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MODELS FOR ASSESSING ENERGY MANAGEMENT OPTIONS

by

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ABSTRACT.

This paper develops models for predicting the effects of energy management technologies on two important system attributes - production cost and the load shape seen by central generation. The models can include arbitrary mixes of central generation, dispersed generation, central storage and dispersed storage, both direct and indirect control strategies for load management can be accommodated, facilities for including the effects of load rebound - the dynamic response of a load to direct control strategies - are provided. The models are built around a Transportation Algorithm. This makes them efficient enough to use interactively.

INTRODUCTION

Emerging energy, control and communication technologies are expanding the range of the structural and operational alternatives available to power system planners [1]. The new structural alternatives span the use of dispersed storage (e.g. batteries and residential thermal stores), dispersed generation (e.g. windmills, small hydro units and solar heaters), cogeneration (the use of residual heat from electricity production for processes such as petroleum refining and apartment building heating) and an extended set of central storage options (e.g. compressed air and batteries).

For each structural alternative there is a multitude of operational alternatives belonging to two major classes - direct control and indirect control. The former covers strategies in which the utility directly adjusts energy flows (e.g. the interruption of supply to water heaters during peak periods); the latter covers strategies that offer customers incentives to adopt desirable usage patterns. An example of an indirect strategy is time-of-day pricing to discourage demand during peak periods.

We will refer to the combination of a structural and an operating alternative as an energy management alternative\* or just an "alternative", for short. We note that such alternatives span both the ranges of activity that are referred to as load management and supply management in the emerging jargon [21].

It is clear that the new alternatives can exert important influences on system behavior. Therefore, key questions facing analysts and planners are: which of the alternatives are best? which of them are most deserving of further consideration, field testing and eventual full scale deployment?

Work on assessment and impact methodologies (see, for instance [C]-[10]) has tended to be alternative-specific or lacking in detail. In this paper we will seek to develop a set of models in the middle ground - models that can accommodate a wide range of alternatives with enough attention to detail to preserve the effects of important phenomena. The objective is to provide the means with which to assess and compare the impacts of arbitrary mixes of structural and operating alternatives. The attributes used will be production cost and load shape (as seen by system components and especially, central generation). In some attributes are the principal indicators of an alternative's economic impacts. This subject will be dealt with further in the next section. Subsequent sections will describe the models and their usage.

PRODUCTION COSTING -  
SOME PRELIMINARY CONCLUSIONS

In this section we will show that the concepts of production costing in conventional networks can be applied, without change, to networks containing significant amounts of storage and dispersed generation. The purposes of this demonstration are to delineate the problem to be tackled in subsequent sections, point out its importance and identify the underlying assumptions.

Consider the problem of optimally expanding a network with additions of central and dispersed generation as well as central and dispersed storage. In form, this problem is similar to the conventional generation-expansion problem (which considers only additions of central generation). In fact, both problems can be stated as:

$$\text{Minimum } \phi(Y, Z) \tag{1}$$
$$Y, Z$$

subject to

$$GH(Z) \leq 0 \tag{2}$$

where:

- Y is a decision vector representing an expansion plan (type, size, timing and location of equipment additions over the planning horizon)
- Z is a decision vector of operating variables (mostly energy flows and prices charged to customers)

\*for more details on alternatives, see [1]-[7].

U is a function vector embodying the economic, technological, regulatory and other constraints that the decision variables must meet

A is the objective function reflecting the concerns of the network's decision makers.

$V_i(Y) = U_i \cup U_i(Y, Z_i)$  is the set of feasible operating variable values in interval  $i$ , given expansion plan  $Y$ .

$\phi$  is chosen to represent the present-worth-cost of the plan and we will assume this to be the case. The expression for present-worth-cost is:

$$\phi = \sum_{i=1}^N r^{i-1} (K_i + C_i) - S \quad (3)$$

$N$  is the number of years in the horizon,  $r$  is the discount factor,  $K_i$  and  $C_i$  are the fixed and variable costs in year  $i$ , respectively, and  $S$  is the salvage cost of the added equipment at the end of the horizon.

In formulation (1), (2), operating and expansion activity are tightly linked. To dissect out the operating activity and replace it by simpler approximations we make the following assumptions.

- (a) Equipment additions can only be made at discrete points in time; say at the end of each year.
- (b) The constraint functions can be partitioned in the following manner:

$$G(Y, Z) = \begin{bmatrix} G_0(Y) \\ G_1(Y, Z_1) \\ \vdots \\ G_N(Y, Z_N) \end{bmatrix} \quad (4)$$

where  $Z^1, \dots, Z^N$  are the subsets of the operating variables in the intervals between equipment additions. A rough, physical interpretation is that the operating variables are not allowed to have long term cumulative effects on the network, and especially no cumulative effects across interval boundaries. By and large this assumption is reasonable but there can be violations, for instance, the energy extracted from a nuclear reactor core in one interval usually affects the energy extraction schedule in subsequent intervals.

(c)  $C_i$ , the variable cost in interval  $i$ , does not depend on the operating variables in other intervals, that is:

$$C_i = C_i(Y, Z_i) \quad (5)$$

The implications are similar to those of the second assumption.

Under these assumptions, the general problem contained in relations (1) and (2) can be rewritten in the form:

$$\text{Minimum}_{Y \in W} \left\{ \sum_{i=1}^N r^{i-1} \left[ K_i + \text{Minimum}_{Z_i \in V_i(Y)} \{ C_i(Y, Z_i) \} \right] - S \right\} \quad (6)$$

Inner Loop  
 Outer Loop

where:

$W = \{Y \in \mathbb{R}^n \mid Y \leq P\}$  is the set of feasible expansion plans

The inner loop represents the optimum value of the variable or production cost of interval  $i$ . Instead of being calculated in full, it is advisable to replace it with an approximation to the optimum production cost over a smaller sampling period. The replacement is for reasons of computational feasibility. In the approximation, the intricacies of the transmission network are neglected in favor of simplified loss representations. An example of a sampling period is a few typical days from each season.

The net effect is the replacement of the inner loop by:

$$\text{Minimum}_{X_i \in U_i} \{ \hat{C}_i(Y, X_i) \} \quad (7)$$

where  $T_i$  is the length of interval  $i$ ,  $t_i$  is the length of its sample period,  $X_i$  is the reduced subset of operating variables over the sample period,  $U_i$  is the corresponding feasible set and  $\hat{C}_i$  is the approximation to the production cost over the sample period.

### Comments

- (a) The form of problem (6) remains the same whether expansions are limited to central generation or extended to include dispersed generation and central and dispersed storage.
- (b) Generation expansion methodologies, e.g. [11], [12], solve the outer loop of (6) given load duration curves and, for each iteration of the outer loop, the corresponding optimum production cost.
- (c) Thus, generation expansion methodologies can be extended to handle expansions of dispersed generation and central and dispersed storage. All that is required is a new modelling methodology for the inner loop to include the impacts of new alternatives on optimum production costs and load shapes.
- (d) Conversely, optimum production costs and load shapes are among the principal indicators of the long term economic impacts of an alternative. Even if they are not used to calculate the precise values of these impacts, they constitute a good pair of metrics for comparing alternatives.

### Whirl Model Features

This section provides an overview of a methodology for solving problem (7), i.e., for calculating optimum production costs and load shapes of simplified networks over sample periods.

We will consider networks whose components are energy sources (generators), energy transmission devices (lines, transformers and switches, for instance), energy stores (batteries and pumped hydro units, for instance) and energy demands (loads). The optimization tools available to minimize the production costs of such networks fall into three categories -- Nonlinear Programming, Linear Programming, and a

specialized version of Linear Programming, a transportation algorithm. The widest class of network structures tend to be the most computationally expensive. In contrast, transportation algorithms tend to be the most versatile but also the fastest (100-200 times faster than a Linear Program). Experiments we have conducted with members of all three classes indicate that the speed advantages of the transportation algorithms outweigh their disadvantages. Consequently, we have chosen a transportation algorithm to serve as the computational core for our models.

The network components are described in generic energy terms. Voltages and currents are not used, only energy flows, losses and various costs. Besides simplifying the descriptions, this approach makes it possible to accommodate hybrid activities such as those of a thermal load that can be simultaneously supplied from electric and solar heaters.

Input Data

- The models require the following data as inputs:
- the configuration of the network.
  - the operating and maintenance costs, the losses and the capacities of each source, transmission device and store. These quantities can be time dependent and, in the case of costs and losses, power dependent. Capacity values may be either hard or soft. In the latter case, they may be linear in emergencies and a penalty cost levied. Charging, discharging and seepage losses may be separately specified for stores.
  - the energy demands of each load as a function of time, and if curtailments are to be studied, the rebound characteristics of the load. (The term rebound refers to the load dynamics that result from demand curtailments and is discussed further in the section on Applications.)

Output

The models calculate the minimum operating cost over the sample time period (typically a day or a week), the associated schedule (the power flows from each device in each interval) and the sensitivities of the minimum operating cost to the device capacities.

TRANSPORTATION ALGORITHMS

Generalized Transportation Algorithms seek to distribute a commodity (energy in our case) from a group of supply centers or sources to a group of demand centers or loads, via intermediate transmission points, so as to minimize the total distribution costs. A necessary condition is that the distribution cost between any two points be linear in the quantity distributed; this can be generalized to piecewise linear convex costs. The law of conservation forms the basic constraint set; at each point, the total incoming quantity plus the supply at the point must equal the total outgoing quantity plus the demand at that point. The problem may conveniently be represented in terms of a network structure; the equivalent network problem is called the shortest-route problem.

We now turn to a formal statement of the problem. Consider a network wherein  $N$  represents the set of all nodes, and  $F$  the set of all arcs. The net supply/demand at node  $n$  is indicated by  $a_n$ , where  $a_n > 0$  for supply and  $a_n < 0$  for demand. Each arc's characteristics are stored in two descriptors, a triplet and a quadruplet, defined as follows:

$$i_r = C_r \cdot n_r \cdot r$$

$$J_f = (r_f \cdot f^x \cdot i_n, f^x \cdot a_x, f)$$

where

- $f$  is the arc number
- $m_f$  is the head node of the arc
- $n_f$  is the tail node of the arc
- $r_f$  is a multiplier giving the outflow from the tail for each unit of inflow to the head
- $c_f$  is the cost per unit of flow into the head; this cost is allowed to be negative
- $x_{\min, f}$  and  $x_{\max, f}$  are the lower and upper bounds on  $x_f$
- $x_f$  is the inflow to the head.

- Let  $I = \{i|f \in f\}$ , be the set of all arc triplet descriptors
- $H(n) = \{f|(n, m_f, r) \in I\}$ , be the set of all arcs with head  $n$
- $G(n) = \{f|(m, n, f) \in I\}$ , be the set of all arcs with tail  $n$
- $C = \sum_f c_f x_f$  be the total transportation cost.

The problem that a transportation algorithm solves is [22]:

$$\begin{aligned} & \text{Minimize} \\ & \quad x_f, f \in F \\ & \text{subject to} \\ & \quad f \in H(n) \quad f \in G(n) \quad x_f = a_n, n \in N \\ & \quad x_f \leq X \leq x_f, \quad f \in F \\ & \quad \min_f - f - \max_f \end{aligned} \tag{8}$$

METHODOLOGY

Talukdar, Morton and their collaborators have developed a set of procedures by which the "inner loop" problem symbolized by (7) can be translated into the transportation problem (8). Portions of the procedures have been reported in a number of scattered works [11]-[17]. In this section we will assemble a coherent summary of the procedures and their conceptual basis.

Problem (7) requires that we minimize  $\hat{C}$ , the production costs (operating + maintenance) over a sample period for the electric network whose configuration is determined by the equipment additions called for by expansion plan  $Y$  up to the  $i$ -th interval of the planning horizon. The power flows from the devices of the network constitute the decision vector,  $X_i$ . The constraint set,  $U^i(Y)$ , specifies that the power must be conserved at the network's nodes and power flows cannot exceed the capacities of its components.

problem (7) is converted into a transportation problem of form (8) by dividing the sample period into subintervals. The power flow in each subinterval are assumed to be flat (unchanging with time) and are represented by the arc flows,  $x_{ij}$ , of the transportation problem. Specific assignments of arcs are discussed below.

### Structural Alternatives

The "basic representation" of a device that does not store energy consists of a collection of arcs and possibly, a source or a demand. The source is included to simulate energy producing activity, the demand, for energy consumption. The arc cost factors are set to reflect operating and maintenance costs, the multipliers, to reflect efficiencies and losses. Arc bounds are set to reflect device capacity limits and ratings. The "basic representation" of each such device is replicated for each subinterval of time.

Storage devices are represented by collections of arcs that interconnect the subintervals. This reflects a stores principal mode of activity - allowing the energy delivered at one time to be used at another. Charging, discharging and seepage losses are included via the multipliers on these connecting arcs; costs, if any, through their cost factors, and capacity limits through the arc bounds.

### Direct Control Strategies

Direct control operating strategies, that is, curtailments and interruptions of supply to the loads, are represented by fictitious sources and/or stores. The "fictitious energy" supplied by these entities in the models corresponds to the amount of energy not served in the real network.

When a load's energy demands are curtailed in one time interval, it often responds by increasing its demands in subsequent intervals. The phenomenon is called "rebound" or "payback". Constantopoulos and Talukdar [15], [16] have shown that simple models are sufficient to capture the essence of this phenomenon. These models have the form:

$$D_{k+1} = d_{k+1} + \sum_{m=0}^W \beta_m (D_{k-m} - P_{k-m}) \quad (9)$$

where  $d_k$  is original power demand in interval  $k$ ,  $D_k$  is the inflated demand resulting from curtailments in prior intervals,  $P_k \leq D_k$  is the power actually supplied in interval  $k$ , the  $\beta$ 's are constants depending on the load's characteristics and the lengths of the intervals, and  $W$  is the number of intervals over which the load's "memory" extends. The  $\beta$ 's are called rebound factors.

The rebound model of type (9) can be included in the transportation problem formulation (8) with the aid of arcs that carry energy backwards in time, that is, from subinterval  $k$  to subintervals  $k-1$ . The multipliers on these arcs are set to the reciprocals of the rebound factors.

### Indirect Control Strategies

The class of indirect strategies centered around pricing policy is accommodated via the cost factors of the arcs used in load representations. The two major components of these policies are time-of-day rates and market-place-like-activity [17]. Time-of-day rates are directly included by the cost factors for the loads

in the appropriate subintervals. The term, "Market-place-like-activity" is used to refer to the class of activities in which customers with surplus energy are allowed to sell it back to the network. These sales can be simulated by adding arcs that permit flows from the loads to the networks. The cost factors on these arcs are set to the negative of the price at which the network buys energy from customers'.

To use these features, the models would have to be supplied data on demand elasticities and the responses of customers to being placed in market-place-like situations. These data could be obtained either from extrapolations of studies, e.g. [23], or from experiments on small but representative samples. With these data, the models could be applied to determine the effects of pricing policies on production costs and load shapes.

### Restrictions

The principal restrictions on the models are:

1. Costs and losses must be either linear or convex in power.
2. The electric network must be radial.
3. There is no provision for cross constraints between the arc flows in the Transportation Problem. In other words, one cannot include a constraint of the form:  $x_1 = ax_2$ , where  $a$  is a given constant.

To relax the first restriction would require the use of Nonlinear Programming. Fortunately, operating costs tend to be convex so there is seldom any need to relax it. The main exception occurs in attempting to study concave pricing policies, i.e. policies that provide discounts for large purchases of energy.

The second restriction accrues from the use of generic energy models for the components of electric networks. These models dispense with voltages. When applied to nonradial networks they allow power flows that cannot be duplicated in the electric network. This is not a serious limitation. As in the case of generation expansion studies, we are concerned only with estimates of total transmission and distribution losses and can approximate these losses with a few radially deployed elements.

The third restriction is by far the most troublesome. It excludes an important energy management alternative - cogeneration. In cogenerators, there are two outputs, electricity and usable heat. Their amounts are not separately adjustable. Therefore, to include cogeneration would require a removal of the third restriction which, in turn, would seem to require the replacement of the Transportation Algorithm with a Linear Program. This would impose significant running time penalties. We are attempting to find a better way to handle cogeneration;

### APPLICATIONS

#### Production Cost Minimization and Load Shape Development

The primary model functions, to minimize production costs and generate the associated load shapes, are directly accommodated. One need merely define the objective function,  $C$ , as:

$$C = C = \sum_{f \in R} c_f x_f \quad (d)$$

where C is the production costs and the c's are the equipment forced outage rate; arc deterministically represented by capacity derating. More elaborate outage models are possible but their use cannot be justified. Prediction of the expected outage rates for emerging technologies and indeed, many other of their quantities, are not sufficiently reliable.

Besides the primary functions, the models lend themselves to other related functions, two of which are outlined below.

#### Peak Minimization

The objective here is to minimize the demand peak seen by any selected component or group of components (e.g. central generation) in the network. This is done by using permissible curtailment levels and other available components to the fullest extent possible to meet the load peaks with prescribed reserve margins.

This problem has several interesting features. Curtailments can produce new peaks (via rebound phenomena) that equal or exceed the original peaks. Also, when mixes of storage devices with very different loss characteristics are present, the optimal order for charging and discharging them is far from obvious.

The problem may be solved iteratively by starting out with a sufficiently large capacity for the component in question and then reducing it on successive iterations until the problem is on the verge of turning infeasible. Clearly, the average demand over the sample period is a lower bound on the peak demand.

#### Multiojective Problems

The objective of minimizing production cost is in conflict with the objective of minimizing load curtailment (unserved energy). The models can be used to determine the best (Pareto efficient) tradeoffs between these conflicting objectives by using the cost factors and arc bounds to simulate the "Weighting" and "Additionally Constrained" methods of Multiobjective Optimization.

Let S denote the amount of energy curtailed, M the set of arcs used in representing the fictitious sources needed to simulate curtailments (c.f. the section on technology) and h the set of all other arcs. To implement the "Weighting Method", the objective function is chosen as follows:

$$C : (\hat{C} + \lambda_1 S) = \sum_{f \in R} c_f x_f + \sum_{f \in M} \lambda_1 x_f \quad (u)$$

and the associated transportation problem solved for several values of  $\lambda_1$ . Each solution corresponds to a point on the convex part of the Pareto surface. Points on nonconvex parts are obtained from the "Additionally Constrained Method". C is chosen to be  $\hat{C}$  but the additional constraint  $S \leq a_1$  is imposed and satisfied through iteration. For each value of  $a_1$  we obtain a new point on the surface. Further details and some examples may be found in [13], [15], [16].

#### EXAMPLE

As an illustration, consider the hypothetical system shown in Figure 1 which has been synthesized from data in [11-12]. Components representative of "central technologies" have been lumped at node 1. Attached to node 2 are aggregated equivalents or a number of "dispersed technologies". The link between nodes provides a rough approximation to transmission losses.

Data on the central generation and the storage are given in Tables I and II. The operating and maintenance costs of the wind-electric and solar-thermal units have been neglected. Their total time dependent capacities along with the load demand are given in Figure 2. Many of the data are representative though some of the penetrations are optimistic. Therefore, we caution the reader to pay no special attention to the penetrations - the example is intended primarily to illustrate model features.

All costs are relative; power has been normalized w.r.t. peak load demand; energy has been normalized w.r.t. the diurnal energy requirement of the total load.

#### Results

The code for the models automatically formulates and solves the transportation problem. When 12 two-hour intervals are used for the system in Figure 1, the equivalent transportation network contains about 260 arcs. On a modestly sized, time sharing system (DEC-20) the total turn-around-time for solving a problem of this size usually is 1-2 minutes for a base-case and a little less for a change-case. Thus, the code can be used interactively and has been in such use at CMU for some time.

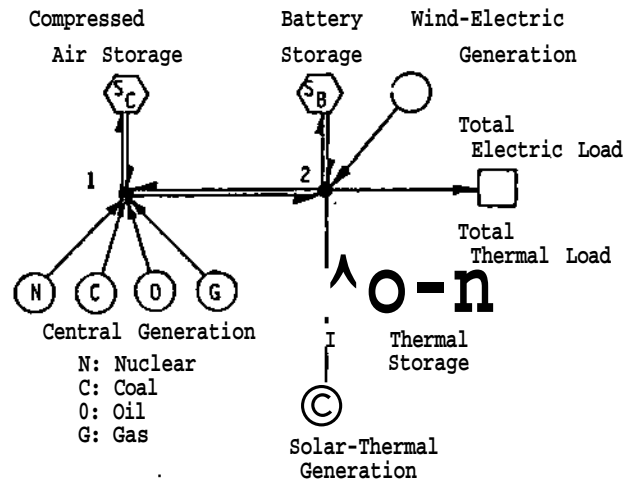


Figure 1. A Study System.

—•— : Only one-way-flows allowed  
 ==\*sr : Two-way-flows are possible

\*With a large, fast computer we would expect CPU times of the order of a second for problems of this size.

;\*p'dmcm rcaulta lor the example ay:iloim .ITV i:hom  
 in T:hl<sup>3</sup> ill mxl ir'lguma 2 nnd j. 'oomc comments on  
 them follow.

- (a) Optimum 3chodul<:i (load ;h:pc:i) .iro Kvw trom  
 obvioua. Thic Is n ch;ir;jctrrlatlc of nyatcma  
 which contain either mixru ol 'itor;<<' technologies  
 or one technology with noticeable losses.
- (b) In explanation of the rcaulta obtained for' the  
 minimization of peak-demand-on-ccntral-gcnration  
 (Hpjure 3), vc note that the output from this  
 generation has been flattened by the liberal use  
 of the oil-burning-eoraprcssed-air storage.
- (c) In changing objectives, the change in production  
 cost is less dramatic than the change in peak  
 demand on central generation and the changes in  
 schedules (load snapes). This appears to be a  
 characteristic of systems with noticeable amounts  
 of directly controllable storage.
- (d) A zero entry for a sensitivity indicates a  
 capacity-surplus, of the associated component.  
 Thus, the production cost of the example system  
 cannot be reduced by increasing S<sup>^</sup>, the amount of  
 central storage, but can be reduced by increasing  
 S<sub>B</sub> and S<sub>J</sub>.
- (e) An examination of the listed sensitivities  
 indicates that increasing solar-thermal generation  
 capacity is the most effective mran3 for reducing  
 tne production-cost-minimum of the example. This  
 is to be expected - the dispersed generation is  
 being used directly to meet load. From the point  
 of view of production costs, it is equivalent to a  
 load reduction (conservation). With other mixes  
 of components, other avenues become more  
 effective. For instance, if solar-thermal  
 capacity were increased much further, the thermal  
 storage would become the most important factor.
- (f) In seeking ways in which to change capacities, a  
 reasonable approach is to follow the gradient  
 vector whose elements are the individual  
 sensitivities. These sensitivities can easily be  
 augmented to include capital (fixed) costs and the  
 resulting gradient vector used by an expansion  
 planning program.

TAIUE I: CENTRAL GENERATION

Type	Quantity in X	Relative Operating Cont per Unit of Energy Produced
Nuclear	U.6	I
Coal	30.1	* 3.83
Oil	44.0	6.5
Gas	14.3	12.33

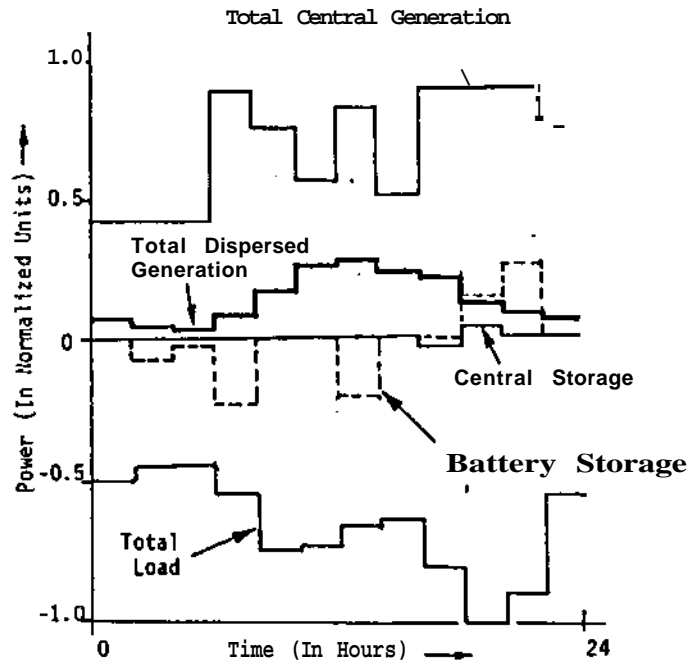


Figure 2. Schedules (load shapes) for the case in which production cost is minimized.

TABLE II: STORAGE DEVICE DATA

TYPE	Capacity in Normalized Energy Units	Maximum Charging Rate	Maximum Discharging Rate	Charging Efficiency (%)	Discharging Efficiency (%)	Relative Operating Cost	
						Charging	Discharging
Thermal	0.0215	∞	∞	100	100	0	0
Battery	0.0522	.229	.243	88	88	.18	.18
Compressed Air	0.1228	.275	.275	260*	45	.714	13.1*

\* A Compressed Air Storage unit burns oil during its discharge cycle [20]. This accounts for its high operating cost and its apparent ability to "discharge" more energy than it has stored.



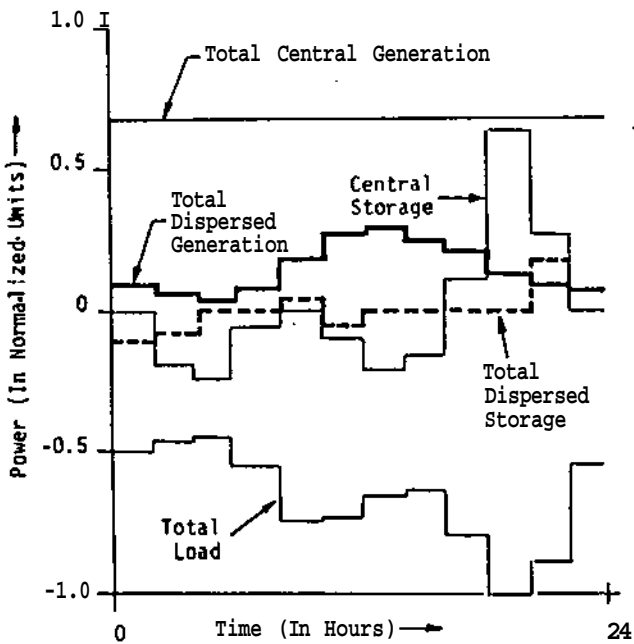


Figure 3. Schedules (load shapes) for the case in which the peak demand on the central generation is minimized.

#### CONCLUSIONS

This paper has described the features, methodology and limitations of a set of models for calculating the impacts of energy management alternatives on system production costs and load shapes. As far as we can determine, the models can include a wider range of alternatives with greater fidelity than other procedures reported in the literature. Moreover, they use a genuine optimizing procedure, not a heuristic. Consequently, they can be relied upon in unfamiliar situations where heuristics may break down. To their discredit, they need more computing time than most heuristics. However, their computing times are not long enough to keep them from being used interactively, unless cogeneration is combined with a complex mix of other technologies.

The models can be used to probe large, unfamiliar sets of management scenarios and as standards in the development and verification of heuristics for the intensive investigation of the nuances of specific alternatives.

Two problems for further investigation are the development of efficient ways to handle cogeneration and the embedding of the models in branch-and-bound or other codes for expansion planning.

#### ACKNOWLEDGEMENT

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TABLE III: ISPECIMEN RESULTS

Objective	Attribute		Some Sensitivities			
	e	P	$\frac{\partial \hat{C}}{\partial s_c}$	$\frac{\partial \hat{C}}{\partial s_b}$	$\frac{\partial \hat{C}}{\partial s_T}$	$\frac{\partial \hat{C}}{\partial a_T}$
			C	B	T	T
Minimize f	1.00	0.925	0	-1.14	-2.61	-7.18
Minimize P	1.11	0.685	X	0	-2.2	-14.27
Minimize R	1.28	1.079	0	-	-	-

**Legend**

$\hat{C}$  : Relative Production Cost  
P : Relative Peak Power Demand on Central Generation  
R : Amount of Oil Used in the Central Generation and Storage  
 $s_c$  : Compressed Air Storage Capacity (in normalized energy units)  
 $s_b$  : Battery Capacity (in normalized energy units)  
 $s_T$  : Thermal Storage Capacity (in normalized energy units)  
 $a_T$  : Solar-Thermal Generating Rating (in normalized energy units)

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