HEAT INTEGRATION IN BATCH PROCESSING

by

J.A. Vaselenak, I.E. Grossmann, A.W. Westerberg

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Jane A. Vaselenak, Ignacio E. Grossmann, Arthur W. Westerberg

Department of Chemical Engineering

Carnegie-Mellon University

Pittsburgh, PA 15213

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Abstract

At a particular time in a batch process, some tanks with a batch of material may require heating while some other tanks may need cooling. This work addresses the problem of determining the maximum heat integration between the tanks to reduce the utility consumption. Depending on the processing requirements, the exchange of heat can present three distinct temperature profiles in time that are similar to cocurrent, countercurrent, and combination cocurrent/countercurrent temperature profiles. For the cocurrent case, a heuristic is given for deciding the processing order of exchanges when multiple hot and/or cold batches are present. This heuristic is shown to lead to the optimal answer for certain instances in which the desired final temperatures of the tanks do not restrict the exchanges; i.e., the matches proceed until the $(AT)_{min}$ requirement is satisfied. For cases in which the desired final temperatures limit some of the exchanges, a mixed-integer linear programming formulation is proposed. Examples are presented to illustrate the application of the two methods.

Introduction

Used in the manufacture of food, Pharmaceuticals, and fine chemicals, batch processing is an important production scheme. A significant proportion of the world's chemical production, weighted by its economic value, is manufactured in **batch facilities** (Rippin, 1983). Systematic methods for design of these plants mainly **has dealt with selection of** capital equipment to minimize investment cost, (See Loonkar **and** Robinson (1970), Sparrow et **al.** (1975), Grossmann and Sargent (1979), Suhami **and Mah (1982), for** example.)

However, energy conservation also can be an important goal in **the** design of such **processes. Knopf et al. (1982) have included correlations for energy consumption in the design of batch plants. Lundberg and Christenson (1979) have examined waste heat recovery In the food procesing industry. On the other hand, most of the design methods for heat integration that have been proposed are for continuous processes (Nishida, et al., (1981), Linnhoff, et al., (1983)), although opportunities for heat integration are present in batch processes. In particular, at any time in a batch process, some vessels may require heating while others may need cooling. An interesting question that then arises is: what is the best way to assign the vessels for heat exchange to minimize the utilities needed?**

Previous studies have derived expressions for the temperature profile for heating or cooling of an agitated, liquid batch. In the 1940's various researchers have examined

the dynamics of various systems:

a) jacketed vessels or vessels with internal heating coils, (Bowman et al., 1940)

b) batch vessels with external countercurrent heat exchangers, (Fisher, 1944)

c) batch vessels without agitation using external heat exchangers, (Kern, 1950)

d) batch vessels with the addition of liquid to the tank. (Chaddock and Sanders, 1944)

e) same as d) with heat of solution effects and external exchangers, (Chaddock and Sanders, 1944)

Troupe (1952) and Fernandez-Seco (1953) have analyzed the exchanger area needed to cool a liquid batch in a given amount of time. A nomograph for heating time and exchanger area has been produced by Unger (1962). It should be noted that all of the previous work has studied the behavior of a single tank, ignoring other batch vessels that may need heating of cooling.

This paper addresses the problem of predicting the maximum heat integration in batch processes. Three arrangements with receiving tanks will demonstrate different temperature profiles in time. Cocurrent and combination cocurrent/countercurrent temperature profiles with respect to time are seen, while countercurrent exchange with respect to exchanger length also is shown. The equations describing the cocurrent case are analogous to those for the draining of tanks. This study presents a heuristic for deciding the pairing of the hot tanks and cold tanks to transfer maximum heat for this case. This rule provides a fast, good answer to a large combinatorial problem and is unusual since a series of local decisions provide a global optimum. However, this heuristic is limited to problems with target temperatures that do not restrict the heat exchange of each match. For cocurrent exchange with restrictive target temperatures, a mixed-integer linear program improves the solution provided by the heuristic. For the combination cocurrent/countercurrent case, a rule for deciding which fluid should use the receiving tank is presented. Countercurrent heat exchange, which results when each fluid in a match flows to a receiving tank, is not examined here since this type of exchange has been reported extensively in the literature, as cited above.

Theory

Definition of Terms and Assumptions

In this section key terms will be defined and the assumptions used for this analysis will be explained. The system under study consists of two or more vessels of fluid. The "hot" tanks contain those liquids which require cooling. Conversely, the "cold" tanks contain the liquids that need to be heated. Hot tanks can exchange heat with cold tanks only; heat exchange between two hot tanks or between two cold tanks is not permitted. Also, a batch must maintain its integrity during an exchange; it cannot be divided into two or more vessels for the exchange, unless specified by the processing requirements. The terms cocurrent and combination cocurrent/countercurrent refer to temperature profiles of the fluids with respect to time rather than distance. While the two fluids flow in a countercurrent manner through the exchanger, it is the return of the fluids to the starting tanks or to receiving tanks that determines the type of temperature profile experienced by the liquids. This behavior will be demonstrated in this section. Also, the length of time for a given exchange is not explicitly considered in this work. The time required for a given exchange depends upon the flow rates of the fluids, the area of the heat exchanger, and the minimum approach temperature for heat exchange $(AT)_{roin}$. . These values are chosen by the design engineer to reflect the trade-off between time and the costs of the heat exchangers and the pumps, a cost versus benefit analysis not addressed in this study. Finally, the minimum approach temperature $(AT)_{mm}$ is assumed to be a specified value.

The following simplifications are used in this analysis.

1. The fluid in each tank is well-mixed and its temperature is uniform.
2. The specific heat capacity of each fluid is a constant.
3. No phase changes occur.
4. The rate of flow of each fluid through the heat exchanger is constant.

5. The fluids contact in a countercurrent manner in the heat exchangers.

6. The heat exchanged in a match is determined by heat balances and is limited only by $(\Delta T)_{min}$.

7. Heat losses and leaks are negligible.

Cocurrent Heat Exchange

Cocurrent heat exchange occurs when both the hot and cold fluids return to their original tanks after passing through the heat exchanger. In this case the processing requirements specify that the materials must remain in their tanks while being heated or cooled as shown in Figure 1A. For instances in which the fluids cannot be pumped easily, for example, food products, heating and cooling media are used to accomplish the exchange, demonstrated in Figure 1B. The analytical solutions to the dynamic equations for this system are the following.

$(FC_p)_h \geq (FC_p)_c$

$$T_h(t) = \frac{(VC_p)_c\,[T_c^i + (\Delta T)_{min}^{exch}] + (VC_p)_h\,T_h^i}{(VC_p)_h + (VC_p)_c} + \frac{(VC_p)_c\,[T_h^i - T_c^i - (\Delta T)_{min}^{exch}]}{(VC_p)_h + (VC_p)_c}\exp\left\{-\frac{(VC_p)_h + (VC_p)_c}{(VC_p)_c}\,\frac{(FC_p)_c}{(VC_p)_h}\,t\right\} \quad (1)$$

$$T_c(t) = \frac{(VC_p)_c\,T_c^i + (VC_p)_h\,[T_h^i - (\Delta T)_{min}^{exch}]}{(VC_p)_h + (VC_p)_c} - \frac{(VC_p)_h\,[T_h^i - T_c^i - (\Delta T)_{min}^{exch}]}{(VC_p)_h + (VC_p)_c}\exp\left\{-\frac{(VC_p)_h + (VC_p)_c}{(VC_p)_c}\,\frac{(FC_p)_c}{(VC_p)_h}\,t\right\} \quad (2)$$

$(FC_p)_h \leq (FC_p)_c$

$$T_h(t) = \frac{(VC_p)_c\,[T_c^i + (\Delta T)_{min}^{exch}] + (VC_p)_h\,T_h^i}{(VC_p)_h + (VC_p)_c} + \frac{(VC_p)_c\,[T_h^i - T_c^i - (\Delta T)_{min}^{exch}]}{(VC_p)_h + (VC_p)_c}\exp\left\{-\frac{(VC_p)_h + (VC_p)_c}{(VC_p)_h}\,\frac{(FC_p)_h}{(VC_p)_c}\,t\right\} \quad (3)$$

$$T_c(t) = \frac{(VC_p)_c\, T_c^i + (VC_p)_h\, [T_h^i - (\Delta T)_{min}^{exch}]}{(VC_p)_h + (VC_p)_c} - \frac{(VC_p)_h\, [T_h^i - T_c^i - (\Delta T)_{min}^{exch}]}{(VC_p)_h + (VC_p)_c} \exp\left\{ - \left[\frac{(FC_p)_h}{(VC_p)_h} + \frac{(FC_p)_c}{(VC_p)_c} \right] t \right\} \qquad (4)$$

where

$T_h(t)$ is the temperature of the hot tank at time t

$T_c(t)$ is the temperature of the cold tank at time t

T_h^i is the initial temperature of the hot tank

T_c^i is the initial temperature of the cold tank

$(\Delta T)_{min}^{exch}$ is the minimum approach temperature of the two fluids in the exchanger. The effects of the heat transfer coefficients and the exchanger area are lumped into $(\Delta T)_{min}^{exch}$.

F is the flowrate of a fluid through the heat exchanger

V is the volume of liquid in the tank

$(FC_p)_h$ is the heat content per degree per time for the hot tank

$(FC_p)_c$ is the heat content per degree per time for the cold tank

$(VC_p)_h$ is the heat content per degree for the hot tank

$(VC_p)_c$ is the heat content per degree for the cold tank

Displayed graphically in Figure 1C, these equations demonstrate how this arrangement leads to temperature profiles in time that resemble those of cocurrent flow. Analyzing the initial and final states of the system, the first law of thermodynamics is the following.

$$(VC_p)_h\,(T_h^i - T_h^f) + (VC_p)_c\,(T_c^i - T_c^f) = 0 \qquad (5)$$

where T_h^f is the final temperature of the hot tank

T_c^f is the final temperature of the cold tank

If the fluids are pumped through the exchangers until the tank temperatures are separated by $(\Delta T)_{min}$, the equations can be rearranged as shown. These results agree with equations 1 to 4 as $t \rightarrow \infty$, allowing $(\Delta T)_{min} = (\Delta T)_{min}^{exch}$.

$$(\Delta T)_{min} = T_h^f - T_c^f = \text{the difference in the final temperatures of the tanks} \qquad (6)$$

$$T_h^f = \frac{(VC_p)_h\,T_h^i + (VC_p)_c\,(T_c^i + (\Delta T)_{min})}{(VC_p)_h + (VC_p)_c} \qquad (7)$$

$$T_c^f = \frac{(VC_p)_h\,(T_h^i - (\Delta T)_{min}) + (VC_p)_c\,T_c^i}{(VC_p)_h + (VC_p)_c} \qquad (8)$$

This heat exchange configuration can be compared to the draining of one tank into another. A mass balance around the two tanks of the same liquid can be written as follows.

$$\rho\,A_1\,(h_1^i - h_1^f) + \rho\,A_2\,(h_2^i - h_2^f) = 0 \qquad (9)$$

where ρ is the density of the liquid

A_1 is the cross-sectional area of tank 1

A_2 is the cross-sectional area of tank 2

h_1^i is the initial height of tank 1

h_2^i is the initial height of tank 2

h_1^f is the final height of tank 1

h_2^f is the final height of tank 2

Under the force of gravity the liquid would transfer until the levels of the tanks were equal.

$$h_1^f = h_2^f = \frac{A_1 h_1^i + A_2 h_2^i}{A_1 + A_2} \qquad (10)$$

Equations 8 and 10 have a similar form with the comparisons heat content per degree of temperature, (VCp), to area, A, and temperature, T, to height, h. The two cases are identical when $(\Delta T)_{min}$ is zero or when $(\Delta T)_{min}$ is grouped with T_h^i or with T_c^i. Thus, this analogy provides an alternative representation for this problem.

Countercurrent Case

Countercurrent heat exchange occurs when the processing requirements allow the fluids to transfer from their original tanks to receiving tanks while being heated or cooled, as shown in Figure 2A. At steady state the temperature of the supply vessel remains constant as does the temperature of the final tank. Therefore, this system behaves as a semi-continuous process and results in a typical countercurrent heat exchange. Application of the first law of thermodynamics yields the following equation.

$$(FC_p)_h (T_h^i - T_h^f) + (FC_p)_c (T_c^i - T_c^f) = 0 \qquad (11)$$

If $(FC_p)_h$ is greater than $(FC_p)_c$, then a pinch point will occur at the hot end of the exchanger as shown in Figure 2B. Setting the hot outlet temperature equal to the cold inlet temperature plus a design margin, $(\Delta T)_{min}$, yields the next equation.

$$T_h^f = T_c^i + (\Delta T)_{min} \qquad (12)$$

$$T_c^f \ll T_c^j \bullet \text{-Sfik}_{(FC)_{PC}} (T_h' - t_c - (AT)_{min}) \quad (13)$$

If $(FC_p)_h$ is greater $(FC_p)_c$, then the pinch occurs at the cold end of the exchanger, as seen in Figure 2C. Using a similar analysis the following results.

$$T_c^f = T!_h - (AT)_{mm} . \quad (14)$$

$$T'_h = T^*_h - \frac{(PC\ )}{(FC\ )_{ph}} \text{-LIni-} (T'_h - T_c^J - (AT)_{mm}) \quad (15)$$

If $(FC\ )_{ph}$ is equal to $(FC\ )_{pc}$, then the pinch occurs throughout the exchanger as demonstrated by Figure 2D, and these two equations are true.

$$T_c^f = t_h - (AT)_{min} \quad (16)$$

$$T'_h = T_c^j - (AT)_{mm} . \quad (17)$$

If the flow rates are constant and proportional to the original volume of the tanks, (VC_p) can be substituted for (FC_P). Since this configuration of the two batch tanks and two receiving tanks results in the same countercurrent heat exchanger temperature profiles as found in continuous processing, this case is not analyzed in further detail here since many researchers have studied this problem. (See Nishida et al. (1981), or Linnhoff et al. (1983), for example.)

Cocurrent/Countercurrent Case

Figure 3A shows the scheme that results in temperature profiles that are intermediate to those of the cocurrent and countercurrent cases. In this case the processing requirements restrict the cold fluid to return to its original tank, while the hot fluid is permitted to transfer to a receiving tank. If the cold fluid is restricted from recirculating, a heat exchange fluid can be used in its place, similar to the case shown in Figure IB. An energy balance of the tank with the recirculating liquid, labeled as the cold tank, yields this ordinary differential equation:

$$^{(V(}VC \frac{dT_c^i}{dt} = (FC_p)_c (T_c^o - T_c^i) \quad (18)$$

An energy balance of the exchanger gives the next equation.

$$(FC_p)_h (T_h^i - T_h^o) - (FC_p)_c (T_c^o - T_c^i) = 0 \tag{19}$$

wnere

T_h^* is the temperature of the hot fluid entering the exchanger at time t

T_c^i is the temperature of the cold fluid entering the exchanger at time t

T_h^o is the temperature of the hot fluid exiting the exchanger at time t

T_c^o is the temperature of the cold fluid exiting the exchanger at time t

Since the hot fluid passes through the exchanger only once, it should be cooled as much as possible. If the flowrates are set with $(FC_p)_c$ greater than $(FC_p)_h$, the cold end of the exchanger is the limiting side and the desired cooling can be accomplished. This situation is demonstrated in Figure 2C.

$$T_h^o = T_c^i + (\Delta T)_{mm} \tag{20}$$

Substituting Equations 19 and 20 into 18 and integrating from t=0 to $t=t_f$ yields the following integral

$$\int_{T_c^i(t=0)}^{T_c^i(t=t_f)} \frac{dT_c^i}{T_h^i - [T_c^i + (\Delta T)_{mm}]} = \int_0^{t_f} \frac{(FC_p)_h \, dt}{(VC_p)_c} \tag{21}$$

$$T_c^i(t=t_f) = T_h^i - (\Delta T)_{mm} - [T_h^i - (T_c^i(t=0) + (\Delta T)_{mm})] \exp\left\{-\frac{(FC_p)_h t_f}{(VC_p)_c}\right\} \tag{22}$$

Letting the final time t_f equal the time to drain the hot tank, the result is given below.

$$t_f = \frac{V}{F_h} = \tau \tag{23}$$

$$T_c^i(t_f=\tau) = T_h^i - (\Delta T)_{mm} - [T_h^i - (T_c^i(t=0) + (\Delta T)_{mm})] \exp\left\{-\frac{(VC_p)_h}{(VC_p)_c}\right\} \tag{24}$$

Time averaging of the temperature of the receiving tank yields the following equation for the final temperature of the hot fluid.

$$\left\langle T_h^o \right\rangle \Big|_{t=\tau} = T_h^i - \frac{(VC_p)_c}{(VC_p)_h} \left[T_h^i - \left(T_c^i(t=0) + (\Delta T)_{min} \right) \right] \left[1 - \exp\left\{ - \frac{\tau f}{(VC_p)_c} \right\} \right] \quad (25)$$

where $\left\langle T_h^o \right\rangle$ is the temperature of the hot fluid in the receiving tank at time t

These profiles are displayed in Figure 3B.

These results show that the fluid with the larger (VC_p) should transfer to the spare tank, if this option exists. A difference in total heat transferred of about ten per cent or less is found for the two arrangements for typical values for batch processing.

This section shows that countercurrent exchange will provide the maximum heat transfer between two tanks, if spare tanks are available and if the processing requirements will allow the fluids to leave their original tanks. If countercurrent exchange is not possible, then the combination cocurrent/countercurrent configuration is preferred to the cocurrent arrangement, if feasible. This reasoning shows a trade-off between the cost of a spare tank and the value of the energy saved.

Design Procedures for Maximum Heat Integration

The matching of tanks for cocurrent heat exchange is divided into two cases for analysis. In the first case the contents of each individual tank cannot reach their desired temperatures without additional heating or cooling using utilities. This possibility is realistic if the target temperatures *are* far from their initial temperatures. For this case, a heuristic that depends on the initial temperatures only is presented to assign tanks for heat exchange. This heuristic can be proven to lead to the optimal answer for the one hot tank/n cold tanks problem and for the two hot tanks/two cold tanks problem. If the target temperatures are limiting, i.e., one or more tanks can reach its desired temperature without the use of a utility, the heuristic does not provide the best solution at all times. To obtain a better solution the tank sequencing problem is formulated as a multi-period, mixed-integer linear program with each pairing of tanks considered to be one time period.

Cocurrent Heat Exchange With Non-limiting Target Temperatures

In this case, a simple procedure can be developed based on the following heuristic rule: Match the coldest hot tank with the warmest cold tank that are feasible. The algorithm for using the heuristic is listed below.

1. Order the hot tanks in order of increasing temperature with hot 1 being the name of the coldest hot tank, etc.

2. Order the cold tanks in order of decreasing temperature with cold 1 being the name of the hottest cold tank, etc.

3. In order for heat to flow from the hot tank to the cold tank, the temperature of the hot tank must be greater than that of the cold tank plus the specified minimum difference in temperature ($(\Delta T)_{min}$). If this match violates the temperature constraint, check cold 2 to see if it is a feasible match for hot 1, etc.

4. Once a feasible match is found, calculate the outlet temperatures of the tanks using the results from the theory section.

$$T_h^f = \frac{(VC_p)_h T_h^i + (VC_p)_c \left(T_c^i + (\Delta T)_{min}\right)}{(VC_p)_h + (VC_p)_c}$$

$$T_c^f = \frac{(VC_p)_h (T_h^i - \Delta T_{min}) + (VC_p)_c T_c^i}{(VC_p)_h + (VC_p)_c} \quad \text{or} \quad T_c^f = T_h^f - \Delta T_{min}$$

5. Update the temperatures of the tanks using T_h^f and T_c^f above.
6. Continue with the next cold tank and repeat steps 3 to 5 until all of the cold tanks are examined.
7. Repeat steps 3 to 6 for hot 2, hot 3, etc., until all of the hot tanks are considered.

This algorithm will work also if in step 3, cold 1 is compared to hot 1, hot 2, etc., and if the remaining cold tanks are checked in the same fashion as cold 1.

This heuristic is illustrated for the two hot and two cold tanks shown in the heat content diagrams of Figure 4. The ordinate is temperature and the abscissa, heat content per unit of temperature (VC_p). Since the temperature of hot 1 is less than that of hot 2, hot 1 is the first hot tank for matching. Conversely, cold 1 will be the first cold tank for heat exchange because its temperature is higher than that of cold 1. The heat exchanged between hot 1 and cold 1 is represented by the area labeled I. The subsequent matches are shown in the figure by the II, III, and IV. The desired final temperatures are described as "non-limiting" since they did not constrain the outlet temperatures of any of the matches, evidenced by the use of utilities to reach the temperature goals of the tanks.

This heuristic is unusual since a sequence of local decisions provide a global optimum. Another way to word the heuristic is to match the hot and cold tanks that are closest in temperature. A qualitative interpretation of this rule is that the temperature difference between the hot and cold tanks, the driving force for heat transfer, is conserved. Clearly, the main advantage of this heuristic is that it provides a very fast procedure to evaluate heat integration in a batch process, since it avoids the enumeration of all possible match combinations.

Validity of the Heuristic Procedure

In the appendix, a proof for the heuristic for the case of one hot tank/n cold tanks (or n hot tanks/one cold tank) is presented. The first part of this proof (Appendix 1) shows that the optimal total amount of heat exchanged is not transferred if a hot tank matches with more than one cold tank at the same time (or conversely, if a cold tank matches with more than one hot tank at the same time). The second part of the one hot tank/n cold tanks proof (Appendix 2) uses this result to analyze two sequences of cold tanks that differ by a pair-wise interchange, i.e., the order of two consecutive cold tanks, cold tank i and cold tank (i+1) are reversed in the two sequences. If the initial temperature of cold i is higher than that of cold (i+1), then the sequence that matches the hot tank with cold i before cold (i+1) transfers more energy than the alternative sequence. Thus, a series of pair-wise interchanges will produce an exchange sequence of cold tanks ordered from highest initial temperature to lowest. Using a similar reasoning, this analysis will order a set of hot tanks from coldest to hottest for a m hot tanks/1 cold tank problem.

Proving the heuristic for the more general problem of m hot tanks and n cold tanks is a difficult task. If each pair of hot and cold tanks matches just once and if only one hot tank and only one cold tank can match at the same time, then fourteen different sequences for heat exchange between two hot tanks and two cold tanks exist. (4! or twenty-four are possible, but only fourteen are unique.) The heuristic for this case can be shown to be optimal by calculating algebraically the total heat exchanged for all the possible configurations and by comparing the resulting expressions. These results are shown in Vaselenak (1985). The three hot tanks/two cold tanks (or two hot tanks/three cold tanks) problem has 192 different sequences from a possible 6! or 720. Since the problem grows exponentially as more tanks are added, algebraic comparison of all the possible sequences becomes cumbersome. At this time, the optimality of the heuristic has not been proved for the three hot tanks/two cold tanks case and for larger problems.

(n summary, the proposed heuristic procedure provides a design tool to evaluate quickly various systems for heat integration without considering every alternative. Since the number of alternatives increases quite rapidly, the heuristic is a useful design tool to assess quickly the possibilities for energy conservation in batch processes.

Example of the Heuristic

The potentials for heat integration are demonstrated in the problem consisting of ten hot tanks and ten cold tanks shown in Table 1. Eighty-one per cent of the cooling needs and eighty-eight per cent of the heating requirements can be saved by sequencing the tanks according to the heuristic. These results are displayed also in Table 1.

Cocurrent Heat Exchange With Limiting Target Temperatures

A limiting target temperature is a desired final temperature that can be reached using the heat integration without a hot or cold utility. In the previous section, a match proceeds until the outlet temperatures are separated by $(\Delta T)_{min}$. In this section, desired outlet temperatures are reached before the $(\Delta T)_{min}$ requirement becomes active.

For tanks B, C, X, Y, Z shown in Table 2, the estimate provided by the heuristic is not the optimal value, as demonstrated in Tables 3 and 4. For instances in which a better answer is desired, the following multi-period, mixed-integer linear program (MILP) is proposed.

The basic idea of the MILP is as follows. Firstly, we consider that the sequence of matches takes place in a series of T time periods of operation. Our goal is to maximize the total energy exchanged between the hot tanks and the cold tanks. This aim can be expressed as maximizing the heat lost by the hot tanks or as minimizing the sum of the temperatures of the hot tanks after T time periods, weighted by their respective $(VC_p)_i$'s.

$$\max \sum_{i=1}^{Nhot} (VC_p)_i \, (T_i^0 - T_i^T) \qquad (26)$$

$$\left(\text{or } \max \sum_{j=1}^{Ncold} (VC_p)_j \, (T_j^T - T_j^0) \right)$$

where

T_i^{t-1} = initial temperature of hot tank i in time period t

T_j^{t-1} = initial temperature of cold tank j in time period t

T_i^t = final temperature of hot tank i in time period t

T_j^t = final temperature of cold tank j in time period t

T* = outlet temperature of the hot tank considered at time t

Nhot = the number of hot tanks

Ncold = the number of cold tanks

t = index of the time period, t = 1, 2, ..., T

T = the total number of time periods or the number of matches

For any period t, denoting Q_i^t = heat lost by hot tank i in time period t and Q_j^t = heat gained by cold tank j in time period t, then the following total heat balance must hold in each period.

$$\text{s.t.} \quad \sum_{i=1}^{Nhot} Q_i^t - \sum_{j=1}^{Ncold} Q_j^t = 0 \qquad t = 1, 2 \ldots T \tag{27}$$

where Q_i^t, $Q_j^t \geq 0$

Heat balances for the processing streams must hold in each time period.

$$(VC_p)_i \, T_i^t - (VC_p)_i \, T_i^{t-1} + Q_i^t = 0, \qquad \forall i, \; t = 1, 2 \ldots T \tag{28}$$

$$(VC_p)_j \, T_j^t - (VC_p)_j \, T_j^{t-1} - Q_j^t = 0, \qquad \forall j, \; t = 1, 2, \ldots T \tag{29}$$

Also, the desired target temperatures, T_i^{target} for the hot tanks and T_j^{target} for the cold tanks, must not be violated.

$$T_i^t \leq T_i^{target} \qquad i = 1, 2 \ldots Nhot \tag{30}$$

$$T_j^t \geq T_j^{target} \qquad j = 1, 2 \ldots Ncold \tag{31}$$

In order to restrict only one match between a hot tank and a cold tank per time period, we define the 0-1 variables that must satisfy these constraints.

$$\sum_{i=1}^{Nhot} y_i^t = 1 \qquad t = 1, 2, \ldots, T \qquad (32)$$

$$\sum_{j=1}^{Ncold} y_j^t = 1 \qquad t = 1, 2, \ldots, T \qquad (33)$$

If y_i^t is 0, then hot tank i does not exchange heat in period t. Therefore, its corresponding Q_i^t must be zero for that period. On the other hand, if hot tank i is permitted to match with a cold tank in period t, then y_i^t for the hot tank is equal to 1 and its Q_i^t can be any value less than its upper limit U_h. Similarly, a value of 0 for y_j^t indicates the absence of cold tank j from matching in period t, and a value of 1 indicates its presence. Also, the same restrictions for Q_j^t apply as for Q_i^t. These limitations are shown below.

$$Q_i^t - U_h y_i^t \le 0, \qquad \forall i, \; t = 1, 2, \ldots, T \qquad (34)$$

$$Q_j^t - U_c y_j^t \le 0, \qquad \forall j, \; t = 1, 2, \ldots, T \qquad (35)$$

where

U_h = upper bound for Q_i = max $[(VC_p)_i \cdot (T_i^o - T_i^{target})]$ for all hot tanks i

U_c = upper bound for Q_j = max $[(VC_p)_j \cdot (T_j^o - T_j^{target})]$ for all cold tanks j

If T^t is the outlet temperature of the hot stream that exchanges heat at period t and $(\Delta T)_{min}$ is the minimum temperature approach, then the feasibility of the selected match is insured by the following constraints.

$$(VC_p)_i \cdot (T_i^{t-1} - T^t) - Q_i^t - U_h y_i^t \ge - U_h \qquad \forall i, \qquad t = 1, 2, \ldots, T \qquad (36)$$

{}fC Ji {JX - (AT>m.n " T\~J) " Q! " Uc y) *' Uc' Vj. t - 1. 2 . . . .T (37)

If for a given pair (i,j), the corresponding binaries are set to one in a given time *period,* these constraints *ensure* that *ihQ* heat transferred, $\hat{u}^{t}_{i} = Q^{t}_{j}$, is feasible. That is, T_i^{*}~\ the inlet temperature of hot tank i, must be greater than $T_j^{!}$"\ the inlet temperature of cold tank j, since all variables are assumed to be positive. For the hot and cold tanks not selected in the match for that period, these constraints become redundant.

The problem defined above is a multi-period, mixed-integer linear program. The efficiency of the branch and bound search procedure can be increased by further constraining the problem.

Since the heuristic provides an initial estimate for the total energy exchanged, only sequences of matches that improve this lower bound should be examined, where the variable $CX_{heuristic}$ represents the total heat exchanged between the hot tanks and the cold tanks using the heuristic.

$$\sum_{i=1}^{Nhot} \sum_{t=1}^{T} Q_i^t \geq Q_{heuristic}$$

Next, since each hot tank/cold tank pair can match only once throughout the T time periods, the following constraints are imposed.

Z\j * y[• yj - 1 Vu Vj. t = 1, 2. . . . T (38)

$$\sum_{t=1}^{T} Z_{ij}^{t} \leq 1 \qquad \forall i,j \qquad (39)$$

where

Z_{ij}^{t} = a continuous variable (bounded by 1) that is 1 if a match exists between hot tank i and cold tank j in time period t;

0, otherwise

Finally, since redundant sequences may occur when assigning matches in different time periods (e.g., hot 1/cold 1 in period 1 and hot 2/cold 2 in period 2 is equivalent to hot 2/cold 2 in period 1 and hot 1/cold 1 in period 2), the ideas suggested by Pho and Lapidus to eliminate redundant sequences are used to produce these constraints.

For i = 1, 2, ..., (Nhot-1) and for j = 1, 2, ... (Ncold-1),

$$Z_{ij}^{t} + Z_{(i+k),(j+l)}^{t+1} \leq 1 \qquad t = 1, 2, ..., (T-1) \qquad (40)$$

$$k = 1, 2 .... (Nhot-i)$$

$$l = 1, 2 .... (Ncold-j)$$

(Our implementation of the ideas of Pho and Lapidus (1973) substitutes hot tanks for their cold streams and vice versa.) The user can alter easily any of these conditions, but elimination or relaxation of any of the integer constraints will add more combinations to the set of feasible sequences, and thus, the solution will require more CPU-time.

This MILP formulation has been applied to the problem of two hot tanks (B, C) /three cold tanks (X, Y, Z) given in Table 2, considering a maximum of 6 matches (time periods). The MILP is composed of 112 continuous variables, 30 integer variables, 36 equality constraints, and 150 inequality constraints. The objective function is to minimize the final temperature of the hot tanks, weighted by their (VC_p)'s. This example was solved using LINDO, a linear programming package developed by Schrage (1981), which uses a depth-first, branch and bound search technique for the integer variables. This problem required the enumeration of 51 branches and 2081 pivots to locate the optimal solution shown in Table 4 and used about 2,5 minutes of CPU-time on a DEC-20 computer.

The addition of hot tank A of Table 2 increased significantly the time required for a solution. For this 3 hot tanks/3 cold tanks problem, 110 continuous variables, 30 integer variables, 35 equality constraints, and 203 inequality constraints are required in the MILP problem. As found by the heuristic solution displayed in Table 5, only five matches are specified in this formulation to reduce the branching. Evaluation of approximately 350 branches with about 16,000 pivots required almost sixteen minutes of CPU-time to find a solution with a total heat exchanged (Q) greater than 95% of the maximum possible total heat exchanged (min $\{Q_{heating}, Q_{cooling}\}$).

Our CPU-times can be reduced since we did not exploit the structure of the MILP. Using a method to solve time-staged linear programs, for example, Dantzig (1980), or using a more efficient enumeration procedure could improve the time required for a solution, but we did not explore these possibilities. Although this MILP grows unwieldy for problems larger than three hot and three cold tanks, it is our opinion that most practical problems will be of this size or smaller. Also, solution of this MILP may be unnecessary if the heuristic solution is near the known maximum value of total heat exchanged, min $\{Q_{heating}, Q_{cooling}\}$.

Summary

This paper presents the first work to our knowledge that addresses the problem of heat integration in batch-type facilities. Because of processing requirements, three configurations of batch vessels and receiving tanks are shown to result in three different temperature profiles for the fluids: *cocurrent,* countercurrent, and combination cocurrent/countercurrent.

The heuristic, match the hot and cold tanks that are closest in temperature, provides a very good sequence for the exchanges for the cocurrent case with non-restrictive target temperatures. For one hot tank/n cold tanks and two hot tanks/two cold tanks problems without temperature limitations, the heuristic solution can be shown to be optimal. For problems with restrictive target temperatures, a mixed-integer linear program is presented to improve the heuristic result. The methods presented in this paper provide useful tools for* assessing the potentials for energy conservation in batch processing.

Bowman, R. A.; Mueller, A. C; Nagle, W. M. *Trans. A.S.M.E.. 62.* 283-294.

Chaddock, R. E.; Sanders, M. T. *Trans. A. I. Ch. £.,* *40(21.* 203-210.

Dantzig, G. B. *Time-Staged Methods in Linear Programming.* Technical Report 80-18, Stanford University Systems Optimization Laboratory, June 1980.

Fernandez-Seco, M. *Chemical Engineering. 60(12).* 172-173.

Fisher, R. C. *industrial and Engineering Chemistry. 36(10).* 939-942.

Grossmann, I.E., and Sargent, R.W.H. *Ind. Eng. Chem. Proc. Des. Dev., 18(2).* 343-348.

Kern, Donald Q. *Process Heat Transfer.* New York, New York: McGraw-Hill Book Company 1950.

Knopf, F. Carl, Okos, Martin R., and Reklaitis, Gintaras V. *Ind. Eng. Chem. Proc. Des. Dev.. 21(1).* 79-86.

Linnhoff, B., et al. *User Guide on Process Integration for the Efficient Use of Energy.* Rugby, England: The Institution of Chemical Engineers 1983.

Loonkar, Yashwant R., and Robinson, Jerome D. *Ind. Eng. Chem. Proc. Des. Dev.. 9(4),* 625-629.

Lundberg, Wayne L, and Christenson, James A. *Applications of Thermal Energy Storage to Waste Heat Recovery in the Food Processing Industry,* final report EC-77-C-01-5002, Department of Energy, 1979.

Nishida, N.; Stephanopoulos, G.; Westerberg, A. A *I. Ch. E. J.. 27(3).* 321-351.

Pho, T. K.; Lapidus, L *A. I. Ch. E. J.. 19(6).* 1182-1189.

Rippin, David W. T. *Computers and Chemical Engineering. 7(3).* 137-156. also presented at CHEMCOMP Symposium; Antwerp, May 1982.

Schrage, L *User's Manual for UNDO.* Palo Alto, California: The Scientific Press 1981.

Sparrow, Robert E., Forder, Graham J., and Rippin, David W.T. *Ind. Eng. Chem. Proc. Des. Dev.. 14(3).* 197-203.

Suhami, Iren, and Mah, Richard S.H. *Ind. Eng. Chem. Proc. Des. Dev.. 21(1).* 94-100.

Troupe, Ralph A. *Chemical Engineering. 59(9).* 128-131.

Unger, T. A. *Chemical Engineering. 69(15).* 119-122.

Vaselenak, J. *Studies in the Retrofit Design and Scheduling of Batch Processes.* PhD thesis, Carnegie-Mellon University, 1985.

Appendix 1

Part A

Part A shows that the maximum total heat exchanged does not occur if one tank is permitted to transfer to more than one tank at the same time for the first match, while the remaining matches are done one tank at a time. The one hot tank/n cold tanks arrangement is displayed in Figure 5. The numbering of the cold tanks is arbitrary; that is, no ordering of the initial temperature of the cold tanks or of the VC_p's of the cold tanks is specified.

First Match

Match hot 1 with all of the cold tanks. Stop when the temperatures of hot 1 and cold 1 are equal. The final temperatures of the other cold tanks are free variables but must be less than or equal to the final temperature of the hot tank.

$x_1^0 > y_i^0$

$A_1 = \left(VC_p\right)_{hot\ 1}$

$B_i = \left(VC_p\right)_{cold\ i}\ \ i = 1, 2, ..., n$

x_1^0 = initial temperature of hot 1

y_i^0 = initial temperature of cold i, i = 1, 2, ..., n

x_1^j = temperature of hot 1 after the jth match

x_1^1 = common temperature that hot 1 and cold 1 reach

y_i^1 = temperature of cold i after the first match

$x_1^1 - y_1^1 \geq 0$ for all i = 1, 2 . . . . n

$A_1 (x_1^0 - x_1^1)$ = heat lost by hot 1

$B_1 (x_1^1 - y_1^0)$ = heat gained by cold 1

$KB_1 (x_1^1 - y_1^0)$ = heat gained by all other cold tanks

$$= B_2 (y_2^* - y_2^0) + B_3 (y_3^1 - y_3^0) + \dots + B_n (y_n^1 - y_n^0) \qquad (41)$$

$K \geq 0$

where K represents the ratio of the heat gained by all cold tanks but cold 1 to the heat gained by cold 1

Equation 41 can be solved for y_2^1.

$$y_2^1 = \dots \qquad (42)$$

The overall heat balance yields the following result.

$$A_1 (x_1^0 - x_1^1) = B_1 (x_1^1 - y_1^0) + KB_1 (x_1^1 - y_1^0) \qquad (43)$$

$$x_1^1 = \frac{A_1 x_1^0 + B_1 y_1^0 (1 + K)}{A_1 + B_1 (1 + K)} \qquad (44)$$

Match hot 1 with each remaining cold tank individually.

Second Match: Match hot 1 with cold 2 until their final temperatures are the same.

$$A_1 (x_1^1 - x_1^2) = B_2 (x_1^2 - y_2^1) \qquad (45)$$

$$x_1^2 = \frac{A_1 x_1^1 + B_2 y_2^1}{(A_1 + B_2)} \qquad (46)$$

Third Match: Match hot 1 with cold 3 until their final temperatures are the same.

$$A_1 (x_1^2 - x_1^3) = B_3 (x_1^3 - y_3^1) \tag{47}$$

$$x_1^3 = \frac{A_1 x_1^1 + B_3 y_3^1}{(A_1+B_3)} \tag{48}$$

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.

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Nth Match: Match hot 1 with cold n until their final temperatures are the same.

$$A_1 (x_1^{n-1} - x_1^n) = B_n (x_1^n - y_n^1) \tag{49}$$

$$x_1^n = \frac{A_1 x_1^{n-1} + B_n y_n^1}{(A_1 + B_n)} \tag{50}$$

Equations 46, 48, 50, and similar equations represent a system of (n-1) equations in 2(n-1) unknowns: $x_1^2, x_1^3, \ldots, x_1^n, y_2^1, y_3^1, \ldots, y_n^1$. Elimination of the (n-2) variables x_1^2, x_1^3, ..., x_1^{n-1}, can occur by substituting equations 46, 48, and similar ones into equation 50.

$$x_1^n = \frac{A_1^{(n-1)} x_1^1 + A_1^{(n-2)} B_2 y_2^1}{(A_1+B_2)(A_1+B_3)\ldots(A_1+B_n)} \tag{51}$$

$$+ \ldots + \frac{B_n y_n^1}{(A_1+B_n)} \left[\prod_{i=2}^{j-1} (A_1+B_i)\right]$$

If $\{x_1^1, y_3^1, y_4^1, \ldots, y_n^1\}$ is chosen as the set of independent variables, x_1^2 and y_2^1 can be expressed in terms of these variables using equations 44 and 42.

$$x_1^n = \frac{A_1^{(n-1)} (A_1 x_1^1 + B_1 y_1^1 (1+K))}{(A_1+(1+K)B_1)(A_1+B_2)\ldots(A_1+B_n)} \tag{52}$$

$$+ \frac{A_1^{(n-2)} B_2}{(A_1+B_2)(A_1+B_3)\ldots(A_1+B_n)} \left\{ y^0 + \frac{KB_1}{B_2} \left[\frac{A_1 x_1^0 + B_1 y_1^0 (1+K)}{A_1 + (1+K)B_1} - y_1^0 \right] \right\}$$

$$+ \frac{A_1^{(n-2)} B_2}{(A_1+B_2)(A_1+B_3)\ldots(A_1+B_n)} \left\{ \frac{B_3}{B_2} (y_3^1 - y_3^0) + \cdots + \frac{B_n}{B_2} (y_n^1 - y_n^0) \right\}$$

$$+ \sum_{j=3}^{n} \left\{ \frac{A_1^{(n-j)} B_j y_j^1}{(A_1+B_2)(A_1+B_3)\ldots(A_1+B_n)} \left[\prod_{i=2}^{j-1} (A_1+B_i) \right] \right\}$$

Next, equation 52 is differentiated with respect to K. After collecting terms, the following results.

$$\frac{\partial x_1^n}{\partial K} = \frac{A_1^{(n-1)} B_1^2 (x_1^0 - y_1^0)}{(A_1 + (1+K)B_1)^2 (A_1+B_2)(A_1+B_3)\ldots(A_1+B_n)} > 0 \qquad (53)$$

Equation 53 shows that x_1^n, the final height of hot 1, is a monotonically non-decreasing function of K, a parameter that indicates the heat flow to cold *2,* cold 3, ... , cold n, based on the heat flow to cold 1. Since minimizing x_1^n will yield the maximum total heat flow from hot 1, the smallest value of K, K=0, should be selected.

Therefore, Part A shows that the optimal amount of heat is not transferred if one tank exchanges with more than one tank at a time, while the remaining exchanges are done one at a time.

Part B

The goal of Part B is to show that the maximum total heat exchanged does not occur if one tank is permitted to transfer to more than one tank at a time, for any of the matches.

For the system of one hot tank and n cold tanks shown in Part A, the cold tanks are assumed to receive energy with cold tank 1 being the first to reach the same temperature as hot 1, cold tank 2 being the second, etc., cold tank n being the final. Once a cold tank attains the same temperature as the hot tank, the cold tank is eliminated from the analysis since it cannot receive any more heat from the hot tank (thermodynamically infeasible). Each match represents the heat balances of the system until at least one cold tank is eliminated (or completed). If more than one cold tank is eliminated during a match period, then the match periods for the completed cold tanks are omitted from the analysis. The numbering assignment of the cold tanks is arbitrary.

step 1: Once the order of cold tanks to be matched to completion is selected, then the final two cold tanks, cold tank (n-1) and cold tank n, must be matched in an optimal manner to give an optimal result. From the result of Part A, the optimal heat exchange occurs if the two cold tanks individually match with hot 1.

step 2: Next, the final three cold tanks to be matched are analyzed. From the result of Part A, cold tanks (n-1) and n should not receive any heat while cold tank (n-2) is exchanging heat with the hot tank. After cold tank (n-2) is finished, the final two tanks individually exchange heat with hot 1 as determined in step 1.

step 3: The final four cold tanks are analyzed using a reasoning similar to that used in step 2. This process is repeated until all the cold tanks are analyzed.

This application of Bellman's principle of optimality shows that the maximum energy exchanged occurs when each cold tank individually matches with the hot tank.

Appendix 2

The purpose of this appendix is to find the optimal order for exchange of n cold tanks with one hot tank. First, a general expression for $x_1''\left(1, 2, ..., n\right)$, the final temperature of hot 1 after n matches with the cold tanks in the arbitrary sequence $\left(1, 2, ..., n\right)$ is determined. Each cold tank matches individually with hot 1, as justified by Appendix 1. Also, the nomenclature of Appendix 1 is used here.

In the following two sequences the order of tanks i and (i+1) is interchanged.

sequence 1 = $\left(1, 2, ..., (i-1), i, (i+1), (i+2), ..., n\right)$

sequence 2 = $\left(1, 2, ..., (i-1), (i+1), i, (i+2), ..., n\right)$

The final temperature of the hot tank using sequence 1 is found by setting K=0, $y_3^1 = y_3^0$, $y_4^1 = y_4^0$, ..., $y_n^1 = y_n^0$ in the expression for x_1'' in Appendix 1. The result is displayed below.

$$x_1''\left(1, 2, ..., i, (i+1), ..., n\right) = \frac{A_1^n x_1^0 + A_1^{(n-1)} B_1 y_1^0}{(A_1+B_1)(A_1+B_2)...(A_1+B_n)} \tag{54}$$

$$+ \sum_{j=2}^{n} \left\{ \frac{A_1^{(n-j)} B_j y_j^0}{(A_1+B_1)(A_1+B_2)...(A_1+B_n)} \left[\prod_{k=1}^{j-1} (A_1+B_k) \right] \right\}$$

If cold i and cold (i+1) are interchanged in the sequence, the following results.

$$x_1''\left(1, 2, ..., (i-1), (i+1), i, (i+2), ..., n\right) = \frac{A_1^n x_1^0 + A_1^{(n-1)} B_1 y_1^0}{(A_1+B_1)(A_1+B_2)...(A_1+B_n)} \tag{55}$$

$$+ \sum \left\{ \frac{A_1^{(n-j)} B_j y_j^0}{(A_1+B_1)(A_1+B_2)...(A_1+B_n)} \left[\prod_{k=1}^{j-1} (A_1+B_k) \right] \right\}$$

$$+ \frac{A_1^{(n-i)} B_{(i+1)} y_{(i+1)}^0}{(A_1+B_1)(A_1+B_2)...(A_1+B_n)} \left[\prod_{k=1}^{i-1} (A_1+B_k) \right]$$

$$+ \frac{A_1^{\{n-(i+1)\}} B_i y_i^0}{(A_1+B_1)(A_1+B_2)...(A_1+B_n)} \left[\prod_{k=1}^{i-1} (A_1+B_k) \right] (A_1 + B_{(i+1)})$$

$$+ \sum_{j=i+2}^{n} \left\{ \frac{A_1^{n-j} B_j y_j^0}{(A_1+B_1)(A_1+B_2)...(A_1+B_n)} \left[\prod_{k=1}^{j-1} (A_1+B_k) \right] \right\}$$

Subtracting equation 55 from equation 54 yields the following.

Let A = x_1''(i. 2, ... i. (i+1). . . . .n)- x_1''(1, 2. . . . .(i-1). (i+1), i. (i+2). . . . .n)

$$A = \frac{A_1^{n-(i+1)} B_i B_{i+1} (y_{i+1}^0 - y_i^0)}{(A_1+B_1)(A_1+B_2)...(A_1+B_n)} \left[\prod_{k=1}^{i-1} (A_1+B_k) \right] \le 0 \qquad (56)$$

Since $y_i^0 \le y_{(i+1)}^0$, A ≤ 0, and the final temperature of the hot tank using sequence one is colder than that using sequence two. This result shows that sequence one transfers more energy than sequence two. Also, for a pair-wise interchange in a sequence, the preferred order is to match the cold tank with a higher inlet temperature before the alternative cold tank.

This result holds for any two consecutive tanks in the sequence. Thus, by a series of pair-wise interchanges, the cold tank with the highest inlet temperature will be the first match, the second warmest cold tank, the second match, etc. Therefore, the optimal sequence of cold tanks for any 1 hot tank/n cold tank problem is to order the cold tanks from highest initial temperature to lowest.

$(\Delta T)_{min}$ is incorporated easily into this analysis. Replacing y_k^0 with $y_k^0 - (\Delta T)_{min}$ in the equations for x_1^n will account for the minimum difference in approach temperatures, (k = 1, 2, ... n) Also, x_1^"^ $(y_K^\circ \cdot (AT)_{mm})$ for a match between hot tank 1 and cold tank k to occur. The result, equation 56, remains unchanged by these additions.

Table 1: Example of the Heuristic for 10 Hot Tanks and 10 Cold Tanks

Tank	VC_p (kJ/°C)	$T_{initial}$ (°C)	$T_{desired}$ (°C)	Q_{needed}	$T_{heuristic}$	$Q_{heuristic}$
H1	1.8	202	90	201.6	94.7	193.1
H2	1.6	208	80	204.8	102.1	169.5
H3	2.3	215	110	241.5	112.8	235.0
H4	1.5	215	85	195.0	120.4	142.0
H5	0.8	224	75	119.2	124.6	79.5
H6	1.6	228	130	156.8	133.8	150.7
H7	1.3	230	130	130.0	141.8	114.7
H8	1.8	236	150	154.8	153.3	148.9
H9	1.7	245	150	161.5	164.3	137.3
H10	2.0	250	100	300.0	177.8	146.4
				1865.2		1517.1

Tank	VC_p (kJ/°C)	$T_{initial}$ (°C)	$T_{desired}$ (°C)	Q_{needed}	$T_{heuristic}$	$Q_{heuristic}$
C1	1.0	138	250	112.0	241.8	103.8
C2	2.1	128	240	235.2	234.3	223.3
C3	1.8	123	230	192.6	227.4	187.8
C4	1.4	116	210	131.6	221.5	147.7
C5	0.8	114	250	108.8	218.0	83.2
C6	2.2	106	210	228.8	206.9	221.9
C7	1.3	102	200	127.4	199.8	127.1
C8	1.1	97	250	168.3	193.5	106.1
C9	2.0	92	200	216.0	181.2	178.4
C10	1.5	80	210	195.0	171.8	137.7
				1715.7		1517.1

Table 2: Tank Data

Tank	VC_P (kJ/°C)	$T_{initial}$ (°C)	$T_{desired}$ (°C)	needed
A	I.Q	400	150	250
B	1.4	350	125	315
C	1.3	325	175	195
				760 = $Q_{cooling}$
X	2.0	100	175	150
Y	1.5	90	200	165
2	1.6	50	275	360
				675 = $Q_{heating}$

$(\Delta T)_{min} = 0$ °C

Table 3: Hot Tanks B, C / Cold Tanks X, Y, Z Heuristic. Solution

match sequence: C/X; C/Y; B/Y; B/Z

outlet temperatures (°C):

$T_B = 150$ $T_{..} = 175$

$T_j = 175$ $T^*_Y = 200$

$T_2 = 150$

$Q_{heuristic} = 475$ kJ

Table 4: Hot Tanks B, C / Cold Tanks X, Y, Z MILP Solution

match sequence: C/Z; B/Z; B/X; B/Y; C/X

outlet temperatures (°C):

$T_B = 125$ $T_X = 167.3$

$T_C = 175$ $T_Y = 125$

$T_Z = 251.8$

$Q_{MILP} = 510$

$Q_{max} = \min\{315 + 195, 150 + 165 * 360\} = \min\{510, 675\} = 510$

Table 5: Hot Tanks A. B, C / Cold Tanks X, Y, Z Solution Data

Heuristic Solution

match sequence: C/X; C/Y; B/Y; B/Z; A/Z

outlet temperatures (°C):

T = 246.2	T = 175
T^*_B = 150	T^*_Y = 200
T^o_C = 175	T^*_Z = 246.2

$Q_{heuristic}$ = 628.9 kJ

MILP Solution with $Q_{MILP} \le$ 95% of Q_{max}

match sequence for 5 time periods: A/X; B/Z; C/Y; A/X; B/X

outlet temperatures (°C):

T_A = 156.1	T^*_X = 175
T^A_B = 175	T^*_Y = 199.1
= 199.1	T_Z = 261.8

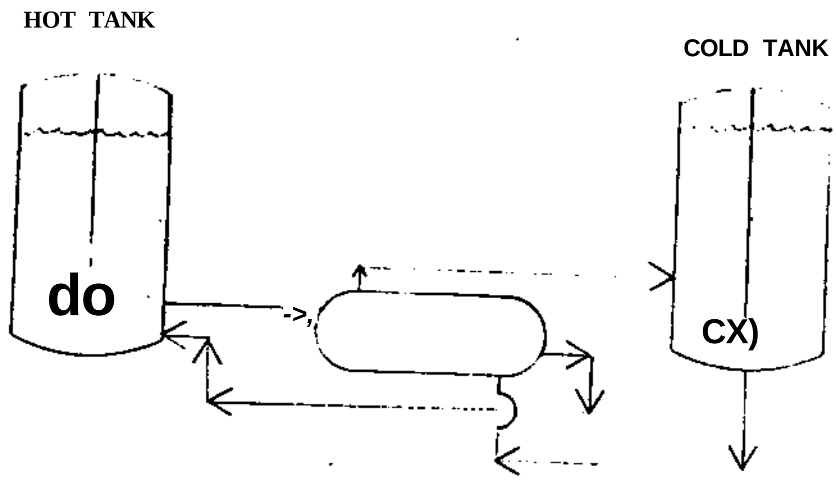
Q_{MILP} > 652.6

Q_{max} = min {760, 675J = 675

95% of Q_{max} = 641.3

$(\Delta T)_{min}$ = 0°C

COCURRENT HEAT EXCHANGE



Fijure 1A

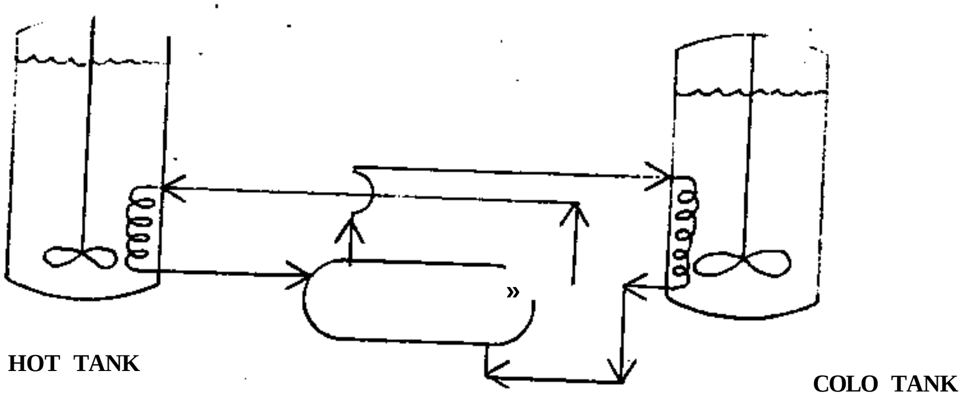


Figure IB

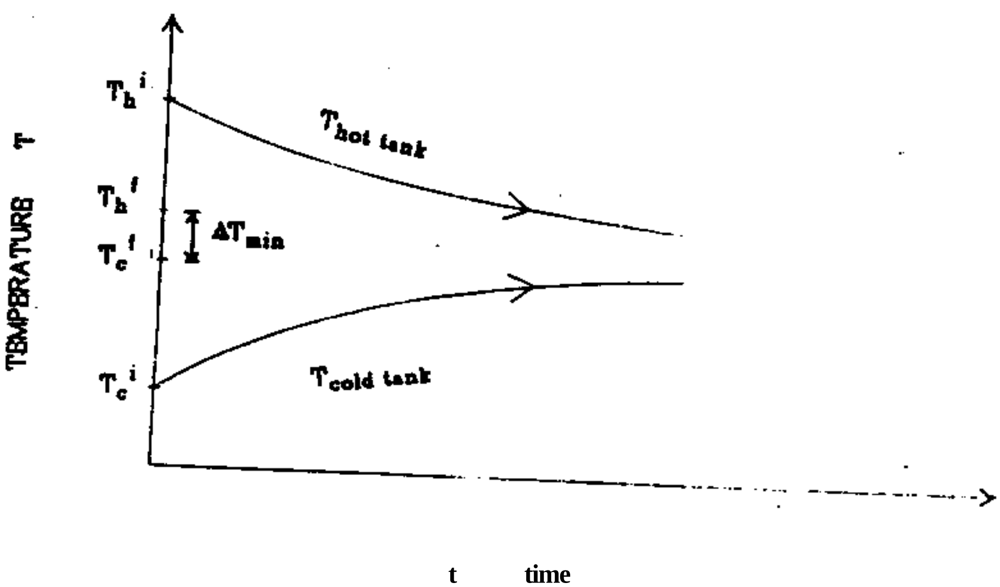
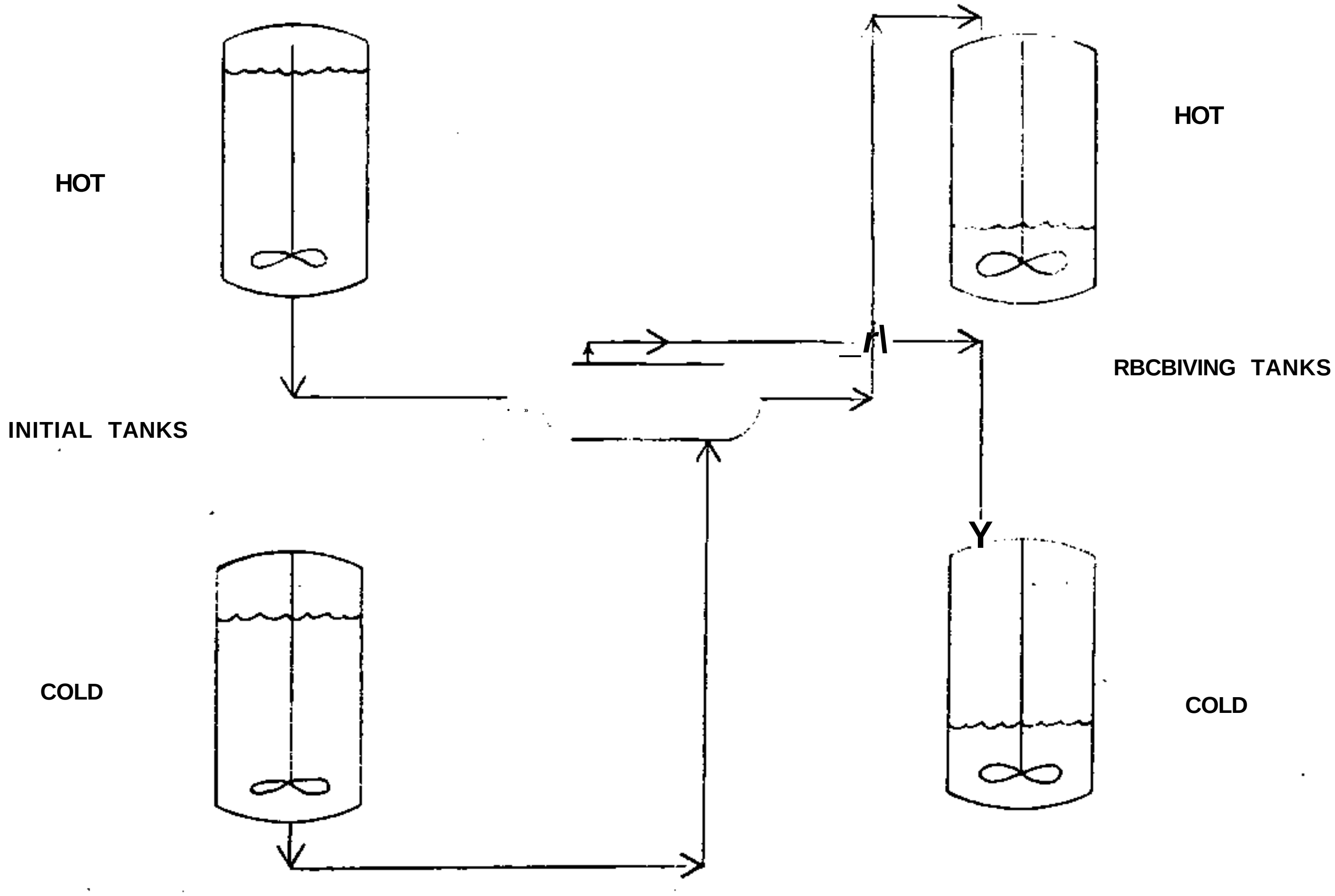


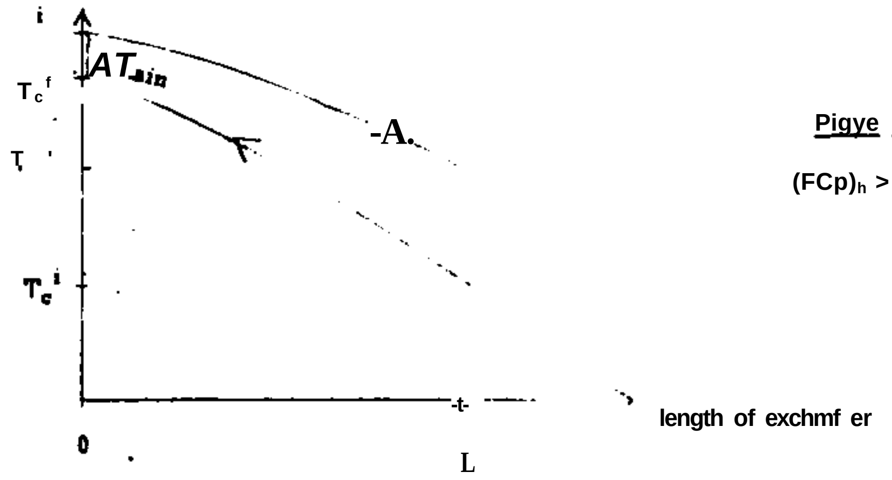
Figure 1C

COUNTERCURRENT CASE



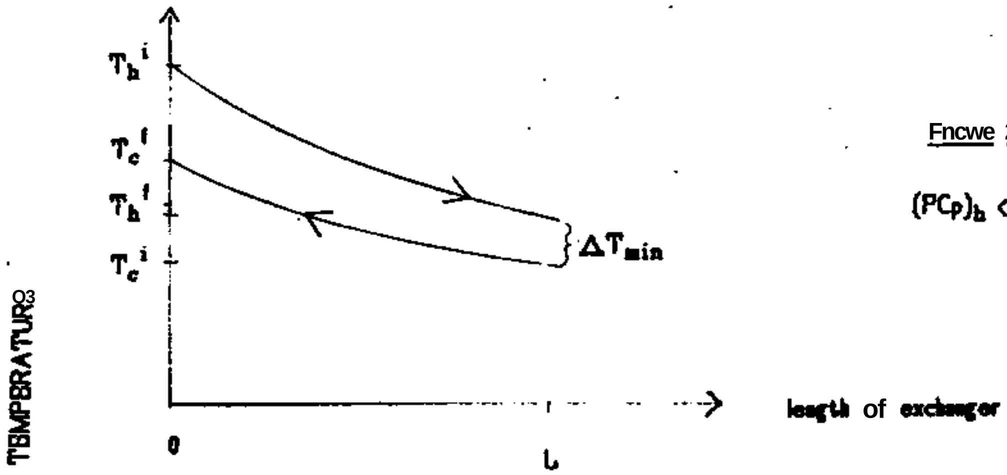
Pifwe 2A

TEMPERATURE PROFILES - COUNTERCURRENT CASE



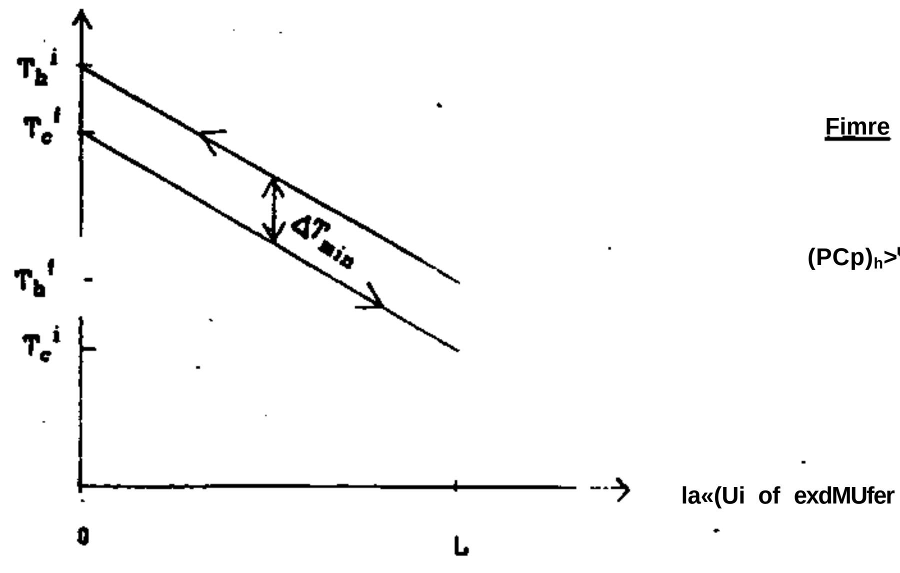
Pigye 26

$(FCp)_h > (FCp)_0$



Fncwe 2C

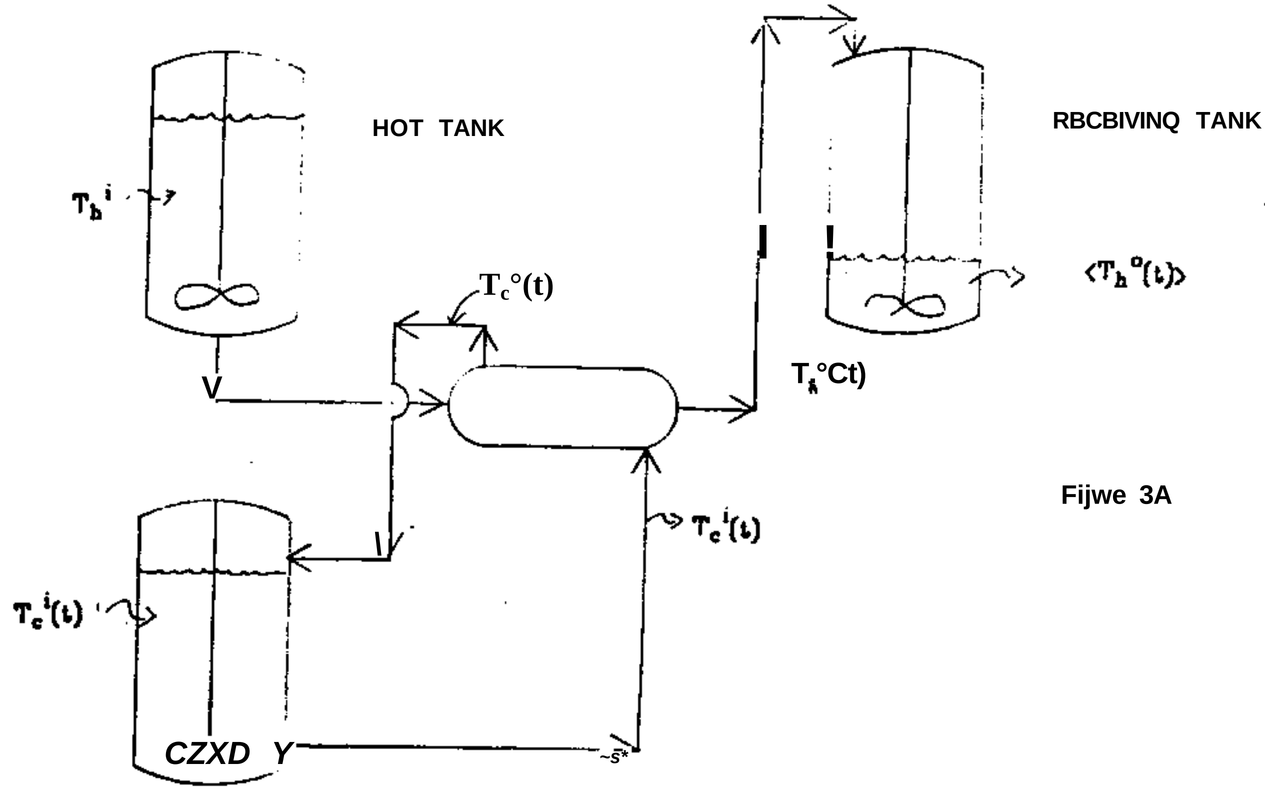
$(PCp)_h < (PCp)_c$



Fimre 2D

$(PCp)_h >= (PCp)_c$

COCURRENT/COUNTERCURRENT CASE



Fijwe 3A

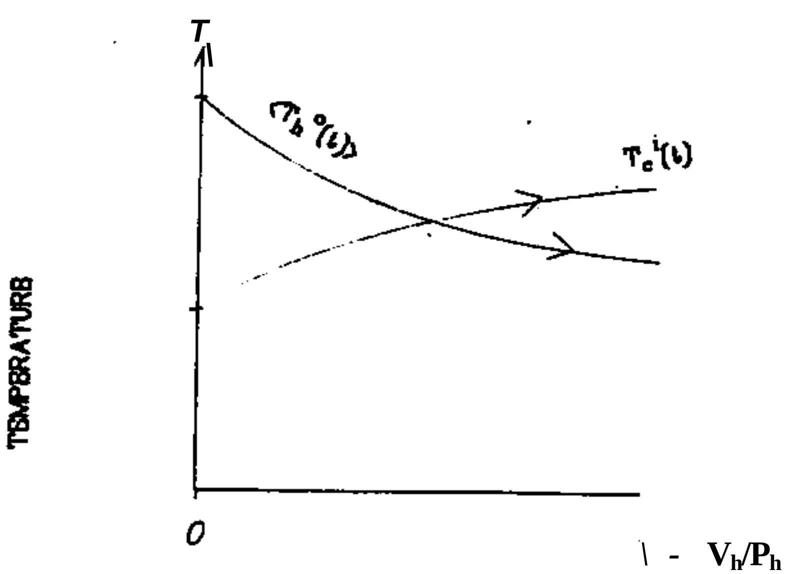
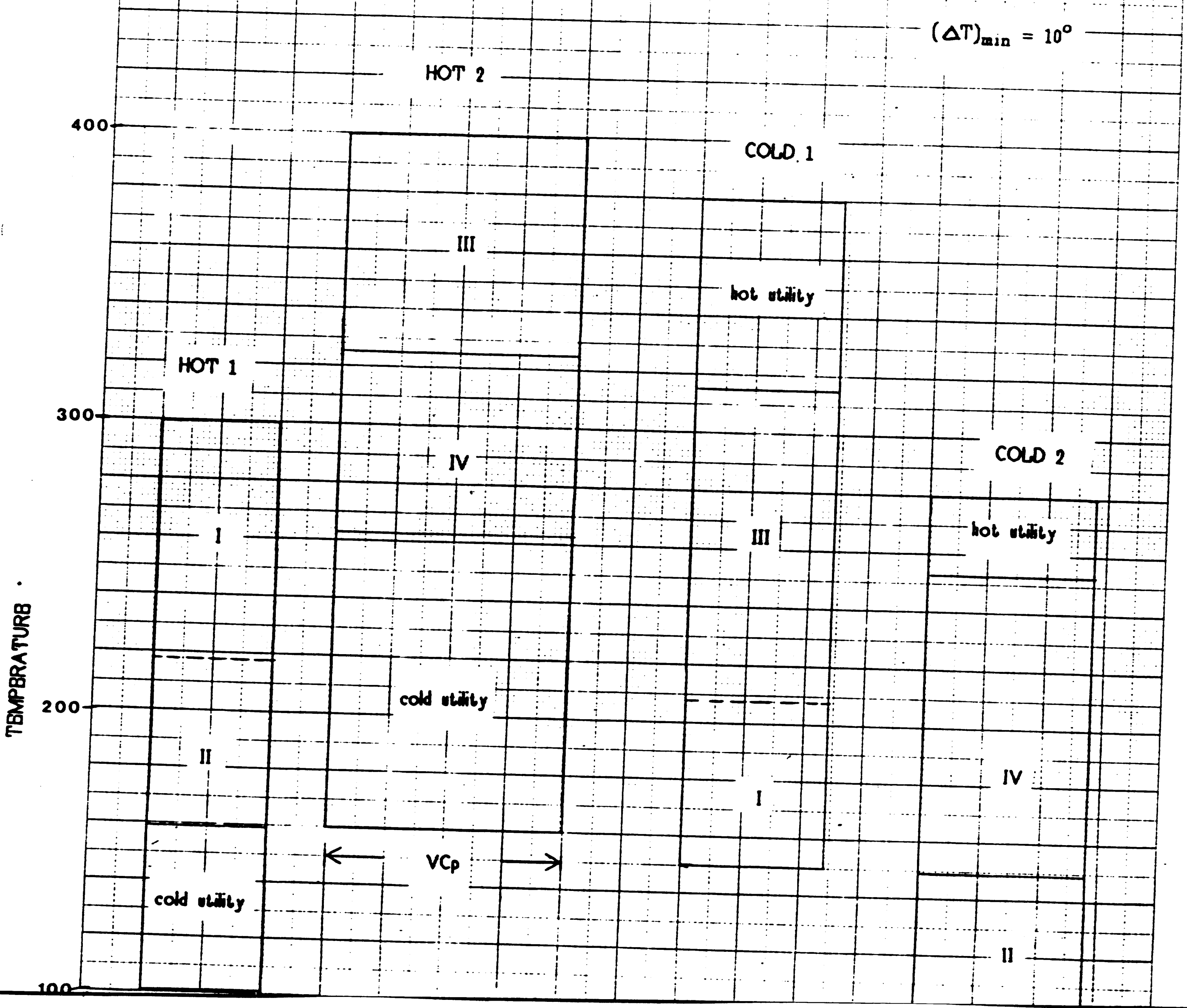


Figure 38

Figure 4

EXAMPLE USING THE HEURISTIC



ONE HOT TANK / $_N$ COLD TANKS ARRANGEMENT

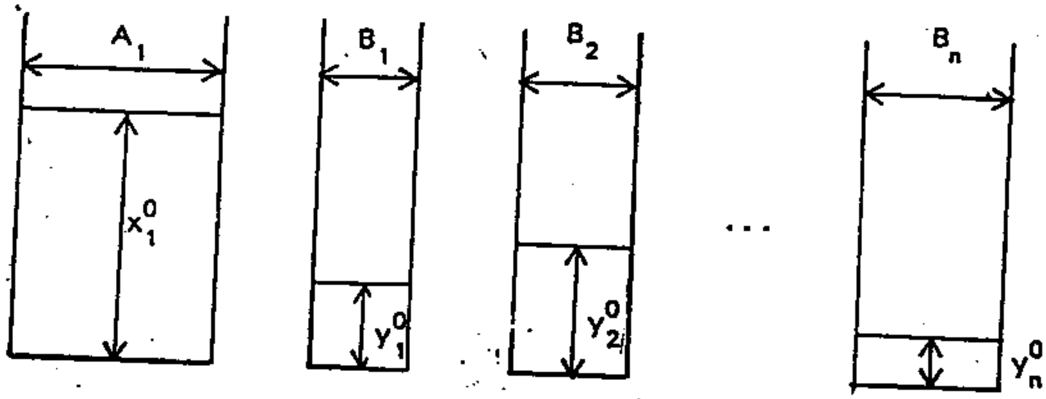


Figure 5