions given earlier, it is clear that (i) $F \cap H = \emptyset$, (ii) $F \cup H = L$, and (iii) $|F| = p - 1$. The elements of F and H will be denoted by f and h, respectively. For each final link f we define the following:

$$H_f = \{h \mid f \text{ lies on the unique path connecting the endpoints of } h\}$$

Note that, by the definition of the network hierarchy, each high-usage h belongs to some subset H_f, and that $H_f = \emptyset$ is possible for some final link if $q < (p^2-p)/2$.

We consider a planning horizon of t' periods. Let $T = \{1,2,...,t,...,t'\}$. Furthermore, we assume that there exists s' alternate transmission systems. Let $S = \{1,2,...,s,...,s'\}$. We shall assume, for the moment that, only one system s unit can be installed in a specific period on a specific link. We thus designate the following binary variable:

$$x_{stm} = \begin{cases} 1 & \text{if system } s \text{ is installed on link } m \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

We also assume at this point that, the supply of transmission systems is unlimited.

Once the system $\bar{s}$ is installed on link m in period $\bar{t}$, i.e., given that $x_{\bar{s}\bar{t}m} = 1$, circuits of that system can be installed on that link in periods $t > \bar{t}$. Hence, let y_{stm} denote the number of system s circuits installed on link m in period t. We assume that the circuit capacity of each transmission system is finite, and we denote this number by a_s.

Let c_s^f be the fixed charge of installing system s on any link in the network and let c_s'' be the linear cost of installing a circuit in system s. Also let $w_t = (1 + r)^{-t}$ be a discounting factor for year t with an annual interest rate r.

Then, the total discounted cost throughout the planning horizon to be minimized is the objective function below:

$$\sum_{m\in L} \sum_{s=1}^{s'} \left(c_s' \sum_{t=1}^{t'} w_t x_{stm} + c_s'' \sum_{t=1}^{t'} w_t y_{stm}\right) \qquad (1)$$

Let d_{tm} be the circuit demand on link m in period t. The number of circuits on a final link f in period $\bar{t}$ must be at least that of the sum of $d_{\bar{t}f}$ and those routed from the high-usage links in H_f. This, once again, is prescribed by the network hierarchy. Circuit requirements of a final link can only be met by circuits installed on that link only. We shall assume that $d_{(t+1)h} \geq d_{th}$, $\forall t \in T$ and $\forall h \in H$. That is, circuit requirements of high-usage links are nondecreasing in time. This assumption will be relaxed later on. Circuit requirements of the final links can be represented as follows:

$$\sum_{s=1}^{s^f} \sum_{t=1}^{\bar{t}} y_{stf} \geq d_{\bar{t}f} + \sum_{h \in H_f} \left(d_{\bar{t}h} - \sum_{s=1}^{s^f} \sum_{t=1}^{\bar{t}} y_{sth} \right), \quad \forall \bar{t} \in T \text{ and } \forall f \in F \qquad \text{(2a)}$$

Since the circuits installed on a high-usage link can only be used for the traffic of that link we need the following constraint:

$$\sum_{s=1}^{s'} \sum_{t=1}^{\bar{t}} y_{sth} \leq d_{\bar{t}h}, \quad \forall \bar{t} \in T \text{ and } \forall h \in H \qquad \text{(2b)}$$

We note that (2b) would not be valid if we had not assumed earlier that circuit requirements of high-usage links do not decrease by time. Relation (2b) guarantees that the summation term in the right-hand-side of (2a) is always non-negative. Thus, circuits installed on a high-usage link can not be used to meet — partially or wholly — the circuit requirements of any other link, whether high-usage or final.

We, next, need constraints on the number of circuits that can be installed on link m. This number is subject to the type(s) of transmission systems that have been installed, or are being installed in the current period on that link. Hence,

$$\sum_{t=1}^{\bar{t}} y_{stm} \le a_s \sum_{t=1}^{\bar{t}} x_{stm}, \quad \forall s \in S, \ \forall t \in T, \text{ and } \forall m \in L \tag{3}$$

And finally, we have,

$$x_{stm} = 0,1, \quad \forall s \in S, \ \forall t \in T \text{ and } \forall m \in L \tag{4}$$

$$y_{stm} \ge 0, \quad \forall s \in S, \ \forall t \in T \text{ and } \forall m \in L \tag{5}$$

We note here that y_{stm}'s are not required to assume integer values for they are, in real-life, sufficiently large and rounding them off does not affect the model significantly.

We shall refer to (1)-(5) as <u>Program P</u>. <u>Program P</u> has $[(s^f+1)t^f q]$ structural constraints and $(2s^f t^f q)$ variables, exactly one-half of which are 0-1 variables. In <u>Section 4</u>, we shall discuss methodologies for solving <u>Program P</u> and present numerical examples.

It should also be pointed out that the model presented here assume that there exists no circuits on any link of the network prior to the first period of the planning horizon. In reality, of course, this is not the case. However <u>Program</u> P can easily accommodate this by replacing d_{tm} with (d_{tm} - number of existing circuits on link m).

We shall now relax some of the restrictive assumptions that we made in constructing <u>Program P</u>. First of all, suppose that circuit requirements of high-usage links may decrease by time. We can update the circuit requirements of final links -- that is, replace (2a) -- with the following:

$$\sum_{s=1}^{s'} \sum_{t=1}^{\bar{t}} y_{stf} \geq d_{\bar{t}f} + \sum_{h \varepsilon H_f} (\max \{0, d_{\bar{t}h} - \sum_{s=1}^{s'} \sum_{t=1}^{\bar{t}} y_{sth}\}) \quad \forall \bar{t} \varepsilon T \text{ and } \forall f \varepsilon F \qquad (6)$$

Note that the second term of the right-hand-side of (6) is always nonnegative for

$$d_{\bar{t}h},\ y_{sth} \geq 0,\ \forall \bar{t} \varepsilon T,\ \forall h \varepsilon H \text{ and } \forall s \varepsilon S$$

Thus, the circuit requirements of a final link can only be met by circuits installed on that link. The extra circuits on a final link f, then, are there to cover the circuit requirements of high-usage links whose alternate routes contain f. Furthermore, since we have

$$\max \{0,\ d_{\bar{t}h} - \sum_{s=1}^{s'} \sum_{t=1}^{\bar{t}} y_{sth}\} \qquad (6.1)$$

in the right-hand-side of (6), the excess circuits on a high-usage link can not be used to cover the circuit requirements of any other link, whether high-usage or final. Thus, the circuits on any high-usage link are used exclusively for the traffic of that link. That is, if in period $\bar{t}$ a high-usage link h has $\alpha_{\bar{t}h}$ circuits but the circuit requirement of h in period $(\bar{t}+1)$ is less than $\alpha_{\bar{t}h}$, i.e.,

$$d_{\bar{t}h} - d_{(\bar{t}+1)h} > 0,$$

those circuits will be idle in period $(\bar{t}+1)$. They can, of course, be used in period $t'' > \bar{t}$ if $d_{t''h} > d_{(\bar{t}+1)h}$. The fundamental assumption (also made by Yaged [14]) here is that circuits, once installed, can not be removed and installed on some other link. Otherwise, we should replace (6.1) with the following:

$$\max \{0, \sum_{h \in H_f} d_{\bar{t}h} - \sum_{s=1}^{S_f} \sum_{t=1}^{\bar{t}} y_{sth} \} \qquad (6.2)$$

The above relation would not require a revision of the objective function (1) because it is reasonable to assume that cost of removing circuits on a specific link and installing them on some other link does not exceed that of installing new circuits.

Also note that replacing (2a) with (6) would make (2b) redundant.

Secondly, instead of restricting x_{stm}'s to assume only 0-1 values, we can allow

$$x_{stm} \geq 0 \text{ and integer, } \forall s \in S, \forall t \in T \text{ and } \forall m \in L \qquad (7)$$

Thus, more than one system s (for instance, cable) can be installed in period t on link m. Furthermore, we can assume that there exists limited supplies of transmission systems (for instance, satellites), say a_s units, throughout the planning horizon. We would then need the following constraint:

$$\sum_{m \in L} \sum_{t=1}^{t'} x_{stm} \le a_s, \quad \forall s \in S \tag{8}$$

Finally, we can add "parity constraints" (see, for instance, [8]) such as

$$\sum_{t=1}^{t'} y_{stm} \ge k \sum_{s=1}^{s'} \sum_{t=1}^{t'} y_{stm} \quad \forall s \in S \text{ and } \forall m \in L, \text{ where } 0 < k < 1.$$

4. SOLUTION METHODOLOGIES AND NUMERICAL EXAMPLES

We developed and coded (by the third author) a branch-and-bound procedure for solving Program JP given in Section 3^. At each vertex of the search tree, i.e., after setting some variable x_{stm} equal to 1, a subproblem is solved for each period of the planning horizon. Each one of these subproblems is a "single commodity flow problem" and is solved using the "flow augmenting method" (see [4], for instance). We thus obtain network theoretic upper and lover bounds at each stage. The computations are terminated when either (i) the optimal solution is found or (ii) time allocated is consumed.

We now present a numerical example. The telecommunications network considered is depicted in Figure 1, and is taken from the field. The planning horizon is 10 years and separated into 3 periods. t*= 1 is the base year; t = 2 is the fifth year and t * t? » 3 is the tenth year. The circuit requirements for the first period are taken from [9]. The requirements for the remaining two periods are obtained by using appropriate levels of average annual growth rates. These circuit requirements are given in Table 1. Three alternative transmission systems are considered. Table 2 contains the installation cost of each system, the per unit cost of each system s circuit, and the maximum circuit capacity of each system. The discount factor is 10%.

TABLE 1

Circuits Demand for Each Link

Link	Link Number	t»l	t*2	t=3
$(P_r P_2)$	1	35	60	70
(P_2, P_5)	2	21	42	63
$(P_r P_3)$	3	92	184	184
$(P_r P_4)$	4	58	99	174
(P_3, P_6)	5	47	80	188
(P_3, P_7)	6	47	80	177
$(P_3 . P_8)$	7	59	100	177
$(P_r P_5)$	8	2	5	8
(P_2, P_6)	9	17	34	68
(P_2, P_7)	10	17	39	51
(P_3, P_5)	11	7	14	21
(P_4, P_5)	12	18	31	72
(P_5, P_6)	13	18	31	72
(P_5, P_7)	14	18	36	54
(P_5, P_8)	15	18	41	72

TABLE 2

Costs and Upper Bounds of Systems

System	Fixed (c^1) Charge	Linear (c") Cost	Upper Bound (a_s) (max. capacity)
1	530000	3100	30
2	870000	1070	90
3	1400000	277	270

The solution, obtained by implementing the branch-and-bound procedure described above, is presented in Table 2- T&e same solution was obtained when the discount factor was changed, first, to 5%, and, then to 15%. Also note that, no systems are installed in the last period.

Given the magnitude of computation time for the above example and other runs of the same problem for different constants, we have started devising a "heuristic methodology" for solving Program £. This procedure is not formally completed yet. However, our initial efforts show that such a methodology could prove very useful.

The heuristic solution for the above numerical example is shown in Table 4^. Note that total cost is smaller than that of the branch-and-bound procedure. We are not ready to claim the superiority of either one of the methods as of now. These will be reported in the near future. Also to be reported in the near future is a generalization of the model given here.

TABLE 3

Approximate Optimum Solution
(Branch-and-Bound)

Link Number	t=1	t=2	t=3
1	3	1	0
2	3	0	0
3	3	1	0
4	3	0	0
5	3	0	0
6	3	0	0
7	3	0	0
8	0	0	0
9	0	2	0
10	0	2	0
11	0	1	0
12	0	0	0
13	0	2	0
14	0	2	0
15	0	2	0

N.B. (1) Numbers in the table are indices of systems installed

(2) z* * 13,872,230; an approximate optimal solution.

(3) $\frac{2^* - \text{the value of the optimum}}{2^*} \leq$ ^5

TABLE 4

Approximate Optimum Solution
(Heuristic)

Link Number	t-1	t=2	t=3
1	3	0	0
2	3	0	0
3	3	2	0
4	3	0	0
5	3	0	0
6	3	0	0
7	3	0	0
8	0	0	0
9	0	0	0
10	0	1	0
11	0	0	0
12	0	0	2
13	0	0	2
14	0	0	0
15	0	0	2

(1) Numbers in the table are indices of systems installed.

(2) 2* - 12,203,954

ACKNOWLEDGEMENT

We have benefited from comments and criticisms of Professor Alan J. Goldman (Johns Hopkins University) on an earlier version of this paper.

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