NETWORK FLOW MODELS FOR HEAT EXCHANGER NETWORK SYNTHESIS: PART 3 - SOLUTIONS WITH STREAM SPLITTING AND CYCLIC STRUCTURE

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Jaime Cerda and Arthur W. Westerberg

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Jaime Cerda*

and

Arthur W* Westerberg

Department of Chemical Engineering
Carnegie-Mellon University
Pittsburgh, PA.15213

This work was funded by NSF Grant No- CPE-78O189

* Current Address: INTEC
Guemes 3450
3000 Santa Fe
Argentina

ABSTRACT

This paper shows how to draw the heat exchanger networks which correspond to the minimum match, minimum utility solutions discovered in Part 2, even if the networks require the splitting of streams and/or a cyclic structure. The network flow representation of the synthesis problem can admit several alternate optimal solutions involving the same set of active stream/stream matches (c_i, h_j) and energy flows q_{ij}, but only one of them is a clear representation of the network design for which it stands. To find it, this paper presents a row assignment rule. If the neat exchanger network found is cyclic or includes stream splitting, the optimal solution tableau provided by the row assignment rule indicates the matches which are to be performed in multiple units, the process streams to split and their split ratios and the arrangement of the heat exchangers in the network. Based on that information, a rather simple systematic procedure is presented to draw a preliminary network*. Additional merging rules show how to reduce the number of units in this initial network. Several examples are solved where stream splitting or cyclic structures are required.

INTRODUCTION

As remarked by Nishida et al. (1977), most of the proposed synthesis techniques have no explicit provision either for the use of stream splitting or for the generation of a cyclic network in which a stream is exchanged more than once with another stream. However, both cyclic structure and stream splitting are ordinary features of economic networks for industrial problems such as a crude unit preheat train. In the past, this type of problem was chosen by several authors to test their proposed network synthesis algorithms. Thus, Kobayashi et al. (1971) studied the synthesis of a heat exchange system around the topping unit in a refinery. Crude oil was the only cold stream to be heated and several hot streams such as naptha, kerosene, gas, oil, etc. were to be cooled. A similar type of problem was later used by Cena et al. (1977) to illustrate their optimal assignment synthesis method. In both cases, the economic network design was highly cyclic and included splitting of the cold stream at several different points in the network.

Two algorithmic-evolutionary approaches account also for the use of stream splitting and/or cyclic matches. They are due to Nishida et al. (1977) and Linnhoff and Flower (1978). Recently, Flower and Linnhoff (1980) developed the thermodynamic combinatorial (TC) approach which is capable of discovering all of the network designs requiring minimum utility usage and comprising a minimum number of units as given by Hohmann's rule. A major disadvantage of the method is that it will yield no answer if cyclic structures and/or stream splitting are features of the economic networks. Such a situation arises in practical problems where full heat recovery is difficult to achieve unless certain streams are split or more than the minimum number of units must be used. Part 2 presented a systematic method to synthesize a maximum energy recovery network where a minimum number of stream/stream matches is accomplished. It did this by introducing the following "network flow" related problems described briefly for the convenience of the reader.

Fully partitioned problems

- PI: Minimum utility problem, with streams partitioned
- P2: Minimum match/minimum utility problem, with streams partitioned
- $\overline{P2}$: P2 but with integer constraints ignored on y_{ij}
- P3: $\overline{P2}$ with variables y_{ij} substituted out

Merged problems

- $P2^{f}$ P2 but with merged surrogate constraints — i.e. streams not partitioned. Leads to much reduced problem size.
- $\overline{P2^{1}}$: $P2^{l}$ but with integer constraints ignored on y_{ij}
- P4 $\overline{P2^{1}}$ with variables y_{ij} substituted out.

Once a maximum energy recovery basis, i.e. a basis for problem P4, has been discovered, the next step is to draw the related network structure. Since P4 is a relaxation of P2 there generally are several alternate optimal solutions to the fully partitioned problem P2 giving the same basis for merged problem P4« Among them there always exists one $(q_{ik,jk^*})^*_{opt}$ which is a clear representation of the network design. Besides the ordering of the active matches in the network, such a solution tableau $(q_{ik,jk^*})^*_{opt}$ can indicate if either stream splitting or multiple units for an active match is required in the network due to thermodynamic constraints.

Usually, an optimal solution to problem P2 includes several positive elementary matches $q_{ik,jl}$ between a hot and a cold stream (c_i, h_j) at different temperature levels (k or l). The P2-optimal tableau $\{q_{ik,jl}\}_{opt}$ which is a picture of the network structure describes a match as a sequence of positive elementary matches of the form:

$q_{i(k+1),j(l+1)}$; $q_{i(k+1),jl}$; $q_{ik,jl}$; $q_{ik,j(l-1)}$; ... . Such a sequence implies that the temperature level of each process stream involved in the match (c_i, h_j) generally varies along the heat transfer unit which implements it. The process of merging all or some of those positive elementary matches $q_{ik,jl}$ into a single larger one that can be accomplished in a heat exchanger is obviously much simpler if the appropriate solution tableau $\{q_{ik,jl}\}_{opt}$ is used.

The Row Assignment Rule

In order to develop a picture of the network design structure associated with a maximum energy recovery basis, a simple computational procedure is suggested which we call the *row assignment rule*. The merged q_{ij}-tableau describing the basis is the type of information the row assignment rule needs to derive the sought P2-optimal tableau.

At the bottom of Table 1 a merged q_{ij}-tableau shows a maximum energy recovery basis for problem 5SP1. Above it is an associated P2-transportation tableau where the row assignment rule is implemented. In it, rows and columns have been ordered as shown is Part 2 of this paper. Starting with the first row, from left to right, the row assignment procedure allocates heat flow units exclusively to active routes carrying energy to that destination until its demand is met. In the course of the assignment process, however, one cannot exceed the values shown in the merged q_{ij}-tableau. For the first row c_{34}, for instance, there is a single active route available (c_{54},S). The procedure allots as many flow units to that route as possible, i.e. min (130, 888) = 130. In doing so, the match (c_5,S) has been made active. Therefore, any further heat flow demand by cold stream c_5 in the following rows should be satisfied by "columns" standing for the utility S until the specified value $q(c_5,S)$ from the q_{ij}-tableau is reached. The next row c_{14} includes a sole active route (c_{14}, h_{24}) to which min(114, 731, 615) = 114 is alloted.

When row c_{13} is considered the first available active route involving h_2 is (c_{13}, h_{24}). We can allocate up min(1151, 731 - 114, 615 - 114 = 501) = 501 units for this match (c_1, h_2). This assignment scheme is continued up to satisfying the last row demand. In this way one obtains the P2-solution tableau shown in Table 1.

If for thermodynamic reasons the splitting of a cold stream is required, the assignment procedure will be unable to meet the row demand by only using the "columns" standing for the hot stream already in operation. Thus such a cold stream will require matching with additional hot streams as many as necessary. First, one should consider those hot streams not already matching with another cold stream, proceeding in the order the hot streams show up in the tableau. If no such hot streams exist or they cannot completely meet the heat flow demand, then hot streams which have already started matching other cold streams must be used. When two hot streams match a cold stream in parallel the assignment procedure for the next rows considers first the one which started matching in the earliest row.

In the same way, one can define a *column assignment rule.* Usually both the row and the column assignment rules provide the same P2-optimal tableau. Rarely will both rules fail to provide a feasible P2-tableau. If no feasible P2-tableau results, the heat flow demand for a certain row will require additional auxiliary heating (see Table 1), To get the desired P2-optimal tableau all heat flow units assigned to matches involving any of the two extra auxiliary sources must then be systematically removed. We are facing a utility usage problem where the minimum utility requirement is equal zero (Cerda et al., 1981). For this problem, the row or column assignment rule would provide a very good initial solution, one which is generally an optimal one. In Table 1 there was no need for such extra auxiliary sources, and the row assignment rule developed a P2-optiraal tableau.

Drawing the Network Design Structure

By keeping track of the active elementary matches involving a particular cold (or hot) process stream in the P2-optimal tableau provided by the row assignment rule, one can note the route by which it travels through the heat exchanger network. It starts from its highest temperature interval and goes to its lowest one, moving from left to right. For instance, Table 1 suggests that the stream Cj matches last with h^. It matches immediately before with h_2. It also indicates that hot stream h_4 is matched in the network first with c_3 and then with c_1. While keeping track of a process . stream, one could merge those adjacent positive elementary matches which comprise the same partner hot (or cold) stream. However, the merging procedure is not so simple because there are certain constraints which prevent certain elementary matches from being merged.

Every time the P2-optimal tableau provided by the row assignment rule includes active elementary matches among a cold (or hot) and two or more hot (or cold) process streams, with <u>all</u> of the streams at the <u>same</u> temperature interval, *a ApJULt o£ the common CJOJXL (OX. hot) tiyieam may, be. mc&MCviy, fx>/i the rvetwo/ik de*<Lgn to be fieaAiJbte.* In Table 1, that situation arises in a single occasion, i.e. h^ giving thermal energy

simultaneously to c_{53} and c_{3}. All three streams are at level 3. Therefore, a split of the stream h_4 may be needed. In case the split point is really required, the merging procedure should initially *ki~p* the elementary matches which may cause it to avoid developing an infeasible network. We avoid merging matches (c_{53}, ^43) and (c_{13}, h_{43}) with others for the moment.

To enhance the merging process among the *eMLgMbZz* elementary matches, one can switch the positions of $q_{IK,J*}$ and q_{mpjl} along the h_j-route if $I > k$ and $I > p$. Such a move is always thermodynamically feasible. In the same way, two active elementary matches $q_{ik,jl}$ and $q_{iK,rs}$ along the c_i-route can switch positions if $k < 1$ and $k < s$. For example, we could interchange the order for the (c, h_{24}) and (c_{33}, h_{24}) matches along the h_4-route in Table 1 if it were useful to do so.

Testing Possible Stream Split Points (I)

Generalized pictures of the special cases which may require one to split the process stream c_{il} in the network is shown in Figure 1. If the structure depicted in Figure 1 admits the sequential rearrangement shown in Figure 2, the potential split point is unnecessary and the procedure to merge elementary matches can be applied to matches $q_{,J}$ and q too. In Figure 2, the hot stream h_j is the one matching later at higher temperatures with the cold stream c_i The difference between Figure 2 and Figure 1 is that we match c_{il} with h_{jl} after a match with h_{kl} rather than in parallel. The first match with h_j is obviously safe in Figure 2 if it was in Figure 1. However, the stream c_t exits this first match with h_{jl} somewhat heated, to T^*. If c entering the second match at T^* is 100 hot to be counter to the exit temperature for $h_{..}$ in the second match, then the match in sequence rather than in parallel is not feasible. Thus the sequential arrangement given in Figure 2 is feasible if and only if

$$T^*_{j\ell} - T^*_{i\ell} \geq \Delta T_m$$

where ΔT_m is the minimum allowed approach temperature. At the other points in the subnetwork, the temperature driving force is always greater than or equal to ΔT_m in Figure 2 if it was in Figure 1. From Figure 2, it follows that

$$T^*_{j\ell} = T^{(\ell)}_{out} + \Delta T_m - (1/F_{h_j}) \left(q_{i\ell,j\ell} + \sum_c q_{c\ell,j\ell} \right)$$

and

$$T^*_{i\ell} = T^{(\ell)}_{in} + (1/F_{c_i})\ (q_{i\ell,k\ell})$$

where F_{h_j} and F_{c_i} are the heat capacity flow rates of the process streams h_j and c_i respectively. Furthermore, the summation is extended over all the cold streams other than c_i exchanging heat with h_j at temperature level ℓ

$$T^*_{j\ell} - T^*_{i\ell} = T^{(\ell)}_{out} - T^{(\ell)}_{in} + \Delta T_m - (1/F_{h_j}) \left(q_{i\ell,j\ell} + \sum_c q_{c\ell,j\ell} \right) - (1/F_{c_i})\ q_{i\ell,k\ell} \geq \Delta T_m$$

Then

$$\Delta T^{(\ell)} - (1/F_{h_j}) \left(q_{i\ell,j\ell} + \sum_c q_{c\ell,j\ell} \right) - (1/F_{c_i})\ q_{i\ell,k\ell} \geq 0$$

where:

$$\Delta T^{(\ell)} = T_{out}^{(\ell)} - T_{in}^{(\ell)}$$

$T_{out}^{(\ell)}$ and $T_{in}^{(\ell)}$ are the outlet and the inlet temperature for the temperature interval ℓ. Then:

$$(1/F_{h_j}) \left(q_{i\ell,j\ell} + \sum_{c} q_{c\ell,j\ell}\right) + (1/F_{c_i})\, q_{i\ell,k\ell} \leq \Delta T^{(\ell)} \qquad (1)$$

It inequality (1) is not violated, active elementary matches $q_{i\ell,k\ell}$ and $q_{i\ell,j\ell}$ can be arranged in a sequence on the c_i-route, and the merging procedure can be applied to them too. A similar result is possible:

$$(1/F_{h_j})\, q_{i\ell,j\ell} + (1/F_{c_m}) \left(q_{m\ell,j\ell} + \sum_{h} q_{m\ell,h\ell}\right) \leq \Delta T^{(\ell)} \qquad (2)$$

where c_m is the second (colder) stream to be matched against hot stream h_j in the sequential arrangement. The sum in inequality is over all other hot streams exchanging heat with c_m at level ℓ.

If the inequality (2) is not violated, the elementary matches q. ._ and q.a .a can be arranged in a sequence along the h_j-route; and the merging procedure can be applied to them too. Otherwise, the stream h_j must be split in the network.

We are facing the latter case in Table 1 where the hot pseudostream h_4^3 matches with the cold pseudostreams c_5^3 and c_1^3. Since c_5^3 matches earlier with h_4^3, c_1 plays the role of c^1 in inequality (2), i.e. it will be the later match which may not be thermodynamically possible. Both elementary matches (*Vj, C53) and ^43' ci3^ can be placed in a sequence if inequality (2) is not violated:

$$(1/F_{h_4})\, q(c_{53},h_{43}) + (1/F_{c_1})\, q(c_{13},h_{43})$$

$$= (1/13.29)\ (558) + (1/11.4)\ 650 = 99 < \Delta T^{(3)} = 195 - 94 = 101$$

The temperature intervals for the problem 5SP1 are given in Table 2. Therefore, elementary matches (c_{53}, h_{43}) and (c_{13}, h_{43}) can be sequentially arranged and the merging process can include them. At every process stream route, now, all of the elementary matches for the same pair (c_i, h_j) can be combined into a single one which is accomplished in a single heat transfer unit, as we already knew (see Part 2).

In order to illustrate the use of inequalities (1) and (2) to derive the network design structure from the (P2)-optimal tableau, several example problems are now solved. In all of them, the split ratios in which to divide the process streams are obtained from the (P2)-tableau. We will find that the network structures which are so developed may be improved by reducing further the number of heat exchangers in the structure.

The Four-Stream Problem Introduced by Linnhoff and Flower (1978a)

The four-stream problem reported by Linnhoff and Flower (1978a) as test case No. 2 is a particular instance where Hohmann's lower bound on the number of units in a network fails. The relevant data for the problem and the set of temperature intervals to partition the process streams are listed in Tables 3 and 4. The solution of problem (P3) and the subsequent implementation of the procedure to find additional minimum match networks produced the two feasible solutions described in Tables 5 and 6. Since the first of them, that is solution No. 1, minimizes the (P3)-objective functic- it was used as the starting solution in the search for other minimum match solutions to the network synthesis problem. The introduction of the non-basic cells (c_3, H) and (C, h_2) were ruled out by the initial test. The single alternative remaining is to move heat units around the cycle (c_1, h_2> c_3«, h_4) until one reaches the upper-bound for another match in the cycle, i.e. (c_1, h_4). Note the cycle cannot be eliminated. By moving the heat units as just described, solution No. 2 was found. The (P2)-solution tableau produced by the row*assignment rule for both minimum match solutions are also shown in Tables 5 and 6, respectively.

Before attempting to carry out the elementary match merging procedure, one should analyze the (P2)-solution tableau to look for possible process stream split points. For solution No. 1, the (P2)-tableau suggests two potential process stream split points. One of them is where cold stream c_1 exchanges heat with hot streams h_2 and h_4 at temperature interval 2. The other one where stream h_4 matches cold streams c_1 and c_3, also at level 2. The use of inequality (I) proves that the sequential arrangement shown in Figure 2 is infeasible and the split point for c_1 is unavoidable:

$$(1/2)\ (160) + (1/3)\ (80) - 106.66 > AT^{(2)} .- 140 - 60 \gg 80$$

Inequality (2) also shows that a split point for h_4 should appear in the network:

$$(1/4)\ (80) + (1/2.6)\ (182) * 90 > AT^{(2)} \gg 80$$

The merging procedure suggested before should not be applied to the elementary matches (c_{12}, h_{22}^' ^c_{12}* h_{42}^ and ^c_{32}* h_{42}^ causing the split points for streams ^_ and h_4 at the temperature level 2. Such a merging procedure consists in keeping track of each process stream from its inlet to its outlet temperature level in Table 5 and merging those adjacent positive elementary matches which comprise the same partner process stream and do not cause any split point. The implementation of the elementary match merging scheme yields a set of eight combined matches each of which can be accomplished in a single heat exchanger. In this way, the network design structure shown in Figure 3 is obtained. It is a cyclic, split network.

For the process stream to split, the flow rate fraction going through each branch of the sub-network comprising the heat transfer units arranged in parallel is proportional to the amount of heat exchanged along it. Thus, the fraction of stream c_1 required in heat exchanger No. 5 is given by:

$$80/(160 + 80) - (1/3) \ll 0.333$$

while the fraction of h_4 through the same unit is:

$$80/(80 + 182) - 0.308$$

However, the number of units in the network shown in Figure 3 can be further reduced by allowing *ov&ihecLtijig,* of the cold stream c_1 along one of its branches in the network. Such a possibility appears when heat transfer units, like heat exchangers 2 and 7, which immediately follow the merging point of the divided process stream carry out the same type of match (c_i, h_j) accomplished by one of the units arranged in parallel. That is the case for the pairs of units 2 and 3, and 6 and 7, which perform the matches (c_1, h_2) and (c_3, h_4), respectively. The merger of units like 2 and 3 is made *tvhLie keeping, the. ptaation o{L the. -ttsiexim c_1 going through that branch in the. mtwoik unchanged.* In this way that portion of c_1 is overheated (see Figure 4). By doing the same procedure, heat exchangers 6 and 7 can be combined into a single unit but the mixer is in this case not strictly required. Although it improves the temperature driving force in units 1 and 5, the mixer may be removed from the network by simply adjusting the split ratio for stream h_4 (see Figure 4). Figure 4 depicts a maximum energy recovery network comprising the least number of units that is capable of reaching the specifications for Linnhoff and Flower's test case No. 2.

For minimum match solution No. 2, the (P2)-solution tableau in Table 6 indicates that streams h_4 and c_3 may have to split at temperature level 2 to make the network feasible. The use of inequality (2) to test whether or not such a split point for h_4 is really required shows that

$$(1/4)\ (240) + (1/2.6)\ (22 + 160) = 130 > \Delta T^{(2)} = 80$$

and h_4 should be split. Inequality (1) is also violated when applied for c_3:

$$(1/2)\ (160) + (1/2.6)\ (22) = 88.46 > \Delta T^{(2)} = 80$$

and therefore the network will have to contain a pair of process stream split points. The merging procedure applied to the active elementary matches, except those involved in the split points, leads to the network structure shown in Figure 5. The number of units in the network can be reduced by two when the merging process is completed as just described for the minimum match solution No. 1. No overcooling or overheating are strictly needed and the additional joining of units is achieved by merely adjusting the split ratios for h_4 and c_3 (see Figure 6).

The Sixth Maximum Energy Recovery Basis for Problem 5SP1

Through the searching technique introduced in Part 2 of this paper, six maximum energy recovery bases were found for Problem 5SP1. As said there, one of them was not reported by Flower and Linnhoff (1980) because it does not represent an unsplit network design. Using the row assignment rule and subsequently the merging procedure, a network design can be derived for this alternative.

The analysis of the P2-optimal tableau developed through the row assignment rule and shown in Table 7 indicates four possible stream split points. Two of them are related to hot stream h_2 because of its heat exchange with cold streams c_5 and c_1 at temperature levels 4 and 3. Inequality (2) assures that both are required in the network. Here, it is convenient to introduce another easy-to-prove rule for merging active elementary matches:

> Two split points required for a hot (or cold) process stream (h_2) at successive temperature levels (levels 4 and 3) can always be merged into a single split if (1) the split point at a lower temperature (level 3) is at least caused by the same set of cold (or hot) streams (streams c_1 and c_5) causing the one at a higher temperature (level 4), (2) between split points the stream (h_2) to split only exchanges heat with some or all of the streams causing the splitting.

All the active elementary matches for each pair (c_i, h_j) at and between the upper and lower split points can be merged into a single match and then the combined matches are arranged in parallel. Thus, active elementary matches (c_{54}, h_{24}), (c_{53}, h^) and (c_{53}, h^) are merged into a single match and arranged in parallel with the combination of (c_{14}, h_{24}) and (c_{13} h_2^). Such a procedure <L* aAvaysi tkejunodynarriicaLLy, feasible.
Each combined match can be performed by a single heat exchanger.

The third split point which might be required is for stream c_1 at level 3, and it is due to hot streams h_2 and h_4. The use of inequality (1) verifies that the split is needed.

(1/16.62) (829 + 567) 4- (1/11.4) (584) - 135.22 > AT$^{(3)}$ « 101

Table 7 also indicates a possible split point for stream h_4 at level 3 caused by cold stream c_1 and c_3. However, inequality (2) shows that the active elementary matches (c-^, ^43) anc* ^ciV ^43^ can ^e secluent:ially arranged:

(1/13.29) (584) + (1/12.92) (249) » 63.21 < AT$^{(3)}$ » 101

Next, we merge adjacent active elementary matches not causing split points and involving the same pair (c_i, h_j). An interesting situation arises when the elementary matches between process streams c_3 and, h_4 are joined as well as those between c_1 and h_4. A cyclic sub-network is generated where stream h_4 matches successively with c_3, then with c_1, again with c^ and finally with c_1. There is an easy way to check if such a cyclic substructure is unnecessary based upon the P2-tableau. We switch the positions of rows c1<? and c_{32} and reapply the row assignment rule to generate the P2-optimal tableau shown in Table 8, one which does not include a cyclic sub-network. The merging procedure yields the network design depicted in Figure 7 which contains one more unit than Hohmann's lower bound.

Testing Possible Stream Split Points (II)

Sometimes, the splitting of a cold stream c_i in a network may be required because it exchanges simultaneously heat with three or more hot streams at the same temperature level ℓ. For instance, with h_j, h_k and h_m. To check if the match $(c_{i\ell}, h_{m\ell})$ can be arranged in a sequence with respect to the matches $(c_{i\ell}, h_{j\ell})$ and $(c_{i\ell}, h_{k\ell})$ as shown in Figure 8, one must verify whether the inequality (1) when applied to both pairs $(c_{i\ell}, h_{m\ell}/h_{j\ell})$ and $(c_{i\ell}, h_{m\ell}/h_{k\ell})$ is obeyed. If not so for one of them, for instance $(c_{i\ell}, h_{m\ell}/h_{j\ell})$, it may still be possible that a substructure showing $(c_{i\ell}, h_{m\ell})$ and $(c_{i\ell}, h_{k\ell})$ in a sequence but both in parallel to $(c_{i\ell}, h_{j\ell})$ be feasible (see Figure 9). Before using inequality (1) to check the feasibility of such a sequential substructure one should multiply F_{c_i} by the factor α given by:

$$\alpha = \frac{q_{i\ell,k\ell} + q_{i\ell,m\ell}}{q_{i\ell,k\ell} + q_{i\ell,m\ell} + q_{i\ell,j\ell}}$$

where α is readily derived from Figure 9. Similar conclusions are drawn if the stream to split is to be cooled.

The Four-Stream Problem 4SP2 (Ponton and Donaldson, 1974)

The four-stream problem 4SP2 introduced by Ponton and Donaldson (1974) is another example where both goals, the least utility usage and the minimum number of heat exchangers in the network, can only be reached in a split network. The set of relevant data and the temperature intervals to partition the process streams for this problem are listed in Tables 9 and 10. By using the synthesis procedure explained in Part 2 one finds a unique feasible maximum energy recovery basis (see Table 11). The row assignment rule provides the P2-solution tableau also shown in Table 11.

The P2-optimal tableau suggests that stream c_1 may have to be split at level 1 due to its heat exchange with hot streams h_2, h_3 and h_4. On applying the row assignment rule the hot stream h_4 first matches with c_1; then, we should first verify if match (c_{11}, h_{41}) can be arranged in a sequence with the other two elementary matches at level 1, For the pair (c_{11}, h_{41}) and (c_{11}, h_{21}) inequality (1) is disobeyed:

$$(1/36.93)\ (2564) + (1/10.55)\ (1709) = 231.42 > \Delta T^{(1)} = 199$$

but an opposite result is attained for the other pair:

$$(1/36.93)\ (2564) + (1/26.38)\ (2506) = 164.42 < \Delta T^{(1)} = 199$$

Therefore, an arrangement similar to that shown in Figure 10 is not feasible. However, there still is a chance that a sub-network like that in Figure 11 be feasible. As said before, one should first multiply F_{c_1} by:

$$\frac{2564 + 2506}{2564 + 2506 + 1709} = 0.748$$

to apply inequality (1). Then:

$$(1/36.93)\ (1/0.748)\ (2564) + (1/26.38)\ (2506) = 181.78 < \Delta T^{(1)} = 199$$

Inequality (1) is satisfied and the sub-network shown in Figure 9 is feasible. Therefore, elementary matches (c_{11}, h_{21}), (c_{11}, h_{31}) and (c_{11}, h_{41}) are to be skipped by the initial merging procedure and arranged as shown in Figure 9. The result is depicted in Figure 10. The subsequent merger of units 2 and 4, and 3 and 5 yields the network illustrated in Figure 11. Both mergers are feasible only if portions of c_1 are overheated and the mixer is strictly required. It is important to remark that the merger of units is made without changing the flow rate of c_1 in each branch of the network.

If elementary matches (c^** ^21^* ^cll^f ^31^ an(* ^ll* ^41^ are all arranged in parallel, one can successively derive the heat exchanger networks depicted in Figures 12 and 13 following the same procedure applied before. Figures 11 and 13 show split heat exchanger networks comprising a minimum number of active matches. Both were already reported by Linnhoff and Flower (1978b).

The General Network Design Synthesis Algorithm

New steps are to be added to the network design synthesis algorithm proposed in Part 2 of this paper to make it capable of dealing with situations where stream splitting or multiple units for an active match are mandatory to achieve maximum energy recovery. Their goals will be explained while developing the network structure for the lowest cost solution to the five-stream example introduced by Cena et al. (1977). Mass flow-rates, supply/target temperatures as well as the thermal properties within the range of interest for each process stream in such a test problem are listed in Tables 12 and 13. Table 14 indicates the necessary set of temperature intervals to partition the process streams, while Table 15 lists the set of new hot and cold pseudostreams. Upper bounds on the amount of heat to assign to each match are displayed in Table 16.

By solving the linear transportation problem P3 described in Part 2, the solution shown in Table 17 is found. Such a solution contains a cycle which proves to be unbreakable by using the Reverse Stepping Stone Method. Thus, active matches (c_1, h_1) and (C, h_1) must stay in the cycle even for the relaxation problem P4.

An alternate solution is still possible. We reject one of the remaining active matches (C, h_2) because it has a negative cycle value associated with it. Therefore, we have a single choice, i.e. to remove

(c_1, lu_2) from the cycle. However, the amount of heat involved in match (c_1, lu_2) can at most be reduced to 880 kw if the network is to achieve maximum energy recovery. The new cyclic maximum energy recovery basis is shown in Table 18.

No new optimal solution to P2 can be discovered using the searching technique introduced in Part 2 because by bringing either the match (C, h_4} or (C, h_3) into the basis an upper-bound secondary constraint for the relaxation problem P4 is violated. We now develop the network for the P2-optimal tableau in Table 18, which is the lowest cost network found for this problem and which comprises the fewest matches.

The new steps to be incorporated into the network synthesis algorithm given in Part 1 are:

Step 6: After discovering a maximum energy recovery basis, use the row assignment rule to develop the P2-optimal solution which is a clear representation of the network design.

Step 7: Analyze the P2-solution tableau found in Step 6 to look for possible stream split points.

Table 18 suggests that it might be required to split cold stream c_1 at. four different points in the network. At level 6, c_1 exchanges heat with hot streams h_4 and h_3 which are also at that temperature level. At level 5, the same situation repeats itself with c_1 receiving heat from h^, tu_3 and h«. It seems also necessary to split c_1 again at level 4 because of its matches with hot streams h_4, h_3 and h_1. Finally, c_1 should be partitioned at level 2 due to its heat exchange with h_4 and h_3.

Step 8: Verify if the stream split points discovered in Step 7 are really required by using the inequalities (1) and/or (2).

None of the four stream split points found in Step 7 for the five-stream test problem can be avoided without violating inequality (1).

In each case, unsuccessful further tests were made to establish whether or not a simpler split point may be achieved through the sequential arrangement shown in Figure 9.

Step 9: If possible, merge two split points required for a hot (or cold) process stream at successive temperature levels.

While drawing the network design corresponding to the sixth maximum energy recovery basis for Problem 5SP1, the conditions under which this merging rule can be applied were given. In the five-stream test problem the first and second split points can be joined because all the hot streams causing the lower split point, i.e. h_3 and h_4, originate the higher split point too. Then, all the active elementary matches for each pair (c_i, h_j) at and between the first and second split points are merged into a single match and the two combined matches are arranged in parallel.

Step 10: On the route travelled by a hot (or cold) stream through the network, join all adjacent elementary matches $q_{ik,jl}$ not causing stream splitting. Each process streaa route *is* provided by the P2-optimal tableau found in Step 6. (See Figure 14.)

Step 11: Make additional merging of heat exchangers by allowing overheating of cold or overcooling of hot streaas in the network.

Frequently, one can still lessen the number of heat exchangers by allowing the overheating (or overcooling) of portions of the split cold (or hot) process stream. The merging process should start from the exit (or inlet) section of the partitioned cold (or hot) stream and move toward its inlet (or exit) section in the following way:

(a) Merge all the heat exchangers performing matches between the same pair of process streams (c_i, h_j) that are at and after the last split point. While doing so, the split ratio of the stream to partition is kept unchanged. If the merging process is feasible, go to (b). Otherwise, stop.

Figure 17(a) shows that the merger of heat transfer units obtained by performing Step 11(a) on the network design in Figure 16 is feasible. The number of units is reduced by one, down to ten.

(b) Merge the heat exchangers carrying out matches between the same pair of process streams (c_i, h_j) that are at two successive split points. While doing so, the split ratio of the stream to partition at the split point at lower temperature is kept unchanged. Keep doing (b) until the merging process fails to be feasible or the lowest split point has been reached.

Figure 17(b) shows that Step 11(b) is successfully applied to the network design in Figure 16 causing the merging of the second and third split points. The number of units is decreased by two, down to eight. Further merging fails to be feasible.

In Cena et al.'s best network, the hot stream h_2 reaches the process specifications by exclusively transferring thermal energy to the auxiliary cooling source (Cena et al., 1977). In other words, the match (c_1, h_2) does not show up in that network. From Tables 16 and 17, however, it is clear that the minimum utility requirement can be achieved only if the cold stream c_1 receives an energy flow at temperature level 5 of 880 kw from hot stream h_2. Otherwise, both the heating and cooling demand increase by 880 kw. Therefore, the new network design shown in Figure 16 permits one to reduce the fuel cost by $231,932, i.e. 11.2% of Cena et al.'s network total cost. Data cost given in Cena et al. (1977) were used to estimate the fuel cost reduction. Such a saving does not include those derived from the smaller electrical energy consumption due to the lower cooling requirement as well as those coming from the fact that the new network design comprises two fewer units.

CONCLUSIONS

1. A systematic procedure is presented which permits one to derive the network design associated to each optimal solution of the network synthesis problem P2.

2. The method can be applied even if the solution stands for a network design which is cyclic and/or includes stream splitting. If so, the P2-optimal solution tableau indicates the matches which are to be performed in multiple units, the process streams to split and their split ratios as well as the arrangement of heat exchangers in the network.

3. By incorporating such new features into the algorithm introduced in Part 2, an improved network synthesis method is defined which is then applied to several example problems where economic networks are cyclic and/or must split certain process streams. When the five-stream problem introduced by Cena et al. (1977) is solved, a new lowest cost network is found which contains two fewer units and demands a lower amount of utilities.

4. The new synthesis algorithm handles network designs associated with any of the maximum energy recovery bases for a given synthesis problem, even if some of them stand for cyclic and/or split networks.

5. As in Part 2, the new network synthesis algorithm does not require the process streams to make use of the utilities in its last matching step to reach the output temperature.

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LIST OF TABLES

TABLE 1: The Implementation of the Row Assignment Rule for a Feasible Tree-Type Solution to Problem 5SP1.

		888	731	1396	1342	3S5	120	10,000
		H	h_{24}	h_{23}	h_{43}	h_{42}	h_{41}	h
130	c_{54}	130		I	I	I	I	
114	c_{14}		114	I	I	I	I	
1316	c_{53}	758			558	I	I	
1151	c_{13}		501		650	I	I	
1137	c_{33}		116	1021		I	I	
331	c_{12}				134	197	I	
575	c_{32}			375			I	
308	c_{11}					188	120	
10,000	c_F							10,000

q_{ij}-Tableau

		888	2127	1847
		H	h_2	h_4
1446	c_5	888		558
1904	c_1		615	12S9
1512	c_3		1512	

TABLE 2: Set of Temperature Intervals to Partition the Cold anvl Hot Streams for the Test Problem 5SP1.

Temperature Intervals (k)	Cold Stream Intervals	Hot Stream Intervals
1	38-65	48-75
2	65-94	75-104
3	94-195	104-205
4	195-239	205-249

TABLE 3: Set of Data for the Test Case No. 2 (Linnhoff and Flower (197[illegible]))

Streams	F_i (kW/°C)	T_{in} (°C)	T_{out} (°C)	Q_i (kW)
c_1	3.0	60	180	360
h_2	2.0	180	40	-280
c_3	2.6	30	130	260
h_4	4.0	150	40	-440

$\sum Q_i = 100$

TABLE 4: Set of Temperature Intervals to Partition the Process Streams for the Test Case No. 2 (Linnhoff and Flower (1978a)).

Temperature Intervals (k)	Cold Stream Intervals	Hot Stream Intervals
1	30-60	40-70
2	60-140	70-150
3	140-170	150-180
4	170-180	180-190

TABLE 5: Minimum Match Solution No. 1 **for** Test Case No. 2 (Linnhoff aiui **Flower,** 1978a)).

		60	60	160	320	60	120
		H	"23	"22	"42	"21	h_{41}
30	c_{14}	30	I	I	I	I	I
90	c_{13}	30	60	I	I	I	I
240	c_{12}		0	$\boxed{160}$	80	I	I
182	c_{32}				182	I	I
78	c_{31}				18	60	
160	W	I			40		120

q_{ij}-Tableau

		60	280	440
		H	*h*	h_4
360	#	60	$\boxed{220}$	80
260	c_3		60	200
160	C			.160

TABLE 6: Minimus Match Solution No. 2 for Test Case No. 2 (Linnhoff and Flower, 1978a)

		60	60	160	320	60	120
		H	»23	"22	h_{42}	"21	h_{41}
30	c_{14}	30	I	I	I	I	I
90	c_{13}	30	60	I	I	I	I
240	c_{12}		0		J240	I	I
182	c_{32}			160	Z.-J2 ●	I	I
78	c_{31}			-	18	60	
160	C	I			40		120

q_{ij}-Tableau

		60	280	440
		H	hj	h_4
360	c_1	60	60	$\boxed{240}$
260	c_3		220	40
160	C			160

TABLE 7: (P4)-Tableau for the Sixth Tree-Type Solution to the Problem 5SP1 Obtained Through the Row Assignment Rule.

		888	731	1396	1342	385	120
		*	*24	h_{23}	h_{43}	h_{42}	h_{41}
130	c_{54}		130	I	I	I	I
114	c_{14}		114	I	I	I	I
1316	c_{53}		487	829		I	I
1151	c_{13}			567	584	I	I
1137	c_{33}	888			249	I	I
331	c_{12}				331		I
375	c_{32}				178	197	I
308	c_{11}					188	120

q_{ij}-Tableau

		888	2127	1847
		H	h_2	h_4
1446	c_S		1446	
1904	c_1		681	1223
1512	c_3	888		624

TABLE 8: Another (P4)-Tableau for the Sixth Tree-Type Solution to the Problem 5SP1 Obtained Through the Row Assignment Rule

		888	**731**	1396	1342	385	120
		H	h_{24}	h_{23}	h_{43}	h_{42}	h_{41}
130	c_{54}		**130**	I	I	I	I
114	c_{14}		**114**	I	I	I	I
1316	c_{53}		**487**	**829**		I	I
1151	c_{13}			**567**	584	I	I
1137	c_{33}	888			**249**	I	I
375	c_{32}				**375**		I
331	c_{12}				**134**	197	I
308	c_{11}					**188**	120

TABLE 9: Set of Data for Problem 4SP2 (Ponton and Donaldson, 1974).

Streams	F_i (kw/°C)	T_{in} (°C)	T_{out} (°C)	Q_i (kW)
c_1	36.93	-4	216	8125
h_2	10.55	260	43	-2289
h_3	26.38	221	110	-2928
h_4	15.83	205	43	-2564
			$\sum Q_i$ =	341

TABLE 10: Set of Temperature Intervals to Partition the Process Streams for Problem 4SP2

Temperature Intervals (k)	Cold Stream Intervals	Hot Stream Intervals
1	-4;195	4;205
2	195;211	205;221
3	211;250	221;260

TABLE 11: (P4)-Tableau for the Only Tree-Type Solution to the Problem **4SP2** Obtained Through the Row Assignment Rule.

		341	411	169	422	1709	2506	2564
		H	^ 3	h_{22}	h_{32}	''21	h_{31}	h_{41}
185	c_{13}	**185**		I	I	I	I	I
591	c_{12}	**156**	**411**	24		I	I	I
7349	c_{11}			**145**	422	T709	2506	2564

q_{ij}.-Tableau

		341	2289	2928	2564
		H	h_2	h_3	h_4
8125	c_1	341	2289	2928	**2564**

TABLE 12: Set of Data for the Five-Stream Test Problem Introduced by Cena et al. (1977).

Streams	Flow rate (t/h)	T_{in} (°C)	T_{out} (°C)
c_1	219.0	30	350.0
h_1	180.3	149	37.4
h_2	21.4	210	38.0
h_3	58.5	271	88.0
h_4	79.0	310	80.0

TABLE 13: Specific Heats for Each Process Stream Within the Range of Interest.

Streams	Temperature Range (^oC)	Specific Heat ($kcal/kg^oC$)
c_1	$30 \le T \le 198$	0.580
	$198 \le T \le 350$	0.833
h_1	$92.6 \le T \le 149$	1.259
	$T = 92.6$	$\lambda = 35.04$
	$37.4 \le T \le 92.6$	1.291
h_2		0.580
h_3		0.600
h_4		0.590

TABLE 14: Set of Temperature Intervals to Partition the Process Strearas for the Five-Stream Test Problem Based on AT_m=12°C.

Temperature Intervals (k)	Cbld Stream Intervals	Hot Stream Intervals
1	25.4-30	37.4-42
2	30-80.6	42-92.6
3	80.6-80.6	92.6-92.6
4	80.6-137	92.6-149
5	137-198	149-210
6	198-259	210-271
7	259-298	271-310
8	298-350	310-362

TABLE 15: Set of New Cold and-Hot Pseudostreams for the Five-Stream Tc
Problem.

Streams	C_p (kcal/kg°C)	F_t (kW°C)	T_{in}C°C)	T_{out} <°0	a_{ik} or $b_{j.z}$ (kW)
c_{18}	0.823	212.03	298	350	11026
c_{17}	0.823	212.03	259	298	8269
c_{16}	0.823	212.03	198	259	12934
c_{1S}	0.580	147.63	137	198	9005
c_{14}	0.580	147.63	80.6	137	8326
c_{12}	0.580	147.63	30	80.6	7441
"14	1.259	263.83	149	92.6	-14854
h_{13}	(condensation)		92.6	92.6	- 7343
h_{12}	1.291	270.53	92.6	42	-13689
"11	1.291	270.53	42	37.4	- 1245
"2S	0.580	14.43	210	149	- 880
"24	0.580	14.43	149	92.6	- 814
"22	0.580	14.43	92.6	42	- 730
h_{21}	0.580	14.43	42	38	- 58
"36	0.600	40.79	271	210	- 2488
h_{35}	0.600	40.79	210	149	- 2488
"34	0.600	40.79	149	92.6	- 2301
"32	0.600	40.79	92.6	88	- 196
h_{47}	0.590	54.17	310	271	- 2113
h_{46}	0.590	54.17	271	210	3304
"45	0.590	54.17	210	149	- 3304
"44	0.590	54.17	149	92.6	- 3055
"42	0.590	54.17	92.6	80	- 683
				7 Qi =	- 2544

TABLE 16: Match Upper Bounds and Cost Coefficients for the Five-Stream **Test Problem.**

Match	U_{ij} (kw)	$(1/U_{ij})\times 10^4$	min (a_i, b_j)
$(c_r S)$	26657	0.375	26657
(c_1, h_1)	15767*	0.634	37131
$(c, .h_2)$	2424*	4.125	2482
{C,.bj)	7473	1.338	7473
$(c_v h_4)$	12459	0.803	12459
(W, h_1)	29201	0.342	29201
(W, h_2)	2482	4.029	2482
(W, h_3)	7473	1.338	7473
CV,V	12459	0.803	12459

TABLE 17: Optimal Solution to the Transportation Problem (P4) for the Five-Stream Test Problem.

	H	h_{47}	h_{46}	h_{36}	h_{45}	h_{35}	h_{25}	h_{44}	h_{34}	h_{24}	h_{14}	h_{13}	h_{42}	h_{32}	h_{22}	h_{12}	h_{21}	h_{11}
c_{18}	11026	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
c_{17}	8269		I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
c_{16}	7362	2113	3304	155	I	I	I	I	I	I	I	I	I	I	I	I	I	I
c_{15}				2333	3304	2488	880	I	I	I	I	I	I	I	I	I	I	I
c_{14}							0	3055	2301	814	2156	I	I	I	I	I	I	I
c_{12}											5832		683	196	730		I	I
c											6866	7343				13689	58	1245

q_{ij}-Tableau

		H	h_4	h_3	h_2	h_1
		26657	11776	7473	2482	37131
57001	c_1	26657	11776	7473	2424	7988
29201	c				58	29143

TABLE 18: Another Minimum Match Solution to the Five-Stream Test Problem.

	H	h_{47}	h_{46}	h_{36}	h_{45}	h_{35}	h_{25}	"44	h_{34}	h_{24}	h_{14}	h_{13}	h_{42}-	h_{32}	h_{22}	h_{12}	h_{21}	h_{11}
c_{18}	11026	I	I	I	i	I	I	I	I	I	I	I	I	I	I	I	I	I
c_{17}	8269		I	I	i	I	I	I	I	I	I	I	I	I	I	I	I	I
c_{16}	7362	2113	[3304	T551	*	I	I	I	I	I	I	I	I	I	I	I	I	I
c_{15}				2333	3304	2488	880	I	I	I	I	I	I	I	I	I	I	I
c_{14}	•						0	3055	TS01)		2970	I	I	I	I	I.	I	I
c_{12}											6562		683	196]				
C										814	5322	7343			730	13689	58	1245

q_{ij}-Tableau

		H	h_4	V	h_2	h_1
57001	c_1	26657	11776	7473	880	9532
29201	C				1602	27599

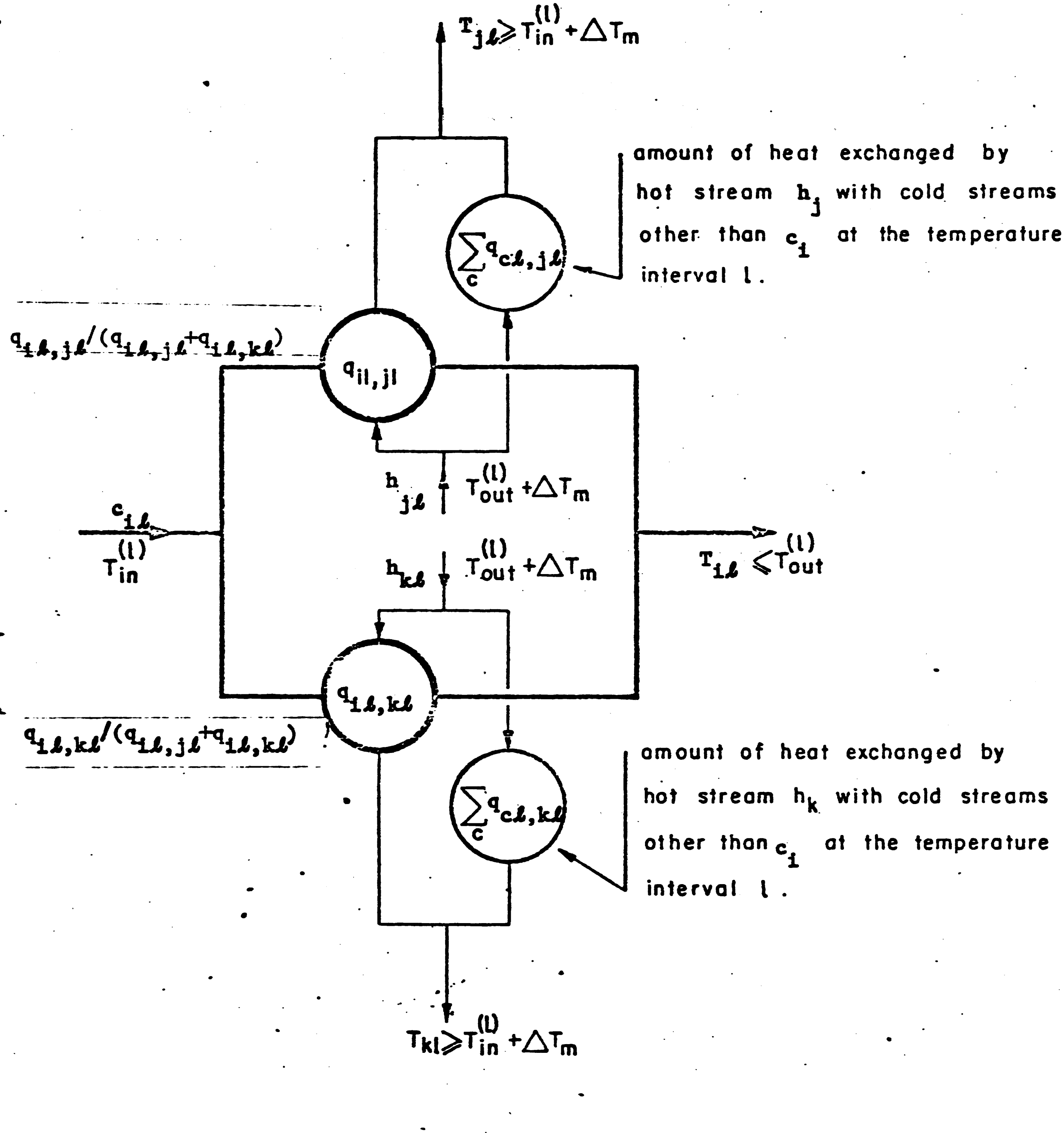
LIST OF FIGURES

11. An Alternate Betwork Design Associated to the P2-Optimal Solution for Problem 4SP2.

12. Preliminary Network Design Associated to P2-Optimal Solution No. 2 for the Five-Stream Test Problem Introduced by Cena et al. (1977).

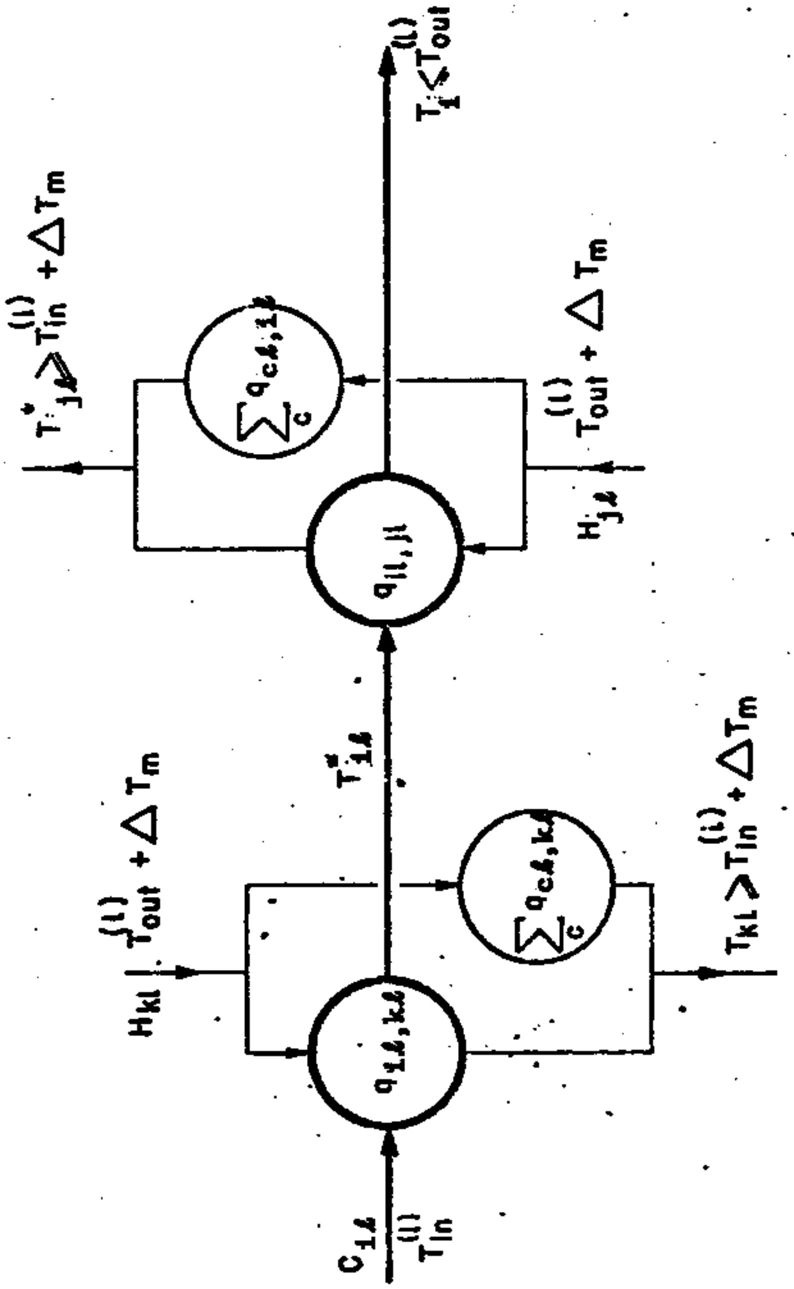
13. Merging Heat Transfer Units by Applying Step 11 of the New Network Synthesis Algorithm.

14. Inproved Network Design Associated to P2-Optimal Solution No. 2 for the Five-Stream Test Problem Introduced by Cena et al. (1977) •

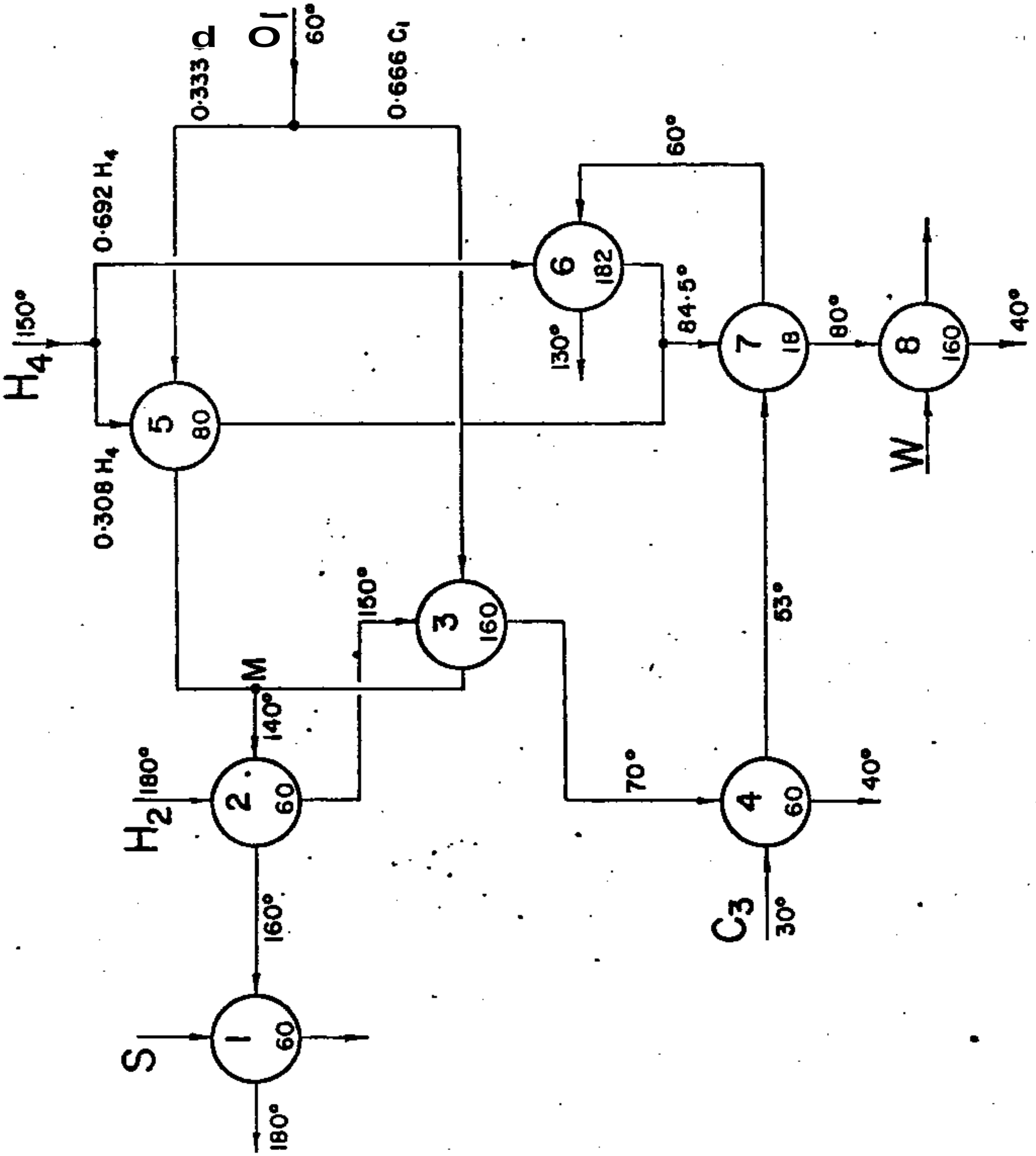


$T_{j\ell} \geq T_{in}^{(l)} + \Delta T_m$
amount of heat exchanged by hot stream h_j with cold streams other than c_i at the temperature interval l.
$\sum_c q_{c\ell,j\ell}$
$q_{i\ell,j\ell}/(q_{i\ell,j\ell}+q_{i\ell,k\ell})$
$q_{il,jl}$
$h_{j\ell}$ $T_{out}^{(l)} + \Delta T_m$
$c_{i\ell}$
$T_{in}^{(l)}$
$h_{k\ell}$ $T_{out}^{(l)} + \Delta T_m$
$T_{i\ell} \leq T_{out}^{(l)}$
$q_{i\ell,k\ell}$
$q_{i\ell,k\ell}/(q_{i\ell,j\ell}+q_{i\ell,k\ell})$
amount of heat exchanged by hot stream h_k with cold streams other than c_i at the temperature interval l.
$\sum_c q_{c\ell,k\ell}$
$T_{kl} \geq T_{in}^{(l)} + \Delta T_m$

FIG. 1



a 2 to



H_4 150°
0.692 H_4
0.333 C_1
O_1 60°
0.666 C_1
60°
6
182
84.5°
130°
7
18
80°
8
160
40°
W
5
80
0.308 H_4
3
160
150°
M
140°
H_2 180°
2
60
70°
4
60
40°
53°
C_3 30°
160°
S
1
60
180°

S_{co}

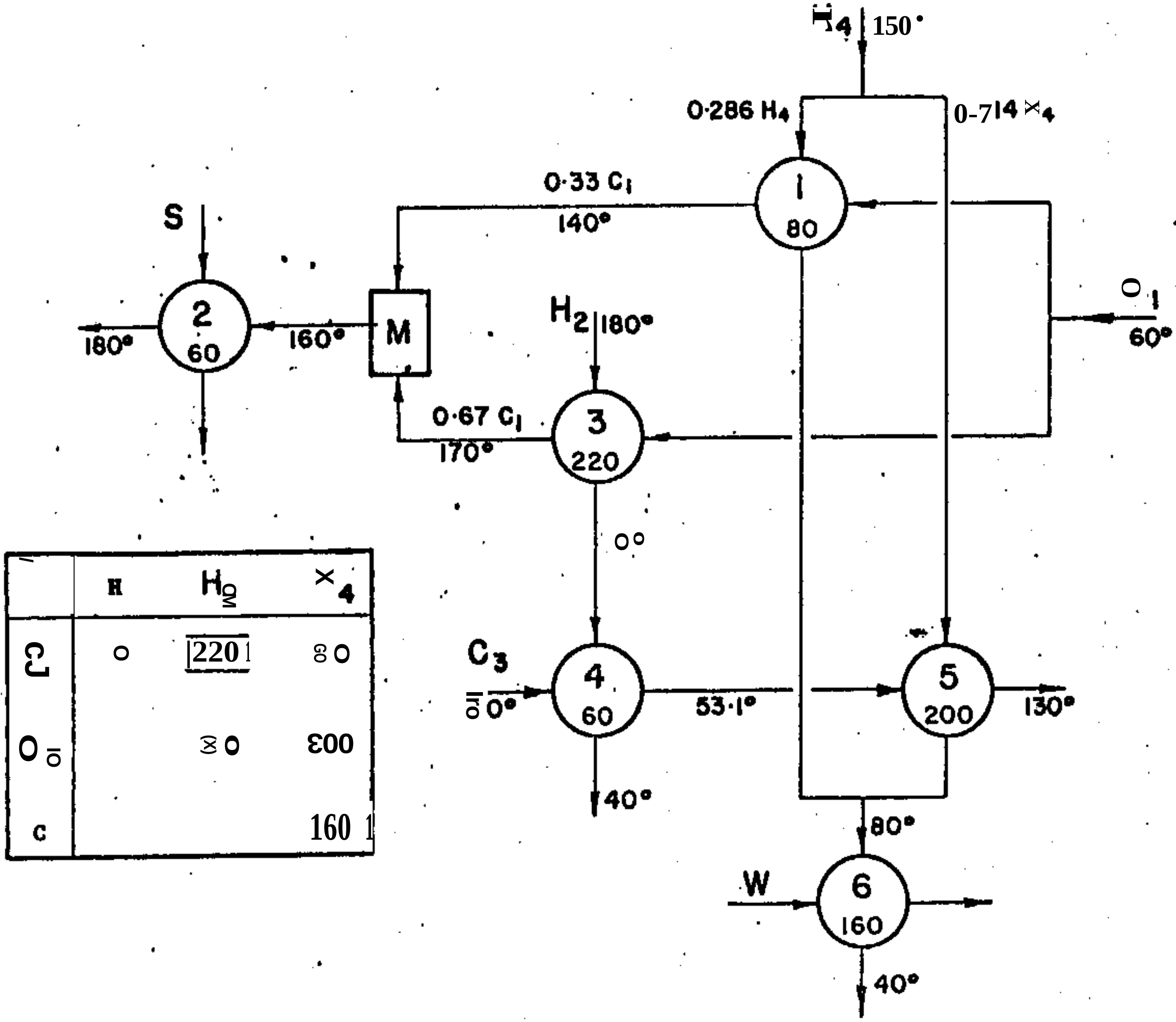

H4 150°
0·286 H4
0-714 X4
0·33 C1
140°
1
80
O1
60°
S
2
60
180°
160°
M
H2 180°
3
220
0·67 C1
170°
4
60
C3
40°
53·1°
5
200
130°
80°
W
6
160
40°

FIG 4

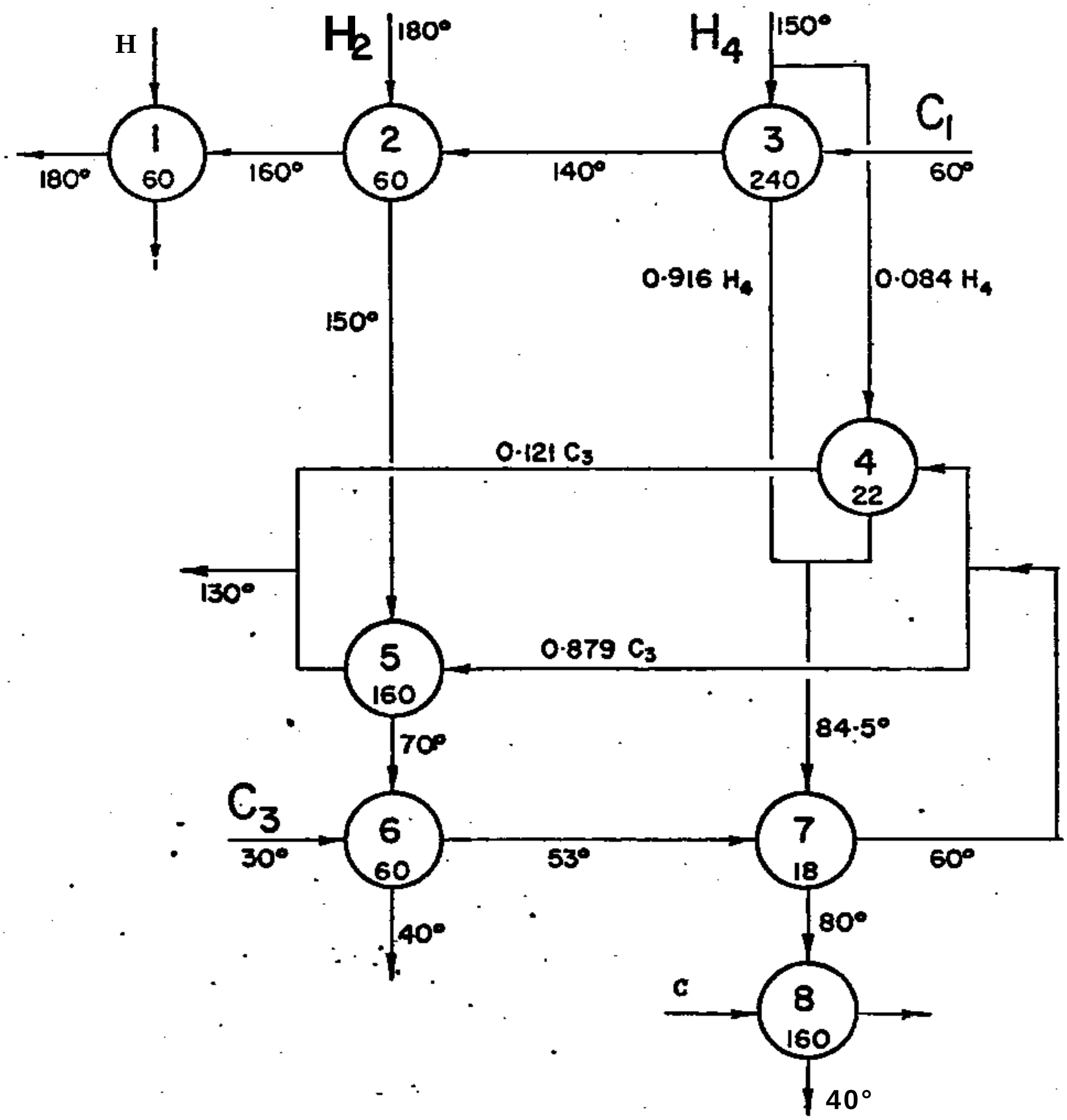

H
H_2
180°
H_4
150°
C_1
1
60
2
60
3
240
180°
160°
140°
60°
150°
0·916 H_4
0·084 H_4
4
22
0·121 C_3
130°
5
160
0·879 C_3
70°
84·5°
C_3
6
60
7
18
30°
53°
60°
40°
80°
C
8
160
40°

FIG 5

	H	X	X_4
C_1	[illegible]	[illegible]	240
C_3		[illegible]	40
[illegible]			180

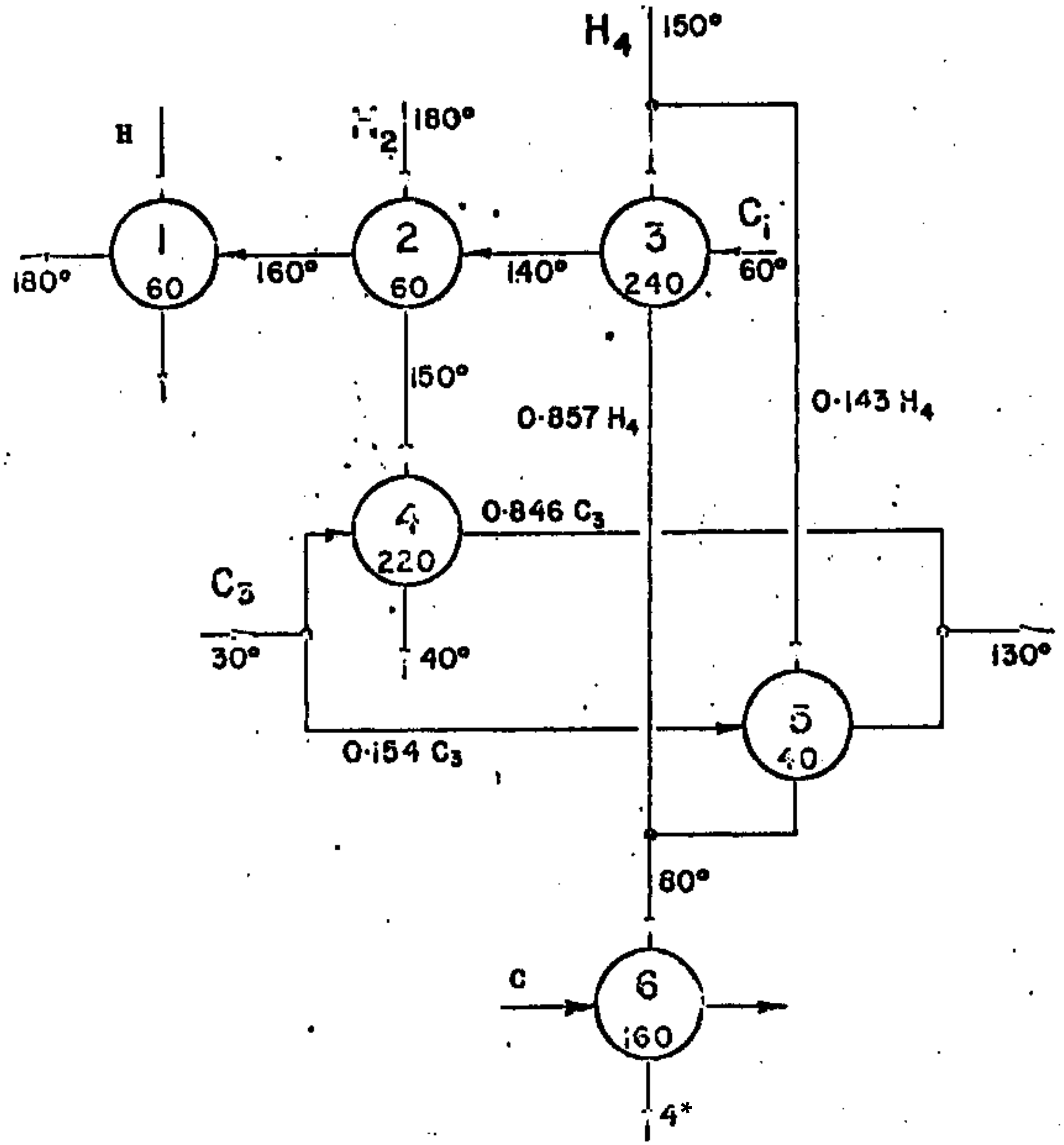
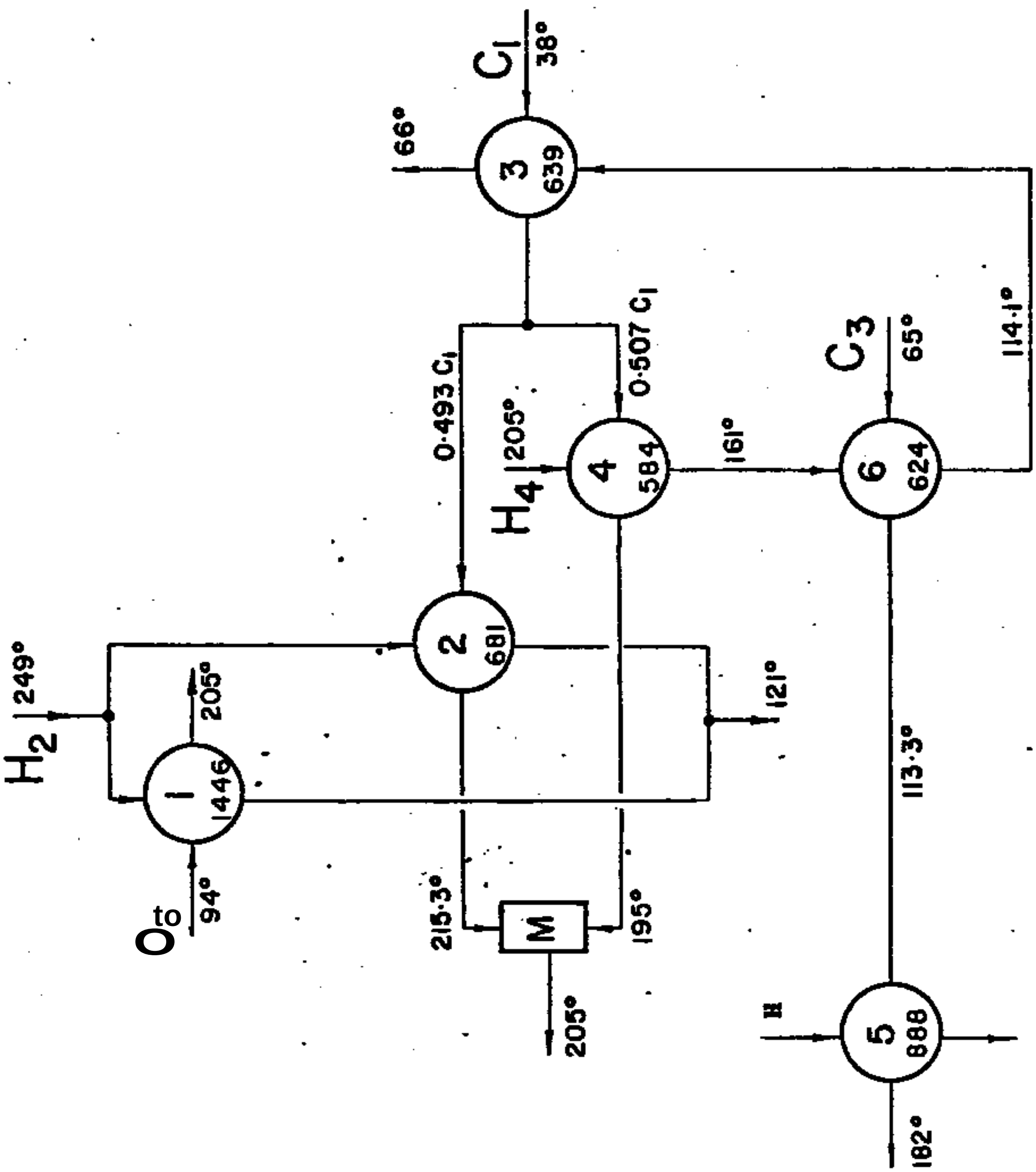
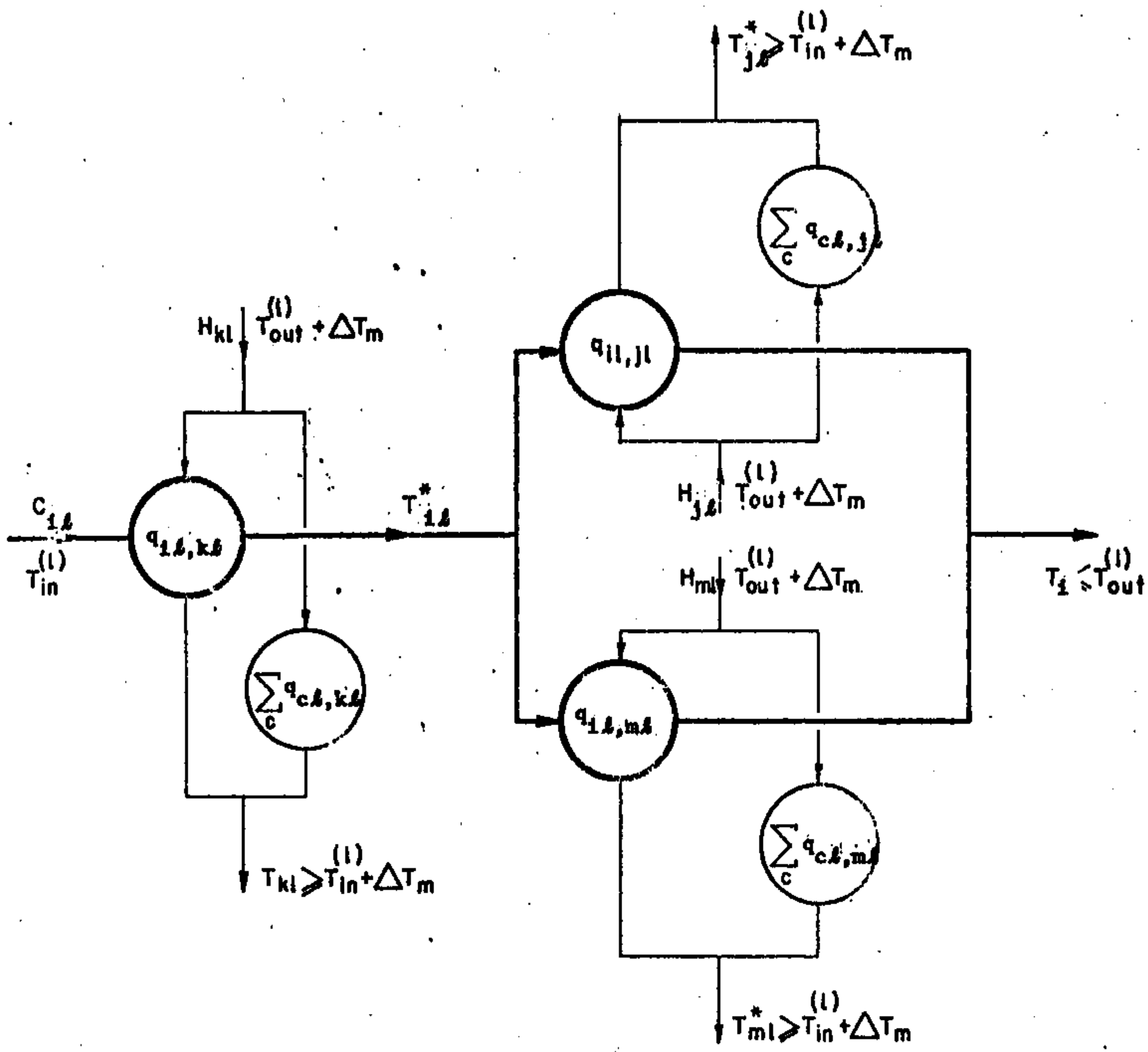


FIG 6



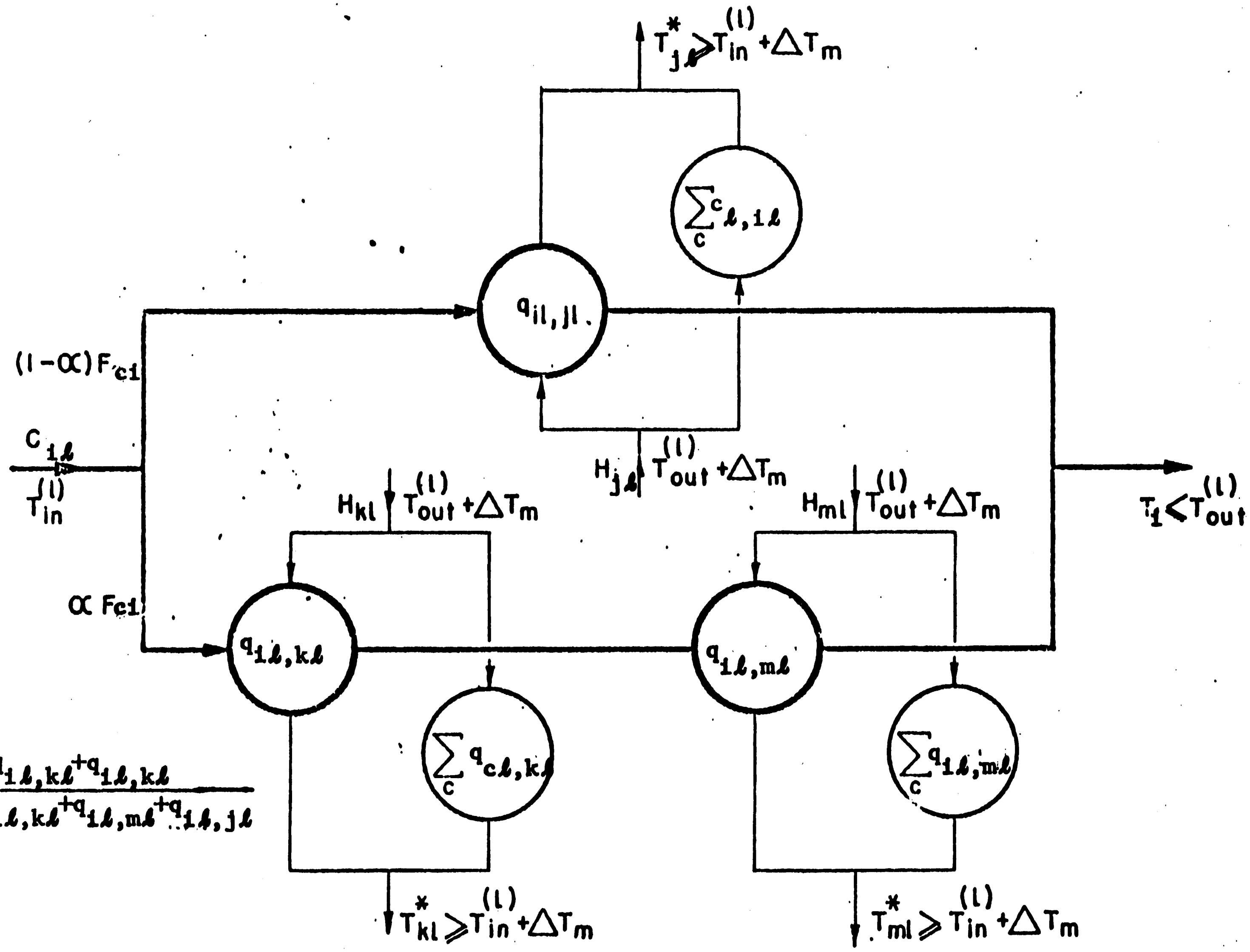
H_2
249°
205°
1
1446
94°
2
681
3
639
C_1
38°
66°
0·493 C_1
0·507 C_1
H_4
205°
4
584
161°
C_3
65°
6
624
114·1°
121°
215·3°
M
195°
205°
113·3°
H
5
888
182°

FIG 7



$C_{i\ell}$
$T_{in}^{(l)}$
$q_{i\ell,k\ell}$
H_{kl} $T_{out}^{(l)} + \Delta T_m$
$\sum_c q_{c\ell,k\ell}$
$T_{kl} \geq T_{in}^{(l)} + \Delta T_m$
$T_{i\ell}^*$
$q_{il,jl}$
$\sum_c q_{c\ell,j\ell}$
$T_{j\ell}^* \geq T_{in}^{(l)} + \Delta T_m$
$H_{j\ell}$ $T_{out}^{(l)} + \Delta T_m$
H_{ml} $T_{out}^{(l)} + \Delta T_m$
$q_{i\ell,m\ell}$
$\sum_c q_{c\ell,m\ell}$
$T_{ml}^* \geq T_{in}^{(l)} + \Delta T_m$
$T_i \leq T_{out}^{(l)}$

FIG 8



$$\alpha = \frac{q_{i\ell,k\ell} + q_{i\ell,k\ell}}{q_{i\ell,k\ell} + q_{i\ell,m\ell} + q_{i\ell,j\ell}}$$

FIG 9

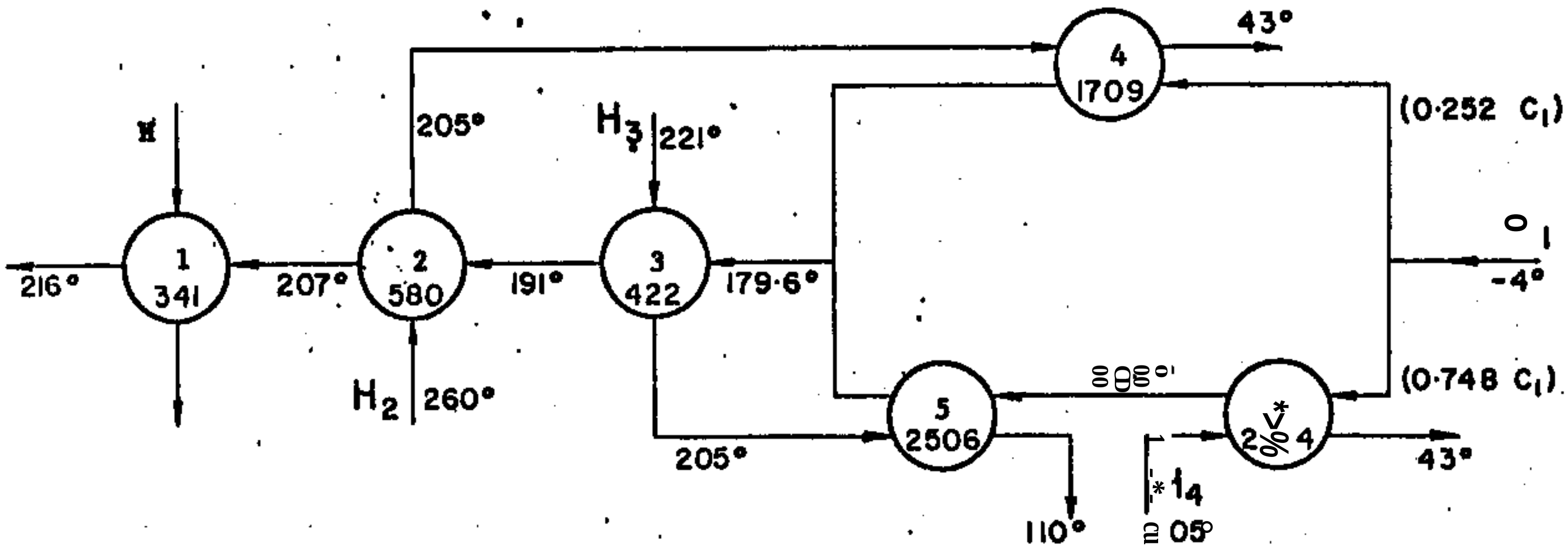

1
341
216°
207°
2
580
205°
H2
260°
191°
3
422
H3
221°
179·6°
205°
4
1709
43°
(0·252 C1)
0
-4°
(0·748 C1)
5
2506
110°
43°

FIG 10

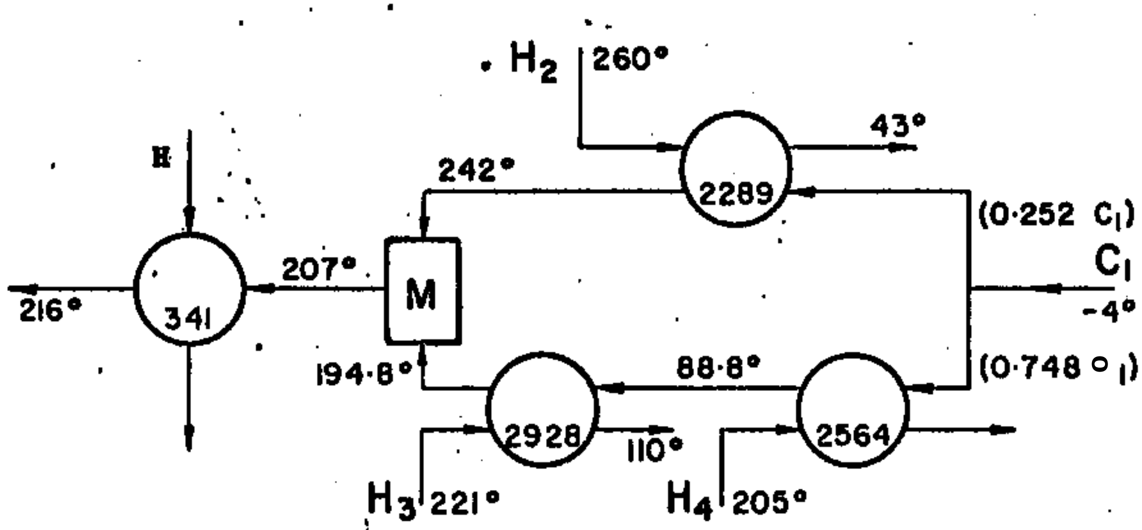
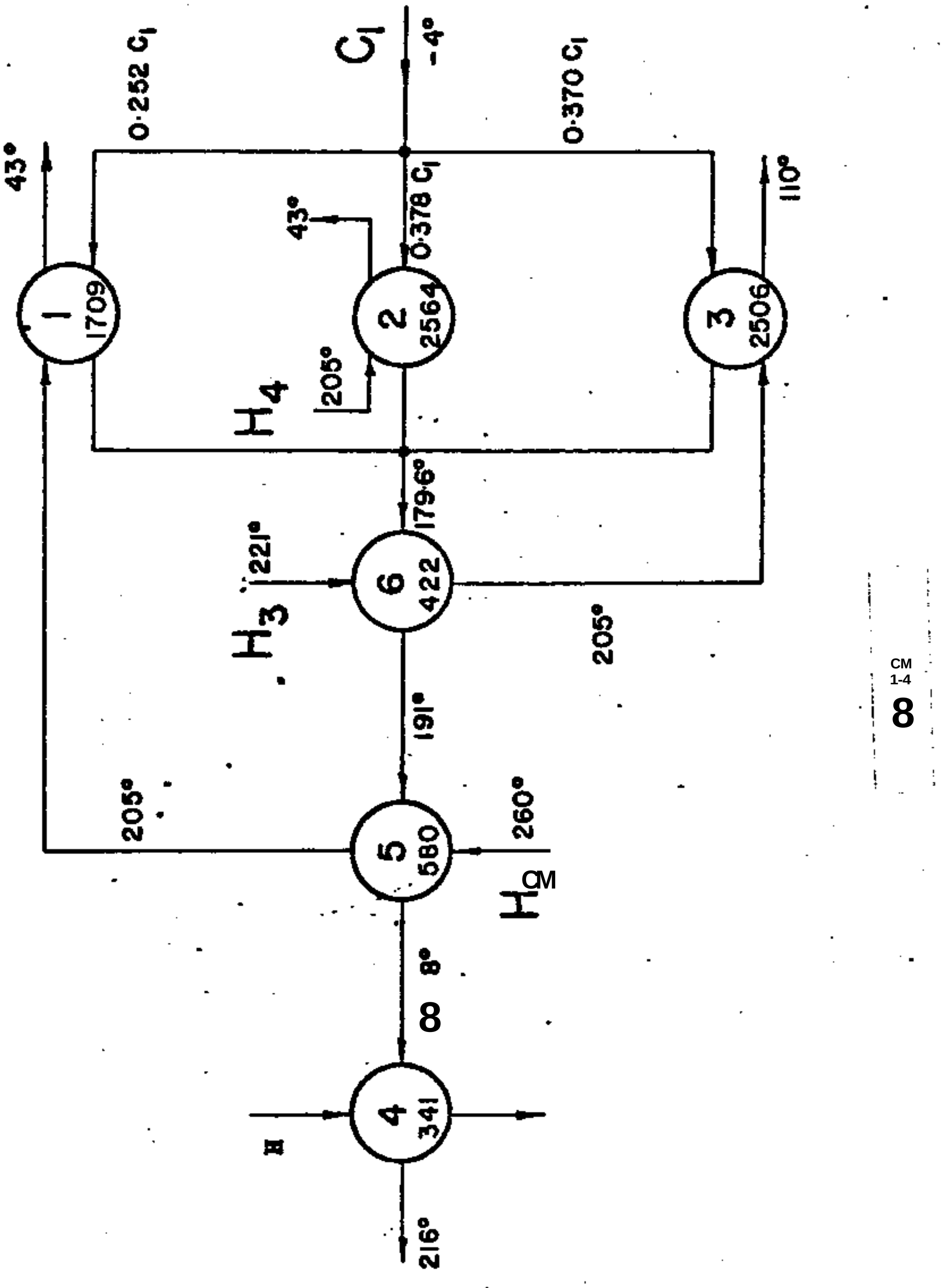
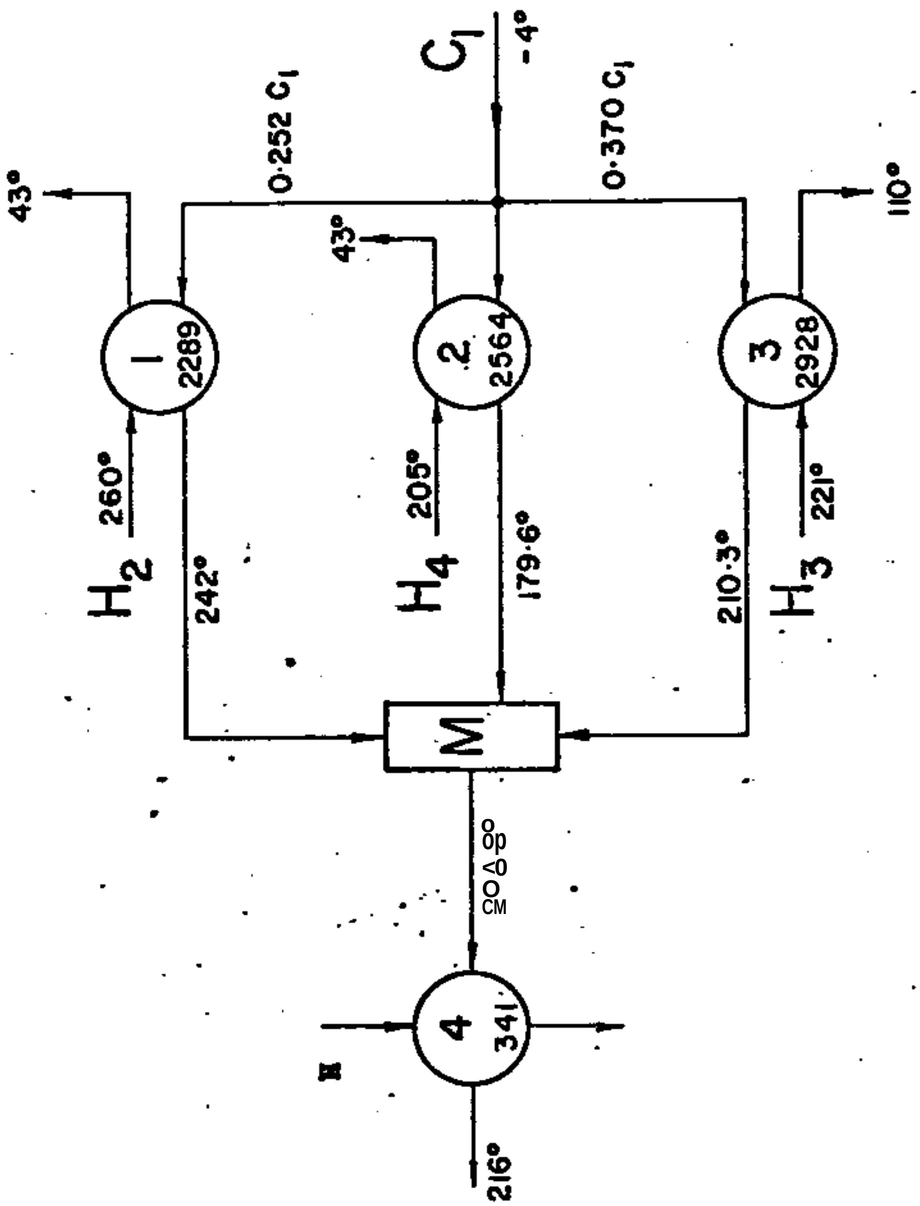

H$_2$
260°
43°
2289
242°
(0·252 C$_1$)
C$_1$
-4°
(0·748 C$_1$)
H
207°
M
216°
341
194·8°
2928
88·8°
2564
110°
H$_3$
221°
H$_4$
205°

FIG 11



C_1
-4°
0·252 C_1
0·370 C_1
0·378 C_1
43°
43°
110°
1
1709
2
2564
3
2506
H_4
205°
179·6°
6
422
221°
H_3
205°
191°
5
580
205°
260°
H_{CM}
8°
8
4
341
H
216°
CM
1-4
8



C1
-4°
0·252 C1
0·370 C1
43°
43°
110°
1
2289
2
2564
3
2928
H2
260°
242°
H4
205°
179·6°
H3
221°
210·3°
M
4
341
216°

S 13

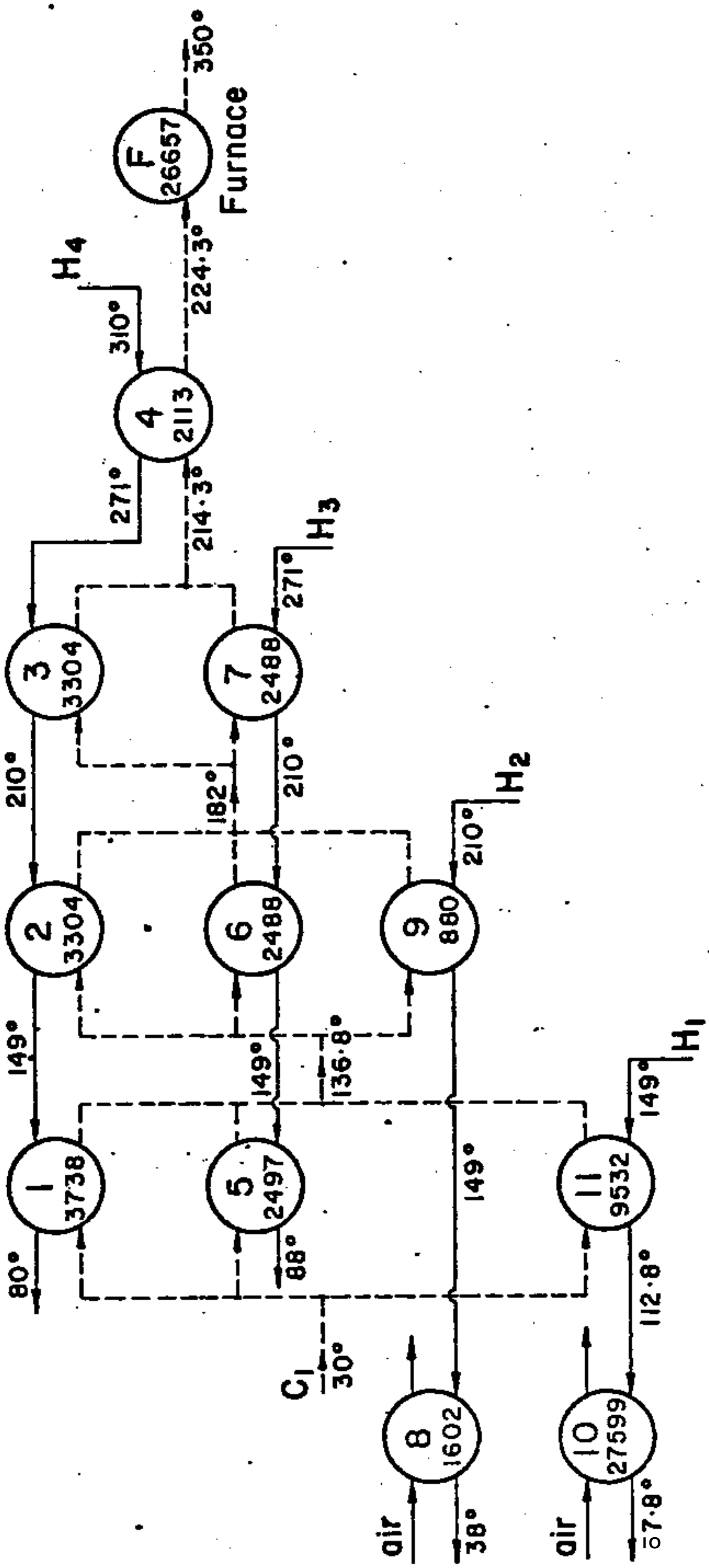

F
26657
Furnace
350°
224.3°
H4
310°
4
2113
271°
214.3°
H3
271°
3
3304
7
2488
210°
182°
210°
H2
210°
2
3304
6
2488
9
880
149°
149°
136.8°
H1
149°
1
3738
5
2497
149°
11
9532
80°
88°
112.8°
C1
30°
8
1602
10
27599
air
38°
air
7.8°

FIG 14

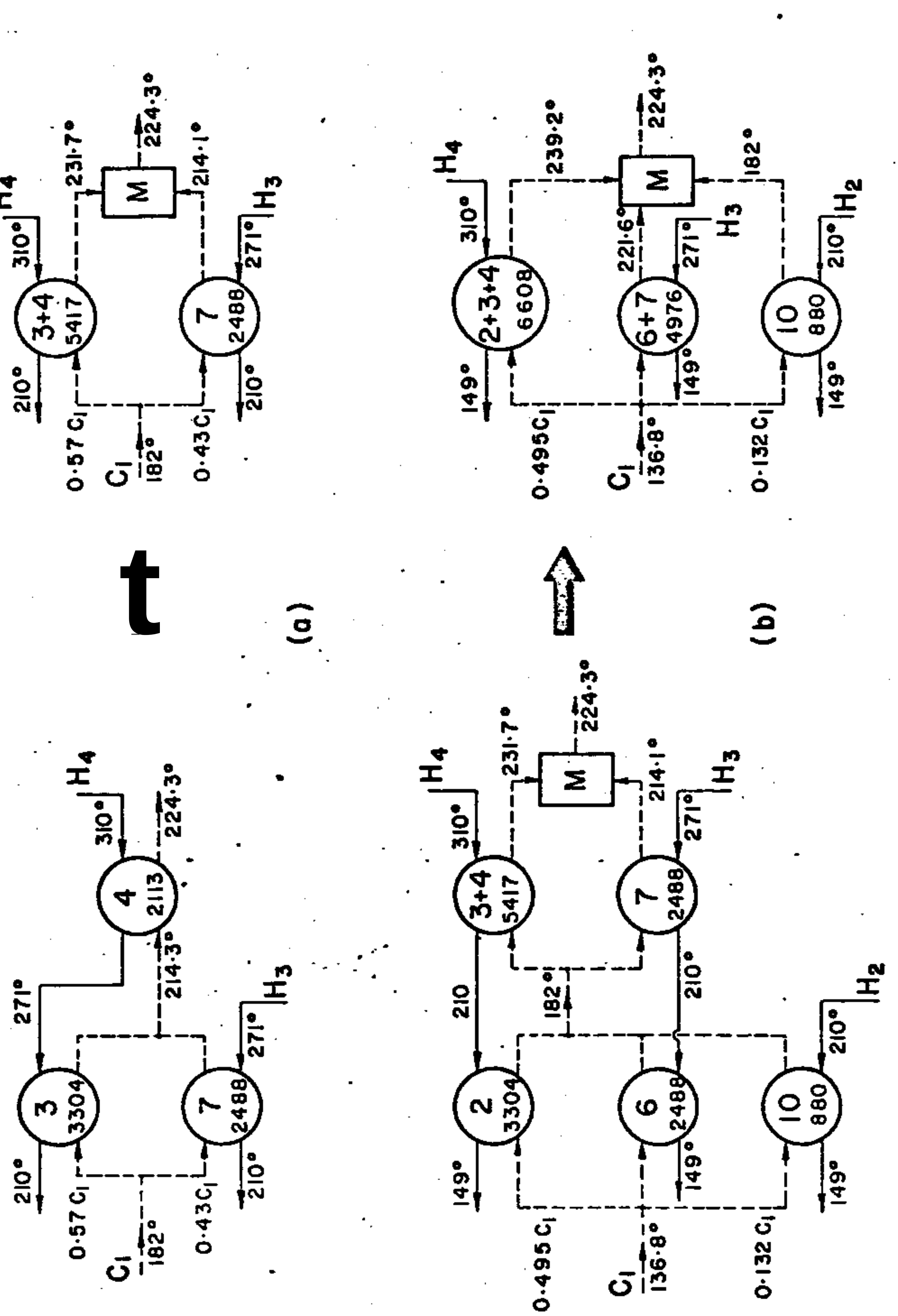

(a)
(b)
3+4
5417
7
2488
M
H4
H3
H2
310°
271°
210°
231·7°
214·1°
224·3°
0·57 C1
0·43 C1
C1
182°
3
3304
4
2113
214·3°
2+3+4
6608
6+7
4976
10
880
239·2°
221·6°
182°
149°
0·495 C1
136·8°
0·132 C1
2
6

FIG 15

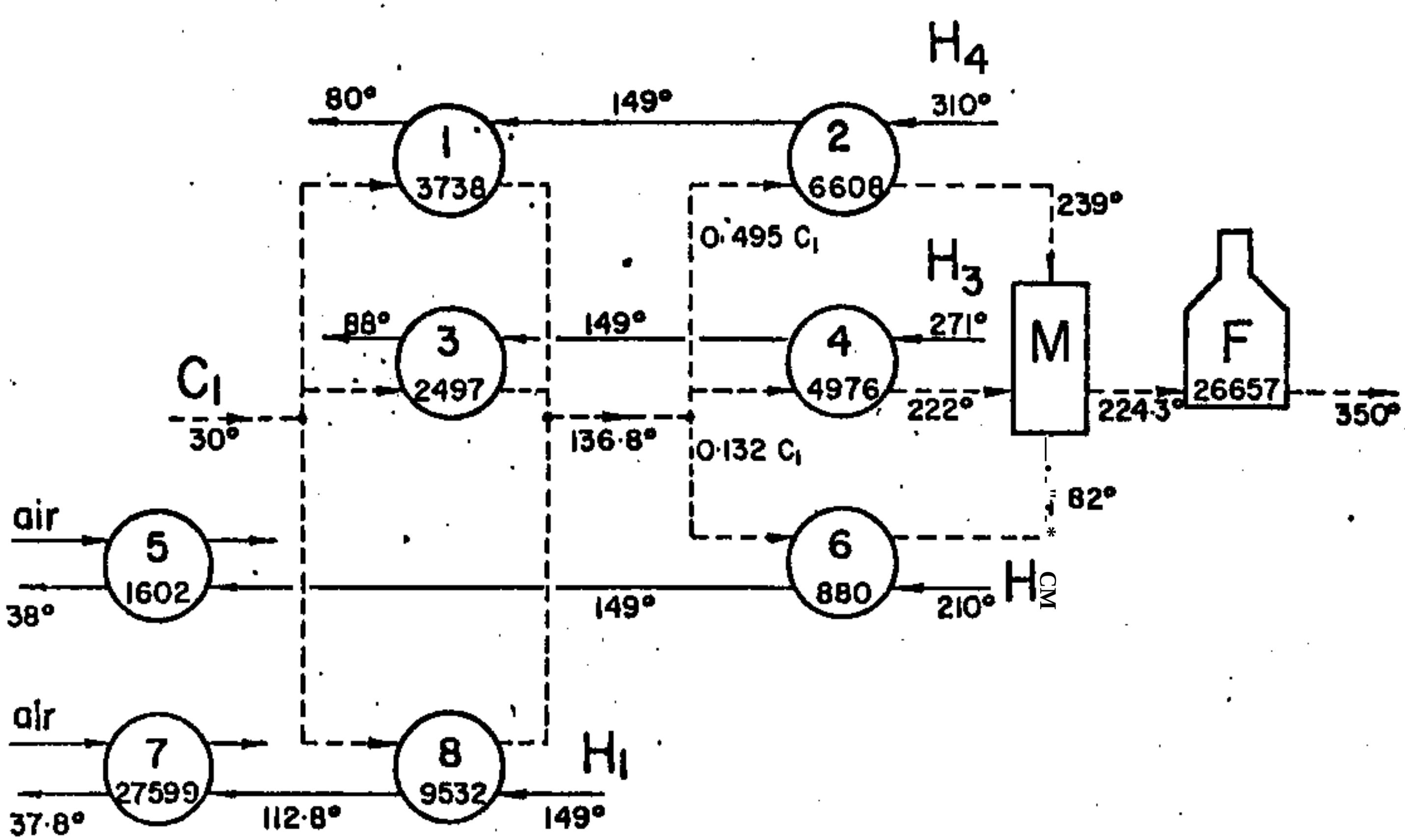

H_4
80°
149°
310°
1
3738
2
6608
239°
0·495 C_1
H_3
88°
3
2497
149°
4
4976
271°
M
F
26657
C_1
30°
136·8°
222°
224·3°
350°
0·132 C_1
82°
air
5
1602
6
880
38°
149°
210°
H
air
7
27599
8
9532
H_1
37·8°
112·8°
149°

FIG 3

NOTATION

a_i = thermal energy flow required by the cold stream i, kw.

a_{ik} = thermal energy flow at temperature level k required by the cold stream i, kw.

$b_{j\ell}$ = thermal energy flow at temperature level ℓ to be removed from hot stream j, kw.

C = cold utility stream

c_i = primitive cold process stream i, dimensionless.

c_{ik} = primitive cold process stream i at temperature level k, dimensionless.

F = heat capacity flow, kw/$^{\circ}$C.

H = hot utility stream.

h_j = primitive hot process stream j, dimensionless.

$h_{j\ell}$ = primitive hot process stream j at temperature level ℓ, dimensionless.

q_{ij} = thermal energy flow exchanged in the match (c_i, h_j), kw.

$q_{ik,j\ell}$ = thermal energy flow exchanged in the match (c_{ik}, $h_{j\ell}$), kw.

U_{ij} = upper bound on the thermal energy flow exchanged in the match (c_i, h_j), kw.