LOCALLY TESTABLE EVENTS AND SEMIGROUPS

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ABSTRACT

It is an open problem, posed by Papert and McNaughton (1970), to find an algorithm for determining whether a regular event is locally testable. In this paper, we present a partial solution. First, we characterize locally testable events algebraically in terms of their semigroups. Then we find an effective necessary condition for local testability and prove that it is also sufficient for a large class of finite semigroups including all finite regular semigroups. We conjecture that the condition is necessary and sufficient for all regular events.

List of Symbols

Σ^* - set of all words over the alphabet Σ

Σ^+ - set of all non-null words over Σ

S^1 - semigroup S with an identity element adjoined

Ⓛ, Ⓡ, Ⓙ, Ⓗ (the circle denotes script letters) - the Green relations

$\in$ - set membership

$s*t$ - multiplication in J^o

LOCALLY TESTABLE EVENTS AND SEMIGROUPS

1. INTRODUCTION

A regular event E is locally testable iff for some k, membership in E is determined by a testing of the first k letters of the word, the last k letters of the word and all other solid subwords of the word of length k, but without regard to the order or number of these subwords. Let LT denote the class of locally testable events. Locally testable events can be thought of as a generalization of definite events and are easily seen to be non-counting and thus by the results of Schützenberger and Papert-McNaughton, star-free. Their importance stems from the fact that any regular event is a homomorphic image of a locally testable event. Furthermore, by a result of Chomsky and Schützenberger any context-free language is a homomorphic image of the intersection of a Dyck language and a locally testable event.

Given k, it is easy to check whether a given regular event is k-testable. Papert and McNaughton (1970) have suggested the much harder problem of finding an algorithm for deciding whether a given regular event is locally testable, for some k. This paper provides a partial solution to this problem. Our strategy is to replace the infinite event E by its finite semigroup S(E) and using algebraic techniques to obtain conditions that can be checked in a finite number of steps. We define the notion of a locally testable semigroup and prove that a regular event E is locally testable if and only if S(E) is locally testable. Using this result we derive an effective necessary condition for local testability (theorem 1). We conjecture that this condition is also sufficient. We were unable to prove this for all regular events but only for a subclass (theorem 2 and the following remark). Finally, we state without proof a characterization of the locally testable semigroups in terms of wreath product decompositions.

We assume that the reader is familiar with regular events and with elementary semigroup theory. The proofs of the theorems require results from the semi-local theory of semigroups (Clifford and Preston (1962), chapter 3 or Krohn, Rhodes and Tilson (1968)) which are reviewed briefly.

2. PRELIMINARIES

In this section we recall some basic definitions and establish notation. We abbreviate "if and only if" as "iff."

Let Σ be a finite non-empty alphabet and Σ^+ the set of all non-null words over Σ. For technical reasons we will consider only non-null events, i.e., subsets of Σ^+. This involves no essential loss of generality. Throughout this paper, E will denote a non-null regular event. E is <u>star-free</u> iff E belongs to the smallest family of events containing the singletons $\{\sigma\}$, $\sigma \in \Sigma$ and closed under the boolean operations.

A <u>machine</u> or <u>input-output map</u> is a map $f: \Sigma^+ \to Y$, where Y is a finite, non-empty set. The <u>semigroup</u> f^S of a machine f is the quotient of the free semigroup Σ^+ modulo the equivalence relation $\equiv$ defined by $w_1 \equiv w_2$ iff for all $w,w' \in \Sigma^*$, $f(ww_1w') = f(ww_2w')$. For each event $E \subseteq \Sigma^+$ the characteristic function χ_E of E is a machine $\chi_E: \Sigma^+ \to \{0,1\}$. We call χ_E^S the <u>semigroup</u> of E and denote it by S(E). McNaughton and Papert (1968) is an excellent reference on regular events and their semigroups.

Finally we need some basic definitions and results about semigroups. A <u>monoid</u> is a semigroup with an identity element. Let S be a semigroup. All semigroups considered are finite unless it is explicitly stated otherwise. S^1 will denote the monoid obtained from S by adjoining an identity element if S does not have an identity to begin with. An element $e \in S$ is called <u>idempotent</u> iff $e^2 = e$. A <u>band</u> is a semigroup all of whose elements are idempotent. The Green equivalence relations on S are defined as follows.

$s \circledR t$ iff $sS^1 = tS^1$ i.e., there are elements $u,v \in S^1$ such that $su = t$ and $tv = s$

$s \, Ⓛ \, t$ iff $S^1s = S^1t$

$s \, Ⓙ \, t$ iff $S^1sS^1 = S^1tS^1$.

$s \, Ⓗ \, t$ iff $s \circledR t$ and $s \, Ⓛ \, t$

The R-relation has a more intuitive definition in terms of the Graph G of S (see McNaughton and Papert (1968), pp. 304-307) s Ⓡ t iff there are (directed) paths in G from s to t and from t to s.

A semigroup is <u>combinatorial</u> or <u>group-free</u> iff all its subgroups are trivial. As is well-known, S is combinatorial iff there is a positive integer $p = p(S)$ such that $s^p = s^{p+1}$ for all $s \in S$ iff no two distinct elements of S are Ⓗ-related.

Let S be a semigroup. An <u>ideal</u> is a non-empty set $I \subseteq S$ such that for all $x \in I$, $s \in S$, $xs, sx \in I$. If S has a zero element 0 such that for all $s \in S$, $s0 = 0s = 0$, then $\{0\}$ is an ideal. S is <u>simple</u> iff it has no proper ideals. S is 0-<u>simple</u> iff it has no proper ideals except $\{0\}$. A well-known theorem of Rees states that for any 0-simple combinatorial semigroup there are positive integers m,n and an n×m matrix C over $\{0,1\}$ such that S is isomorphic to a set $(A \times B) \cup \{0\}$ $A = \{1,2,\ldots,m\}$, $B = \{1,2,\ldots,n\}$ with the multiplication

$$(i_1,j_1)(i_2,j_2) = \begin{cases} (i_1,j_2) & \text{if } C_{j_1 i_2} = 1 \\ 0 & \text{if } C_{j_1 i_2} = 0 \end{cases}$$

Equivalently, the element with "coordinates" (i,j) can be represented by a m×n matrix M_{ij} having 1 at the ij-th entry and all other entries being zero. The product of $M_{i_1 j_1}$ and $M_{i_2 j_2}$ is the ordinary matrix product $M_{i_1 j_1} C \, M_{i_2 j_2}$.

Let α be one of the Green relations Ⓛ, Ⓡ, Ⓙ, Ⓗ. The equivalence classes modulo α are called α-classes. We can define a partial ordering $\leq$

on the $\textcircled{J}$-classes of S by $J_1 \leq J_2$ iff $S^1 x S^1 \subseteq S^1 y S^1$ where $x \in J_1$, $y \in J_2$. A $\textcircled{J}$-class J is minimal (0-minimal, respectively) iff $S^1 x S^1$ is minimal (0-minimal, respectively) for $x \in J$.

If S is simple and combinatorial, then S is isomorphic to $A \times B$ with the multiplication $(i_1, j_1)(i_2, j_2) = (i_1, j_2)$.

Let I be an ideal of S. S is a *nilpotent extension* of I iff there is a positive integer n (the *degree of nilpotence*) such that for all $s_1, s_2, \ldots, s_n \in S$, the product $s_1 \ldots s_n$ belongs to I. S is *nilpotent* iff S has a zero element 0 and is a nilpotent extension of $\{0\}$.

An element $s \in S$ is *regular* iff it has a "pseudo-inverse" $t \in S$ such that sts = t. A $\textcircled{J}$-class is *regular* iff all its elements are regular; otherwise it is called *null*. A semigroup is *regular* iff all its $\textcircled{J}$-classes (or equivalently all its elements) are regular.

3. NECESSARY AND SUFFICIENT CONDITIONS FOR LOCAL TESTABILITY

We begin with a precise definition of local testability.

DEFINITION (Papert-McNaughton 1970)

Let k be a positive integer. For $w \in \Sigma^+$ of length $\geq k+2$, let $L_k(w)$, $R_k(w)$ and $I_k(w)$ be, respectively, the prefix of w of length k, the suffix w of length k and the set of interior solid subwords of w of length k.

Let E be a regular event over Σ. E is k-testable iff for all $w,w' \in \Sigma^+$ of length $\geq k+2$, if $L_k(w) = L_k(w')$, $R_k(w) = R_k(w')$ and $I_k(w) = I_k(w')$, then $w \in E \Leftrightarrow w' \in E$. E is locally testable iff it is k-testable, for some k. Let LT denote the class of all locally testable events over Σ.

Next, we define the algebraic counterpart of the preceding definition.

DEFINITION

Let k be a positive integer. A finitely generated semigroup is k-testable iff for any products $s = s_1s_2\ldots s_m$, $t = t_1t_2\ldots t_n$ in S, $m,n \geq k+2$, if

$$(s_1,s_2,\ldots,s_k) = (t_1,t_2,\ldots,t_k),$$

$$(s_{m-k+1},\ldots s_m) = (t_{n-k+1},\ldots,t_n) \qquad \text{and}$$

$$\{(s_i,s_{i+1},\ldots,s_{i+k-1}): i = 2,3,\ldots,m-k\} = \{(t_j,t_{j+1},\ldots,t_{j+k-1}): j = 2,3,\ldots,n-k\}$$

(we say that $(s_1,\ldots,s_m)$, $(t_1,\ldots,t_n)$ have the same k-test vectors), then $s = t$.

S is local or locally testable iff it is k-testable for some k. Let LTS denote the class of locally testable semigroups.

LEMMA 1. *A k-testable semigroup is finite and combinatorial with* $p(S) \leq k+2$. *If, furthermore,* S *is regular, then* $p(S) \leq k$.

PROOF. Let $s_1,\ldots,s_m$ be a set of generators for S and assume S is k-testable. Then every formal word of length $> m+2$ in the free semigroup over $\{s_1,\ldots s_m\}$ multiplies to the same element as some word of length $m+2$. Hence there are only finitely many elements in S.

Next, by k-testability, $s^{k+2} = s^{k+3}$, thus S is combinatorial with $p(S) \leq k+2$.

Finally, assume that S is regular. Then for $s \in S$, there are idempotents $e_1, e_2 \in S$ such that $e_1 s^k e_2 = s^k$ (see Rhodes and Tilson (1968) fact (2.22)). Furthermore, by k-testability, $e_1 s^k e_2 = e_1 s^{k+1} e_2$, thus $s^k = e_1 s^{k+1} e_2$ and so s^k belongs to the two-sided ideal generated by s^{k+1}. Since s^{k+1} belongs to the ideal generated by s^k, s^k and s^{k+1} generate the same two-sided ideal and are thus $\mathcal{J}$-equivalent. But then by standard results (see Rhodes and Tilson (1968) section 1) $s^{k+1} = s^k$. Since s is arbitrary, it follows that $p(S) \leq k$ and the lemma is proved.

LEMMA 2. *The class of locally testable semigroups is closed under subsemigroups, homomorphic images and finite direct products* (LTS *is thus a "pseudo variety" of semigroups*).

PROOF. Closure under subsemigroups and finite direct products is immediate. Assume S is k-testable and let h be an onto homomorphism $S \to V$. Let $(s_1,\ldots,x_m)$, $(y_1,\ldots,y_n) \in V^+$, $m,n \geq k+2$ have the same k-test vectors. For each element $x_i \in V$ pick an arbitrary but fixed $s_i \in S$ such that $h(s_i) = x_i$

and similarly for y_j pick an arbitrary but fixed t_j usch that $h(t_j) = y_j$. Then it is easy to see that $(s_1,\ldots,s_m)$ and $(t_1,\ldots,t_n)$ have the same k-test vectors. Thus $s = s_1 \ldots s_m = t_1 \ldots t_n = t$ and consequently $x_1 \ldots x_m = h(s) = h(t) = y_1 \ldots y_n$, so V is k-testable.

LEMMA 3. A simple or 0-simple combinatorial semigroup S is k-testable for all $k > 1$.

PROOF. Observe that in a 0-simple semigroup a product $s_1 \ldots s_m$ equals 0 iff for some i, $s_i s_{i+1} = 0$. Furthermore, if a product $s = s_1 \ldots s_m$ is not zero, then the A-coordinate of s equals the A-coordinate of s_1 and the B-coordinate of s equals the B-coordinate of s_m. Since S is combinatorial, every element is determined by its A and B coordinates.

LEMMA 4. A nilpotent extension of degree n of a k-testable ideal I $n \leq k+2$ is k-testable.

PROOF. Any product $s_1 \ldots s_m$, $m \geq k+2$ lies in I which is k-testable. Q.E.D.

PROPOSITION 1. Let E be a non-null regular event. Then E is locally testable iff S(E) is locally testable.

PROOF. Assume S(E) is k-testable and let $w = (\sigma_1,\sigma_2,\ldots,\sigma_m)$, $w' = (\tau_1,\tau_2,\ldots,\tau_n) \in \Sigma^+$, $m,n \geq k+2$. Suppose $L_k(w) = L_k(w')$, $R_k(w) = R_k(w')$ and $I_k(w) = I_k(w')$, then the words

$$([\sigma_1],[\sigma_2],\ldots,[\sigma_m]) \text{ and } ([\tau_1],[\tau_2],\ldots,[\tau_n]$$

of $S(E)^+$ will have the same k-test vectors. Thus $[\sigma_1\ldots\sigma_m] = [\tau_1\ldots\tau_n]$

and so $w \in E \Leftrightarrow w' \in E$ and E is k-testable. Conversely, assume E is k-testable. Let $(s_1,\ldots s_n)$, $(t_1,\ldots,t_m) \in \Sigma^+$, $m,n \geq k+2$ have the same k-test vectors. Pick an arbitrary but fixed representative in Σ^* for each equivalence class in S(E). Consider the two words $w, w' \in \Sigma^+$ obtained from $(s_1,\ldots,s_m)$ and $(t_1,\ldots,t_n)$ respectively by replacing each s_i and t_j by their representatives. It is easy to see that w and w' have the same k-test vectors for any $x,y \in \Sigma^*$, $L_k(xwy) = L_k(xw'y)$, $R_k(xwy) = R_k(xw'y)$ and $I_k(xwy) = I_k(xw'y)$. Thus by k-testability $xwy \in E \Leftrightarrow xw'y \in E$, i.e., $[w] = [w']$. But w represents $s_1 \ldots s_m$ and w' represents $t_1 \ldots t_n$. Hence $s_1 \ldots s_m = t_1 \ldots t_n$. Q.E.D.

Next, we derive an effective necessary condition for local testability and prove that it is also sufficient for all events whose semigroups are regular.

First, we recall a few more standard definitions. A matrix M over $\{0,1\}$ is row-monomial (column-monomial, respectively) iff every row (respectively column) of M contains at most one non-zero entry.

A combinatorial semigroup S is a right (respectively left)-mapping semigroup iff S has a minimal or 0-minimal ideal $I = J^o$ and S is isomorphic to a semigroup or right (respectively left) translations on J^o. I is called the distinguished ideal and J the distinguished Ⓙ-class of S. A right (respectively left)-mapping semigroup is isomorphic to a semigroup of row (respectively column)-monomial matrices in a natural way (Rhodes and Tilson (1968) (2.12)-(2.15), Krohn, Rhodes and Tilson (1968)(2.5), (2.14) (2.15)).

If J is a (J)-class, J^o is the set $J \cup \{0\}$ with multiplication

$$s*t = \begin{cases} st \text{ if } st \in J \\ 0 \quad \text{otherwise.} \end{cases}$$

Recall that a band is a semigroup all of whose elements are idempotent.

THEOREM 1. _Let S be a local semigroup. Then for every idempotent_ $e \in S$, _eSe is a commutative band. Furthermore, no two distinct elements of Se are (R)-related and no two distinct elements of eS are (L)-related._

PROOF. Let S be k-testable.

Let exe, eye $\in$ eSe. The products $e^k xe^k ye^k$ and $e^k ye^k xe^k$ have the same k-test vectors. Thus $(exe)(eye) = e^k xe^k ye^k = e^k ye^k e^k = (eye)(exe)$, and eSe is commutative.

Similarly, the products $e^k xe^k xe^k$ and $ee^k xe^k e$ have the same k-test vectors. Thus $(exe)^2 = exexe = e^k xe^k xe^k = ee^k xe^k e = exe$ and eSe is a band.

The second statement of the theorem will follow from the next lemma.

LEMMA 5. _Let S be a finite semigroup. Assume that for every idempotent_ $e \in S$, _eSe is a commutative band. Then for every idempotent_ $e \in S$, _no two distinct elements of Se are (R)-related and no two distinct elements of eS are (L)-related._

PROOF. If S contains a non-trivial subgroup G then the identity element e of G is an idempotent and $eGe = G \subseteq eSe$, contradicting the hypothesis that eSe is a band. Thus S must be combinatorial.

Let xe,ye $\in$ Se and assume xe Ⓡ ye. Since the Ⓡ-relation is a left congruence, xe Ⓡ ye implies exe Ⓡ eye. Furthermore, by hypothesis, eSe is commutative and hence no two distinct elements of eSe are Ⓡ-related. Thus exe = eye.

Assume ye is an idempotent. Then y(eye) = ye since eye = e(ye) it follows that eye Ⓛ ye. Then by the dual of lemma (3.15)(i) of Clifford and Preston (1962), exe Ⓛ xe. Thus ye Ⓛ eye = exe Ⓛ xe. Consequently xe Ⓗ ye. Since S is combinatorial, it follows that xe = ye. By symmetry the same argument applies if xe is an idempotent. Thus no Ⓡ-class of Se of cardinality ≥ 2 can contain an idempotent. Hence if xe Ⓡ ye and xe $\neq$ ye, then the Ⓙ-class J containing xe and ye must be null (Rhodes and Tilson (1968) (1.13) and (1.18)(b)).

For every null Ⓙ-class in a finite semigroup there is a unique $\leq$ - minimal regular Ⓙ-class J_1 such that for all a $\in J_1$ there is some b $\in$ J such that ba $\in$ J (see Rhodes and Tilson (1971)(2.7) and (2.8)). Since (xe)e = xe $\in$ J, e $\in J_1$. Since xe Ⓡ ye, then is u $\in$ Se such that (xe)u = ye. Thus u $\in J_1$. Now, J_1 is regular. Thus by the argument in the preceding paragraph, no two distinct elements of J_1 are Ⓡ-related. Consequently J consists of a single Ⓛ-class and so u Ⓛ e. Furthermore, eu $\in J_1$ since (xe)(eu) = (xe)u = ye $\in$ J. Thus eu = e. But (xe)(eu) = ye while (xe)e = x, contradiction. A dual argument proves that no two distinct elements of eS are Ⓛ-related.

REMARK. It can be shown that the converse of lemma 5 holds for all finite regular semigroups. However, it is not true in general as can be shown by easy counter examples.

Observe that the conditions of theorem 1 are effective, since they can be determined from the multiplication table of S.

LEMMA 6. *Let S be a right (respectively, left)-mapping semigroup such that for all idempotents* $e \in S$, *eSe is a commutative band. Then S is locally testable.*

PROOF. Let S be right-mapping. Let I be the distinguished ideal and J the distinguished J-class of S. The hypothesis implies that S is combinatorial. Thus S is isomorphic to a semigroup of row-monomial matrices over $\{0,1\}$.

Let e be an idempotent of S. By lemma 5, no two distinct elements of Je are Ⓡ-related. It follows that e, as a row-monomial matrix, must have rank 1. But the row-monomial matrices in J have rank 1 and J is the unique minimal or 0-minimal J-class of S. It follows easily that $e \in J$. So all the idempotents of S are in I. It follows that S is a nilpotent extension of I. By lemma 3 I is k-testable for all k. Thus by lemma 4, S is locally testable. The result for a left-mapping semigroup follows by a dual argument. Q.E.D.

THEOREM 2. *Let S be a regular semigroup such that for every idempotent* $e \in S$, *eSe is a commutative band. Then S is locally testable.*

PROOF. By proposition (2.6)(b) of Krohn, Rhodes and Tilson (1968) (essentially the Schützenberger-Preston representation), S is isomorphic to a subsemigroup of a direct product of right-mapping and left-mapping semigroups. The theorem follows by lemmas 2 and 6.

REMARK. The proof of theorem 2 goes through for finite semigroups S such that for every null Ⓙ-class J of S, $N_J(S)$ (see definition (2.5) of Krohn, Rhodes and Tilson (1968)) is locally testable. For example, if for all null Ⓙ-classes J of S, $N_J(S)$ is nilpotent and if for all idempotents $e \in S$, eSe is a commutative band, then S is locally testable.

Finally, we mention without proof a characterization of the locally testable semigroups in terms of wreath product decompositions.

An element $x \in S$ is a *right zero* iff for all $s \in S$, $sx = x$. S is a right-zero semigroup iff every element of S is a right-zero.

THEOREM 3. *Let* S *be a finite semigroup. Then* S *is locally testable iff* S *is a homomorphic image of a subsemigroup of a wreath product* BwN, *where* N *is a nilpotent extension of a right-zero kernel and* B *is a commutative band.*

The proof will appear in Zalcstein (1971).

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13. ABSTRACT

It is an open problem, posed by Papert and McNaughton (1970), to find an algorithm for determining whether a regular event is locally testable. In this paper, we present a partial solution. First, we characterize locally testable events algebraically in terms of their semigroups. Then we find an effective necessary condition for local testability and prove that it is also sufficient for a large class of finite semigroups including all finite regular semigroups. We conjecture that the condition is necessary and sufficient for all regular events.

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ERRATA and ADDENDA

LOCALLY TESTABLE EVENTS AND SEMIGROUPS

Page 7, lines 4 and 5:

Correct to read: Then there is $p \geq m^k$ such that every formal word of length $> p$ in the free semigroup over $\{s_1, \ldots, s_m\}$ multiplies to the same element as some word of length p. Hence there are

Page 8, line 13:

Delete $n \leq k+2$

Correct to: is $n(k+2)$-testable.

PROOF. Any product $s_1 \cdots s_m$, $m \geq n(k+2)$ equals a product $x_1 \cdots x_p$, $m \leq np$, where $x_1 = s_1 \cdots s_n, \ldots, x_i = s_{(i-1)n} \cdots s_{in}, \ldots, x_p = s_{(p-1)n} \cdots s_m$. Since S is a nilpotent extention of I of degree n, $x_i \in I$, $1 \leq i \leq p$. The lemma follows easily by the k-testability of I.

Page 12, line 14:

Delete "It follows easily that"

Add: We claim that any row-monomial matrix in S of rank 1 belongs to J. To prove the claim, observe that a row-monomial matrix of 0's and 1's of rank 1 is determined by a pair (x,y) where x is a set of row indices (the indices of those rows that contain a non-zero entry) and y is a column index (the index of the column that contains all non-zero entries). Furthermore, it is easy to see that (x,y) can serve as Rees coordinates. Precisely, if T is the subsemigroup of S consisting of all matrices of rank ≤ 1, then T is isomorphic to a Rees matrix semigroup over $\{0,1\}$ with A = set of x-coordinates of elements of T, B = set of y-coordinates of elements of T, and with structure matrix C, where

$$C_{yx} = \begin{cases} 1 & \text{if } x \in y \\ 0 & \text{otherwise} \end{cases}$$

T is thus simple or 0-simple and it follows that T = J or T = J∪{0}, and the claim is proved. Thus,

Page 13, line 2:

Correct to: semigroups S_i, satisfying the condition that eS_ie is a commutative band. The theorem follows by lemmas 2 and 6.

Page 14, line 5:

Correct to: Semigroups," vol. 1, ...