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## Specifying Functional and Timing Behavior for Real-Time Applications

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### Abstract

We present a notation and a methodology for specifying the functional and timing behavior of real-time applications for a heterogeneous machine. In our methodology we build upon well-defined, though isolated, pieces of previous work: Larch and Real Time Logic. In our notation, we strive to keep separate the functional specification from the timing specification so that a task's functionality can be understood independent of its timing behavior. We show that while there is a clean separation of concerns between these two specifications, the semantics of both pieces as well as their combination are simple.

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## 1 Problem Context

Many computation-intensive, real-time applications require efficient concurrent execution of multiple *tasks* devoted to specific pieces of the application. Typical tasks include sensor data collection, obstacle recognition, and global path planning, in applications such as robotics and vehicular control. Since the speed and throughput required of each task may vary, these applications can best exploit a computing environment consisting of multiple special and general purpose processors that are logically, though not necessarily physically, loosely connected. We call this environment a *heterogeneous machine*.

During execution time, *processes*, which are instances of tasks, run on possibly separate processors, and communicate with each other by sending messages of different types. Since the patterns of communication can vary over time, and the speed of the individual processors can vary over a wide range, additional hardware resources, in the form of switching networks and data buffers are required in the physical heterogeneous machine. Logically, *queues* are used to buffer data; processes dequeue data on queues attached to input ports and enqueue data from queues attached to output ports.

The application developer is responsible for prescribing a way to manage all of these resources. We call this prescription a *task-level application description*. It describes the tasks to be executed, the assignment of processes to processors, the data paths between the processors, and the intermediate queues required to store the data as it moves from source to destination processes. A *task-level description language* is a notation in which to write these application descriptions.

We are using the term "description language" rather than "programming language" to emphasize that a task-level application description is not translated into object code in some kind of executable "machine language." Rather, it is to be understood as a description of the structure and behavior of a logical machine, that will be synthesized into resource allocation and scheduling directives. These directives are to be interpreted by a combination of software, firmware, and hardware in a heterogeneous machine.

We have an initial design of such a description language [1], a compiler for it, and a simulator that takes task descriptions as input. A task description (see Figure 1) contains information about four aspects of a task: (1) its interface to other tasks (**ports**) and to the scheduler (**signals**); (2) its functional and timing behavior, (3) its attributes, and (4) its internal structure, thereby allowing for hierarchical task descriptions. Reference [1] contains a more complete explanation of these and other features of the language. In this paper we focus on only one aspect: the information appearing in the behavior part of a task description.

## 2 Contributions

Formal specifications have been used successfully for specifying the functional behavior of software systems, e.g., individual program modules and abstract data types. These specifications have traditionally been used to verify a program's correctness ("is the right answer computed?"). Often, however, one is interested in not only the functional correctness of a system but also other properties, such as reliability, performance, security, and real-time behavior. Less work has focused on formally specifying these other properties of software systems, let alone their interactions with each other.

To our knowledge no work has addressed the formal integration of the formal specification of functional and timing behavior of software. The main contribution of this paper is exactly this integration of functional and timing specifications as embodied in our task description language.

---

```

task task-name
  ports -- Used for communication between a process and a queue
  port-declarations
  signals -- Used for communication between a user process and the scheduler
  signal-declarations -- A description of the functional and timing behavior of the task
  behavior
  requires predicate
  ensures predicate
  timing timing expression -- Additional properties of the task
  attributes
  attribute-value-pairs -- A process-queue graph describing the internal structure of a task
  structure
  process-declarations
  queue-declarations
  reconfiguration-statements
end task-name

```

Figure 1: A Template for Task Descriptions

---

We combine two separate formalisms: an axiomatic specification language, Larch [11, 12], used to specify functional behavior, and an event expression language used to specify timing behavior. Both are mapped to the same underlying logic, typed first-order predicate logic, so that their combination has a formal semantics.

Two significant aspects of our work are as follows:

- Since the formal semantics is relatively simple (first-order logic), not only can people easily understand our specifications but the specifications themselves can easily be subject to machine analysis.
- We build upon previous well defined and isolated pieces of research and combine them in a meaningful way. Their combination is applied in a context (heterogeneous machines) that itself is of growing interest to those involved in parallel architectures and languages.

### 3 Introduction to Larch

Before we describe the functional and timing specifications of a task, we give a brief introduction to Larch<sup>1</sup>.

Larch uses a two-tiered approach to specifying program modules: a *trait* defines state-independent properties, and an *interface* specification defines state-dependent properties of a program. A trait is written in the Larch Shared Language (LSL), and it provides the assertion language used to express and define the meaning of the predicates of an interface specification.

For a program module, such as a procedure, a Larch interface specification is written in a Larch Interface Language (LIL) and contains predicates about the states before and after the execution of the procedure. The Larch Interface Language to be used is specific to the programming language in which the procedure is written (e.g., C, CommonLisp, Ada, etc.). For this paper we will use a relatively simple interface language, such as would be defined for an Algol-like language.

---

<sup>1</sup>We are keeping this introduction to Larch very short. The reader is encouraged to consult the appropriate references in the bibliography.

---

```

QVals: trait
  introduces
    Empty: → Q
    Insert: Q, E → Q
    First: Q → E
    Rest: Q → Q
    isEmpty: Q → Bool
    isIn: Q, E → Bool
  constrains Q so that
    Q generated by [ Empty, Insert ]
    for all q: Q, e, e1: E
      First(Insert(Empty), e) = e
      First(Insert(q, e)) = if isEmpty(q) then e else First(q)
      Rest(Insert(q, e)) = if isEmpty(q) then Empty else Insert(Rest(q), e)
      isEmpty(Empty) = true
      isEmpty(Insert(q, e)) = false
      isIn(Empty, e) = false
      isIn(Insert(q, e), e1) = (e = e1) | isIn(q, e1)

```

a. A Trait for Queue Values

```

Enqueue = operation (q: queue, e: element)
  ensures qpost = Insert(q, e)

Dequeue = operation (q: queue) returns (e: element)
  requires !isEmpty(q)
  ensures qpost = Rest(q) & e = First(q)

```

b. Interfaces for Queue Operations

Figure 2: A Larch Two-Tiered Specification for Queues

---

Figure 2 depicts a Larch (two-tiered) specification of queues with Enqueue and Dequeue operations. The top part of the specification (Figure 2.a) is a trait written in LSL used to describe values of queues. A trait is akin to an algebraic specification (see Section 7 on Related Work). A set of operators and their signatures following `introduces` defines a vocabulary of terms to denote values of a type. For example, `Empty` and `Insert(Empty, 5)` denote two different queue values. The set of equations following the `constrains` clause defines a meaning for the terms; more precisely, an equivalence relation on the terms, and hence on the values they denote. For example, from the above trait, one could prove that `First(Rest(Insert(Insert(Empty, 5), 6))) = 6`.

The bottom part of the specification (Figure 2.b) contains two interfaces written in our “generic” Larch interface language. They describe the functional behavior of two queue operations, `Enqueue` and `Dequeue` (queue operation names are used to write timing expressions, which are described later in this paper). A `requires` is a pre-condition on the state of an operation’s input data that must be true upon operation invocation; an `ensures` is a post-condition on the state of an operation’s input and output data that is guaranteed to be true upon operation termination. An omitted predicate is taken to be true. The specification for `Dequeue` states that `Dequeue` must be called with a non-empty queue and that it modifies the original queue by removing its first element and returning it.

## 4 Behavioral Information

The behavioral information in a task description is divided into two parts: a functional specification and a timing specification. In the next two subsections we describe informally the syntax and meaning of these two specifications. Section 5 gives the formal meaning, and in particular, the meaning of the combination of functional and timing specifications.

### 4.1 Functional Specifications

#### 4.1.1 Syntax and Meaning

The functional information of a task description (see Figure 1) describes the behavior of the task in terms of predicates about the data in the queues, before and after each execution of the task. It consists of a **requires** clause and an **ensures** clause, together constituting a simple Larch interface specification. LSL is used as the assertion language in the predicates of these clauses.

A **requires** clause states what is required to be true of the data coming through the input ports; an **ensures** clause states what is guaranteed to be true of the data going out through the output ports. If one were to view each cycle of a task as one execution of a procedure, the **requires** and **ensures** are exactly the pre- and post-conditions on the functionality of that cycle.

A task implementation must satisfy the predicates, R and E, of the **requires** and **ensures** clauses. A task implementation is simply a program written in some programming language, e.g., C, CommonLisp, or Ada. Using Hoare-like notation, an implementation, Prog, *satisfies* the (functional) specification if:

$$\{R\} \text{Prog} \{E\}$$

It is up to the task implementor to show that a task implementation satisfies the functional specification as given by the **requires** and **ensures** clauses. This verification can be done formally — standard verification techniques can be used ([13, 14]) and some mechanical tools are available to aid this process ([9, 19, 22, 21]). We defer to Section 5.2 for the definition of the meaning of the predicates in the presence of timing information.

#### 4.1.2 Example

Consider a matrix multiplication task (Figure 3) that takes input matrices from two queues and outputs the result matrix on an output queue. The data traveling through these ports are of type matrix. Matrix values are specified using LSL just as for queue values, so "rows," "cols" and "" would be defined in a trait about matrix values. The **requires** clause states that the task implementor may assume that the number of rows of the matrix entering through the port in1, equals the number of columns of the matrix entering through in2. The **ensures** clause states that the result of multiplying the two input matrices is output through the output port.

### 4.2 Timing Specifications

#### 4.2.1 Syntax and Meaning

The timing information describes the behavior of the task in terms of the operations that it performs on the queues attached to its input and output ports; this is the behavior of the task seen from the outside.

---

```

task multiply
  ports
    in1, in2: in matrix;
    out1: out matrix;
  behavior
    requires rows(First(in1)) = cols(First(in2))
    ensures Insert(out1, First(in1) * First(in2))
end multiply

```

Figure 3: The Functionality of a Matrix Multiplication Task

---

The simplest timing expression is the name of a queue operation, e.g., Enqueue or Dequeue, on a queue attached to a specific port, e.g., in1. The duration of a queue operation or the delay between two operations is described by a time window. Time windows are denoted by a pair of time values  $[T_{min}, T_{max}]$  defining the boundaries of the interval. The time window associated with a queue operation describes the minimum and maximum time needed to perform the operation. Intervals of time between queue operations are denoted by a Delay "operation" whose time window describes the minimum and maximum time consumed by the process in between queue operations.

A composite timing expression denotes the sequential and/or concurrent execution of operations on queues. Sequential composition is denoted by a space between operations; parallel composition is denoted by a "||" between operations. For example,

```
loop (in1.Dequeue[10,15] || in2.Dequeue) delay{*,30} out1.Enqueue
```

is a sequential timing expression that specifies two parallel Dequeue operations on the queues attached to the input ports in1 and in2 followed, after some delay, by an Enqueue on the queue attached to the output port out1. The Delay lasts some undetermined amount of time less than 30 seconds. The Dequeue operation on port in1 takes between 10 and 15 seconds to complete. The other two operations take some implementation dependent default time to complete. The keyword loop denotes a cyclic or repeating task.

An optional guard in a timing expression specifies:

1. the number of times the task is to be executed: "*repeat integer => expression,*" or
2. during what time interval the task is allowed to start: "*during timewindow => expression,*" or
3. the earliest allowable start time: "*after timevalue => expression,*" or
4. the latest allowable start time "*before timevalue => expression,*" or
5. a predicate on the state of the input queues or the current time which must be true before the task is allowed to start: "*when predicate => expression.*"

In our examples, we will often drop the name of the queue operation and use just the name of the port (i.e., "in1" instead of "in1.Dequeue"). Since this paper introduces only two queue operations: Enqueue and Dequeue, and given that the former applies only to input queues and the other applies only to output queues, no confusion should occur as to which operation is implied.



#### 4.2.2 Example

Consider a matrix multiplication task (Figure 4) that takes input matrices from two queues and outputs the result matrix on an output queue. The timing clause states that the task does not start executing until both input queues contain data. Once that condition is satisfied, the task will remove its input data from both input queues concurrently (the Dequeue operations), will operate on the data for between 10 and 15 seconds (this "computation" time is lumped together under the delay operation), and finally will enqueue some output in the output queue. Notice another use of LSL in our specifications: the when condition places a constraint on the state of the queues (not on the state of the data in the queues). We use the trait from Section 3 to define the assertion language for predicates in a when guard.

---

```

task multiply
  ports
    in1, in2: in matrix;
    out1: out matrix;
  behavior
    requires rows(First(in1)) = cols(First(in2))
    ensures Insert(out1, First(in1) * First(in2))
    timing when (~isEmpty(in1) and ~isEmpty(in2)) =>
      ((in1.Dequeue || in2.Dequeue) delay[10,15] out1.Enqueue)
end multiply

```

Figure 4: The Timing of a Matrix Multiplication Task

---

### 5 Formal Meaning of Functional and Timing Specifications

We use Jahanian and Mok's Real-Time Logic (RTL) [15] to give meaning to our timing expressions. Furthermore, we use their logic to give meaning to the combination of our functional and timing specifications. We use four of their notational conventions:

<i>Syntax</i>	<i>Meaning</i>
$\uparrow A$	The start of an operation ("action" in RTL's terminology).
$\downarrow A$	The end of an operation.
$@(E, i)$	The time of the $i^{\text{th}}$ occurrence of event $E$ , where events in our context are the start of an operation or the end of an operation. $@$ is an occurrence function that captures the notion of real-time.
$P(t_1, t_2)$	The interval of time during which the predicate $P$ holds. $P$ holds before or at $t_1$ , from $t_1$ to $t_2$ , and at or after $t_2$ . If $t_1$ and $t_2$ are identical, then $P$ holds at an interval around $t_1$ . For brevity, we will use $P(t)$ when $t_1=t_2$ (i.e., "P holds around time $t$ ").

#### 5.1 Assigning Meaning to Timing Specifications

In this section we describe the meaning of our timing specifications in terms of RTL logic. In the following discussion, we assume  $E$ ,  $E_1$ , and  $E_2$  are arbitrary timing expressions;  $A$ ,  $A_1$ , and  $A_2$  are operations;  $t_1$  and  $t_2$  are times (absolute or relative);  $a_1$  and  $a_2$  are absolute times;  $r_1$  and  $r_2$  are relative times; and  $W$  is a predicate of a when guard.

To simplify the exposition, we introduce a simple rewrite rule: Any timing expression of the form "repeat  $n \Rightarrow E$ " can be rewritten as a sequence of  $n$  occurrences of the unguarded expression  $E$  (" $E E E \dots E$ "). Thus, the only guards we need to consider are before, after, during, and when.

We also introduce the following axioms:

1. For any queue operation A, and for some implementation defined duration T, the following axiom expresses the duration of A:

$$\forall i [ @(\downarrow A, i) - @(\uparrow A, i) = T ]$$

2. For any queue operation A[t1,t2], with a duration defined by the time window [t1,t2], the following axiom expresses the duration of A:

$$\forall i [ t1 \leq @(\downarrow A, i) - @(\uparrow A, i) \leq t2 ]$$

3. For any sequence of queue operations, A1 ... An, the following axiom relates the start and end times of the sequence to the start and end times of the individual operations:

$$\forall i [ @(\uparrow A, i) = @(\uparrow A1, i) \wedge @(\downarrow A, i) = @(\downarrow An, i) ]$$

4. For any parallel queue operations, A1 || ... || An, the following axiom relates the start and end times of the composition to the start and end times of the individual operations:

$$\forall i [ @(\uparrow A, i) = \min(@(\uparrow A1, i), \dots, @(\uparrow An, i)) \wedge @(\downarrow A, i) = \max(@(\downarrow A1, i), \dots, @(\downarrow An, i)) ]$$

5. The last two axioms state that cycles in a repeating task do not overlap. Thus, we cannot have an input operation finish after any of the output operations and we cannot have an output operation start before any input operation starts:

$$\forall i [ \max(@(\downarrow out_1, i), @(\downarrow out_2, i), \dots, @(\downarrow out_j, i)) > \max(@(\downarrow in_1, i), @(\downarrow in_2, i), \dots, @(\downarrow in_k, i)) ]$$

$$\forall i [ \min(@(\uparrow out_1, i), @(\uparrow out_2, i), \dots, @(\uparrow out_j, i)) > \min(@(\uparrow in_1, i), @(\uparrow in_2, i), \dots, @(\uparrow in_k, i)) ]$$

where J and K are the number of output and input queues, respectively.

We assign a meaning to timing expressions by introducing a function,  $M_t$  (Table 1.a), which maps timing expressions to Boolean values,

$$M_t : \text{Timing Expression} \rightarrow \text{Boolean.}$$

We use an auxiliary function,  $op$  (Table 1.b), which maps timing expressions to operations,

$$op : \text{Timing Expression} \rightarrow \text{Operation.}$$

$op$  is needed because "start time" and "end time" are meaningful only for queue operations.

As an example of how to interpret the formalism intuitively, consider the entries for the during guard in Table 1.a. They specify a time window during which the operation is allowed to start. The first value is the earliest start time allowed and must be an absolute time value. The second value is the latest start time allowed and can be an absolute time value or a time value relative to the former. The meaning of the guarded expression is the conjunction of the meaning of the expression proper and a predicate stating the restriction on starting times.

## 5.2 Assigning Meaning to the Combined Specifications

Given a task description of the form:

```

task taskname
.....
behavior
  requires Req ;
  ensures Ens ;
  timing E ;
.....
end taskname ;

```

<u>Timing Expression</u>	<u><math>M_t(\text{Expression})</math></u>
(E1)	$M_t(E1)$
E1 ... En	$M_t((E1 E2) \dots En)$
E1  ...  En	$\wedge M_t(Ei  Ej)$ for all $i \neq j$
E1 E2	$M_t(E1) \wedge M_t(E2) \wedge \forall i [ @(\downarrow op(E1), i) \leq @(\uparrow op(E2), i) ]$
E1    E2	$M_t(E1) \wedge M_t(E2) \wedge \forall i [ @(\uparrow op(E1), i) < @(\downarrow op(E2), i) \wedge @(\uparrow op(E2), i) < @(\downarrow op(E1), i) ]$
when W => E1	$M_t(E1) \wedge \forall i [ W(@(\uparrow op(E1), i)) ]$
before a1 => E1	$M_t(E1) \wedge \forall i [ @(\uparrow op(E1), i) \leq a1 ]$
after a1 => E1	$M_t(E1) \wedge \forall i [ @(\uparrow op(E1), i) \geq a1 ]$
during [a1, a2] => E1	$M_t(E1) \wedge \forall i [ a1 \leq @(\uparrow op(E1), i) \leq a2 ]$
during [a1, r2] => E1	$M_t(E1) \wedge \forall i [ a1 \leq @(\uparrow op(E1), i) \leq a1 + r2 ]$
A[r1, r2]	$\forall i [ @(\uparrow A, i) + r1 \leq @(\downarrow A, i) \leq @(\uparrow A, i) + r2 ]$
A[* , r1]	$\forall i [ @(\downarrow A, i) \leq @(\uparrow A, i) + r1 ]$
A[r1, *]	$\forall i [ @(\uparrow A, i) + r1 \leq @(\downarrow A, i) ]$
A	true

a. Mapping from Timing Expressions to Booleans

<u>Timing Expression</u>	<u><math>op(\text{Expression})</math></u>
loop E1	$op(E1)$
E1 ... En	$op(E1) \dots op(En)$
E1   ...    En	$op(E1)    \dots    op(En)$
G => E1	$op(E1)$ for all guards G (when, before, during, and after).
A [t1, t2]	A
A	A

b. Mapping From Timing Expressions to Operations

Table 1: Assigning Meaning to Timing Expressions

we give meaning to the predicates of the functional specification as related to time (i.e., at what times are these predicates to hold?) via a function  $M_f$  which maps from behavioral specifications to Boolean values:

$$M_f : \text{Predicate} \times \text{Timing Expression} \rightarrow \text{Boolean}$$

<u>Predicate</u>	<u>Timing Expression</u>	<u><math>M_f(\text{Predicate}, \text{Expression})</math></u>
Req	E	$\forall i [ \text{Req}(@(\uparrow op(E), i)) \wedge M_f(E) ]$
Ens	E	$\forall i [ \text{Ens}(@(\downarrow op(E), i)) \wedge M_f(E) ]$

The function  $M_f$  is precisely the link between the functional and timing specifications. This link is characterizable purely in terms of first-order logic.

## 6 Examples

Figure 5 shows our multiply task with functional and timing information together. The figure shows two different multiply tasks, specified to have the same functionality but with different timing behavior. The timing expression in Figure 5.a states that the multiply task first checks that the input queues are non-empty, and if so perform two parallel Dequeue operations followed by an Enqueue operation. The timing expression in Figure 5.b states that the inputs come in sequentially instead of in parallel.

---

```

task multiply
  ports
    in1, in2: in matrix
    out1: out matrix
  behavior
    requires rows(First(in1)) = cols(First(in2))
    ensures Insert(out1, First(in1) * First(in2))
    timing when (~isEmpty(in1) and ~isEmpty(in2)) =>
      ((in1.Dequeue || in2.Dequeue) delay[10,15] out1.Enqueue)

```

a. Parallel Input

```

task multiply
  ports
    in1, in2: in matrix
    out1: out matrix
  behavior
    requires rows(First(in1)) = cols(First(in2))
    ensures Insert(out1, First(in1) * First(in2))
    timing when (~isEmpty(in1) and ~isEmpty(in2)) =>
      (in1.Dequeue in2.Dequeue delay[10,15] out1.Enqueue)

```

b. Serial Input

Figure 5: Matrix Multiplication Task

To further illustrate the richness of our specification language and to show the benefits of cleanly separating the functional from the timing information, we write three alternative descriptions for a task built into our library. This task, `deal`, has one input port and a number of output ports. Data dequeued from the input port is enqueued to one of the output ports, but this can be implemented in a number of ways, as illustrated in Figure 6, below<sup>2</sup>.

<sup>2</sup>Assume that `second(in1)`, `third(in1)`, and `fourth(in1)` as abbreviations for `First(Rest(in1))`, `First(Rest(Rest(in1)))`, `First(Rest(Rest(Rest(in1))))`, respectively, are defined in the trait for queues.

The first example (Figure 6.a) states that we alternate the dequeuing of input and enqueueing of output and ensures that first (second) output queue will see the first (second) item removed from the input queue. The second example (Figure 6.b) states that we dequeue all input before the output operations start, which themselves take place concurrently. It allows for the first dequeued data item to be enqueued on either of the output queues, but ensures that the second dequeued item will not be enqueued to the same as the first. The third example (Figure 6.c) states that input data are dequeued and grouped in pairs before enqueueing them into the output ports. The first pair is enqueued to the first output queue; the second pair, to the second.

---

```

task deal
  ports
    in1: in matrix;
    out1, out2: out matrix;
  behavior
    ensures Insert(out1, First(in1)) & Insert(out2, second(in1))
    timing loop (in1 out1 in1 out2)
end deal

```

a. Alternating Input and Output

```

task deal
  ports
    in1: in matrix;
    out1, out2: out matrix;
  behavior
    ensures [Insert(out1, first(in1)) & Insert(out2, second(in1))] |
            [Insert(out2, first(in1)) & Insert(out1, second(in1))]
    timing loop (in1 in1 (out1 || out2))
end deal

```

b. Concurrent Output

```

task deal
  ports
    in1: in matrix;
    out1, out2: out matrix;
  behavior
    ensures [Insert(out1, First(in1)) & Insert(out1, second(in1))] &
            [Insert(out2, third(in1)) & Insert(out2, fourth(in1))]
    timing loop (in1 in1 in1 in1 (out1 || out2) (out1 || out2))
end deal

```

c. Grouping Data

Figure 6: Deal Task

---

## 7 Related Work

The axiomatic approach to specifying a program's functional behavior has its origins in Hoare's early work on verification [13] and later work on proofs of correctness of implementations of abstract data types [14], where first-order predicate logic pre- and post-conditions are used for the specification of each operation of the type. The algebraic approach, which defines data types to be heterogeneous algebras [2], uses axioms to specify properties of programs and abstract data types, but the axioms are restricted to equations. Much work has been done on algebraic specifications for abstract data types [8, 7, 10, 27, 3, 6, 25, 16]; we use more recent work on Larch specifications [11, 12] for program modules. None of this work addresses the formal specification of timing behavior of systems.

Operational approaches, such as those based on Timed Petri-net models [20, 23], are more commonly used for specifying behavior of real-time systems. Timed Petri-nets can be roughly characterized by whether “operation” time is assigned to the *transitions*, as in the original model by Ramchandani [20], or is assigned to the *places*, as in Sifakis’ model [23]. In addition, both deterministic and stochastic timing are allowed, giving origin to a variety of models for specifying or evaluating performance requirements. This has been illustrated in recent work by Coolahan [4] (places, deterministic), Smith [24] (transitions, deterministic), Wong [26] (places, stochastic), and Zuberek [28] (transitions, stochastic). In contrast, our work takes a more axiomatic than operational approach to specifying timing behavior.

Specification and verification of timing requirements for real-time systems include recent work by Dasarthy [5], and by Lee, Gehlot, and Zvarico [17, 29]. This work as well as that by Jahanian and Mok, whose real-time logic we borrow, all focus on timing properties and not on functional behavior. Either states are left uninterpreted or predicates on states are simplistic, e.g., boolean modes as in Jahanian and Mok’s work. In contrast, since we have a formal means of specifying the functional behavior of tasks and the data on which they operate, we have a more expressive specification language with a richer semantics.

## 8 Summary

Our approach to specifying the functional and timing behavior of real-time applications for a heterogeneous machine has the following characteristics:

- It takes advantage of two well defined, though isolated, pieces of previous work.
- There is a clean separation of concerns between the two specifications.
- The semantics of both specifications as well as their combination are simple.

In our language design, we strove to separate the functional specification from the timing specification so that a task’s functionality could be understood independent of its timing behavior. This separation of concerns gives us the usual advantages of modularity. Different timing specifications can be attached to the same functional specification. Task implementors can focus on satisfying functionality first, timing second. Task validation can be performed separately. For example, one could use formal verification for functionality and simulation for timing.

Since the semantics can be given in terms of first-order predicate logic, our specifications are amenable to machine manipulation and analysis. The algebraic style of Larch traits can be analyzed by rewrite-rule tools, e.g., Reve [18]; the two-state predicates of Larch interfaces and thus, task predicates, can be analyzed by verification systems that support first-order reasoning, e.g., Gypsy, HDM, and FDM [9, 21, 22]; formulae in real-time logic can be mechanically transformed into equivalent formulae in Presburger arithmetic. However, though many of these tools are available, much work is needed to integrate them so our specifications could be machine checked and analyzed.

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