Prediction of Optimal Operating Parameters for Continuous Casting of Billets

by

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Abstract

Process optimization determines process parameters that maximize or minimize (optimize) so aspect of a process (the obieciive function), while ensuring that the process operates wr established limits. In this work a mathematical model that simulates heat flow and solidification continuously cast steel strand is coupled with mathematical optimization techniques to predict opt process parameters lor several aspects ol the continuous casting process The optimizations constrained so that representative process constraints are enlorced A description ol the model optimization method, and the means ol coupling are presented The tormulaiion ol obieciive tune and constraints tor continuous casting ol billets and the predictions resulting iioni optimizing t lormuiattons is also discussed

Introduction

The continuous casting process currently accounts for more than 50% of total world crude steel production.[1] Many applications require steel of a quality level only obtainable through continuous casting. The productivity of a continuous casting operation and the quality of the resulting product are largely dependent on the casting parameters used during the casting process.[2] The operating parameters for the continuous casting process need to be chosen so that a predetermined balance between productivity, product quality and operating costs is optimized. The selection of optimal operating parameters becomes even more important as the use of direct charging of hot strands to rolling operations becomes more prevalent. The temperature distribution and total heat content in the strand must be closely controlled in order to roll high quality products.

The problem of selecting continuous casting process parameters that optimize some function ol the caster operating state falls within the Iramework of problems known as constrained optimization problems. We desire to optimize (maximize or minimize) an objective function (a function used to determine if one operating state is more or less desirable than another), while ensuring that constraints that represent physical limits on the process are obeyed. The casting process is represented by a mathematical model, for reasons of cost and convenience and to allow the optimization process to proceed to the optimal point by paths that may include infeasible states (operating states where one or more of the process constraints are violated). In this work, only heat flow aspects of the continuous casting process are considered, although extensions to stress/strain and other aspects can certainly be made within the Iramework presented here. All of the objective and constraint functions are therefore stated in terms of temperature fields and thermal behavior.

The relationships between the objective function, constraint values and process variables are available only through the use of a numerical heat transfer simulation for a continuous caster. These relationships are nonlinear, hence the optimization problem is a Non-Linear Program (NLP). An optimization technique known as Successive Quadratic Programming[3,4] (SOP) has been used in this study to solve the constrained NLP problems. This technique was chosen because it typically requires fewer function evaluations than other NLP methods[5] and function evaluations (model simulations) have been lound to be quite expensive in terms of both real time and computer time.

The SOP algorithm can be derived from a Newton-Raphson approach applied to the optimally conditions for the nonlinear programming problem. Numerous applications of this method have been made to chemical process optimization problems (see the paper by Biegler[6] for a review) and currently it is the algorithm of choice for solving moderately sized optimization problems based on computationally intensive models. In addition, SOP has excellent constraint handling features and requires only function and gradient information from the process model. Based on the implementation of Biegler and Cuthrell[7] the optimization algorithm is relatively straightforward to apply to general purpose optimization problems with *smooth* objective and constraint functions.

Previous attempts at applying nonlinear constrained optimization techniques to continuous casting processes have been lew. In the work by Larrecq. et.al.[2], a detailed list of process operation and product quality constraints is presented. A gradient method is used to minimize a cost function that represents violated constraints, at constant casting speed, and then the casiing speed is manipulated manually until a maximum casting speed is found. Holappa. et.al.. have used a similar method.[8] Neither group has allowed the casting rate to be a variable in the optimization process even though the casting rate has an extremely important effect on the temperature distribution and metallurgical structure of the cast product.

The optimization system is comprised mainly of two parts, the model and the optimizer. These are shown schematically in Figure 1. The model is further subdivided into a part that calculates the temperature field in a continuous caster and a part that uses the resulting temperature field to calculate

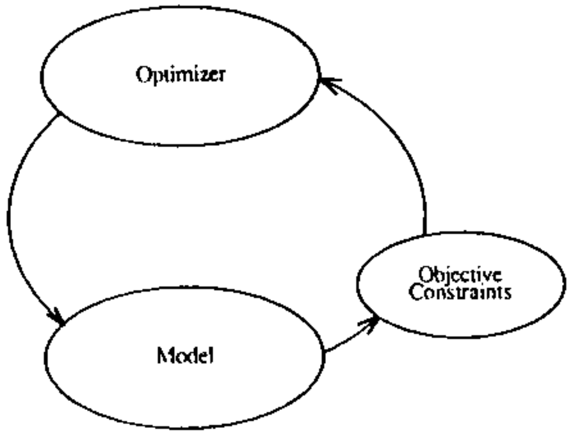

Figure 1: Schematic Representation ol Optimization System

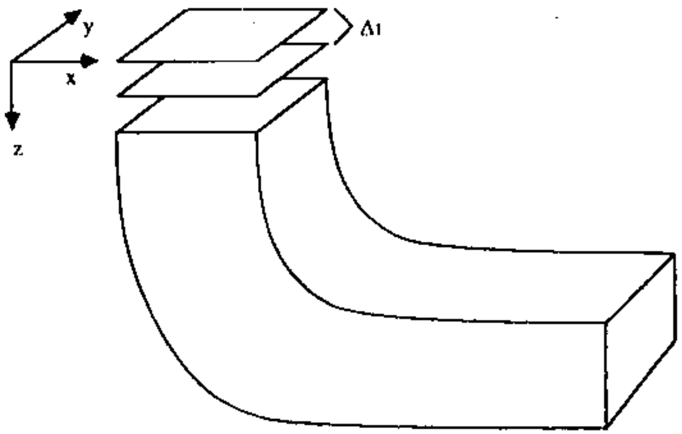

Figure 2: Schematic Representation of Slice Modelling Technique

the values of the objective and constraint functions. The optimizer is "primed" with initial values lor all of the process parameters, and uses the model to calculate the objective function and the values of the constraints. New values for the process parameters are calculated by the optimizer on the basis of the objective and constraint values returned by the model, and a new set of objective and constraints are calculated. This is done iteratively. until a set of equations that describe the optimum point are satisfied. This modular approach, where the model is separated from the optimizer, allows us to easily change models to reflect other phenomena that are considered important to the problem at hand.

Description and Verification of Models

A slice technique, similar to that used by Brimacombe[9]. Mizikar[10] and Perkins and Irving[11] was used to model the temperature field in the continuously cast strand. Heat flow in a two dimensional, iransverse slice moving with the strand was considered. Heat flow by conduction in the direction of strand movement is small compared to the heat flow caused by bulk motion of the strand in this direction, and can be safely ignored.[12] The slice is shown schematically in Figure 2. By calculating the time dependent temperature field in the transverse slice at sufficient positions during the withdrawal of the strand, a three dimensional, steady state temperature field can be calculated for the entire caster

The two dimensional, transient heal flow equation solved in this work is shown in equation (1).

$$\frac{\partial}{\partial x}k(T)\frac{\partial T}{\partial x}+\frac{\partial}{\partial y}k(T)\frac{\partial T}{\partial y}=\rho(T)C_p(T)\frac{\partial T}{\partial t} \quad (1)$$

The position along the caster z is related to time i through the casting rate r, as shown in equation (2).

$$z=n \quad (2)$$

Boundary conditions and constraints are normally stated in terms of position, while the heat flow equations are most easily formulated in terms of time. The thermal conductivity k, heat capacity c_p and density p are allowed to be unrestricted functions of temperature. Convection in the liquid pool is modelled by using an artificially high value for the thermal conductivity, nominally 5 times normal. The effect of convection in the two phase region is modelled as a quadratic function of the fraction liquid as shown in equation (3).

$$k_{eff}=k_{t_S}+(k_{t_L}-k_{t_S})f_L^2 \quad (1)$$

The heat of fusion is accounted for by letting C_p be a strongly varying function of temperature.

To facilitate solution of the nonlinear equations resulting from the discretization of equation (1), the Kirchoff transformation was used.[13] This transformation is shown in equation (4). Use of this transformation removes the dependence of the thermal conductivity on temperature from the left side of the equation, where it appeared within a gradient operator, and puts.all temperature dependencies m one term on the right side of the equation, outside of any differential operators. The transformed heat flow equation actually solved is shown in equation (5).

$$\theta=\int_{T_0}^{T}\frac{k(T')}{k(T_0)}dT' \quad (4)$$

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=C(\theta)\frac{\partial\theta}{\partial t} \quad (5)$$

The initial condition used to solve the heat flow equation is given by equation (6). This condition sets the temperature at the beginning of the simulation to the pouring temperature.

$$T(x,y,0)=T_p \quad (6)$$

In this work, symmetry across the centcrlmes of tho cast piece was assumed, hence equation (5) was solved for only one quarter of the cast section. The boundary conditions applied along the centertines of the strand are given by equations (7).

$$-k\frac{\partial T}{\partial x}=0 \qquad @\ x=0$$
$$-k\frac{\partial T}{\partial y}=0 \qquad @\ y=0 \quad (7)$$

The boundary conditions in the mold have been modelled using several empirical equations found in the literature, and a proprietary relationship developed by Inland Steel using an instrumented mold. Inland Steel is a member of the Center for Iron and Steel Research at Carnegie Mellon University. Since the examples in this study are based on an Inland Steel caster, the mold boundary conditions used were the Inland Steel conditions. They are of the form shown in equations (8). where the heat flux Q was from an experimentally determined table.

$$-k\frac{\partial T}{\partial x}=Q_x(z) \qquad @\ x=X$$
$$-k\frac{\partial T}{\partial y}=Q_y(z) \qquad @\ y=Y \quad (8)$$

The spray zone boundary conditions are shown in equations (9), and are in terms of heat transfer coefficients. No attempt has been made in this work to relate heat transfer coefficients to water flow rates, nozzle type or spray chamber design. These correlations have previously been considered by Mizikar[14] and Muller.[15] Outside of the mold and spray chambers, radiant cooling has been assumed, with boundary conditions given by equations (10).

$$-k\frac{\partial T}{\partial x}=h_x(T-T_0) \qquad @\ x=X$$
$$-k\frac{\partial T}{\partial y}=h_y(T-T_0) \qquad @\ y=Y \quad (9)$$

$$-k\frac{\partial T}{\partial x}=\sigma\varepsilon(T^4-T_0^4) \qquad @\ x=X$$
$$-k\frac{\partial T}{\partial y}=\sigma\varepsilon(T^4-T_0^4) \qquad @\ y=Y \quad (10)$$

The equations have been solved using an alternate direction implicit finite difference scheme, with iterations at each time step to recalculate and average the rapidly changing thermal properties in the vicinity of the solidus and liquidus temperatures. We have found that iteration coupled with the Kirchoff transformation has allowed us to take very large steps in time and reduced the computer time required for accurate solution of these problems significantly.

The model has been verified against a model belonging to Inland Steel that is known to closely represent one of their billet casters. The result of this comparison is shown in Figures 3 and 4. The model has also been used to calculate the surface temperatures in a cast slab using the data from the paper by Larrecq. et.al.[2] In all cases the agreement has been quite satisfactory.

Optimization Method

The general statement of a constrained optimization problem is given by:

$$\text{Minimize } f(p) \quad (11)$$

subject to

$$h_i(p)=0 \qquad i=1,2,\ldots,I$$
$$g_j(p)\le 0 \qquad j=1,2,\ldots,J$$

where p is an n dimensional vector of variables that represent the process operating parameters. $h_i(p)$

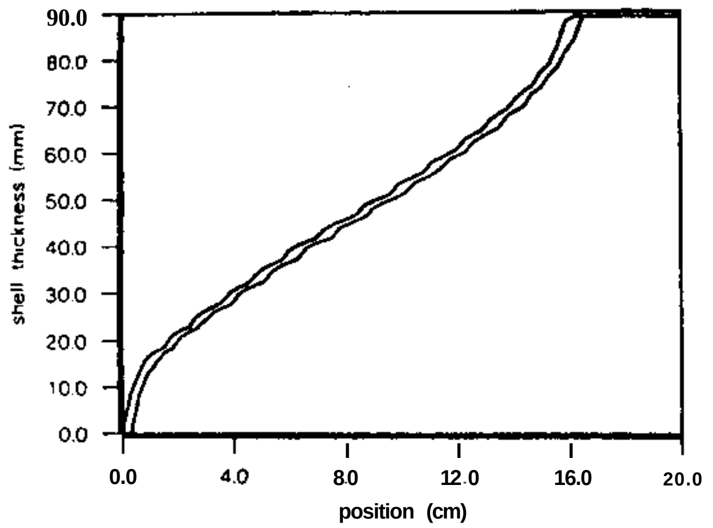

Figure 3: Verification of Model Results - Shell Thickness

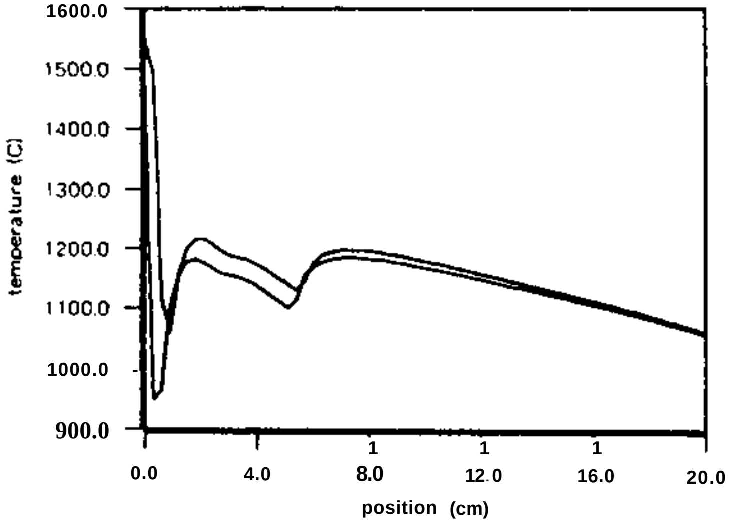

Figure 4: Verification of Model Results • Surface Temperature

are *I* equality constraints. *</>) are *J* inequality constraints and *ftp)* is the objective function. The Inequality constraints are active if *//,)=<> at the point *p*; they are inactive H $g_j(p)>0$. The following conditions are satisfied at the optimum point of a nonlinear, constrained function and are known as the Kuhn-Tucker conditions:

$$\nabla L(p,u,v) = \nabla f(p) - \sum_{j=1}^{J} u_j \nabla g_j(p) - \sum_{i=1}^{I} v_i \nabla h_i(p) = 0 \qquad (12)$$

$g_j(p) \geq 0$ >= 12——7

$h_i(p) = 0$ i= 1 2——I

$u_j g_j(p) = 0$ >= i . 2 — y

$u_j \geq 0$ ý=1.2——y

L is the Lagrangian function, the coefficients v_i are Lagrange multipliers that are applied to the equality constraints, and the coefficients u_j are similar multipliers that apply to the inequality constraints. u_j is positive if constraint *j* can be active, otherwise it is zero.

The Kuhn-Tucker conditions lor optimally are solved using a Successive Quadratic Programming technique. SQP approximates the Lagrangian function. *L,* in equation (12) at a trial point with a quadratic polynomial. The constraints at the trial point are linearly approximated. The solution to the approximate problem, with linear constraints, is easily found, using a pivoting strategy, tor example. This solution is used as a new trial point for another iteration. The method stops when the Kuhn-Tucker conditions are within a specked tolerance.

Gradients of the objective and constraint functions with respect to the process variables are needed for the solution of the Kuhn-Tucker conditions. These gradients have been obtained by perturbation of the independent variables

Formulation of Optimization Problems

Four optimization problems have been solved in this work. The problems differ in the objective functions used and the limiting value lor one of the constraints. The first problem investigates maximum withdrawal rates. Knowledge of the maximum withdrawal rate is needed to maximize caster throughput. Caster scheduling for sequences of uninterrupted casts requires knowledge of both the maximum and minimum casting rates possible and is discussed by Lally. Biegler and Henein.[6] The second and third problem both determine minimum possible casting rates, using different maximum values lor the reheat constraints. The fourth problem addresses maximizing the internal heat contem of the cast strand in preparation for direct charging of the strand to a rolling mill. The variables in the optimization lormulations have been restricted to the withdrawal rate />,. and settings lor the heat transfer coefficients used in the secondary cooling system p_{2_n}. The objective and constraint lunctions have been formulated in terms of these variables and the 3 dimensional temperature field. 7'(x.v.:). predicted Irom these variables

Objective Functions

The formulation of the objective function for the maximum casting rate is given in equation (13) The standard form of a NLP is staled '.o as to minimize the objective function, hence to find a maximum, we minimize the negative of tno objective function. No difficulties are introduced by making the objective function a simple linear function of one of the variables

$$f(p) = -p_1 \qquad (13)$$

To find Hie minimum casting rate, the following objective function is used in the optimization problem

$$f(p) = p_1 \qquad (14)$$

The formulation of the objective for the maximum heat content problem is only slightly more complicated, and is given by the integral In equation (15). This integral Is evaluated over the cross sectional area of the strand at the strand cut off point and approximates the enthalpy of the strand at this point.

$$f(p) = -\int_A T(x,y,Z_{cutoff})\,dx\,dy \qquad (15)$$

The same types of constraints were used in all the problems. The constraints were chosen to represent both strand quality and the mechanical limitations of the machine. The constraints involved bounds on the casting variables, and limits on shell thickness at the mold exit, metallurgical length, surface temperature and surface reheating. Each type of constraint is formulated separately in the following paragraphs.

Bounds

The simplest types of constraints to enforce are simple upper and tower bounds on the casting variables. Mechanical considerations such as maximum and minimum motor speeds, water availability and water pump capacity give rise to upper and tower bounds on the casting rate, and the heat transfer coefficients for each spray zone. The formulation of these constraints is shown In equation (16).

$$p_i^L \le p_i \le p_i^U \qquad i = 1,2,\ldots,n \qquad (16)$$

Shell Thickness

The shell thickness at the end of the mold is required to be greater than some fixed distance d_{shell}^{min}. This requirement is used to prevent breakout conditions to be present in the optimal solutions. This constraint is calculated by first calculating the shell thicknesses at the end of the mold along both transverse centerlines and constraining them to be greater than a fixed value, as in equation (17).

$$d_{shell}^{min} \le (\min x \text{ s.t. } T(x,0,Z_{mold}) < T_S)$$
$$d_{shell}^{min} \le (\min y \text{ s.t. } T(0,y,Z_{mold}) < T_S) \qquad (17)$$

Metallurgical Length

In each case discussed, the point of final solidification of the casting is required to be before the unbending point of the curved strand. While this may not be a requirement for all casting operations. It is a good example of the type of positional constraints that can be applied. It is stated in equation (18) in a manner similar to the shell thickness constraint.

$$z_{unbend} \ge (\min z \text{ s.t. } T(0,0,z) < T_S) \qquad (18)$$

Surface Reheating

When the strand passes from a cooling zone with a high heat transfer rate to one with a lesser heat transfer rate, the surface temperature of the strand increases. This is caused by a relaxation of the large thermal gradients created during the high heat transfer period and subsequent accumulation of enthalpy in the surface of the casting. This reheating effect must be limited, as it causes thermally induced stresses that can result in cracking. Several authors have suggested how much reheating can be tolerated.[2-9]

Originally, the amount of reheat was defined as the greatest difference between the maximum surface temperature after the mold exit, and the minimum surface temperature that occurred prior to the maximum temperature location. This definition of reheat led to a nondifferentiable function that was extremely ill behaved and worked very poorly within the optimization framework. The difficulty

occurs when the maximum and/or minimum surface temperatures abruptly change their locations from one cooling zone to another. Switching of locations leads to discontinuities in the reheat constraint gradients that are used to predict the locations of new trial points for the optimization procedure. Here, if the gradient information is valid for only a limited range (because of the discontinuities), the extrapolated predictions will be inaccurate and the optimization algorithm fails.

An alternative formulation of the reheat constraint was developed which removed these gradient discontinuities from the problem. This treatment entailed writing several reheat constraints, in the following manner. The temperature at the end of the mold and each zone end was recorded. Call these temperatures T_i^{min}. The highest temperature found In each zone was also recorded. Call these temperatures T_i^{max}. There are n_z+1 of these temperatures, where n_z is the number of spray cooling zones. The quantities shown in equation (19) were calculated, and all were required to be less than the maximum reheat allowed. By requiring all to be less than the maximum allowed, it is obvious that the greatest one will also be less than the maximum.

$$T_{reheat}^{max} \ge T_j^{max} - T_i^{min} \qquad i = 1,2,\ldots,n_z+1, \; j = i,i+1,\ldots,n_z+1 \qquad (19)$$

Surface Temperature

The surface temperature is required to be less than a given value at all points of the simulation after the mold exit. This constraint is used to ensure that the solid shell has sufficient strength to contain the molten steel in the center. A separate constraint is used for each cooling zone i in order to make the problem less ill behaved, as is done for the reheat constraints. The constraints are calculated by finding the greatest temperature in each zone outside of the mold and requiring each to be less than a fixed maximum, equation (20).

$$T_{surf}^{max} \ge \max T(0,y,z) \qquad i = 1,2,\ldots,n_z$$
$$T_{surf}^{max} \ge \max T(x,y,z) \qquad i = 1,2,\ldots,n_z \qquad (20)$$

Unbending Temperature

The surface temperature at the unbending point is also important, as a surface temperature within the ductility trough can cause cracking during straightening. The surface temperature at the unbending point is therefore required to be greater than the temperature that marks the onset of the ductility trough. This constraint is stated in equation (21).

$$T_{unbend}^{min} \ge T(X,0,Z)_{unbend}$$
$$T_{unbend}^{min} \ge T(0,Y,Z)_{unbend} \qquad (21)$$

Problem Solutions and Discussion

The geometry of the caster that was simulated in this work is summarized in Table I. The simulated caster is based on a billet caster in operation at Inland Steel. It casts 17.75cm x 17.75cm billets, using a 61cm mold. There are 4 spray cooling zones, with Independently controlled water sprays. The process parameters chosen as optimization variables were the casting rate, p_1, and four heat transfer coefficients, p_2-p_5, that represent the effect of the cooling water sprays on the solidifying strand in each of the four spray zones. The casting rate is specified in units of m/s. and the heat transfer coefficients in units of $kJ/m^2/sec/°C$. The thermal physical properties of the steel were chosen to approximate a 1010 carbon steel and are shown in Table II.

The first problem that has been solved involved determining the maximum casting rate that the caster could be operated at without violating any of the chosen constraints. The objective and

Table I: Summary of Caster Geometry

section size	17.75 x 17.75 cm
mold length	61 cm
spray zone lengths	
zone 1	9 cm
zone 2	38 cm
zone 3	183 cm
zone 4	244 cm
unbending point	20.0 m

Table II: Summary of Steel Thermal Physical Properties

solidus temperature	1477°C
liquidus temperature	1522°C
heat capacity	0.682 kJ/kg°C
heat of fusion	272 kJ/kg
thermal conductivity	
solid	0.0366 kW/m
liquid	0.2622 kW/m (includes convection)
density	
solid	7400 kg/m^3
liquid	7700 kg/m^3
emissivity	0.6

Table III: Summary of Optimization Results

		Rate Problems			Enthalpy Problem	
	Initial	Optimum			Initial	Optimum
problem number ->		1	2	3		4
objective	0.0300m/s	00326m/s	0.0252m/s	0.0201 m/s	1147*C	..96°C
p,m/s	0.0300	0.0326	0.0252	0.0201	0.0300	0.0300
p_2 kJ/m^s/'C	0.900	0.903	0.836	0642	0.900	0.A3A
p_3 *kJ/n^/sTC*	0.600	0.603	0.504	0.563	0.600	0.414
p_4 kJ/m^sTC	0.400	0.417	0.504	0.454	0.400	0295
p_5 *kj/m?/src*	0.350	0.325	0.504	0.310	0.350	0.189
iterations	*	4	28	31	-	10

constraint functions used have been developed in previous sections. A concise statement of the maximum rate problem is given In equation (22).

max $-p_1$

8.1.

$$0.01 \le p_1 \le 0.15$$
$$0.0 \le p_2 \le 2.0$$
$$0 \le p_3 \le 2.0$$
$$0 \le p_4 \le 2.0$$
$$0 \le p_5 \le 2.0$$
$$d^x_{shell} * Urn$$
$$d^y_{shell} \ge 1cm$$
$$z_{met} \le 20.0m$$
$$T^{reheat}_{ij} \le 175°C$$
$$T^{Ux}_i \le 1200°C$$
$$T^{Uy}_i \le 1200°C$$
$$T^x_{unbend} \ge 900°C$$
$$T^y_{unbend} \ge 900°C \qquad (22)$$

Tne second and third problems involved finding the minimum rate at which the caster could be operated. The formulation of the second problem is the same as the maximum rate problem except equation (14) is used as the objective function, in the third problem, the value of the maximum allowed reheat constraint has been relaxed slightly, from 175°C to 200°C. The fourth problem (the "maximum enthalpy" problem) is somewhat different. Here /?, was fixed at 0.03 m/s and equation (15) substituted lor the objective function. Reheat was limited to 175°C. The starting points and solutions to these problems are summarized in Table III, and are discussed separately in the sections which follow.

Maximum Casting Rate

Initially the casting rate was set to 0.03 m/s. Representative values for the heat transfer coefficients were also chosen. The solution of the maximum casting rate problem predicts that this rate can be raised to 0.0326 m/s without violating any of the casting constraints. The solution yields values of the operating parameters that will result in a rate increase of 8.7%. In this case it was found that the binding, or limiting, constraint was the shell thickness constraint at the mold exit - any further increases in casting rate would result in shell thicknesses that were less than the minimum. The operating parameters p_{2_5} (the heat transfer coefficients) are not involved in calculating the shell thickness at the mold exit, hence they are not uniquely determined. They could be further optimized along the lines of the maximum enthalpy problem, H this were desired.

Minimum Casting Rate

The second and third problems calculated minimum feasible casting rates under two sets ol reheat conditions. In the second problem the maximum reheat was limited to 175°C while in the third problem this constraint was relaxed to 200°C The minimum casting rate problems were started from the same initial point. Again, the results are summarized in Table III. The second problem predicts a minimum casting rate of 0.025 m/s. The binding constraints in this case are reheat constraints. At certain points along the strand surtace, the temperature has increased by the maximum amount allowed. This result provides the motivation lor relaxing the maximum reheat value in the third problem. Resolving the problem with the relaxed constraint yields a minimum casting rate of 0.020 m/s. The binding constraint in this result is also a reheat constraint, but it occurs in a different cooling zone than the previous result. It is common (and intuitive) for problems with relaxed constraints to move further in the direction of the optimum than problems with tighter constraints.

Maximum Enthalpy

During the solution of the maximum enthalpy problem, the casting rate was held fixed at 0.03 m/s. This was done to demonstrate the effect that the secondary cooling system has on the heat content of the strand. The average temperature in the strand increased from 1147 °C at the starting point, to 1196 °C at the optimal point. This is an increase in average temperature of approximately 50°C. The increase occurs without violating any of the preset casting constraints, and means that strands can easily be produced with greater heat content. This extra heat is heat that will not have to be supplied in a reheating furnace if the strand is scheduled for hot charging to the rolling mill. The binding constraint in this case is the limit on the metallurgical length. If the secondary cooling is reduced further in order to increase the average temperature, then the strand will not be fully solidified at the unbending point.

Performance

The calculations were all performed on typical engineering workstation class computing hardware*. This small, affordable, computing hardware was chosen to demonstrate that this type of approach is feasible in a process control/design scenario. The optimization problems required between 4 and 31 SOP iterations to reach the optimal points. The first problem required the least iterations, while the third problem required the most. This translates into CPU time requirements of between 62 and 610 minutes respectively. The number of iterations required is a function of the shape and smoothness of the objective and constraint functions, as well as the tolerance to which the resulting·equations must be satisfied. Each iteration required 6 model simulations (5 in the case of the maximum enthalpy problem). The number of simulations for each iteration is a result of calculating the necessary gradient information by perturbations. For each iteration a base point calculation and a perturbation of each variable must be performed. Hence, it is essential that efficient models be used to solve such optimization problems. The continuous casting model described requires approximately 2.5 CPU minutes to execute.

Figures 5 and 6 show the progress of the optimization procedure as a function of SQP iterations for the maximum enthalpy problem. Results from the other problems are similar. In these figures, the constraints have been normalized, and positive constraint values are allowable, while negative values are not. The optimizer initially makes fast progress by taking large steps, and overshoots the constraint limitations. In subsequent iterations the violated constraints are satisfied, and the optimizer fine tunes the solution.

Sensitivity

An additional result of the optimization algorithm is calculation of the shadow prices associated with the constraints. The shadow prices are measures of the sensitivity of the constraints to small changes In the operating parameters. In effect, they are the derivatives of the constraint functions with respect to the process parameters evaluated at the optimal point. They can be used to detect which constraints are most sensitive to changes In the operating parameters, and how sensitive they are. This information is useful in determining how large a safety factor should be used when calculating values for the constraints. For example, if the shell thickness constraint is extremely sensitive to changes in the casting rate, it might be desirable to set the minimum shell thickness rather conservatively to prevent small fluctuations in withdrawal speed from causing a breakout.

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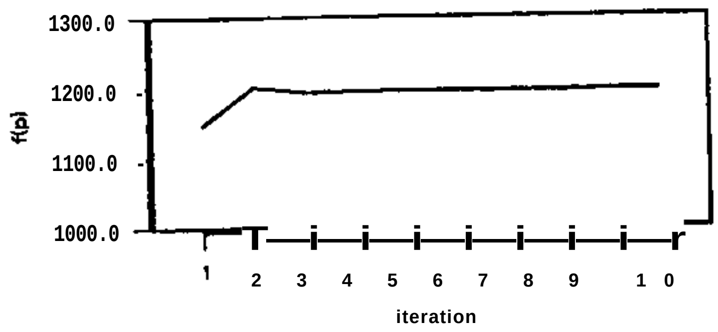

Figure 5: Objective Function Value as a Function of SOP Iterations. Problem 4

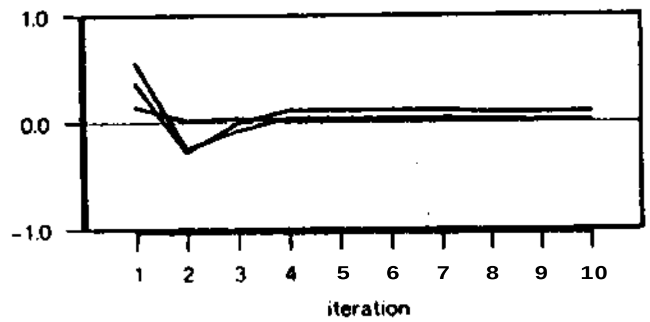

Figure 6: Representative Constraint Function Values as a Function of SOP Iterations. Problem 4

Summary

A method for determining optimal process variables from specified objective functions, subject to constraints on process operation, has been developed. The method uses a mathematical heat flow model for process simulation. Objective and constraint functions are determined from a combination of the process variables and the results of the simulation. The method has been used to solve several example problems concerning the continuous casting of steel billets in an efficient manner on engineering workstation computing hardware.

The developed method is extremely modular and flexible, hence It can easily be applied to variations of these problems, or other problems, simply by changing the definitions of the objective and constraint functions. Other casters can be simulated by changing modelling parameters. If the process to be optimized is dependent on phenomena that are not fully described by a heat flow model, other models (such as stress/strain models) can easily be substituted within this framework. The method does not require a feasible starting point (a point where all of the constraints are satisfied) for the optimization, hence the final predicted optima are insensitive to the initial estimate of the process paramters.

This work is currently being extended to solve problems involving other casters, including slab casters. This will further demonstrate the flexibility and usefulness of the approach. Extensions to other casting operations, with different constraint sets, are also being developed. The method can be used for solving design problems by using an objective that describes the desired design criteria, and adding caster design variables to the optimization problem. It may also be possible to use this type ol optimization technique as part of a real time control system for continuous casting and other plant operations. This type of application will require accurate, high speed models to perform the simulations, as well as alternate algorithms to determine the required gradient information. Solution of the real time control problem will likely benefit from the higher performance computing machinery that te constantly coming available. Also, parallel processing can be applied to the perturbation approach for determining the gradients, as the simulations for each parameter perturbation are independent of each other and can be calculated simultaneously.

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References

1. Continuously Cast Steel, 1977-1986, Iron and Steelmaker. Data originally from the International Iron and Steel Institute

2. M. Larrecq, J. P. Birat, C. Saguez and J. Henry. "Optimization of Casting and Cooling Conditions on Steel Continuous Casters - Implementation of Optimal Strategies on Slab and Bloom Casters". *Application of Mathematical and Physical Models in the Iron and Steel Industry,* AIME, March, 1982, pp. 273.

3. S. P. Han, "A Globally Convergent Method for Nonlinear Programming", *Journal of Optimization Theory and Applications,* July 1977. pp. 297.

4. M. J. D. Powell. "A Fast Algorithm for Nonlinearly Constrained Optimization Problems", *Dundee Conference on Numerical Analysis,* 1977.

5. W. Hock and K. Schittkowski, "Comparative Performance Evaluation for Nonlinear Programming Codes on Hand-Selected and Real Life Test Problems", *Computing,* 1983, pp 335.

6. L T. Blegler, "On the Simultaneous Solution and Optimization of Large-Scale Engineering Systems", *Proceedings ofCEF '87,* 1987, pp. 15.

7. L. T. Biegler and J. E. Cuthrell, "Improved Infeasible Path Optimization of Sequential Modular Simulators - II: The Optimization Algorithm", *Computing and Chemical Engineering,* Vol. 9, No. 3, September 1985. pp. 257.

8. L. Holappa. E. laitinen, S. Louhenkilpi and P. Neittaanmaki, "Optimization of the Secondary Cooling in the Continuous Casting of Steel Billets". *Proceedings of the 24th Annual Conference of Metallurgists,* Vancouver. British Columbia, August. 1985.

9. J. K. Brimacombe. "Design of Continuous Casting Machines Based on a Heat Flow Analysis: State-of-the-Art Review", *Canadian Metallurgical Quarterly,* April 1976. pp. 163.

10. E. A. Mizikar, "Mathematical Heat Transfer Model for Solidification of Continuously Cast Steel Slabs". *Transactions of the Metallurgical Society of AIME,* November 1967. pp. 1747.

11. A. Perkins and W. R. Irving. "Two-dimensional Heat Transfer Model for Continuous Casting". *2nd Process Technology Conference,* 1981, pp. 187.

12. A. W. D. Hills, "Simplified Theoretical Treatment for the Transfer of Heat in Continuous-Casting Machine Molds". *Journal of The Iron and Steel Institute,* January 1965, pp. 18.

13. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids.* University Press, Oxford, 1947.

14. E. A. Mizikar, "Spray Cooling Investigation lor Continuous Casting of Billets and Blooms", *Iron and Steel Engineer,* June 1970, pp. 53.

15. H. Muller and R. Jeschar. "Investigation of the Heat Transfer in a Simulated Secondary Cooling Zone in the Continuous Casting Process". *Arch Eisenhuttenwes.,* 1973. pp. 589.

16. B. Lally, L. Biegler and H. Henein, "A Model for Sequencing a Continuous Casting Operation to Minimize Costs", *Iron and Steelmaker,* October 1987, accepted for publication.