A NOTE ON THE MINIMUM NUMBER OF UNITS FOR HEAT EXCHANGER NETWORK SYNTHESIS

by

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ABSTRACT

Although graph theory shows that the minimum number of exchangers (U_{min}) in a network is usually one less than the number of streams (N - 1), examples have been published, which demonstrate that this limiting case cannot always be achieved. Some of these examples contain a pinch point, for which it is advocated that the 'N-1 target' should be applied on both sides of the pinch.

However, by using a novel arrangement of stream splitting, mixing and by-passes, some of the literature examples can be designed to conform to the N - 1 target.

For networks with similar total areas, those having fewer units will usually be cheaper, so U_{min} networks often have lower capital costs. Although the networks discussed here have somewhat larger total areas, possible applications are discussed where they may be economically attractive.

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INTRODUCTION

In the absence of loops and special subsets, the minimum number of units forming a heat exchanger network is given by Linnhoff et al (1979) as

$$U_{min} = N - 1 \qquad (1)$$

where N is the number of streams (including utilities).

Hohmann (1971) and Hohmann and Lockhart (1976) claimed that a minimum number of units network could be found for any system if parallel stream splitting was used. In later work, while discussing the Pinch Design Method (P.D.M.), Linnhoff and Hindmarsh (1983) point out that the target for U_{min} should be applied to both sides of the pinch, and this was also suggested by Grimes et al (1982). Since in most cases, some streams will cross the pinch, these streams will be counted twice and thus maximum energy recovery (M.E.R.) designs will usually contain more units than the number found when equation 1 is applied to the whole network.

This paper will show however that by using a configuration which allows merging of exchangers from both sides of the pinch, that it is often possible to obtain networks conforming to the U_{min} target given by (1).

DESIGNS WHICH SATISFY THE U_{min} TARGET

The simplest possible example is one having three streams not requiring utilities such as that shown in Fig. 1a in the grid format of Linnhoff and Flower (1978). Analysis of the stream data for $\Delta T_{min} = 20°$, (e.g. by the Problem Table of Linnhoff and Flower (1978) reveals the presence of a pinch point at 344° (hot streams)/324° (cold stream). Network synthesis (e.g. by the P.D.M.) would result in the network shown in Fig. 1a which has three units and so does not achieve the N-1 target.

If a feasible network with two units exists, then the heat duties of the two units are given by the enthalpy changes of streams 1 and 2. Since the temperature range of stream 1 crosses the pinch, the match between streams 1 and 3 must have a temperature difference of 20° at the pinch point. Furthermore because the heat capacities are assumed to be constant, if the match is to be feasible, the temperature approach must be ΔT_{min} (20°) throughout this unit, and the heat capacity flowrates in this match must be equal. Also as stream 2 enters the network and the other heat exchanger unit at the pinch temperature, then it can readily be deduced that the temperature approach must be 20° at the hot side of this unit. Thus the network must have the features shown in Fig. 1b. To make the network feasible, it is necessary, as shown in Fig. 1c, to introduce a further split and mixing of stream 3. This adjusts the inlet temperature to exchanger 1 so that stream 3 enters ΔT_{min} below that of the outgoing stream 1. (For ease of identification, the split heat capacity flowrates are enclosed in rectangles on Fig. 1 and subsequent figures).

Therefore a network having two units which satisfies the N-1 target has been derived. By considering the constraints of heat balance and ΔT_{min} it is easy to show that the split flows for the structure shown in Fig. 1c are unique.

A very similar problem to that of Fig. 1, with ΔT_{min} = 20°, was reported by Linnhoff et al (1979) and is shown as Fig. 2a. For this problem it was claimed that a network could not be devised 'with less than three units whatever stream splitting or other arrangements are made'. However the same structure of Fig. 1c may be used to reduce the number of units to 2 and such a feasible design is given in Fig. 2b. As can be seen from this figure the approach temperatures always exceed ΔT_{min} and so it is not a problem having a pinch point.

For problems having a pinch point such as Fig. 1, the network topolgy considered here has the following features:

1. Exchanger 1 in Fig. 1c crosses the pinch (344°/324°). However the values of C_p are equal for both streams (being numerically equal to 1). Thus it satisfies the conditions in the P.D.M. that above the pinch $(C_p)_C \geq (C_p)_H$, and below the pinch $(C_p)_C \leq (C_p)_H$. Furthermore because this match crosses the pinch and $C_{pH} = C_{pc}$, the LMTD for 1 is equal to ΔT_{min}.

2. At the mixing junction A in Fig. 1c a by-pass stream at the pinch temperature of 324° is mixed with a hotter stream, while at B mixing of a stream at the pinch point occurs with a colder by-pass stream. Thus the P.D.M. criterion that no energy should cross the pinch (i.e. streams with temperatures above the pinch should not transfer heat to or mix with streams having temperatures below the pinch) is satisfied both at A and B. However at both junctions such non-isothermal mixing is thermodynamically highly irreversible.

Problems without a pinch point such as Fig. 2 have similar features, except that the heat capacity flowrates in exchanger 1 in Fig. 2b are not necessarily equal because there is some freedom in how stream 3 is split.

It should be pointed out that the by-passing scheme discussed above has been virtually ignored in previous work on heat exchanger network synthesis. In his thesis (p. 208) Linnhoff (1979) used a similar arrangement in a design for problem 10SP2, but did not apply this scheme to a very similar three stream problem (p.198) to that of Fig. 2. Therefore the general implications that this arrangement can have for reducing the number of units, both in pinched and unpinched networks, have not been reported previously in the literature.

APPLICATION TO MORE COMPLEX PROBLEMS

A four stream problem (with $\Delta T_{min} = 10°$) is given in a recent guide on 'Process Integration for the Efficient Use of Energy' (I.Chem.E. (1982)) and the network from the P.D.M. is given in Fig. 3a. The pinch is at 90°/80° and the design contains six units, which, as there are two utilities, is one more than that given by equation 1. The loop which crosses the pinch involves exchangers 2, 3 and 4 which form a similar scheme to that of Fig. 1. Hence elimination of an exchanger gives the design of Fig. 3b. Here it may be advantageous to locate the heater on the smaller branch of stream 4 . Thus to heat this stream from 80° to 120°, LP steam may be adequate, whereas in the original design higher quality heat is required to raise the temperature of the whole stream from 125° to 135°. (Use of the Grand Composite Curve (I.Chem.E.(1982)) confirms that a lower quality utility is adequate for this problem).

Although it was stated earlier that for pinch problems there is no freedom in how streams are split, there are however alternative structures to that of Fig. 3b. These can be obtained by seeding a network with a dummy exchanger and then breaking the resultant loop as discussed by Grimes et al (1982). A feasible network found by this procedure is given in Fig. 3c. In this design exchanger 3 crosses the pinch but because the target temperature of stream 2 exceeds the inlet temperature of stream 3 by ΔT_{min}, the data for Fig. 3 is a special case. Thus the structure of Fig. 3c is simpler than that of Fig. 3b and is similar to that discussed by Su and Motard (1984). These authors point out that overheating of a branch of a stream and subsequent mixing with another branch can reduce the number of exchangers in a network.

Another four stream problem (with $\Delta T_{min} = 20°$) is considered by Linnhoff and Hindmarsh (1983) and the P.D.M. gives the seven unit network shown in Fig. 4a. The loop containing exchangers 1 and 4 can be broken by the technique considered here giving the 6 exchanger network shown in Fig. 4b. This, however, still exceeds U_{min} (5), but by considering heat loads and necessary matches (i.e. the TC method of Flower and Linnhoff (1980) with the insights of the pinch) it can be shown that no feasible five unit networks exist.

This conclusion has been confirmed by using the mixed-integar version of the trans-shipment linear programming (MILP) model without pinch partitioning (Papoulias and Grossmann(1983)). The minimum number of units predicted by this procedure was six.

Embedded in the MILP trans-shipment model are all feasible configurations for countercurrent heat exchange. Therefore N_{min} for the whole network can be predicted if pinch partitioning is excluded. (Thus allowing the possibility of heat exchangers crossing the pinch). As the solution to this model indicates which pair of streams should exchange heat as well as the corresponding heat loads, this information is helpful in deriving the special type of networks discussed in this paper.

It would therefore appear possible in quite a number of cases to obtain networks having fewer exchangers than those designed by the P.D.M. without increasing the utility requirements or violating the ΔT_{min} constraint. Furthermore in some of the systems considered networks having U_{min} can be designed. However the basic features of the P.D.M. are still retained (e.g. no energy flow across the pinch, choice of C_p in matches at the pinch etc.)

ECONOMIC ASSESSMENT OF THESE DESIGNS

Although the U_{min} design in Fig. 1c is of considerable interest as it is for a pinch problem, at junction A there is thermal degradation of 3 which has been heated nearly 120° above its target temperature. To achieve such a high temperature in 1 with ΔT_{min} throughout, means that this exchanger will have a large area. Hence ΣUA (calculated for counter-current exchangers) for Fig. 1c is 60% higher than that for the equivalent P.D.M. network (Fig. 1a). The investment costs (assumed proportional to $A^{0.6}$) for Fig. 1c exceed those for Fig. 1a by about 20%.

The economic incentive to obtain U_{min} designs is based on work by Hohmann (1971) who found that different topologies for networks with the same utility requirements had similar values for ΣUA. Thus for a given total area, networks will be cheaper if this area is concentrated into as few units as possible. However as discussed above, the design of Fig. 1c contains special features.

The reason why ΣUA is high in Fig. 1c is because the LMTD for exchanger 1 is equal to ΔT_{min}, whereas the average LMTD for the exchangers of Fig. 1a is almost 3 ΔT_{min}. This large difference in LMTD follows from the considerable difference in ΣC_p on either side of the pinch. Thus above the pinch $\Sigma(C_p)_H = 1$ and $\Sigma(C_p)_C = 3$, while below it $\Sigma(C_p)_H = 5$ and $\Sigma(C_p)_C = 3$. Hence the composite curves (e.g. I.Chem.E. (1982)) will be close together only at the pinch, and elsewhere the temperature difference between the hot and cold composites will be much in excess of ΔT_{min}. Therefore Fig. 1c topology will be more appropriate for systems where the composite curves are more nearly parallel. Such an example, from a recent study (Wood (1982)) of a crude oil distillation unit, shows that the composite curves can be reasonably parallel in the neighbourhood of the pinch and may cause almost a second pinch.

A simplified example where there are two pinches is shown in Fig. 5a. The pinches occur at 170°/160° and 70°/60°. As shown, the P.D.M. has five units. However, it is possible to obtain a three unit design as shown in Fig. 5b. This latter design has an estimated investment cost about 15% below that of the conventional design.

Another example with two pinches is shown in Fig. 6a: the pinches being at 350°/340° and 250°/240°. Starting with the feasible design of Fig. 6a, an exchanger may be eliminated giving the design of Fig. 6b. Alternatively from 6a and using the designs discussed here, the two pairs of exchangers 1 and 2, 4 and 5 can be combined giving the design of Fig. 6c. However Fig. 6d achieves U_{min}. In this design part of stream 4 is heated to the pinch temperature 240° in exchanger 3. Exchangers 2 and 1 cross pinch points and in each match the cold inlet temperature is adjusted so that it is ΔT_{min} below the outlet temperature of the hot stream target temperature. With suitable stream data, these same principles can be used to produce even more complex by-passing and mixing structures. The capital costs of the designs in Fig. 6b and 6d are both about 5% below that of Fig. 6a while that of Fig. 6c is about 5% higher.

The designs discussed here are also worth considering for retrofits, especially if advantage can be taken of the possibility of operating with a lower grade source of heat (cf. Figs. 3b and 4b).

It has been the object of this paper to obtain networks which satisfy U_{min} without violating ΔT_{min}. However in practice ΔT_{min} will not be treated as rigidly, indeed there is some evidence, as reported by Challand et al (1981), that violations of ΔT_{min} are necessary to achieve optimum networks. Thus if such violations are allowed it will often be possible to reduce the number of exchangers without resorting to the structures discussed here. For example exchangers are eliminated by breaking loops in the original papers which discuss the designs of Fig. 3a and 4a. Even though these networks violate ΔT_{min}, the process/process match capital costs are slightly below those of Figs. 3b and 4b.

Finally, it should be noted that there are practical limitations of using the objective of minimizing the number of units in a network. As noted by Challand et al (1981), in industrial applications several heat exchanger shells may be required for each predicted unit, thereby increasing the investment cost of the network. Therefore, more detailed analysis may be required to establish the economic desirability of the designs presented in this paper.

CONCLUSIONS

A novel arrangement of stream splitting, mixing and by-passes in heat exchanger networks has been discussed. This arrangement allows designs that have fewer exchangers, and sometimes achieve the 'N-1' target for the minimum number of units even if there is a pinch.

A modified version of the MILP trans-shipment model can be used to predict the true minimum number of units target. It also provides information on stream matches required to derive the network configurations.

Although the designs discussed in this paper require more surface area than conventional designs, they may be economically attractive when the composite curves are approximately parallel. Also these designs have the potential of accommodating lower grade utilities.

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LIST OF FIGURES

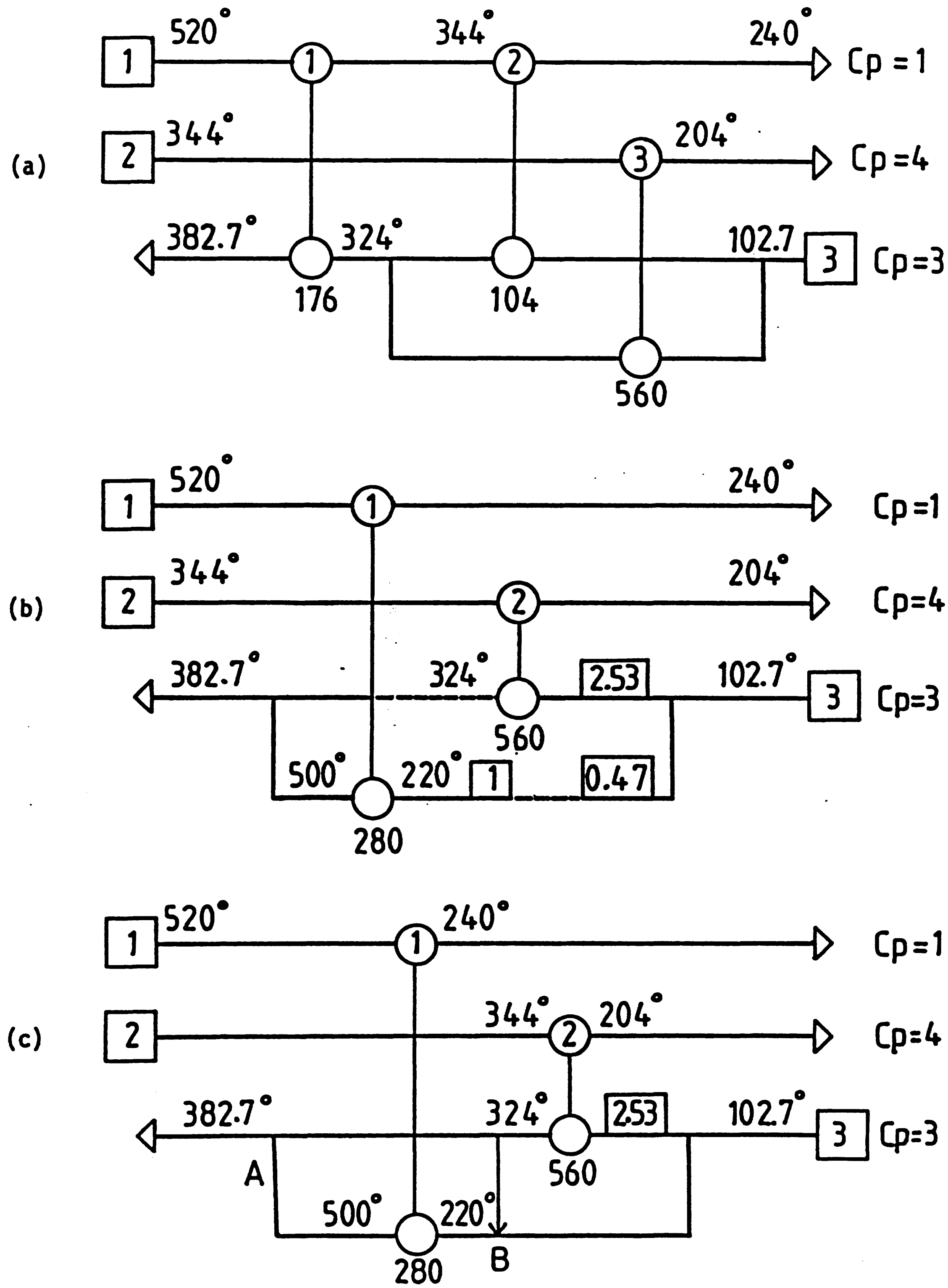
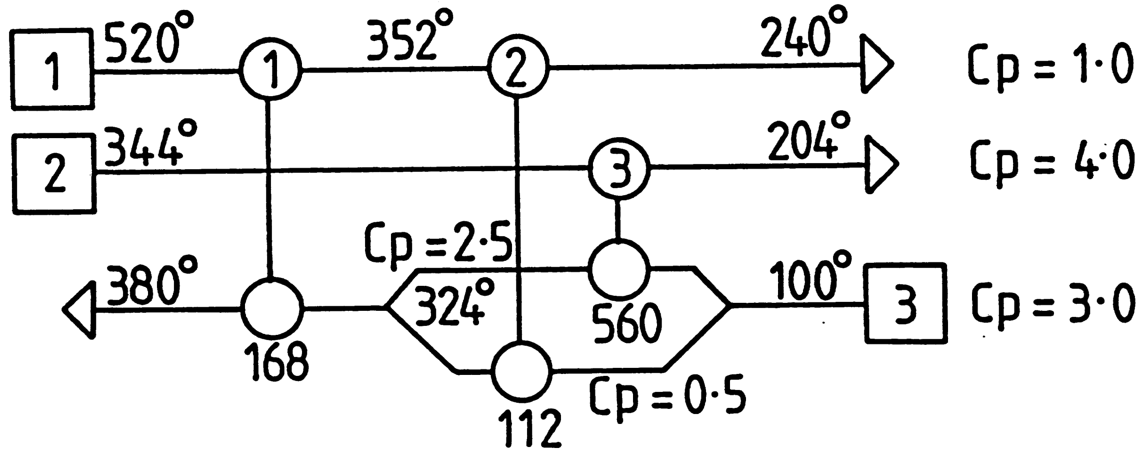


FIGURE 1 (a) A 3 Stream Network with a Pinch
(b) Requirements for a Network Having 2 Units
(c) A 2 Unit Network

(a)



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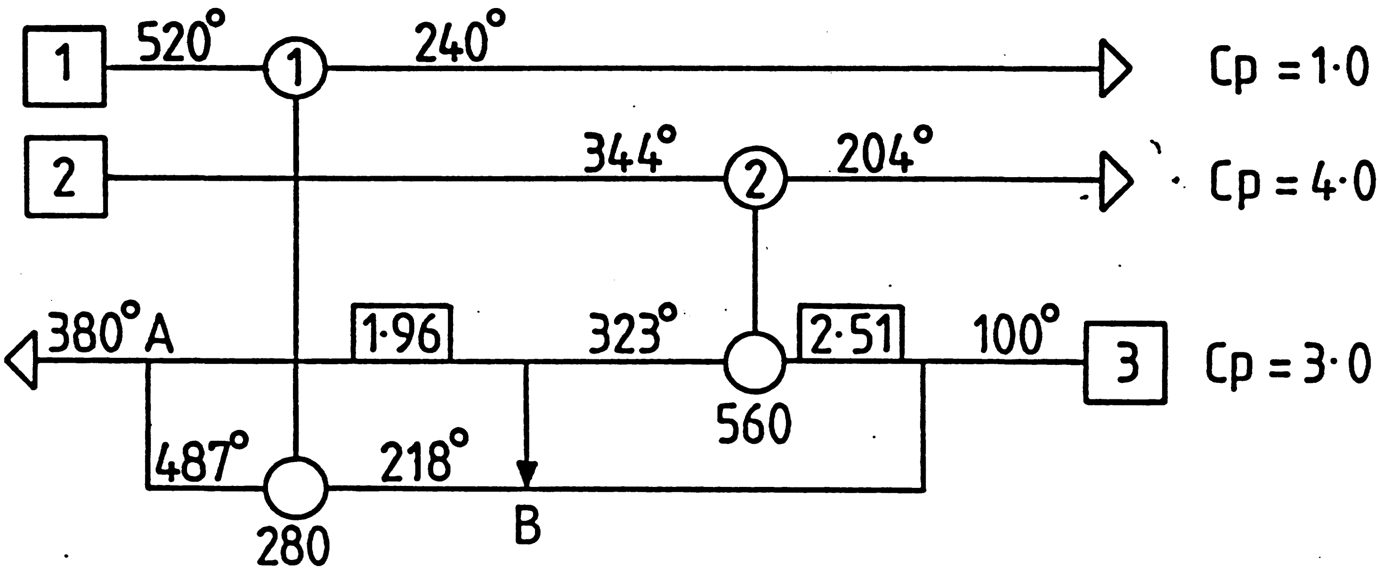


FIGURE 2 (a) 'A System that cannot be solved with the minimum number of units' (Linnhoff et al (1979)

(b) A 2 Unit Network.

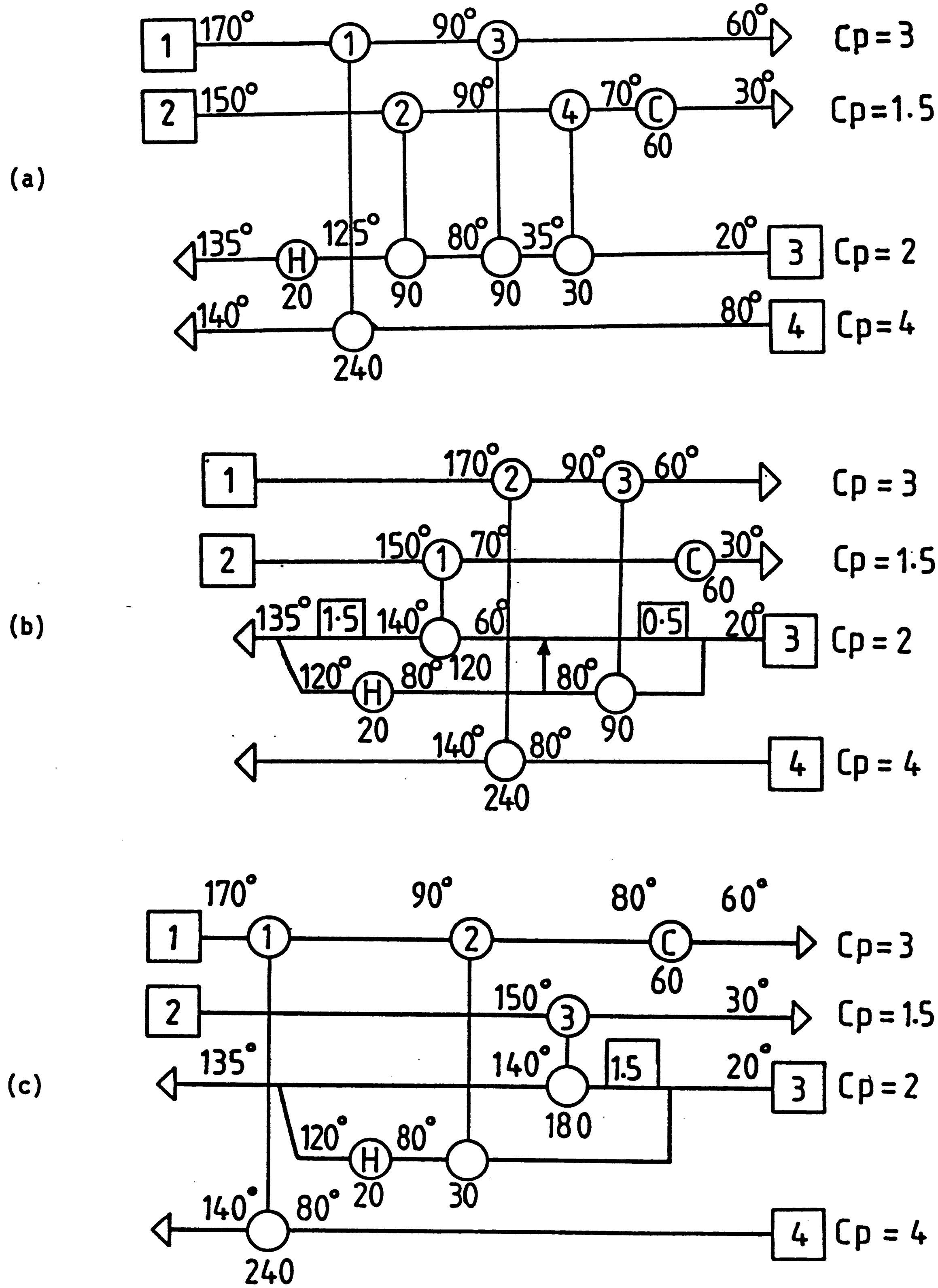


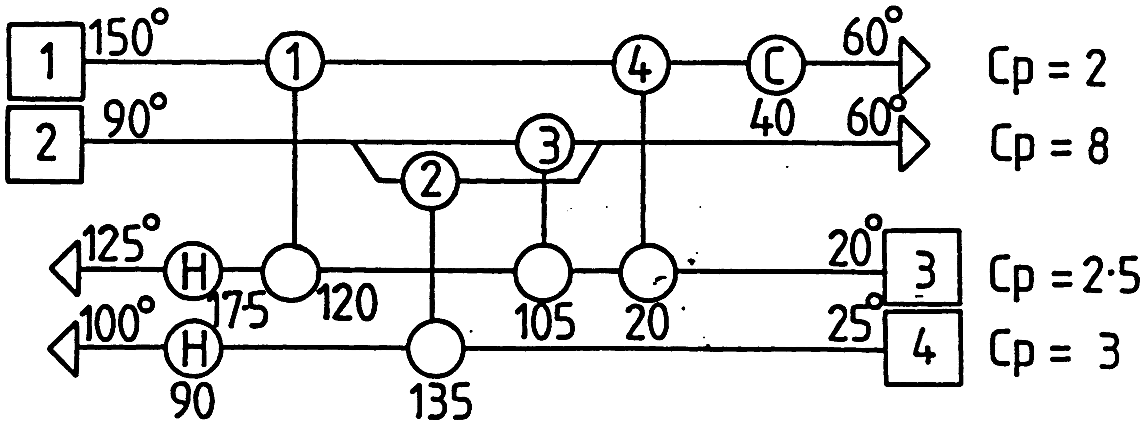
FIGURE 3 4 Stream Network Design

(a) Pinch Design Method

(b) Novel Design Procedure

(c) An Alternative Structure

(a)



(b)

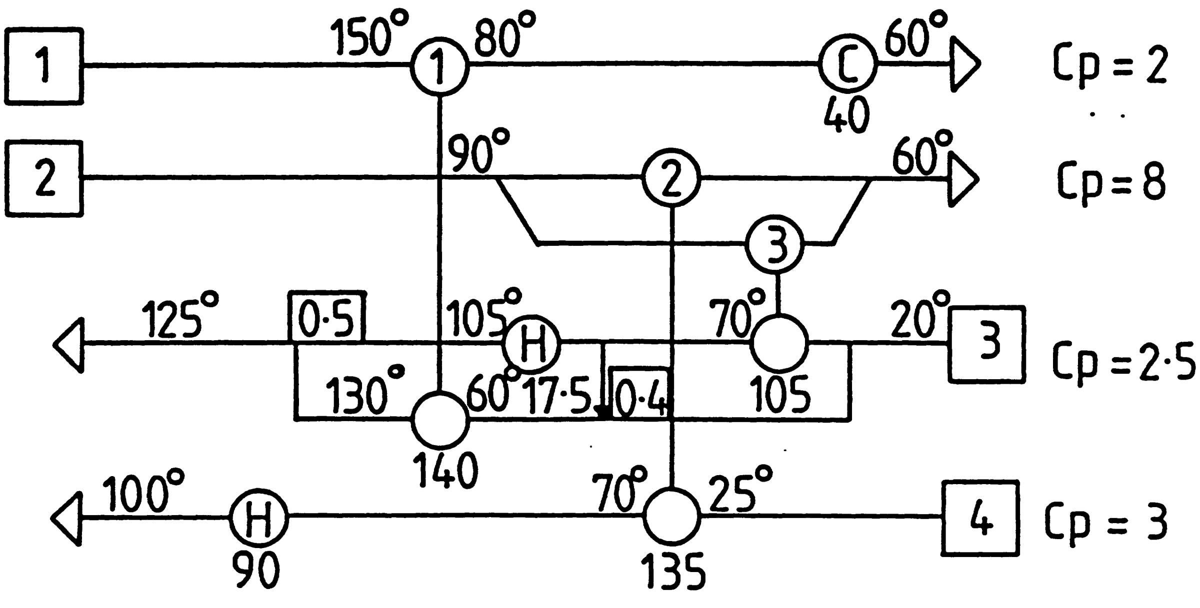
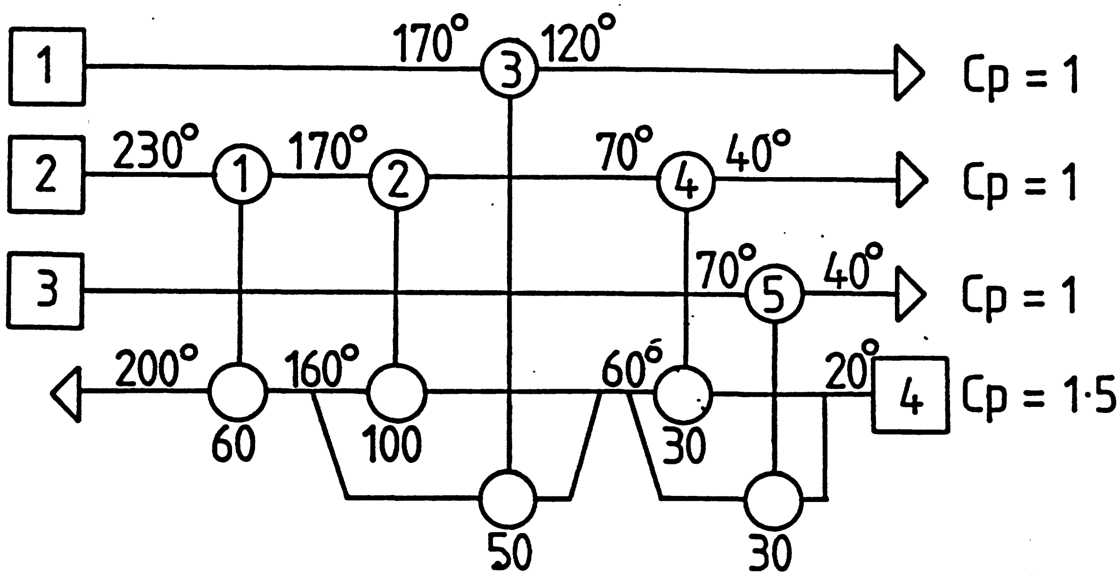
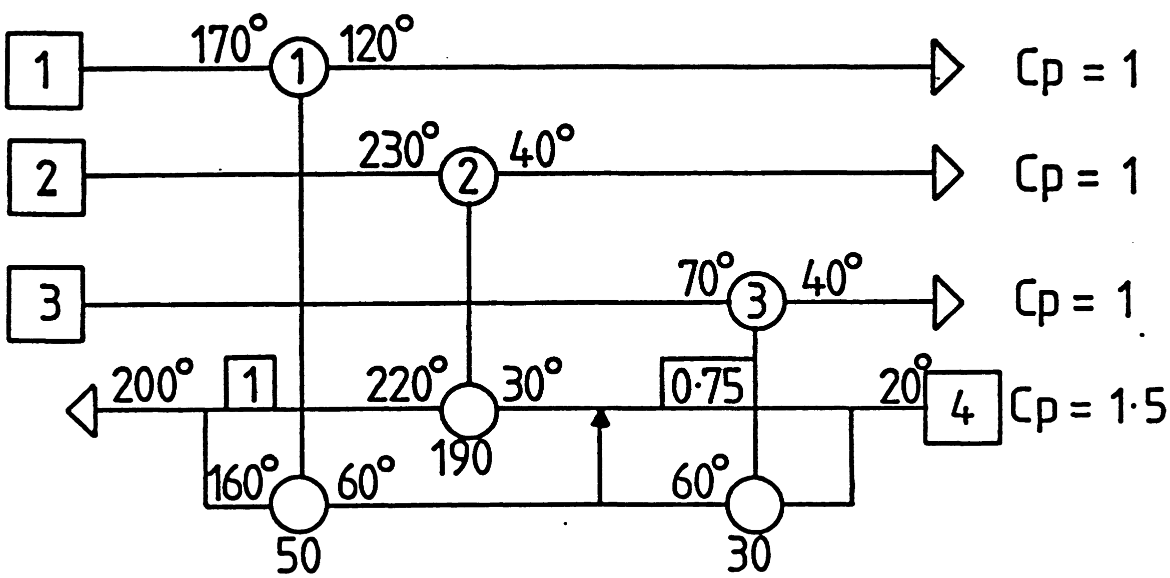


FIGURE 4 A 4 Stream Network Designed by
(a) Pinch Design Method
(b) Novel Design Procedure

(a)



(b)



<u>FIGURE 5</u> A 4 Stream Network Having 2 Pinch Points Designed by

(a) Pinch Design Method

(b) Novel Design Procedure

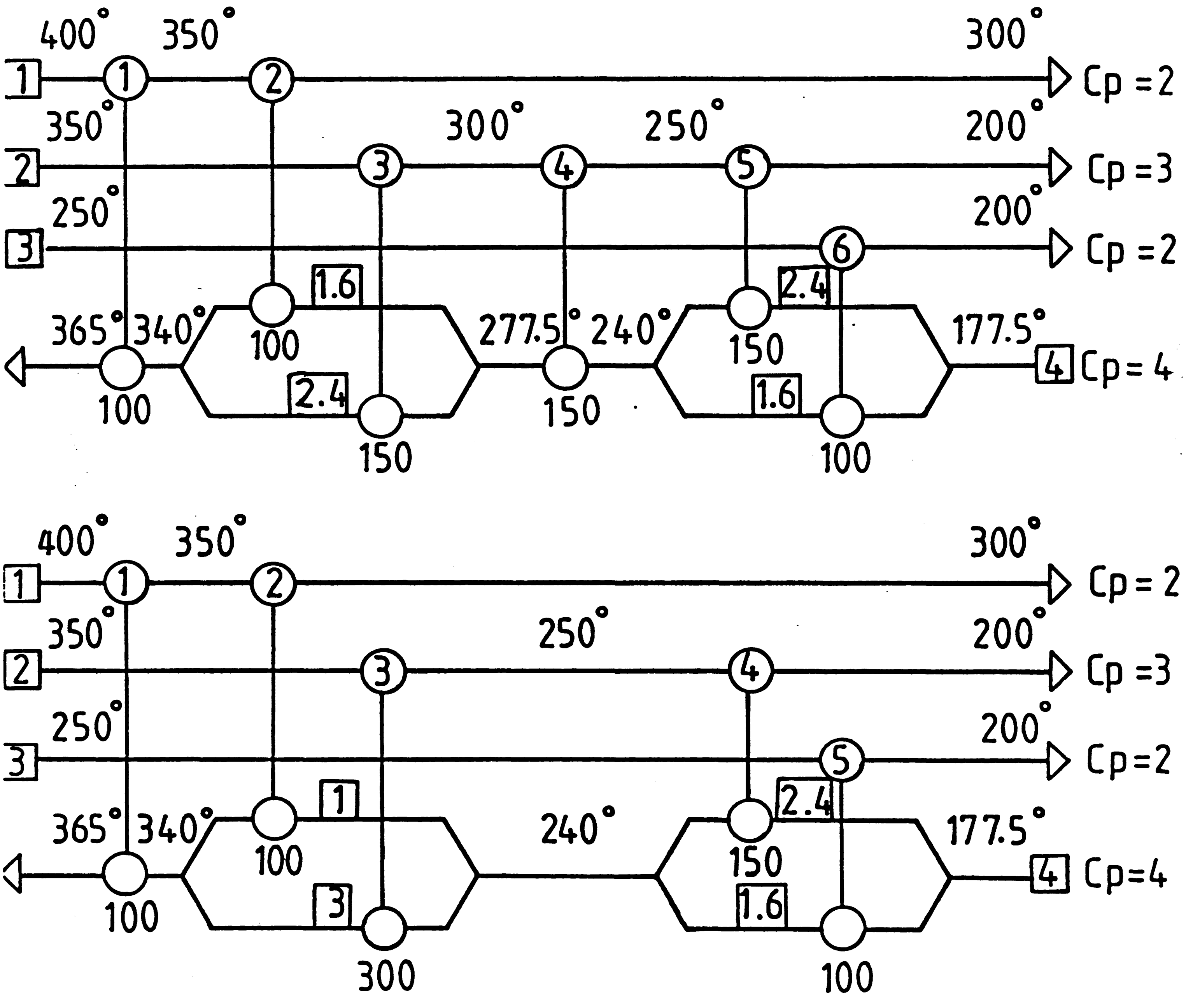
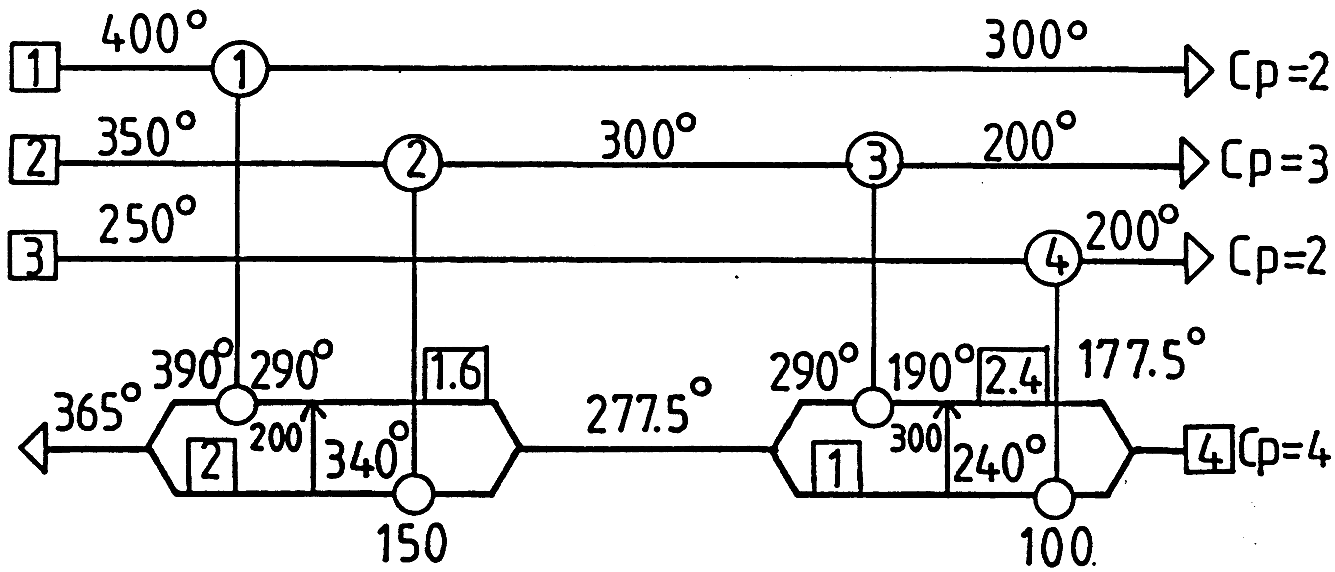


FIGURE 6 Stream Network having 2 Pinch Points

(a) Initial Feasible Network

(b) Traditional Design



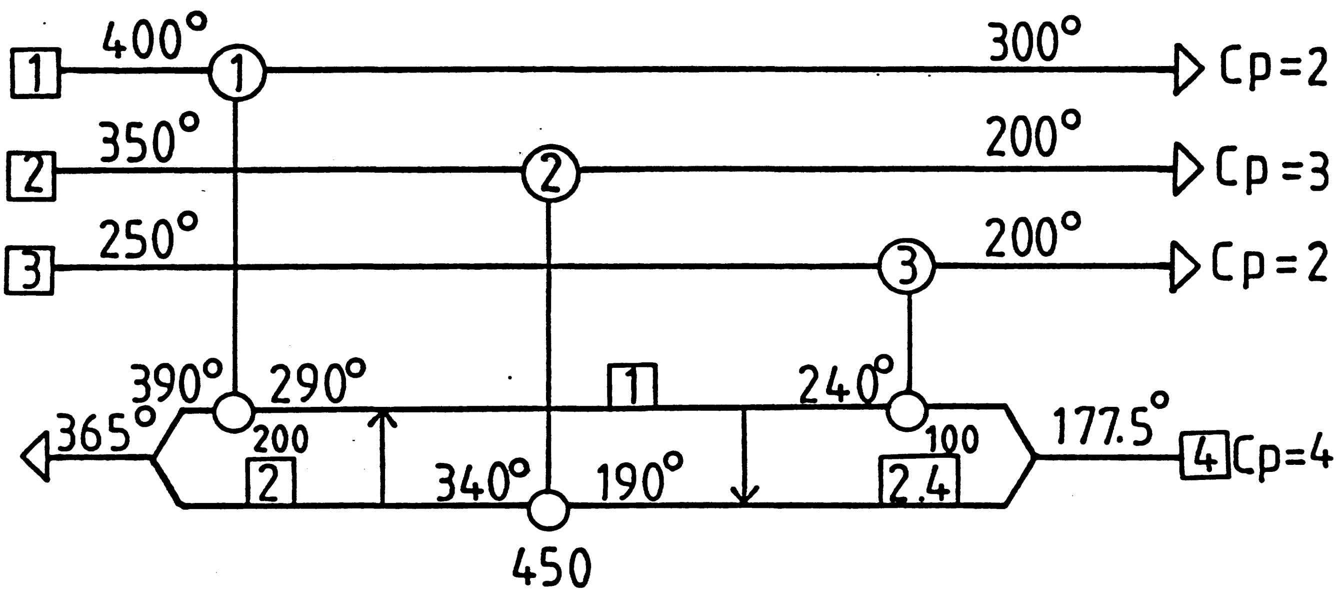


FIGURE 6 (c) By-passing Scheme

(d) New By-passing Scheme