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Reliability Models for Multiprocessor Systems With and Without Periodic Maintenance

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Abstract

Multiprocessor systems, although designed for speed and processing power, lend inherently to redundancy. Appropriately designed distributed intelligence systems that utilize system reconfiguration and graceful degradation can be substantially more reliable than uniprocessor systems. Reliability models for two multiprocessor systems, C.mmp and Cm*, are presented and compared to a single LSI-11 processor.

With the exception of spacebourne systems, most systems may be subjected to tests to ensure proper functioning. When performed regularly, these integrity checks enhance confidence in the system, and its expected mean time to failure. Effect of such periodic maintenance is modeled. The expected life is seen to depend strongly on the efficiency of the tests. The improvement in expected life, however, is observed to be limited by non-redundant parts of a system. Under periodic maintenance, Cm* system offers greater life than C.mmp for tasks allowing considerable redundancy.

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1. Reliability Considerations

1.1. Introduction

Interest in highly reliable systems has increased dramatically in the last five years. This can be attributed to at least three trends :

- Digital systems have bec[illegible]creasingly applied in areas where a failure can lead to catastrop[illegible] consequences.
- System complexity has increased as system performance and capability have increased. Increased complexity in nonredundant system means a less reliable system.
- Decreased hardware cost has expanded the application areas of digital systems. These new application areas require increased unattended system reliability since the users are less sophisticated and the cost of repair personnel easily dominates the system cost.

In order to evaluate and compare systems an accurate reliability model is essential.

It is a common practice in reliability modeling to divide a system under investigation into a number of subsystems or modules. A judicious partitioning leads to a set of modules that are statistically mutually indenpedent. The reliability of a nonredundant system is then merely the product of reliabilities of various modules.

The problem that still remains is that of finding the reliability of individual modules. Historically it was an accepted practice to assume the statistical independence at the gate level, and raise the gate reliability to the number of gates in the module. The present day technology of large scale integration renders this technique obsolete. Although reliability is still a function of the complexity, the complexity may no longer be treated as a simple function of the number of gates. The following subsection outlines the currently acceptable approach. Section 2 presents a comparison of two multiprocessor systems : C.mmp and Cm*. Finally, section 3 discusses the relaibility of system employing periodic maintenance.

1.2. Parts Count Model

Before presenting the parts count model, let us review its basic assumptions. It will be assumed the system is constructed of printed circuit boards. The PC boards hold IC chips that are assumed to be statistically independent. It is further assumed that the reliability of a single module is exponentially distributed or that the failures of a single chip follow the Poisson distribution. In other words,

$$\text{Probability of k failures in time interval } (0,t) = e^{-\lambda t} (\lambda t)^k / (k!) \tag{1.1}$$

$$\begin{aligned} \text{Reliability} &= \text{probability of no failures in } (0,t) \\ &= e^{-\lambda t} \end{aligned} \tag{1.2}$$

With these assumptions, if a system does not contain any redundancy (i.e. every subsystem must function properly for the system to work), the system reliability is also exponential in nature. Furthermore, the failure rate of the system is the sum of failure rates of individual modules.

To estimate the failures rates of chips we will use the data published in the Military Standardization Handbook 217-B [Mil74]. The handbook suggests the following model for the failure rate of a single chip.

$$\lambda = \pi_1 \pi_2 (\pi_T d_1 G^{\alpha} + \pi_E d_2 G^{\beta}) \tag{1.3}$$

where d_1, d_2, α, β and π's are various constants,
G is the number of gates in the chip.

The constants, π_L, π_E, π_T, and π_Q, depend on, respectively, learning (experience in using the particular chip), environment (whether the system is ground based, fixed, spaceborne, etc.), expected junction temperature, and quality control (how rigorously has the chip been subjected to tests and burn-in). Table 1.1 shows the failure rates for chips with various number of gates. The system is assumed to be fixed, ground based; the quality control factor is assumed to be 10 (which is in between the MIL-STD factor of 1.0 and the factor to be used for minimal industrial quality control, 150), and the junction temperature is assumed to be 50°C. The other constants, d_1, d_2, α and β, are fixed and equal to 0.00129, 0.00389, ..67 and 0.35 respectively.

Table 1.1

Gates	Failures in 10^6 hours	Gates	Failures in 10^6 hours
1	0.04342	9	0.10559
2	0.05711	10	0.11037
3	0.06721	11	0.11490
4	0.07553	12	0.11920
5	0.08275	13	0.12332
6	0.08920	14	0.12728
7	0.09508	15	0.13108
8	0.10052	16	0.13475

The estimation of failure rate for a module is best described by an example in the next section.

An interesting phenomenon caa be observed if we plot the failure rate per gate versus the number of gates per ship. As semiconductor components get larger they also become more reliable per function, up to a point of diminishing returns. Figure 1.1 depicts the failure rate per million hours per gate as a function of the number of gates on a chip. The curves marked 65a, 65b were derived from data in [Mil65] circa 1965 while the curve marked 74 was derived from [Mil74] circa 1974. Two trends can be noted. In the 1974 data, gate functions exhibit as much as an order of magnitude decrease in failure rate up to a density of approximately 100 gates. Prior to the 100 gate minimum, packaging and lead failures dominate. Beyond that, gate failure rate increases due to immaturity of the fabrication process.

The 1965 data was incomplete due to the newness of integrated circuits. One study (curve 65a) showed that a failure rate of 0.4 per 10^6 hours was a good approximation for state of the art ICs at that time (one to four gates per IC). Another study examined small functional units composed of discrete components and ICs. Various ten element units showed failure rates of 0.83 to 1.8 per 10^6 hours (curve 65b). While the data is incomplete it is reasonable to assume that gate functions become more reliable with time. In a like manner, the point of minimum failure rate per gate can be assumed to be moving to the right as technology matures. Thus constructing systems from larger components can lead to significantly more reliable systems.

1.3. Example

As an example let us consider the Processor Interface module in C.mmp. Figure 1.2 shows the chip lay-out and list of parts for the Processor Interface. From this data we form Table 1.2.

Table 1.2 Calculation of λ for Processor Interface

No.	Id	Gates	λ
1	74S140	4	0.07553
1	7440	4	0.07553
1	7404	6	0.08920
2	74S138	16	0.13475
6	7438	4	0.07553
13	74S74	10	0.11037

From individual λs in the table, we estimate λ for the Processor Interface to be

$$\lambda = 0.07553 + 0.07553 + 0.08920 + 2(0.13475) + 6(0.07553) + 13(0.11037)$$
$$= 2.39775 \text{ failures}/10^6 \text{ hours.}$$

Thus we arrive at the failure rate for the Processor Interface board. Similar calculations have been carried out for all subsystems of C.mmp to yield the overall system failure rate.

The following section will compare reliabilities of both C.mmp and Cm* (Computer Modules) in both non-redundant and redundant configurations.

2. Reliability Comparison of C.mmp and Cm*

Parts count reliability models were developed for both of the in-house systems at CMU, the multiminiprocessor C.mmp and the computer modules system Cm*. As the conceptual diagrams of Figure 2.1 depict, both are general purpose multiprocessor systems. C.mmp is a general purpose system with a fixed architecture. Up to 16 PDP-11 processors (Pc) can communicate with up to 16 shared memory ports (Mp) thorugh a crosspoint switch (Smp) (Figure 2.1a). Cm*, on the other hand, has a flexible architecture that may be so modified as to afford optimal performance for a given application. Cm* is a modular, multi-micro-processor system based on the LSI-11 processors as depicted in Figure 2.1b. Each Computer Module (Cm) is connected via an interface (S.local) to an intelligent cluster controller, K.map. The clusters can be interconnected via L.inc's. Each Cm can share memory with any other Cm in the network through routing tables in the K.map.

As multiprocessor systems, both C.mmp and Cm* offer potential processing power well beyond that of a single processor. C.mmp has an upper limit of 16 processors (since the existing switch has only 16 ports for processors). In concept, the Cm* architecture is arbitrarily extendible; the only limiting factors are the cost and fundamental limits of the programmed algorithms. When all of the processing power is not required or graceful degradation of processing is tolerable, it is possible to conceive of either C.mmp or Cm* as a potentially redundant architecture. If a task requires the minimal processing capability of, for example, only four processors, then we may view the other processors as stand-by spares or expendables. Assuming that we can detect and locate a faulty component (processor, memory, switch), and the malfunction was not an irrecoverable one, we can then logically replace a faulty component with a stand-by spare or simply exclude it from the system. The structures are thus fault-tolerant and have greater reliability.

To arrive at the reliabilities of multiprocessor fault-tolerant systems, we need to use two levels of modeling. We apply the parts count reliability model to estimate the failure rates of individual modules. Then using the reliabilities of these non-redundant modules, we model the fault-tolerant system to arrive at a system reliability.

2.1. Parts Count Reliability Model

The failure rates for standard IC chips are found in the Military Standardization Handbook (MIL-STD-HDBK-217B). Assuming exponential distribution for reliability and mutual statistical independence, the failure rate of each module is estimated as the sum of the failure rates of its various components. Since the handbook also predicts the failure rates for such components as resistors or printed boards, completeness of the model is assured. The following are the failure rates for various modules of C.mmp and Cm* systems using the parts count reliability model.

	Component	Failure rate (failures per 10^6 hrs.)
C.mmp	PDP-11/40	57.496
	Processor associated circuitry (RELOC, processor interface)	11.414
	Memory box (16K words; core)	54.225
	Memory associated circuitry (Priority decode, etc.) per port	7.14
	Switch	202.403
Cm*	LSI-11 processor	109.0
	Memory (12K words; semiconductor)	203.343
	K.map	178.414

2.2. Redundancy Model for C.mmp

In the absence of data on fault detection/propagation and module replacement capabilities in multiprocessor systems, we use the following simplistic model (giving us the upper bound on potential reliability). If there are N identical components with the reliability of each component R_o, ($R_o = e^{-\lambda t}$ where λ = failure rate), and if a task requires k components, the subsystem can tolerate upto N-k failures, and the reliability of such a subsystem is

$$\sum_{i=0}^{N-K} \binom{N}{i} R_o^{N-i} (1 - R_o)^i \tag{2.1}$$

Thus the reliability of C.mmp with 16 processors and 16 64K-memories, with at least four processors and four memory ports required for the task, is

$$R_s \left(\sum_{i=0}^{12} \binom{16}{i} R_p^{16-i} (1 - R_p)^i \right) \left(\sum_{i=0}^{12} \binom{16}{i} R_m^{16-i} (1 - R_m)^i \right) \tag{2.2}$$

where R_s = switch reliability = e^{-202t}
R = (processor + associated circuitry) reliability = $e^{-68.3t}$
R_m = (memory + associated circuitry) reliability = e^{-224t}

Figure 2.2 shows the reliabilities of a 16-processor C.mmp system as a function of time for tasks requiring various number of processors. The plot for task processors = 16 is the reliability of a totally non-redundant C.mmp. The dramatic increase in reliability as the number of task processors decreases is evident.

2.3. Redundancy model for Cm*

For the Cm* system, we will present a series of system reliability models, each one more accurate than the preceding ones. By following the stepwise refinement, the reader will understand the origin of each term in the final equation. Modeling will be performed at the PMS (processor, memory, switch) level. The components and their failure effects are listed below.

Component	Effect of component failure
LSI-11 processor	Loss of processor, not its associated memory
4K memory	Loss of memory
S.local	Loss of processor and its associated memory
K.map	Loss of cluster
L.inc	Loss of L.inc, possible reduction in processing power and memory capacity

Generally, the above failure effects are pessimistic. However, there are a small number of potential failures that are more severe than indicated. For example, a processor failure might short a bus control line thus disabling the bus and making the local memory inaccessible. The number of such failures is small and their effects will be ignored for the current development.

Consider a single cluster with N LSI-11's. If K are required for a task then the reliability is :

$$R_{sys} = R_{k.m} \left\{ \sum_{i=0}^{N-K} \binom{N}{i} (R_p R_{sl} R_m)^{N-i} \, (1 - R_p R_{sl} R_m)^i \right\} \tag{2.3}$$

where $R_{k.m}$ = reliability of the K.map
R_p = reliability of the processor
R_{sl} = reliability of S.local
R_m = reliability of the 12K memory.

Figure 2.3 represents the reliability for N = 8 and K = 4, 6 and 8. The equation above represents a best case model in that perfect recovery is assumed. To model imperfect recovery a factor called coverage [BourW71] is introduced. Coverage, C, is the conditional probability that the system recovers successfully given that there was a failure. Assuming the system fails the first time recovery or component exhaustion occurs, the system reliability becomes :

$$R_{sys} = R_{k.m} \left\{ \sum_{i=0}^{N-K} \binom{N}{i} (R_p R_{sl} R_m)^{N-i} \, (1 - R_p R_{sl} R_m)^i \, C^i \right\} \tag{2.4}$$

The effect of nonperfect coverage is shown in Figure 2.4. The actual value of the parameter C will be derived from a study of the error detection/recovery features of the Cm* hardware, and is beyond the scope of this paper.

By varying the network topology and requirements, we can vary the resultant system reliability. Consider the two cluster network in Figure 2.1b. Each cluster has eight Cm's and each Cm has 12K of memory in addition to the 4K on board the processor. Assume that at least K processors and l 4K memory modules must function for the network to be performing its task. To assess the reliability we list the following states and the corresponding probabilities.

(i) Both K.maps and L.inc good, $R_l R_{km}^2 R_{2k}$

(ii) One K.map fails, L.inc good, $2R_l R_{km}(1-R_{km})R_{1k}C_{km}$

(iii) One K.map fails, L.inc fails, $2(1-R_l)R_{km}(1-R_{km})R_{1k}C_{km}C_l$

(iv) L.inc fails, both K.maps good, $(1-R_l)R_{km}^2(2R_{1k}-R_{1k}^2)C_l$

where R_l = L.inc reliability
R_{km} = K.map reliability
R_{2k} = reliability of two clusters such that the number of processors is greater than k and number of memories is greater than l
R_{1k} = same as R_{2k} for one cluster
C_{km} = coverage factor for K.map
C_l = coverage factor for L.inc.

Thus the system reliability is given by summing the above states :

$$R_{sys} = R_l R_{km}^2 R_{2k} + 2R_l R_{km}(1-R_{km})R_{1k}C_{km} + 2(1-R_l)R_{km}(1-R_{km})R_{1k}C_{km}C_l + (1-R_l)R_{km}^2(2R_{1k}-R_{1k}^2)C_l \qquad (2.5)$$

We will now derive the one cluster, R_{1k}, and the two cluster, R_{2k}, terms.

For R_{1k} the system fails if there are fewer than K processors or fewer l memories. A processor can be denied to the system through a processor or S.local failure. Similarly, an S.local failure can deny the associated memories to the system. Therefore,

$$R_{1k} = \sum_{i=0}^{8} \binom{8}{i} C_{sl}\ R_{sl}^{8-i}\ (1-R_{sl})^{i}\ R_{proc\,i}\ R_{mem\,i} \tag{2.6}$$

where

R_{sl} = S.local reliability
C_{sl} = Covarage factor for S.local
$R_{proc\,i}$ = aggregate reliability of processors with working S.locals
$R_{mem\,i}$ = aggregate reliability of memories with working S.locals

$$R_{proc\,i} = \sum_{j=0}^{8-i-k} C_{p}^{j} \binom{8-i}{j} R_{p}^{8-i-j}\ (1-R_{p})^{j} \tag{2.7}$$

where C_p = coverage factor for processor.

The equation above indicates that the system only works if at least K processors, whose associated S.local are functioning correctly, are nonfailed. If we assume that the reliability of the 4K memory on the processor board is the same as the other 4K memory and that processor and on-board memory failures are independent, then

$$R_{mem\,i} = \sum_{n=0}^{32-4i-\ell} C_{m}^{n} \binom{32-4i}{n} R_{m}^{32-4i-n}\ (1-R_{m})^{n} \tag{2.8}$$

where C_m = coverage factor for one 4K memory.

By analogy,

$$R_{2k} = \sum_{i=0}^{16} \binom{16}{i} C_{sl}\ R_{sl}^{16-i}\ (1-R_{sl})^{i} \cdot \Big\{ \Big(\sum_{j=0}^{16-i-k} C_{p}^{j} \binom{16-i}{j} R_{p}^{16-i-j}\ (1-R_{p})^{j}\Big). \Big(\sum_{n=0}^{64-4i-\ell} C_{m}^{n} \binom{64-4i}{n} R_{m}^{64-4i-n}\ (1-R_{m})^{n}\Big) \Big\} \tag{2.9}$$

The R_{sys} as calculated from these equations is plotted in Figure 2.5 with various values of K, *l* and all coverage factors assumed to be one. The model can be extended in an obvious manner for a larger number of clusters and/or Cm's. Based on the procedure outlined above, a program is being developed that takes any general PMS structure and minimal component requirements as input, and provides the system reliability as output.

Frequently redundant systems are compared via mission time improvement, MTI [KnoxJ64]. If t_{mi} is the time for which the R_{sys} of system i is above a certain minimum mission reliability, then t_{mi} is called the mission time, and t_{m1}/t_{m2} is the MTI of system one over system two. To compare the single cluster Cm* network against a non-redundant LSI-11 processor, we solve the equation :

$$R_{sys}(t_1) = R_{LSI-11}(t_2) \tag{2.10}$$

Since the MTI is not constant over all values of R_{LSI-11} we plot it as a function of R_{LSI-11} in Figure 2.6.

In Figure 2.7 we plot the MTI of C.mmp over LSI-11. In order to compare the MTI with that of Cm*, the memory size of each port of C.mmp was normalized to 16K words. The Cm* system of Figure 2.6 is seen to offer greater mission times than C.mmp. As work progresses, the reliability model will be refined and compared against actual operational data.

So far we have considered the computer systems as stand-alone systems that fail upon component exhaustion. In the following section, we will investigate the effect of periodic maintenance on mission time.

3. Effect of Periodic Maintenance on Reliability

3.1. Introduction

In an attempt to increase the life of a non-perfect system, various redundancy techniques have been applied. It has been shown that TMR (triple modular redundancy) with stand-by sparing can be used to achieve improvement in reliability and expected life. While the general analysis holds perfectly well for systems performing vital functions in a space mission, it is extremely pessimistic in a more commonly used system where a technician may be able to perform repairs. Even more common is a situation in which certain tests are applied to the system at regular intervals to insure its integrity, followed by any required repair. It is to be expected in most cases that the life span of a system subjected to such maintenance would be greater than that of an identical system left unattended. In the following analysis we estimate the expected life of a non-perfect system under periodic maintenance.

3.2. Life of An Unmaintained System

As has been the common practice, we will assume the failures in a non-redundant system to have an exponential distribution. We will denote the failure rate by λ. The reliability of the system (i.e. the probability that there is no failure during the time interval $(0,t)$) is then $R(t) = e^{-\lambda t}$. The life of the system is estimated as follows.

Let T be the time at which a failure occurs. Then the distribution function $F(t)$ is

$F(t)$ = Prob (the failure occuring in $(0,t)$)

$= \text{Prob}(0<T<t)$

$$= 1 - \text{Prob}(T>t)$$

$$= 1 - R(t) \tag{3.1}$$

The life of the system is the expected value of T, $E(T)$.

And, $$E(T) = \int_0^\infty (1 - F(t))\, dt$$

$$= \int_0^\infty R(t)\, dt \tag{3.2}$$

Note that the above equation is a general one, and may be applied to any function R(t). Also note that according to the equation above, $E(T)$, or the life of the system, is the same as the area under the curve for any function R(t). For the exponential distribution,

$$E(T) = 1/\lambda. \tag{3.3}$$

3.3. Life of A Maintained System

Let us now consider a system being operated under periodic maintenance. We assume that certain tests are performed at a regular time interval, β. Every occasion on which these tests are performed with success (or the failures of the tests are followed by subsequent necessary repairs), there is a lesser probability of failure in immediate future. This leads to an increase in reliability after every maintenance period. Note that this model differs from a repair model. In the former, maintenance and repair are conducted periodically, and the system is considered failed if there is any system failure between maintenance periods. The system is considered unavailable during maintenance and the duration of maintenance is considered unimportant. The latter model only schedules repair after a failure. This distinction is particularly significant in redundant systems where the repair model has a large number of states and is critically sensitive to the repair rate [ShooM68]. We shall consider two models for the improvement obtained by periodic maintenance.

Model I : Improvement through maintenance = d $(1 - R_{sys}(\beta))$, where d is a constant, $0 \le d \le 1$, and corresponds to the percentage of failures detected by the diagnostic procedure. This model assumes that a constant fraction of the lost reliability is recovered. Figure 3.1 shows a hypothetical reliability function using this assumption.

Let $R_i(t)$ = system reliability function during the ith interval.

Then,

$$R_1(t) = R_{sys}(t) \tag{3.4}$$

$$R_2(t) = \{ R_{sys}(\beta) + d\,(1 - R_{sys}(\beta) \}\, R_{sys}(t)$$

$$R_3(t) = \{ R_{sys}(\beta) + d\,(1 - R_{sys}(\beta))\, R_{sys}(\beta) + d\,(1 - R_{sys}(\beta)) \}\, R_{sys}(t)$$

$$= \{ R^2_{sys}(\beta) + d\,(1 - R^2_{sys}(\beta) \}\, R_{sys}(t)$$

And,

$$R_{i+1}(t) = \{ R^i_{sys}(\beta) + d\,(1 - R^i_{sys}(\beta) \}\, R_{sys}(t) \tag{3.5}$$

The area under the (i+1)st segment is

$$A_{i+1} = \int_0^\beta \{ R^i_{sys}(\beta) + d\,(1 - R^i_{sys}(\beta) \}\, R_{sys}(t)\, dt \tag{3.6}$$

$$= \{ R^i_{sys}(\beta) + d\,(1 - R^i_{sys}(\beta) \} \int_0^\beta R_{sys}(t)\, dt$$

For a nonredundant system, $R_{sys}(t) = e^{-\lambda t}$, and

$$A_{i+1} = (1/\lambda)\,(1 - e^{-\lambda\beta})\, \{ e^{-i\lambda\beta} + d\,(1 - e^{-i\lambda\beta}) \} \tag{3.7}$$

The attempt to evaluate life, which is also the sum of all A_i's, yields infinity for an answer. This is due to the fact that under the assumptions, the reliability function reaches a steady state as shown in Figure 3.2. The area under this reliability function is not finite.

Since the tests will not, in general, test all possible system components, there will remain components that are never replaced or repaired during the periodic maintenance. These components will eventually fail. Thus an expected infinite life span is unrealistic.

Model II : In the first model, we assumed the improvement due to maintenance to be a constant. In a more pessimistic model, we may assume that the improvement in reliability due to maintenance at the end of the ith period is a fraction of the reliability lost in the ith period. In other words,

$$R_{i+1}(t) = \{ R_i(\beta) + d\, R_i(0)\, (1 - R_{sys}(\beta)) \}\, R_{sys}(t) \tag{3.8}$$

Writing R_i's explicitly, we have

$$R_1(t) = R_{sys}(t) \tag{3.9}$$

$$R_2(t) = \{ R_{sys}(\beta) + d\,(1 - R_{sys}(\beta)) \}\, R_{sys}(t)$$

$$\begin{aligned} R_3(t) &= \{ R_{sys}(\beta) + d\, R_{sys}(\beta)\, (1-R_{sys}(\beta)) + d\, R_{sys}(\beta) + d^2\, (1-R_{sys}(\beta)) \\ &\quad - d\, R_{sys}(\beta) - d^2\, R_{sys}(\beta)\, (1 - R_{sys}(\beta)) \}\, R_{sys}(t) \\ &= \{ R_{sys}(\beta) + d\,(1 - R_{sys}(\beta)) \}^2\, R_{sys}(t) \end{aligned}$$

And, in general,

$$R_{i+1}(t) = \{ R_{sys}(\beta) + d\,(1 - R_{sys}(\beta)) \}^i\, R_{sys}(t) \tag{3.10}$$

The area under the (i+1)st segment,

$$\begin{aligned} A_{i+1} &= \int_0^\beta R_{i+1}(t)\, dt \\ &= \{ R_{sys}(\beta) + d\,(1 - R_{sys}(\beta)) \}^i \int_0^\beta R_{sys}(t)\, dt \end{aligned} \tag{3.11}$$

Then the life span for the system is

$$\begin{aligned} \text{Life} &= \sum A_{i+1} \\ &= \left(\int_0^\beta R_{sys}(t)\, dt\right) \sum \{ R_{sys}(\beta) + d\,(1 - R_{sys}(\beta)) \}^i \\ &= \left(\int_0^\beta R_{sys}(t)\, dt\right) \{ 1 - R_{sys}(\beta) - d\,(1 - R_{sys}(\beta) \}^{-1} \\ &= \left(\int_0^\beta R_{sys}(t)\, dt\right) (1 - R_{sys}(\beta))^{-1}\, (1 - d)^{-1} \end{aligned} \tag{3.12}$$

Again, for a nonredundant system, $R_{sys}(t) = e^{-\lambda t}$, and

$$\text{life} = 1 \,/\, \{ \lambda\, (1 - d) \} \tag{3.13}$$

The life span is thus improved by a factor of $(1 - d)^{-1}$. If d = 1, the maintenance is perfect and the life span tends to be infinite. If $d = 0$, we revert back to an unmaintained system with life span $(1/\lambda)$. For a detection probability of $d = 0.9$, a fairly modest goal, the expected life increases by a factor of 10.

3.4. Redundant Systems With Maintenance

Until now we have considered a non-redundant system with $R(t) = e^{-\lambda t}$. Where reliabilities of high order (better than one failure in a million hours) are required, improvements through technological advances alone fall short of the objective. System designers have resorted to redundancy to attain higher reliabilities. Triple modular redundancy (TMR) with majority voting is one of the redundancy techniques used. Since such a technique allows the system to tolerate a single failure in any of the three modules, the reliability of a TMR system is

$$\begin{aligned} R_{sys}(t) &= R^3(t) + 3\,R^2(t)\,(1 - R(t)) \\ &= 3\,R^2(t) - 2\,R^3(t) \end{aligned} \tag{3.14}$$

where $R(t)$ = reliability of non-redundant system.

Since we intend to keep the development applicable in general, we will use $R_{sys}(t)$ during the discussion and substitute the expression for a TMR system only to exemplify the results.

3.5. Life of An Unmaintained, Redundant (TMR) System

Recalling that the life span of a system, $E(T)$, is the same as that of the area under the reliability function we can write

$$\text{Life} = \int_0^{\infty} R_{sys}(t)\,dt$$

For a TMR system,

$$\text{Life (TMR)} = \int_0^{\infty} \{\, 3\,R^2(t) - 2\,R^3(t) \,\}\,dt$$

$$= (1/\lambda)\,(\,3/2 - 2/3\,)$$

$$= 5\,/\,(6\,\lambda) \tag{3.15}$$

Again, we now focus our attention to a system under periodic maintenance.

3.6. Life of A Maintained, Redundant (TMR) System

We will again consider the two models. Under the first simple model, we assume that the improvement through maintenance is a constant fraction of the probability of failure, i.e. $d\,(1 - R_{sys}(\beta))$ where β is the period of the maintenance cycle.

We have established that the area under the (i+1)st segment is :

$$A_{i+1} = \quad \{ R^{i}_{sys}(\beta) + d\,(1 - R^{i}_{sys}(\beta)\,\}\int_0^\beta R_{sys}(t)\,dt$$

For a TMR system,

$$R_{sys}(t) = 3\,e^{-2\lambda t} - 2\,e^{-3\lambda t}$$

and

$$\int_0^\beta R_{sys}(t)\,dt$$

$$= \int_0^\beta \{\,3\,e^{-2\lambda t} - 2\,e^{-3\lambda t}\,\}\,dt$$

$$= (3\,/\,2\lambda)\,(1 - e^{-2\lambda\beta}) - (2\,/\,3\lambda)\,(\,1 - e^{-3\lambda\beta})$$

We again note that when we try to sum all A_i's, the results approaches infinity.

Considering the second model that we suggested earlier, where the improvement in reliability is a fraction of the reliability lost in the ith period, we may write

Again we have established that area under the (i+1)st segment;

$$A_{i+1} = \{\,R_{sys}(\beta) + d\,(1 - R_{sys}(\beta))\,\}^i\int_0^\beta R_{sys}(t)\,dt$$

And the life span for the system is

$$\text{Life} \quad = (\int_0^\beta R_{sys}(t)\,dt)\,(1 - R_{sys}(\beta))^{-1}\,(1 - d)^{-1}$$

For a TMR system,

$$\int_0^\beta R_{sys}(t)\,dt = (3\,/\,2\lambda)\,(1 - e^{-2\lambda\beta}) - (2\,/\,3\lambda)\,(1 - e^{-3\lambda\beta})$$

and $R_{sys}(\beta) = 3\ e^{-2\lambda\beta} - 2\ e^{-3\lambda\beta}$.

Substituting these two expressions in the equation for Life we can estimate the life span of a TMR system. As the expression is very complex, the improvement is not obvious in this form. Let us, therefore, consider numerical examples. The following table (Table 3.1) allows the comparison in the life spans of unmaintained and maintained TMR systems.

Table 3.1 Expected life with periodic maintenance; Lamda = 0.0001

Life of an unmaintained TMR system : 8333.33 hours

d	β (hours) 10	100	1000	10000	1000000
0.20	423619.13	48692.87	11958.48	10416.67	10416.67
0.40	564825.50	64923.83	15944.64	13888.89	13888.89
0.60	847238.26	97385.74	23916.96	20833.33	20833.33
0.80	1694476.49	194771.48	47833.92	41666.67	41666.67
0.90	3388952.97	389542.96	95667.84	83333.33	83333.33
0.92	4236191.38	486928.72	119584.80	104166.67	104166.67
0.94	5648254.82	649238.25	159446.39	138888.88	138888.88
0.96	8472382.75	973857.43	239169.60	208333.34	208333.34
0.98	16944762.34	1947714.50	478339.11	416666.60	416666.60

The data conform to the expectations. As the period between maintenance becomes larger, the expected life becomes shorter. We note that after β = 10000 hours, the life is shorter than the period of maintenance. Thus the maintenance has no effect on the performance for any period larger than 10000. The life depends more strongly on d, as seen by the sharp increase of life as d approaches unity. The constant d is a function of how efficient maintenance routines are. Since the expected life is extremely sensitive to the effectiveness of maintenance around d = 0.9, slight improvement in maintenance routines may be rewarded with substantially greater life for the system. This fact is emphasized when we plot the expected life of a TMR system as a function in Figure 3.3.

3.7. Life of C.mmp and Cm* Systems

Now let us examine the effect of periodic maintenance on the two systems under investigation. We will use the general expression for the system life with periodic maintenance (equation (3.12)), namely,

$$\text{life} = \left(\int_0^{\beta} R_{sys}(t)\, dt \right) (1 - R_{sys}(\beta))^{-1} (1 - d)^{-1}$$

For C.mmp with 16 processors and 16 64-K memories, and with at least four processors and four memory ports required for a task, the R_{sys} is given by equation (2.2) :

$$R_s \left(\sum_{i=0}^{12} \binom{16}{i} R_p^{16-i} (1 - R_p)^i \right) \left(\sum_{i=0}^{12} \binom{16}{i} R_m^{16-i} (1 - R_m)^i \right)$$

Substitution in equation (3.12) and numerical evaluation of life yielded the following results.

Table 3.2 Expected life of a maintained C.mmp system :

d	β (hours)			
	100	1000	10000	100000
0.20	6175.80	6175.80	6158.50	5992.46
0.40	8234.40	8234.40	8211.33	7989.95
0.60	12351.60	12351.60	12316.99	11984.93
0.80	24703.19	24703.19	24633.98	23969.85
0.90	49406.38	49406.38	49267.97	47939.71
0.92	61757.98	61757.98	61584.96	59924.63
0.94	82343.97	82343.97	82113.28	79899.51
0.96	123515.97	123515.96	123169.92	119849.27
0.98	247031.88	247031.87	246339.79	239698.49

A significant fact comes to light in this exercise. There is a small improvement in the system life as the period of maintenance is reduced from 100000 to 10000 to 1000. But when it is further reduced to 100 hours, there is no improvement for values of d up to 0.96, and even at higher values of d, improvement is negligible. This is due to the fact that with redundancy, the reliability factors represented by the summations in equation (2.2) are very close to unity. The smaller the period β, the closer these summations get to unity. Consequently, at sufficiently small values of β (in 1000's of hours), the R_{sys} is dominated by and approximates to R_s, the switch reliability. The observation is further strengthened by the fact that for $\beta = 1000$ hours, as d tends to zero, the system life approximates to 4940.63822 hours. This number is very close to the switch life ($1/\lambda_s = 10^6/202.403$ =) 4940.63824 hours.

The expected life of Cm* system under maintenance may be similarly established using equations (2.5) and (3.12). We note, however, that the terms R_{1k} and R_{2k} both represent the redundancies in the system. Using the fact that these terms approach unity for all except extremely large periods of maintenance, we approximate the expression to :

$$\text{life} = \left(\int_0^{\beta} R_{sys}(t)\, dt\right) (1 - R_{sys}(\beta))^{-1} (1 - d)^{-1}$$

where

$$R_{sys} = R_l R_{km}^2 R_{2k} + 2R_l R_{km}(1-R_{km})R_{1k}C_{km} +$$
$$2(1-R_l)R_{km}(1-R_{km})R_{1k}C_{km}C_l + (1-R_l)R_{km}^2(2R_{1k}-R_{1k}^2)C_l$$

or approxiamtely,

$$R_{sys} \cong R_l R_{km}^2 + 2R_l R_{km}(1-R_{km}) + 2(1-R_l)R_{km}(1-R_{km}) + (1-R_l)R_{km}^2$$
$$\cong 2 R_{km} - R_{km}^2 \qquad (3.16)$$

If we let λ_k be the failure rate for the K.map, the life of a maintained Cm* system is approxiamted by

$$\text{life} = \{\lambda_k(1-d)\}^{-1} \{ (1-e^{-\lambda\beta})^{-1} + 1/2 \} \quad (3.17)$$

Note that (3.17) represents the life of a duplicate system under periodic maintenance. The life of a duplicate system without periodic maintenance is

$$\text{life(duplicate system)} = \frac{3}{2\lambda_k} \quad (3.18)$$

To re-emphasize the effect of periodic maintenance, consider the two equations (3.17) and (3.18). For the estimated value for λ_k (from page 8), and d = 0.9, the ratio of the two expressions is as much as 44 for β = 1000 hours, and more than 350 for β = 100 hours.

Numerical evaluation of life as predicted by equation (3.17) leads to results presented in Table 3.3.

Table 3.3 Expected life(approximate) of a maintained Cm* system :

d	β (hours)			
	100	1000	10000	100000
0.20	399708.66	46379.50	11923.38	10509.27
0.40	532944.88	61839.33	15897.84	14012.35
0.60	799417.33	92759.00	23846.76	21018.53
0.80	1598834.62	185518.00	47693.52	42037.06
0.90	3197669.24	371036.00	95387.05	84074.12
0.92	3997086.70	463795.01	119233.81	105092.65
0.94	5329448.61	618393.31	158978.41	140123.53
0.96	7994173.41	927590.03	238467.63	210185.31
0.98	15988343.84	1855179.71	476935.16	420370.54

It is readily noticed that these numbers are considerably higher than those for the C.mmp. While both systems are constrained by the components lacking potential redundancy, namely the switch for C.mmp and the L.inc and K.map for Cm*, the flexibility for structure in the Cm* system allows for some failures of these components. In a failure tolerant environment, the Cm* system would exhibit between 2 to 9 times longer life than the C.mmp system.

4. Conclusions

We set out to establish a realistic reliability model for hardware failures in two multiprocessor systems, C.mmp and Cm*. A parts count reliability model was used to arrive at failure rates of various components of the two systems. To model accurately the potential fault tolerance offered by the multiprocessor systems, various redundancy models were proposed.

Digital systems, apart from those used in space missions, may be subjected to maintenance checks to ensure their integrity. Such checks, although short of complete repair, should increase the confidence in integrity, and hence reliability, of a system. The effect of periodic maintenance was modeled using a parameter d, which signifies the efficiency of the maintenance checks. The system life was shown to have a strong dependence on d.

Finally, the application of the composite models, modeling redundancy as well as periodic maintenance, the two multiprocessor systems were compared. The non-redundant components figured prominently as the bottlenecks. The flexibility of structure in the Cm* system was reflected in its life being considerably longer than that of the C.mmp system.

5. References

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[ShooM68] Shooman, M. L., Probabilistic reliability : an engineering approach, McGraw Hill, 1968.

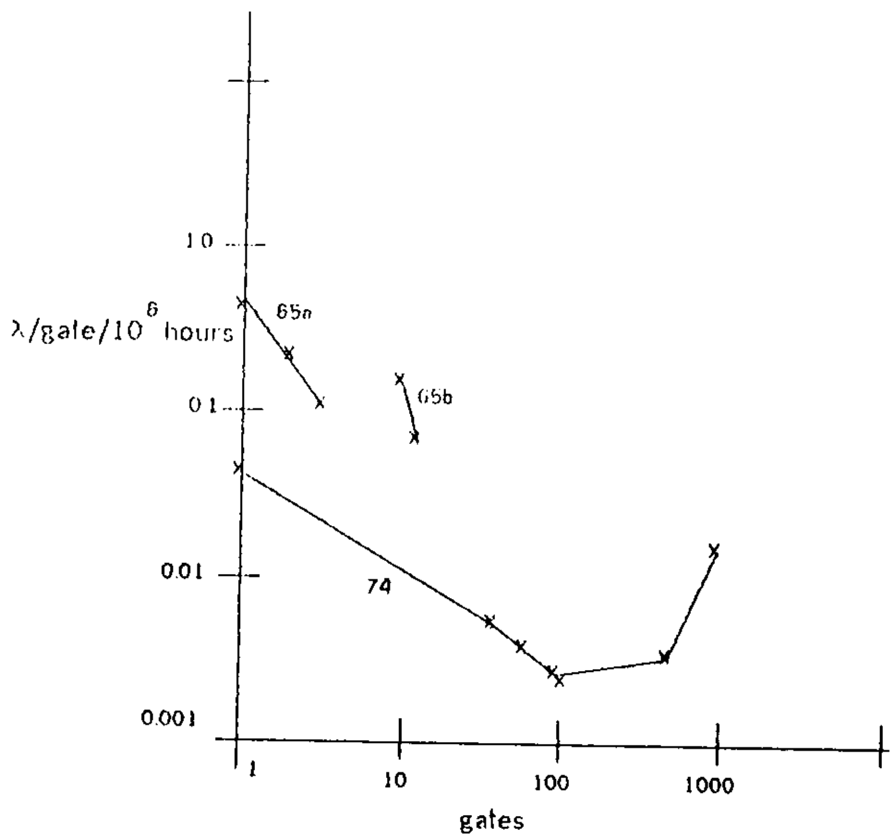
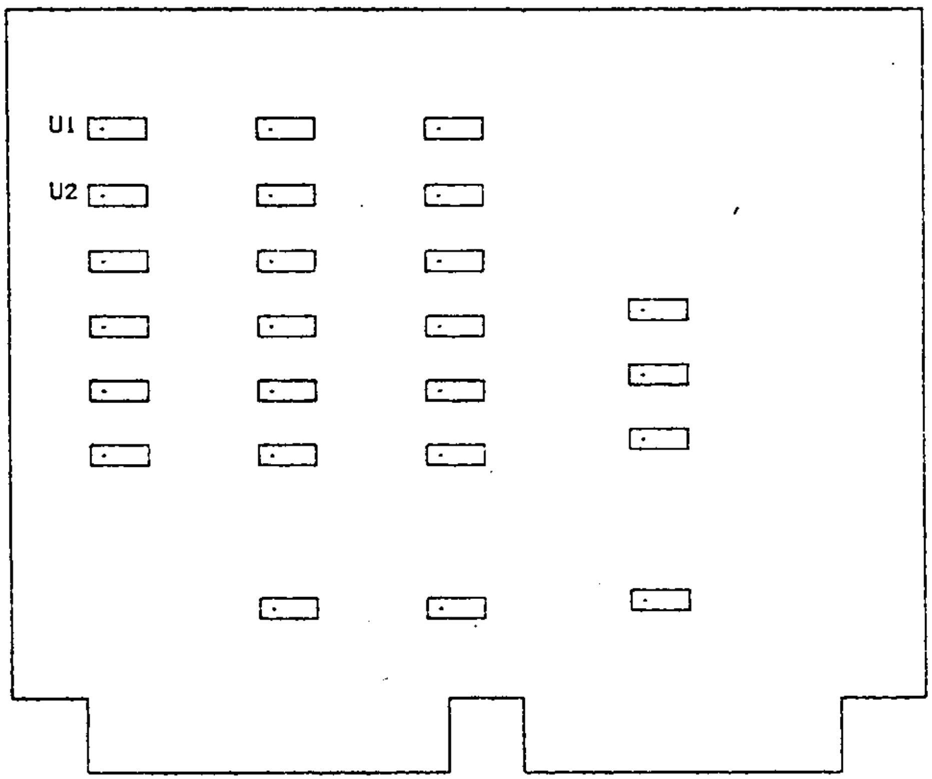


Fig 1.1 The effect of integration on gate failure rate : 1965 (65a, 65b) and 1974 (74) data.

LAY OUT



LIST OF PARTS

U1	SN74S74
U2	SN7438
U3	SN74S74
U4	SN74S74
U5	SN7438
U6	SN74S74
U7	SN74S74
U8	SN7438
U9	SN74S74
U10	SN74S74
U11	SN7438
U12	SN74S74
U13	SN74S74
U14	SN7438
U15	SN74S74
U16	SN74S74
U17	SN7438
U18	SN74S74
U19	SN74S140
U20	SN7440
U21	SN7404
U22	SN74S74
U23	SN74S138
U24	SN74S138

FIGURE 1.2 Processor Interface

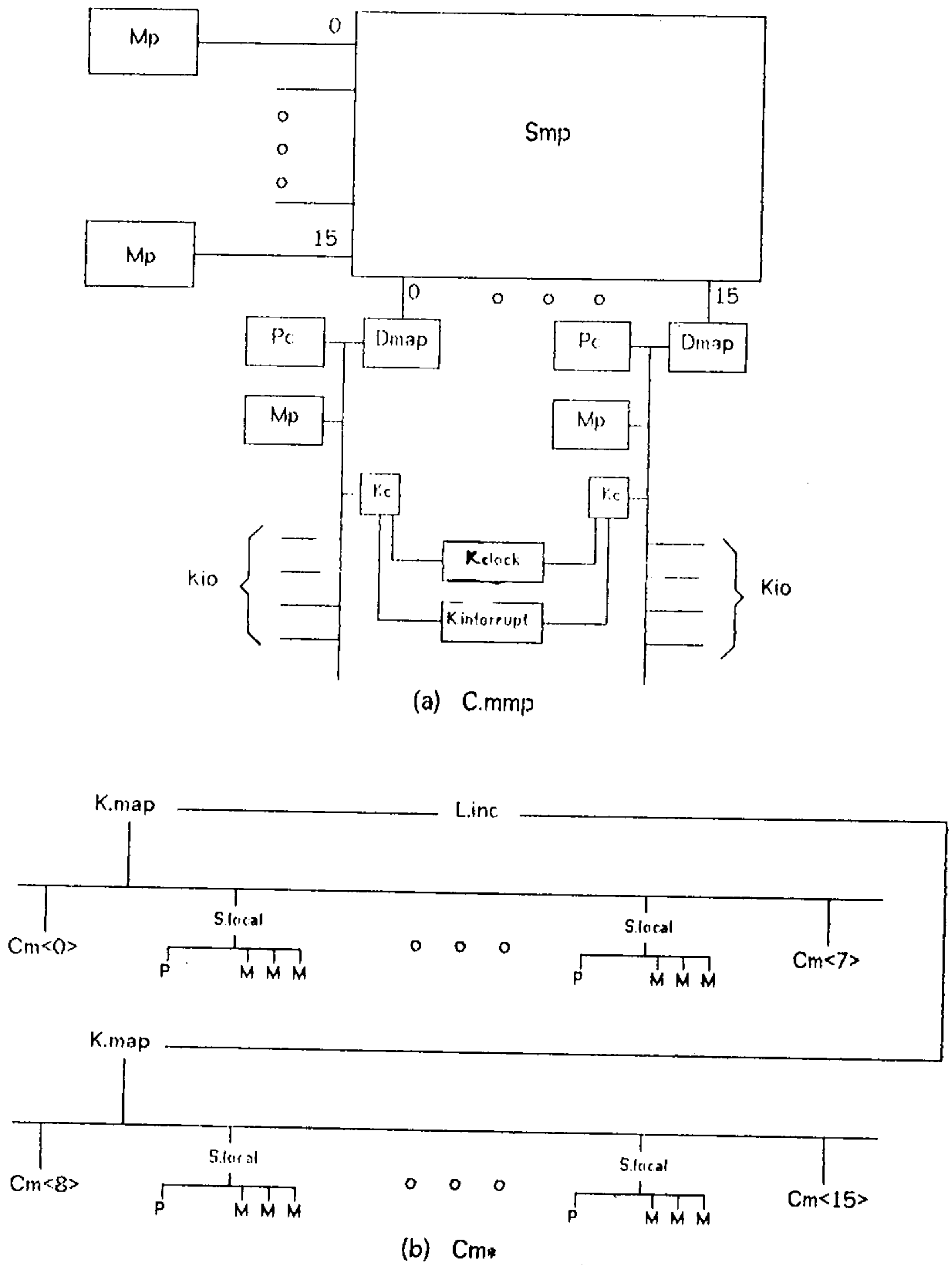


FIGURE 2.1 General structures of the two multiprocessor systems, C.mmp and Cm*

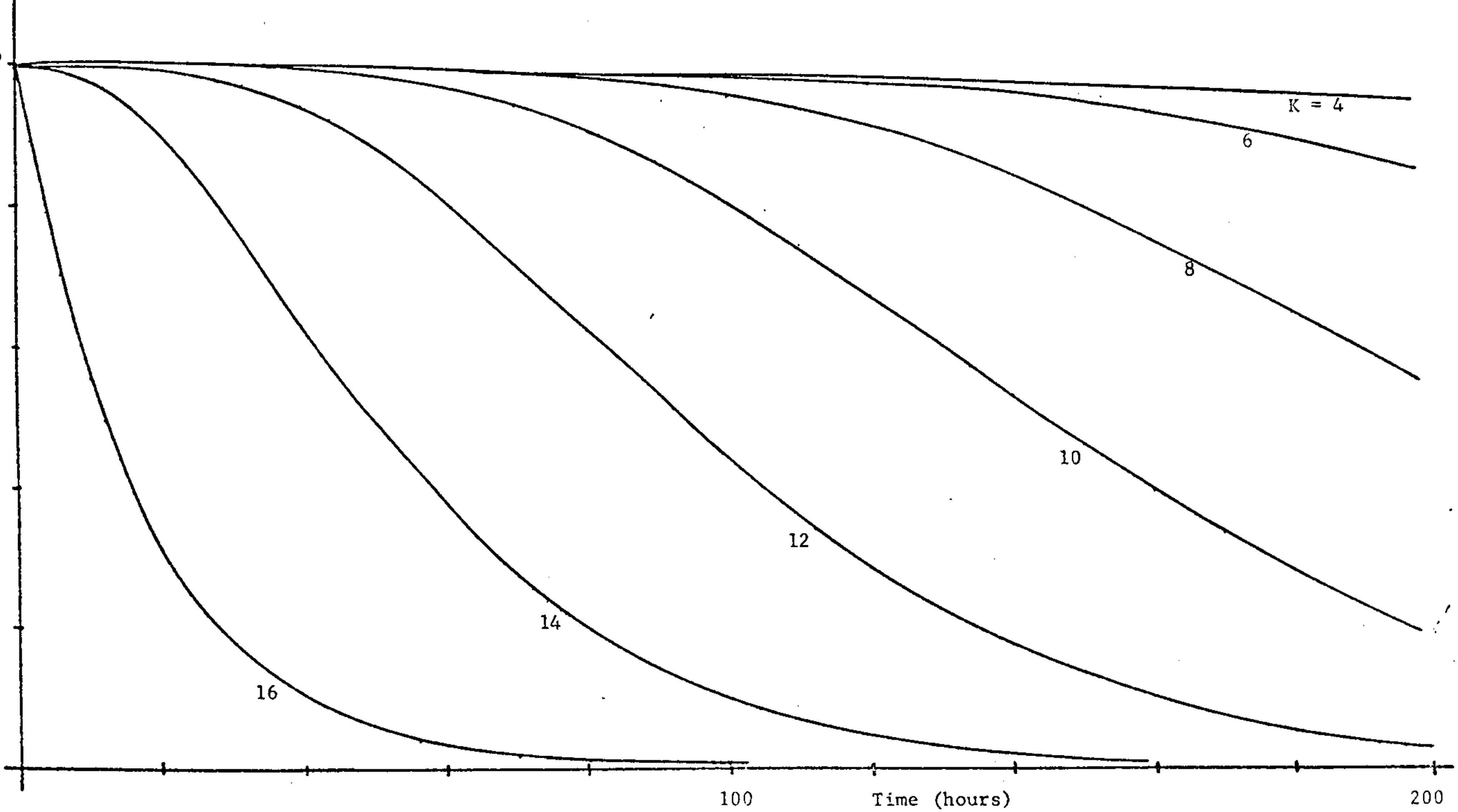


Fig. 2.2 Reliability of C.mmp for a task requiring K processors.

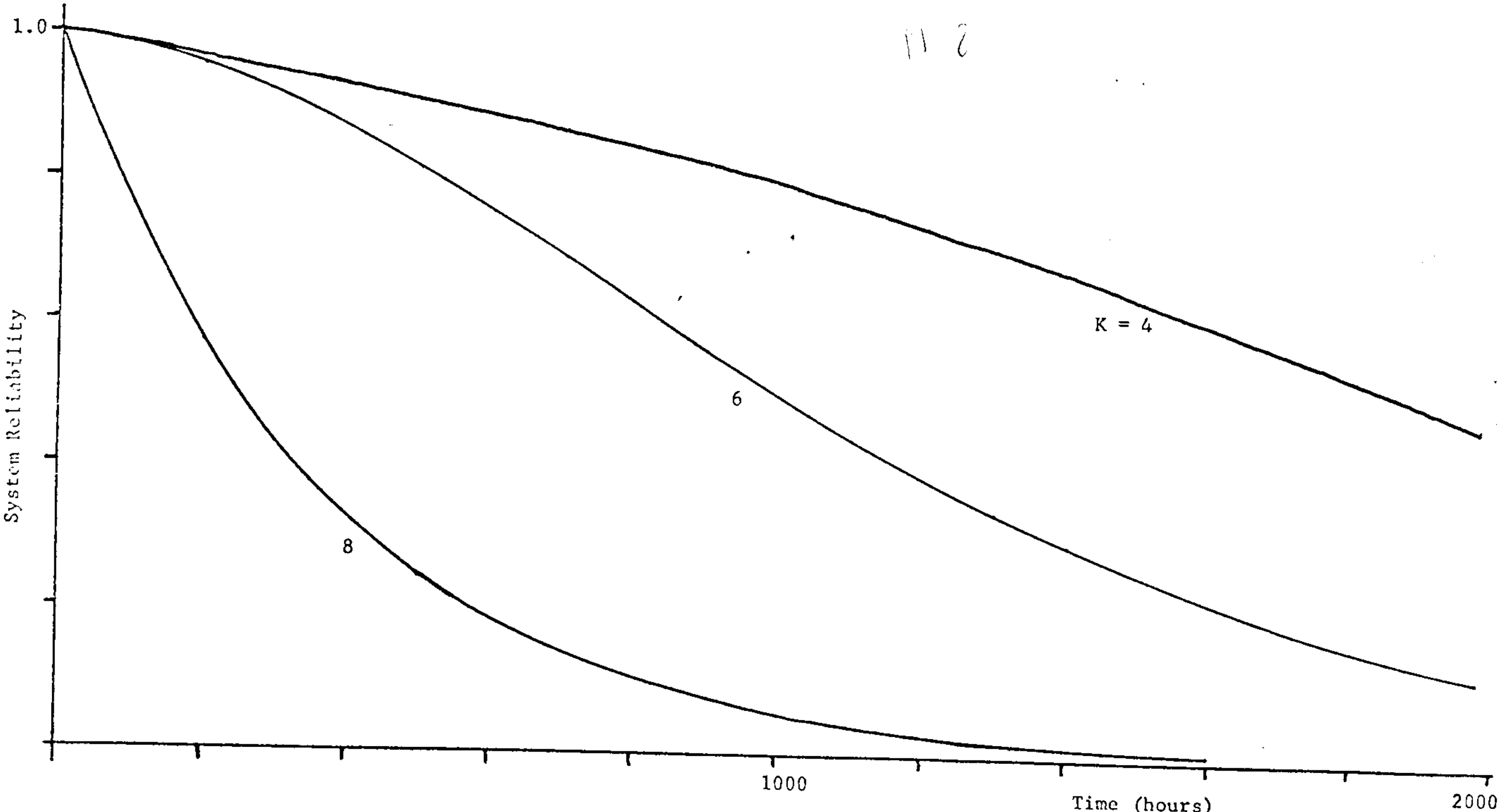


Fig. 2.3 Reliability of Cm* for a task requiring K processors.

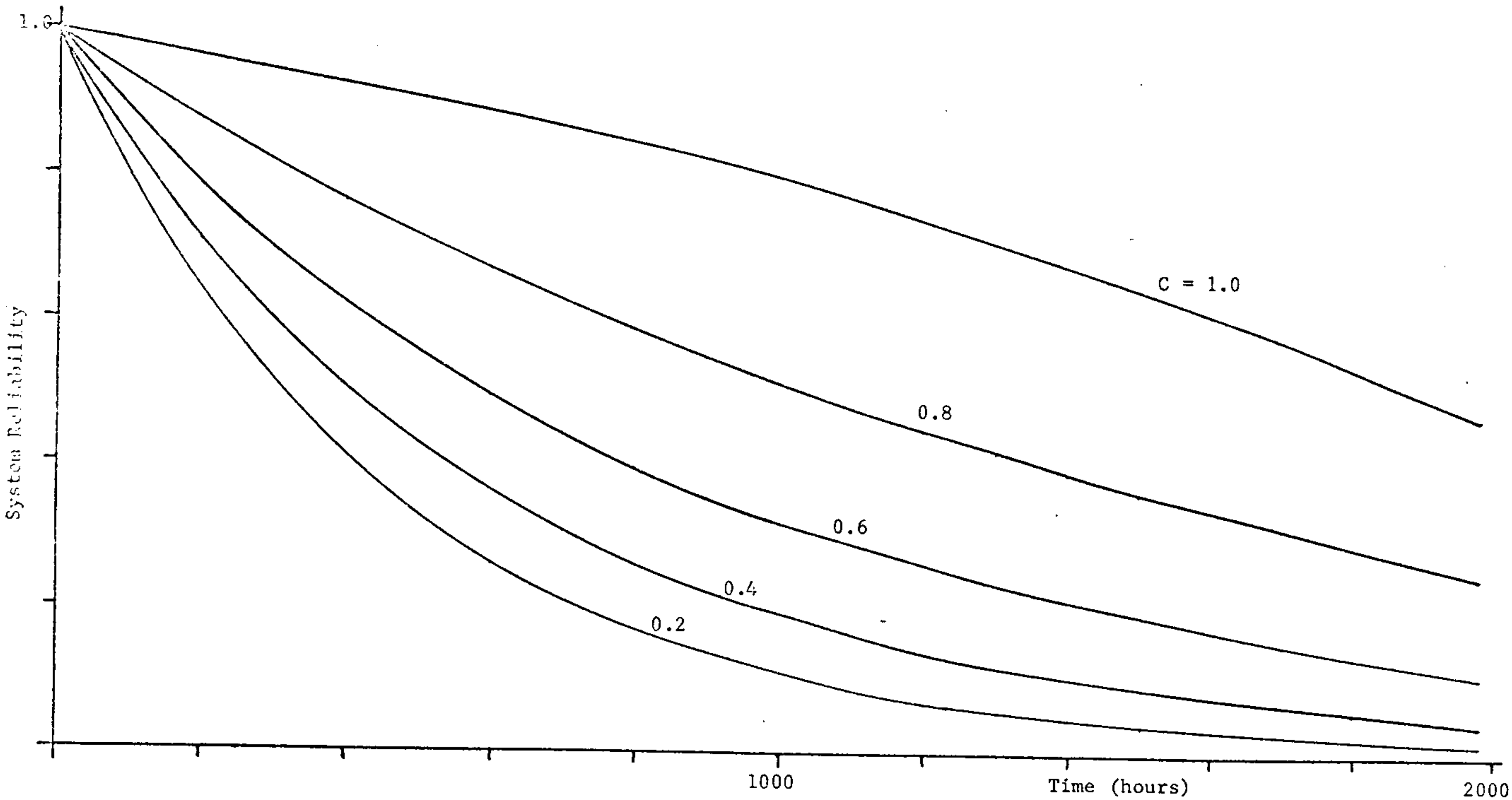


Figure 2.4 Effect of coverage, C, on Cm* reliability (K = 4)

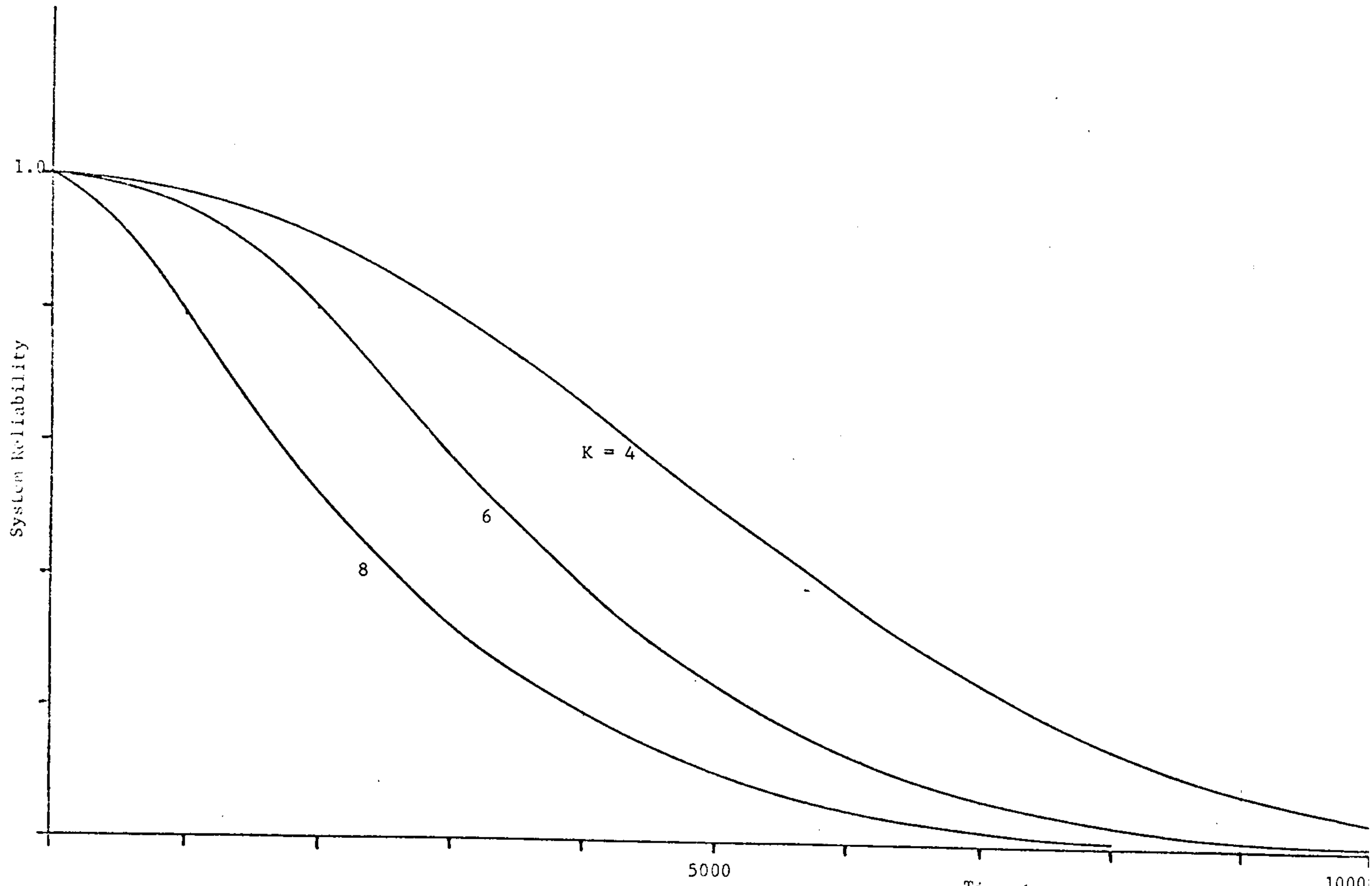


Figure 2.5 Reliability of a two-cluster Cm* system.

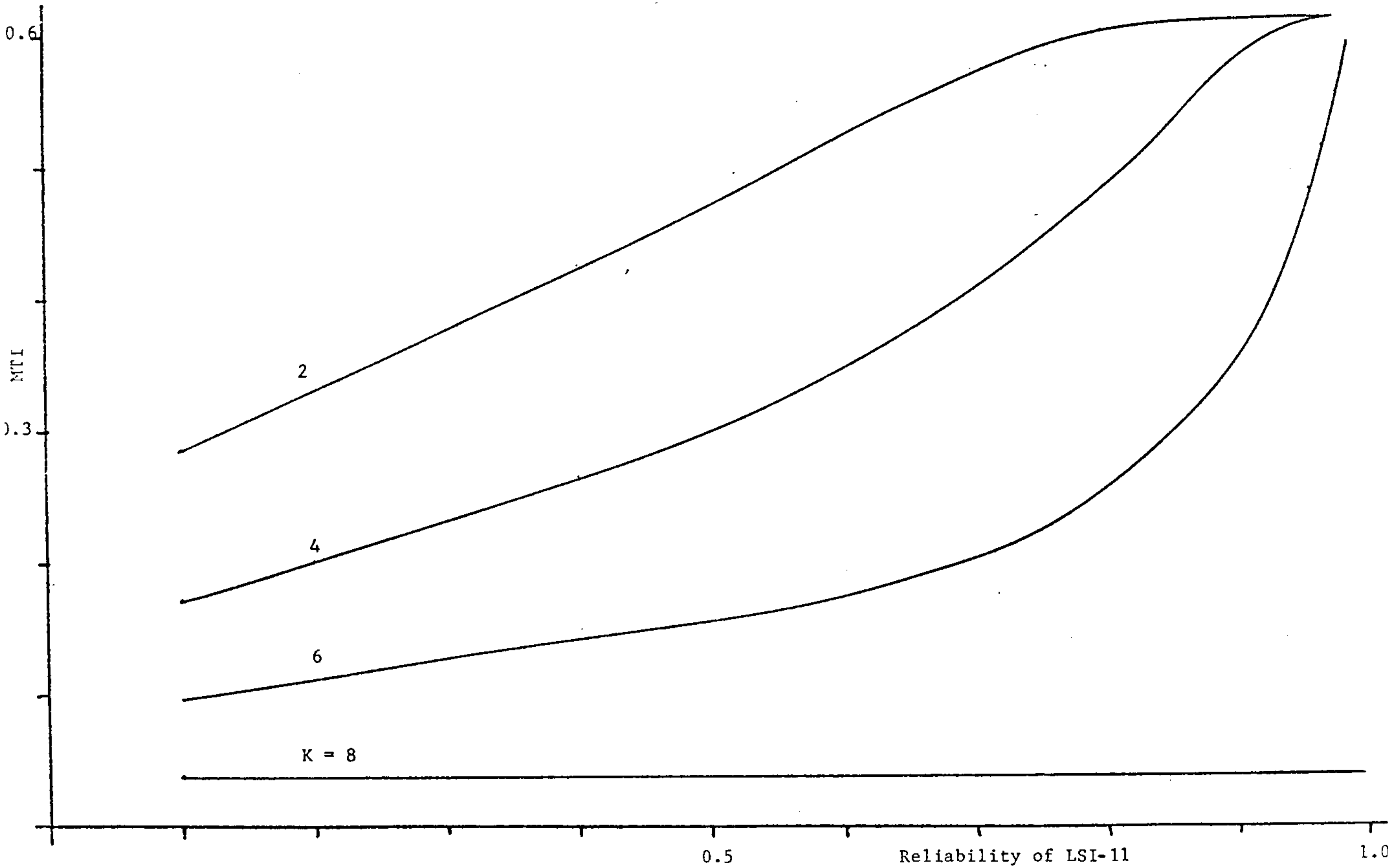


Fig. 2.6: MTI of a single cluster Cm* over LSI-11.

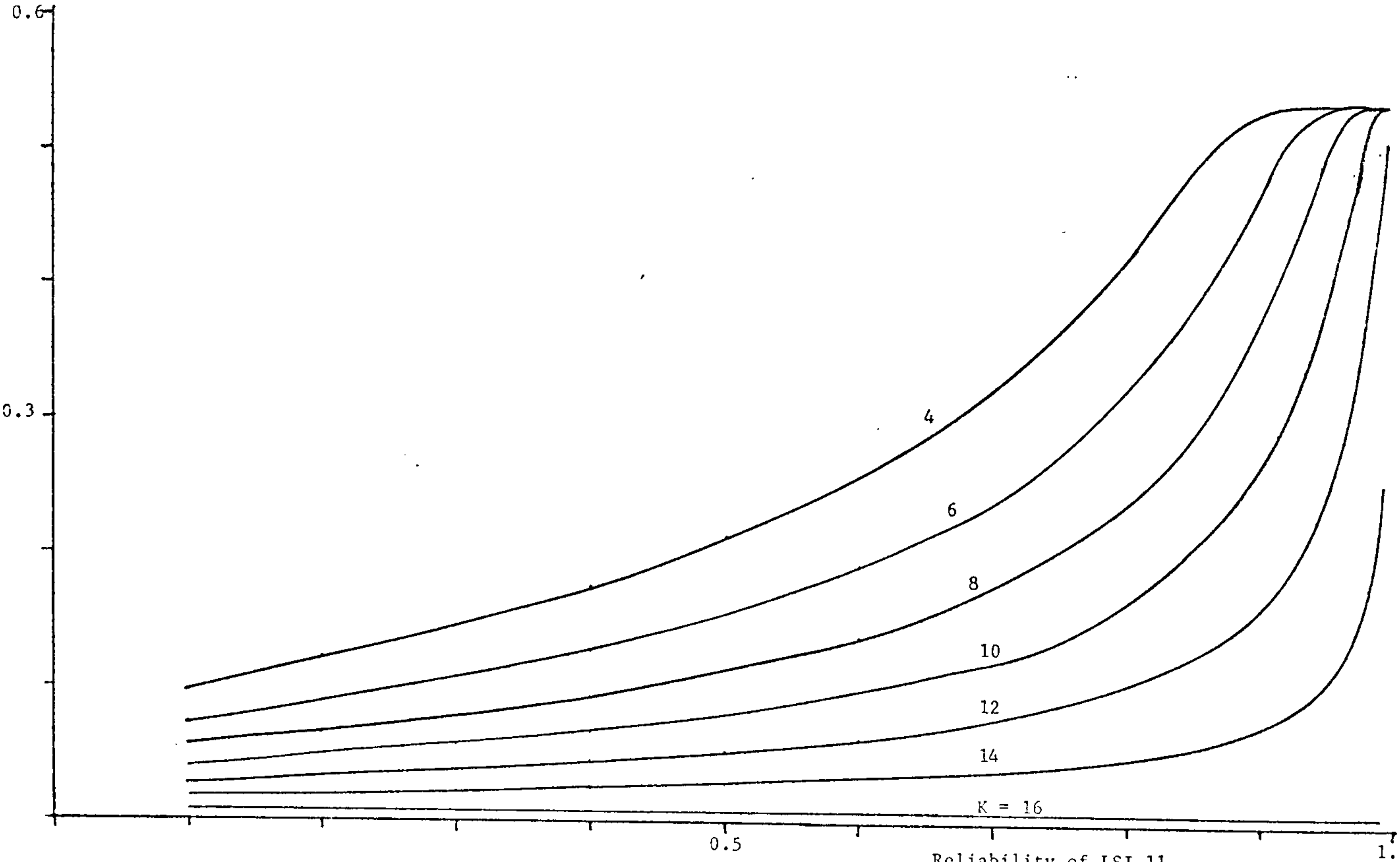


Fig. 2.7: MTI of C.mmp (with normalized memory) over LSI-11.

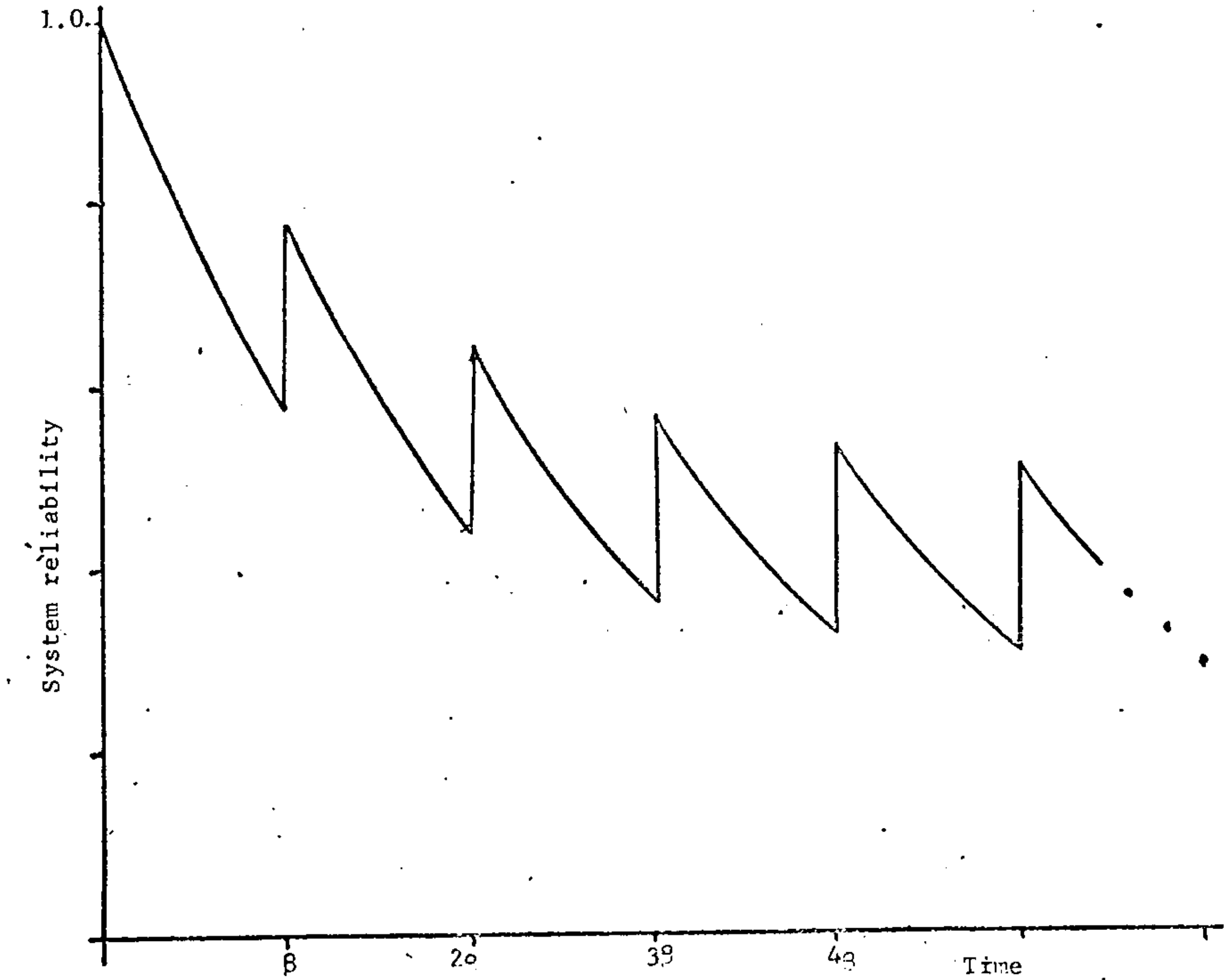


Figure 3.1 : Effect of periodic maintenance on system reliability.

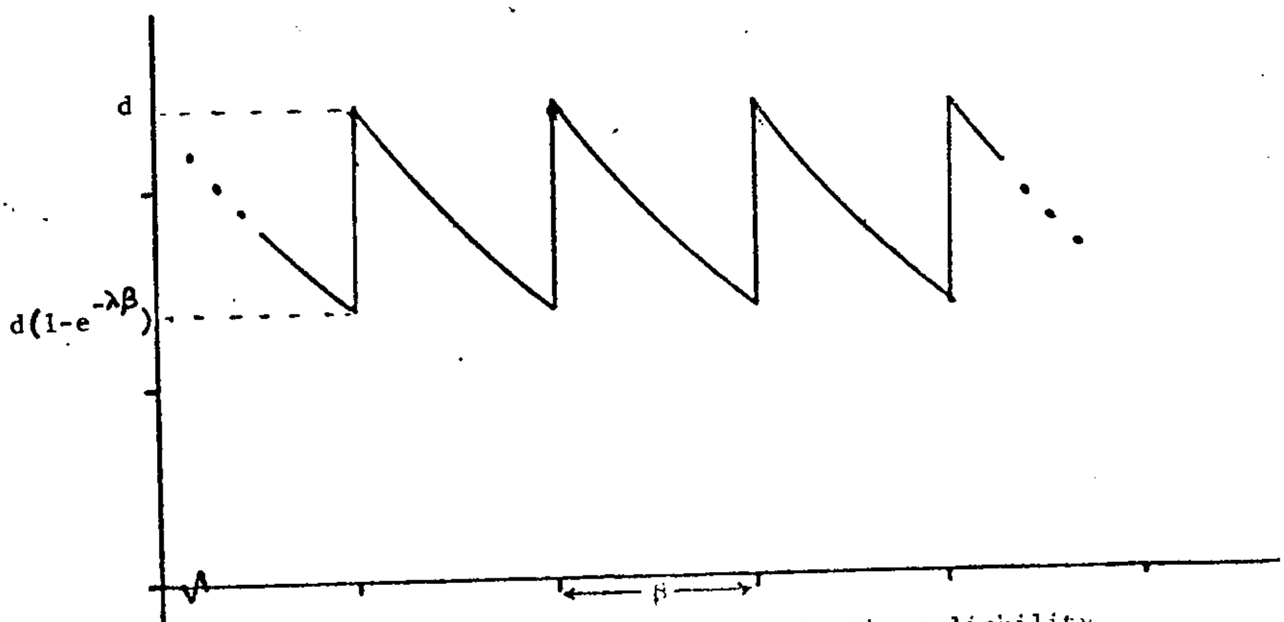


Figure 3.2: Steady state reached by the reliability function in the first model for periodic maintenance.

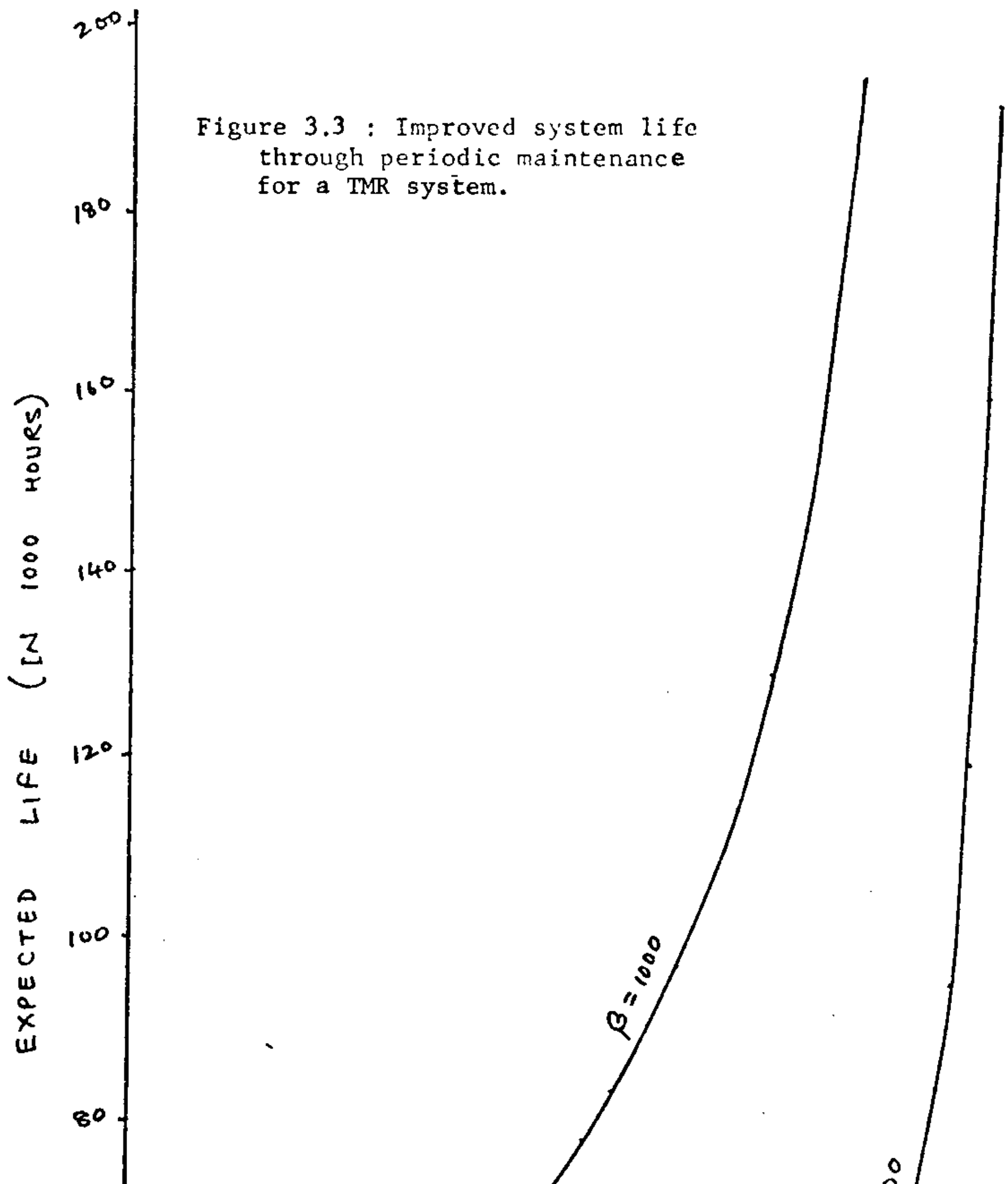


Figure 3.3 : Improved system life through periodic maintenance for a TMR system.