THE SYNTHESIS AND EVOLUTION OF NETWORKS OF HEAT EXCHANGE THAT FEATURE THE MINIMUM NUMBER OF UNITS

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DRC-06-22-81

September 1981

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1981

ABSTRACT

An important problem in the design of chemical processes is that of bringing process streams from their temperatures of availability to the temperatures at which they are needed without undue cost. An important strategy for reducing the cost of doing this is heat recovery: Using the heat available from streams to be cooled to service streams to be heated.

In the absence of nonthermodynamic constraints, it is not difficult to assess the amount of heat recovery possible; and methods have been proposed (Nishida et al., 1971 and 1977; Linnhoff and Flower, 1978) that allow full heat recovery to be systematically obtained. The networks to which these methods lead are, however, more complex than necessary. Typically, therefore, those methods have been augmented with techniques for the evolutionary development of the initial network in order to simplify its structure, usually by minimizing the number of "matches" between streams.

The present work proposes a simple method for exploiting features of maximally simple networks (those that have as few "matches" as possible) in order to design, with greatly reduced effort, such a network that features a high (typically complete) degree of heat recovery. Further exploitation of those features allows a simple method of evolutionary development that makes it possible, in a very few evolutions, to improve the initial network as much as the data allow.

Unlike the other methods offered, the present ones are not hindered by the presence of non-thermodynamic constraints (practicality, safety, operability)• Their generality and enhanced simplicity make

them, more than any others, applicable in an industrial context. Although intended for application by hand, they lend themselves admirably to computer implementation, especially in an interactive mode.

These methods are demonstrated on the standard test examples and prove themselves powerful.

INTRODUCTION

The Problem

An almost universal feature of chemical processes is the heating and cooling of process streams. Synthesizing networks of heat exchange involves devising ways to accomplish such a task at least expense. Several of the more important features of process streams are defined below.

1. Inlet Temperature: Temperature from which a stream is heated or cooled (T_{in}).
2. Target Temperature: Temperature to which a stream is heated or cooled (T_{out}).
3. Heat Capacity Flowrate: Product of the heat capacity and flowrate of a stream (C_P).
4. Heatload: Rate of heat transfer necessary to heat or cool a stream from T_{in} to T_{out} (Q).

C_p can be a function of temperature, but a weak dependence on temperature can be ignored during the preliminary stage of design.

We distinguish between hot and cold streams not on the basis of temperature but on the basis of whether a stream is cooled or heated. Streams to be cooled (T_{out} less than T_{in}) are considered hot; streams to be heated (T_{in} less than T_{out}) are cold,

An example (the simplest) of the standard problems that are used to test synthesis strategies is 4SP1; see data in Figure 1.

Available utilities include steam and cooling water. The minimum approach temperature (T_{min}) is 20^o: No heat may be transferred with less than a 20^o temperature difference as driving force. The unit of C_p is 10,000 heat units per hour per degree: the unit of Q is 10,000 heat units per hour. Added detail and the parameters for estimating the capital and operating costs of a network are given in several sources, e.g., Grimes (1980).

Utilities provide heating and cooling as required. Their flowrates are not given in the problem, but the cost of any network will be a strong function of its utility consumption. A good network consumes as little as possible.

When the only constraint on heat exchange between process streams is the minimum approach temperature, it is possible to assess utility requirements for a problem prior to synthesis. A technique for doing so has been developed independently of one another by Hohmann (1971) and Linnhoff and Flower (1978).

This analysis proceeds by identifying a "pinch Point[11] and evaluating the heating and cooling required above and below it with enthalpy balances. For the case where a number of utilities are available to supply and absorb heat at a number of temperatures, the standard analysis can be easily generalized in terms of a multiplicity of pinch points.

The task can be divided into independent subtasks at the pinch point (in the case of more than two utilities: all pinch points). Any exchange of heat between these subtasks will involve an infringement on the minimum approach temperature somewhere in the network on the use of more (or more expensive) utilities than necessary. The independence of the subtasks has gone unreported but not unnoticed (Linnhoff (1980)).

Analysis reveals that the pinch point in 4SP1 occurs at 480° for the hot streams and 460° for the cold ones. This double figure is due to the minimum approach temperature of 20°.

The subtask above the pinch point is trivial. C3 is heated from 460° to 500° with utility heat. The other subtask is defined in Figure 2. It is simple but not trivial. A network of heat exchangers for this system of streams, the last of which is the needed cooling water, is to be designed.

By designing such a network without resort to extra utilities, complete heat recovery for 4SP1 will have been achieved.

There are infinitely many networks to accomplish the task. An inexpensive one is desired. The investment required to purchase the heat exchangers should be kept low.

The cost of a heat exchanger is approximated as proportional to its heat exchange area taken to the .6 power. The total heat transfer area of many networks is almost the same (Hohmann, 1971). Their cost is, however, very sensitive to the number of units among which this area is distributed.

A stream system involving m hot streams and n cold streams (including utility streams) requires at least m + n - 1 units. That is the number of matches required to allow the sum of the heatloads of the matches involving a stream to be equal to the heatload of the stream. If H_i and C_j exchange $q(H_i,C_j)$ units of heat, then the sum (for all C_j) of $q(H_i,C_j)$ must be $Q(H_i)$. The sum (for all H_i) of $q(H_i,C_j)$ must be $Q(C_j)$. At most m*n(m+n-l) q's can be specified as equal to zero. Only in those extraordinary cases where one of the remaining q's turns out to be zero, can we escape the need for m+n-1 units, one for each non-zero match.

Driving force requirements sometimes necessitate extra units. Thus, Linnhoff (1979) presents a six stream system (four process streams) that defies accomplishment in less than six units. He notes the inadequacy of the traditional formula but offers no new one.

These extra units can be allowed for by dividing the task at its pinch points. If as in Linnhoff's example, two or more streams cross a pinch point, extra units will be required to achieve full heat recovery. The number of units required will be $(m+n-1) + Z\,(S_{P_i} -1)$ where S_{P_i} is the number of streams that cross (not merely abut) the ith pinch point.

This estimate of the minimum number of units is an improvement on that of Hohmann and Linnhoff in that it takes into account driving force considerations as well as those involving heatloads alone. It is not infallible however. Contingencies of the stream data sometimes allow the merging of units above and below a pinch point [as in Linnhoff's solution to 10SP2 (1979)]. Sometimes there is no thermodynamically feasible network involving m+n-1 exchangers for a stream system with m+n members even though there is no pinch point as is evident in the discussions of 5SP1 and 7SP1 to be found later in the present work. Heatload and driving force requirements affect the number of units needed in systematic ways. Both can and should be allowed for. We doubt that any simple formula can account for the effects of the contingencies of stream data.

The synthesis technique that we will offer is intended to be applied to the subtasks and leads to a subnetwork of m'+n'-1 exchangers for a subtask with m' hot streams and n' cold streams. Merging subnetworks will result in $m+n-1 + \Sigma(S_{p_i}-1)$ units for the overall network. The minimum unit network so designed will, typically, feature full heat recovery. A high degree of heat recovery will always be achieved.

The Algorithm - A Summary

The algorithm to be presented comprises the following three major steps:

I. Identify minimum utility requirements and pinch points. Partition the problem into subproblems at the pinch points, each of which is to be solved separately.

II. Following the basic strategy offered by Greenkorn, Koppel and Raghavan (1978), develop a network for each subproblem with a sequence of steps, each of which eliminates entirely one of the streams remaining in the subproblem. By reducing the remaining task purdently at each step, the network developed will feature a high (often complete) degree of heat recovery.

We describe the use of a "search matrix" to aid in the bookkeeping required when selecting the next match at each step.

III. To the result found in Step II, apply an evolutionary method to improve the network. The evolutionary method to be described will create and break "cycles" in the network and is useful for modifying networks which already feature the minimum number of units and full heat recovery as well for those which do not. It will be seen to be a powerful tool for the design engineer.

We now describe and illustrate the algorithm in detail. The algorithm is effective for hand implementation and has been used to solve large problems, as we shall see. The discussion is aimed at the innovative designer who for reasons of his own may be seeking preferred forms to the final structure synthesized.

Task Reducing Matches

Consider the five stream task shown in Figure 2. How shall we, for instance, heat C3 from 240° (T_{in}) to 460° (T_{out})? The required 253.66 units of heat can be supplied by cooling H4 from 480° (T_{in}) to 353.2°. Such a match is thermodynamically feasible, i.e. it is in conformity with ΔT_{min}, and leaves us with the four-stream task given in Figure 3 rather than a five-stream task. C3 disappears, and H4 is replaced by its residual (H4').

The new task, analysis would show, can be accomplished without extra utilities. By designing a minimum unit network to accomplish this task, we will have designed such a network for the original task. We approach the new task in the same way.

We can use Cl, from 218.7° to 320° (T_{out}), to cool H4. This thermodynamically feasible match leaves a two process stream task, plus the cooling water utility which is given in Figure 4. Finally, H2 can be cooled with the residual of Cl and the cooling water required by the overall enthaply balance. The subnetwork is finished.

Upon merging the upper and lower subnetworks, we have the network shown in Figure 5 which is in fact the best network available. Matches have been numbered in the sequence chosen. Heatloads are given beneath the matches. All other figures are temperatures.

Not only does that network feature the minimum number of units, but full heat recovery as well. Further, the design minimizes the investment required to purchase the exchangers. Although this method does not always lead to the best network, it almost always leads to a good one.

Fortuitous insight is not necessary. It is possible to conduct a systematic search for matches that reduce the size of a task.

The task left by a match should satisfy two criteria. The remaining task must involve one less stream, and it should be possible to accomplish that task without extra utilities or more than the minimum number of units.

The first step in finding such a match is to consider each pair of one hot stream and one cold stream in order to determine if a match that leaves a single residual is possible. In the absence of stream splitting, there are exactly two ways to match a pair of streams so as to leave only one residual. These two ways are exemplified in the matches chosen for 4SP1.

Following the ideas in Greenkorn et al (1978) the first match chosen is of Type E. It matches the stream with the smaller heatload (C3) with the extreme temperature range of the stream with the larger heatload (H4). The second match is of Type M. It matches the stream with the smaller heatload (H4) with the moderate temperature range of the stream with the larger heatload (Cl).

Let S_i be the stream with the larger heatload and S_j be the stream with the smaller heatload. In a Type E match, S_j is taken from T_{in} to T_{out} while S_i goes from T_{in} to an intermediate temperature T^1 as determined by a heat balance. In a Type M match, S_j is taken from T_{in} to T_{out} while S_i goes from an intermediate temperature T^f (as determined by a heat balance) to T_{out}.

Both types leave single residuals. Type E leaves S_1 from T^f to T_{out}. Type M leaves S_i from T_{in} to T^f. Were S_i matched with an intermediate region, T'^1 to T'^2, a pair of residuals would remain: S_i from T_{in} to T'^1 and from T'_2 to T_{out}. The task would be made no smaller.

If a minimum unit solution that involves no split streams exists for a task, then an appropriate match exists to be found. The proof of this claim depends on an important feature of the minimum unit network that, by hypothesis, exists. Such a network must be acyclic (Linnhoff, Mason and Wardle, 1979).

The distinction between cyclic and acyclic networks has been variously misdefined in the literature on the synthesis of networks of heat exchange. The best correct explanation is to be found in the discussion of the minimum number of units in Linnhoff and Mason (1979). The definitions offered by, for instance, Hohmann (1971) and Greenkorn, et al. (1978) are not quite correct. We suspect that the difference between cyclic and acyclic networks is one of the many notions that is very simple to grasp intuitively but very difficult to define adequately.

Given a heat exchanger network, it will be possible to trace a path from some units to others by following intervening streams. The difference between a cyclic and an acyclic network is simply that in an acyclic network it is impossible to trace a path that returns to the unit from which it started without retracing one's steps. The absence of "cycles" that would allow returning to the starting point without retracing steps is essential to the following proof.

Call a match that heats or cools S_i from its inlet temperature or to its target temperature an extremal match on S_i. An extremal match on a stream is the first or last match that it encounters. Unless a stream is matched only once, there will be two extremal matches on it.

What must be proven is that there is some S_i that exchanges heat with some S_j in a match that is extremal on S_i and the only match on S_j. Such a match must be of Type E or Type M.

Choose a stream (S_i) at random and find a match in the network hypothesized that is extremal on S_i. Consider S_j the stream that exchanges heat with S_i in that match. Choose, if possible, an extremal match on S_j distinct from the first, which may or may not be extremal on S_i. We can proceed unless S_j encounters exactly one match. Carry the construction out in both directions as far as possible.

This procedure generates a sequence of streams. No stream can appear twice in the sequence; that would indicate a cycle. The sequence must be finite and, hence, has a first and last element. These two streams are matched only once, and those two matches are of Type M or Type E.

The use of stream splitting leads to complex topologies that involve costs not reflected by the standard parameters (Linnhoff and Flower, 1978). Stream splitting is, thus, best avoided unless needed for some particular reason. Certainly, if the double goal of full heat recovery and the minimum number of units is threatened, streams should be split.

If one is willing to split the stream with the larger heatload (S_i), a continuum of intermediate match types (Type I) becomes possible. S_i can be split into two branches having C_p's $x*C_p(S_i)$ and $(1-x)*C_p(S_i)$. The first can be matched with S_j and the second left as a residual. The inlet temperature of the residual will be that for S_j; but C_p and T_{out} will be different than those for the original stream. C_p for the residual will be $(1-x)*C_p(S_j)$; T_{out} for the residual will be $T_{lb}(S_u)$ (+/-)($Q(S_i)$-$Q(S_j)$)/((1-x)*C_p(S^). The choice of sign depends on whether S_I is a hot stream or a cold stream.

If stream splitting is required, this fact can be expected to be apparent from the disproportion between the C_p's of the streams to be matched. The possibility of splitting a stream is easy to exploit; but, in order to keep the exposition simple, that possibility will not be dwelt upon here. The problem under consideration (4SP1) does not require splitting any streams.

The Search Matrix

We are looking not only for a match that is of Type E or of Type M. We want one that does not bias heat recovery either. The utility requirements for the task left by a given match can be assessed with the standard analysis mentioned earlier (Linnhoff and Flower, 1978). It would be tedious, however, to repeat that analysis for every Type E and Type M match that we may discover. Further, this test would not make the choice absolutely safe. The possibility would remain that the reduced stream system, although feasible, might defy accomplishment in a network involving the minimum number of units. We want, instead, a simple heuristic that will allow us to choose a match that is unlikely to bias heat recovery without too much tedious effort.

Matches that have large driving forces and, thus, degrade heat over large temperature ranges, are more likely to offend than those with small driving forces. For each possible match, therefore, we add the driving forces at either end of the match to obtain the mean of the match. If both a match of Type M and a match of Type E are possible, the mean of the Type M match is chosen as it is the lesser of the two. We choose the match with the least mean to reduce the size of the task.

In considering every pair involving a hot stream and a cold stream, we need not consider matches involving the utility. Once the original task has been divided at the pinch point (or, if there are more than two utility streams, every pinch point), each subtask will have exactly one utility. In proving that there will always be an appropriate match to be found, we actually proved the existence of two appropriate matches. Because of this fact, we can safely neglect any given stream in our search without endangering its success. The obvious stream to neglect is the utility stream.

For the bottom half of 4SP1, we must consider four pairs of streams. The results can be displayed as in Figure 6. The entry records the kind of match and its mean. The dash indicates that no match of either type is thermodynamically feasible (satisfies AT_{min}). The asterisk indicates that the least mean possible for a match between the streams is too large to be in the running.

The least mean is 102; we choose to match H2 and Cl with a Type E match. Cl leaves a residual. We evaluate the matches between that residual and the remaining hot streams (Only H4 remains) and, again, choose the match with the least mean.

When no thermodynamically feasible task reducing matches remain, we may be faced with either of two situations. Typically, no task reducing matches remain when the stream system has finally been reduced to, for instance, a set of hot streams and the cold utility that must service them. In that, the simple case, we simply finish the job with the utility.

Sometimes, however, both hot and cold process streams remain although no task reducing match among them is possible. In this case, a second utility must be used. This extra stream allows an extra match. Because the evolutionary rules to be offered require a connected network of exchangers and because any network that requires two utilities in the absence of a pinch point will surely be the subject of evolutionary development, it is important to introduce the needed match.

The greater the heatload of the added match, the smaller the utility loads will be. Thus, it is wise to introduce a large match. We search for the largest amount of heat exchange possible between the remaining process streams, which should be very few in number. One can easily evaluate how much heat transfer is possible between two streams with the formula: $(|T_{in}(S_i) - T_{in}(S_j)| - \Delta T_{min})(C_p(S_j))$ where S_i is the stream with the larger heatload. If heat transfer is possible between any pair of streams, add the match with the potential for the largest heatload at that heatload. S_i will enter the match at $T_{in}(S_i)$ and leave at $T_{in}(S_i) - Q/C_p(S_j)$. S_j will enter at $T_{in}(S_j)$ and leave at $T_{in}(S_i) - \Delta T_{min}$. After adding this match, finish the job with utilities.

This method of choosing successive matches determines an effective function from the problem data to a preliminary network. Anyone correctly employing this method to generate a network for 4SP1 that does not incorporate stream splitting will be brought to the network shown in Figure 7. This network is not quite optimal, but it is very good. The investment necessary to purchase the heat exchange units for this network, as estimated in the standard way, is about 3.4% more than that required by the global optimum. The annual costs, including annualized capital costs, differ by less than 1%.

Evolutionary Improvement

The costs of different networks that display the minimum number of units and complete heat recovery differ, as these figures suggest, only slightly. Any such network is a good one, but it is always desirable to make a good design better.

One clear defect in the above design involves the use of the cooling water. It is unreasonable to cool H4 to 280^{o} with cooling water and use C1 to cool H2 to 200^{o}. If we put a cooler on H2, however, we no longer have a minimum unit network. A cycle (Figure 8) has been introduced. We can remove the cycle, however, without returning to the original network.

The extra unit allows a degree of freedom in allotting heatloads to the units. The units comprising the cycle have been given numbers so as to distinguish between even and odd units. We are at liberty to increase (decrease) all even (odd) units and decrease (increase) all odd (even) units by any given incremental heatload.

If we refuse to consider, as is only reasonable, retrograde heat transfer, i.e. the transfer of heat from cold streams, we can demand that all heatloads be non-negative. Given this constraint, there are two limits to the extent to which heatloads can be altered.

The relative size of the heatloads of the odd units and that of the even units will remain the same as all heatloads are altered. The heatload of the odd unit with the smallest heatload can go to zero, and so can that of the corresponding even unit. The first limit involves the network generated by the synthesis algorithm, and the second involves the global optimum.

Whenever a new match is introduced into a minimum unit network, a cycle appears. There is exactly one way to remove that cycle without returning to the original network, and the way to do so is obvious upon inspecting the heatloads of the units in the cycle. The new network may be better or worse than the original one; it may, in fact, be thermodynamically infeasible. It is not difficult, however, to spot cycles that lead to improved networks.

These cycles are, more exactly, polygons: rectangles, hexagons, octogons, etc. The polygon will always have an even number of both vertices (exchangers) and faces (streams). The vertices can always be divided into even and odd; their heatloads can be increased and decreased alternately.

One can search the original flowsheet for polygons with missing vertices and consider whether the match that would finish the polygon shows promise. One then increases the heatload at the new vertex until a heatload somewhere else in the polygon goes to zero. Two temperatures at each vertex remain the same and, so, can be taken from the original network. Intermediate temperatures are easily interpolated and the therrao-dynamic feasibility of the new network assessed by comparing the temperatures of hot and cold streams.

Although this evolutionary technique was invented with minimum unit networks in mind, it is not restricted to them in its application. The above process can be carried out even though the network under consideration already contains a cycle.

Application

Given any network, the technique of introducing and removing cycles allows the rapid identification of neighboring networks and a continuum of intermediate networks with one extra unit. In the typical case, where the introduction of an extra unit offers no hope of increased heat recovery, the intermediates are of little interest. They have an extra unit and nothing to show for the added investment that unit represents. In cases where the presence of an extra unit allows an increase in heat recovery, however, these intermediate units may represent improvements. Thus, with a problem like 6SP3 (introduced in the discussion of 7SP1 in the present work) where full heat recovery is incompatible with the minimum number of units, the introduction of a match may make attractive alternatives that do not feature the minimum number of units apparent.

This method is intended, primarily, as a tool that will allow the design engineer to identify networks that satisfy whatever criteria may be seen as desirable. The only demonstration of its power that can be given, however, is in lowering the estimated cost of newtorks for the standard test problems. Three cases will be discussed.

5SP1, with a posited non-thermodynamic constraint preventing a crucial match, provides an interesting example of the trade-off between capital costs and operating costs. Heat recovery, in that example, can be improved slightly while continuing to respect a T_{min} of 20° by increasing the number of units; but the network with the minimum number of units and a lower degree of heat recovery is markedly more attractive than that with an extra unit and full heat recovery.

In the discussion of 7SP1, an unconstrained problem that defies full heat recovery in the minimum number of units (6SP3) is introduced. In the case of 6SP3, however, the presence of an extra unit is justified.

Finally, there is an extended discussion of the evolutionary improvement of the output of the synthesis algorithm for 10SP1. In that case, which is more typical of the heat exchanger literature, the method of introducing and deleting cycles quickly reveals a considerable number of networks that are as attractive as any offered in the literature.

These examples are intended to display the goal of the evolutionary method offered: to reveal options. It is up to whoever is using the technique to decide which options are good by whatever criteria are deemed appropriate.

To keep the exposition brief, some details of the reasoning that motivates the evolutions and many of the intermediate and alternative flow sheets that might have been included have been suppressed. A full discussion of these examples is to be found in Grimes, 1980.

5SP1

One shortcoming in the literature on designing networks of heat exchange is its simplistic criterion for the acceptability of a match: A match may be included in a network just in case the inlet and outlet temperatures of the streams matched display adequate driving forces. An acceptable match is one with a driving force adequate to the requirements of ideal countercurrent heat exchange. Most heat exchangers, however, employ multipass designs that place greater demands on temperature relations than countercurrent exchangers* Also, there are sometimes practical reasons why two streams whose temperatures are compatible cannot be allowed to exchange heat. Such complications are rarely given their due in the literature*

One example that disallows certain thermodynamically feasible matches is given by Linnhoff and Flower (1978) in the form of 5SP1 with the added restriction that C_1 and H_4 may not exchange heat although C_1 must bring H, to its target temperature if full heat recovery is to be achieved* Utility requirements for the modified 5SP1 can be determined with the generalization of the method of enthalpy balances given by Cerda and Westerberg (1980). Matching C_1 and H_4 can save $10,432/yr; the cost of the utilities required by the modified 5SP1 is $43,228/yr.

The network for the modified 5SP1 given by Linnhoff and Flower is shown in Figure 9. It features seven units, one more than the minimum number, and recovers all but $37/yr worth of heat* Its cost, including capital charges, is $50,341/yr.

Using the methods presented in this paper, one could approach the modified 5SP1 by means of a constrained search matrix in which the forbidden match is never allowed as feasible. Two evolutions lead from the algorithmic network to that shown in Figure 10. The utility requirements for this network are slightly greater than those for the seven unit network shown in Figure 9. The $118/yr increase in operating cost is, however, more than offset by the $6,254 reduction in capital outlay allowed by the absence of a seventh unit. The cost of the six unit network is $49,834/yr.

This network could also be found by synthesizing a network for the unmodified 5SP1 with an unconstrained search matrix, replacing the match between C_1 and H_4 with a heater and cooler and improving heat recovery with the evolutionary introduction and deletion of matches. The authors prefer the latter approach as it is rarely possible to identify every configuration of circumstances that would make a match unacceptable before the actual synthesis of a network.

The sensitivity of the present evolutionary method to the number of matches facilitates the identification of a minimum unit network with enough heat recovery to make the addition of an extra unit, which would improve heat recovery at best a little, of dubious value. Alternative networks are generated in their completed form thereby allowing a full evaluation of each option. The tradeoff between capital and operating costs can be monitored at each step. The feasibility of each match in light of the indicated exchanger design and the properties of the streams matched can also be evaluated.

Although a seventh unit cannot, in fact, enhance heat recovery enough to be an attractive investment, one might consider some of the seven unit networks available in order to be sure. We could introduce a seventh unit even if no other unit can be deleted to allow a return to six units. In this way two seven-unit networks, both of which represent improvements over that of Linnhoff and Flower, have been identified (Grimes, 1980). One is very much like that in Figure 9, but it achieves the same level of heat recovery at a lower investment cost. The other involves splitting one stream, achieves full heat recovery and has a lower investment cost. Neither, however, has as low an overall annual cost as the network in Figure 10. A seventh unit does not pay its way.

The stream splitting network that achieves "full" heat recovery as defined by the methods of Cerda and Westerberg (1980) requires an investment of $68,087. The investment is not worth it for two reasons. The network in Figure 10 achieves an insignificantly lower degree of heat recovery with considerably less investment. Also, it is possible to achieve more than full heat recovery with less investment.

C_1 may not be matched with H_4, but it can cool C_3 to a point where C_3 is cold enough to bring H_4 to its target temperature. The price of the exchanger in which C_3 is cooled to 130°, as estimated with the standard parameters, is $8,378. The remaining five-stream problem requires no cooling water and can be accomplished in five exchangers. The five-unit network is shown in Figure 11. Capital investment required by those five units and the sixth in which C_1 and C_3 are matched is $67,532. This six-unit network,as might be expected, is less expensive than any of the seven-unit networks. Utility costs, moreover, have been reduced by $10,432/yr over "full" heat recovery. The cost of this final network is $39,706/yr, more than $10,000/yr less than any of the others.

In problems that feature non-thermodynamic constraints on heat exchange, it is sometimes possible to reduce utility consumption by exchanging heat between like streams (cold and cold or hot and hot). This fact makes the mere definition of (as opposed to ascertainment of) minimum utility consumption in such cases problematic. The unorthodox ploy of exchanging heat between like streams may be useful only infrequently; but its rewards on the occasions where it is of use can be dramatic. Orthodoxy in the present case would increase annual costs by 25%.

<u>7SP1</u>

In all but one of the standard test problems the algorithm in this paper leads to an initial network that achieves the double goal of full heat recovery and the minimum number of units. The problem on which it fails is 7SP1, and the reason for its failure is illuminating.

The first match chosen by the synthesis algorithm heats C_6 to its target temperature (410°) from its inlet temperature (350°) and cools H_2 to about 397° from its inlet temperature (440°). This choice of match does not bias heac recovery. Both 6SP3, as we dub the remaining problem, and 7SP1 require only cooling water. Nonetheless, the fate of the algorithm is sealed by the choice of this match because 6SP3 defies full heat recovery in a network of six units. No minimum unit network for 7SP1 contains the chosen match.

This fact about 6SP3 depends in complex ways on the details of the stream data. A demonstration will illuminate the complexity of the dependence. To help the reader follow the proof, the stream date is given in Figure 12. The inlet temperature of H_2 has been rounded off and the heatload adjusted accordingly.

Full heat recovery for 6SP3 entails the absence of utility heating. To achieve full heat recovery, then, H_3 must bring both C_1 and C_5 to their target temperatures. It is the only hot stream whose inlet temperature is sufficiently greater than their target temperatures. But, because the heatload of H_3 is smaller than the heatload of either cold stream, both must be matched twice. Four matches, none of which involves C_4, are thus accounted for.

Because of the small heat capacity flowrate of C_1 it cannot cool either H_1 or H_6 from 200° to 150°. As no other cold stream has an inlet temperature below 180^{o}, coolers are required for both H_2 and H_6. Six matches are accounted for, and none involves C_4.

Any network that achieves full heat recovery must have seven or more units: two in which C_1 is matched, one in which C_4 is matched, two in which C_5 is matched and two coolers. Complete heat recovery requires more than the minimum number (six) of units.

The example of 6SP3 saves us from the tempting inference that the modified 5SP1 defies full heat recovery in the minimum number of units because of the presence on non-thermodynamic constraints on heat exchange. One important difference between the modified 5SP1 and 6SP3 is that in one (the modified 5SP1) the inclusion of an extra unit to enhance heat recovery is not justified economically while in the other (6SP3) it is. The only way to come to such conclusion is, of course, to consider the options available. A network for 6SP3 that achieves full heat recovery by allowing an extra unit is shown in Figure 13.

Other stream systems will defy full heat recovery in the predicted minimum number of units. The complex nature of the reasons for the impossibility of full heat recovery with the minimum number of units in the present case strongly suggests that no acceptably simple criterion will infallibly predict the number of units needed to allow full heat recovery. Such criteria give only goals, i.e. heuristic estimates of what Hohmann (1971) called the "quasi-minimum" number of units. Our estimate of the quasi-minimum number is an improvement over Hohmann's in that it makes allowance for the extra units required by the presence of pinch points. Further improvement is surely possible, but the goal of infallibility seems quixotic.

The objective of our algorithm is to choose matches in sequence until a minimum unit network that achieves full heat recovery takes form. The assured success of such an algorithm requires an infallible criterion for the number of units needed for full heat recovery. As an infallible criterion seems unlikely, we conclude that no method of choosing matches in sequence will necessarily lead to a minimum unit network that achieves full heat recovery whenever applied to a stream system for which such a network is to be had.

The decision not to incorporate repeated analyses of utility requirements into the algorithm as a safeguard against choosing a match which would bias full heat recovery can now be appreciated in its proper light. If that safeguard were to insure success, the additional effort would, perhaps, be worthwhile. As success cannot be assured, however, the initial synthesis is best kept as effortless as possible. Effort should be expended where it does the most good, in the ensuing evolutionary development.

The algorithm given in this paper usually leads to a minimum unit network that achieves full heat recovery before evolutionary development, but such development will be desirable in order to improve the initial network. When the algorithm fails, as it sometimes will, it produces a minimum unit network that features a high incomplete degree of heat recovery. The algorithm can be relied upon only to generate an attractive point of departure for evolutionary development.

The user of the algorithm will be led to the network shown in Figure 14 for 7SP1. That network is highly non-optimal due to the presence of 84 units of unneeded utility heating and a like amount of extra cooling water to preserve the overall heat balance. In addition to this defective network, however, the user of the algorithm will achieve some insight into the contingencies of the stream data by repeatedly comparing the temperatures, heat capacity flowrates and heatloads of each hot stream with those of each cold stream. Three important facts will thereby have become evident.

1. C_6 should be heated by H_3 rather than H_2.

2. C_1 must be brought to its target temperature by H_3.

3. H_2 rather than H_3 should heat C_5 to its target temperature.

A match between C_5 and H_2 can be introduced at once; deleting the match between C_5 and H_7 returns us to the original number of units. In a second evolution, C_6 is matched with H_3 and the match between C_6 and H_2 deleted, Finally a match between C_1 and H_7 can be introduced and that between C_5 and H_2 deleted. These three evolutions allow us to dispense with utility heating to give the network shown in Figure 15.

This network has been identified as the global optimum for 7SP1 (Boland and Linnhoff, 1978). It is the optimum given two constraints.

1. No process stream is split.

2. The cooling water meets H_2 and H_7 in parallel rather than series.

Relaxing either constraint allows the cost to be lowered (Grimes, 1980). If the stream of cooling water is not split but, instead, meets first H_7 and then H_2, the necwork shown in Figure 16 becomes possible. That network features somewhat less flexibility but offers a lower cost.

10SP1

Although the initial network for 10SP1 (Figure 17) features both the minimum number of units and full heat recovery, that network is decidedly non-optimal. The cost per annum is about 1.4% greater than necessary, but the increased annual cost results entirely from investment costs that are more than 6% greater than necessary.

One source of unnecessary expense in the network is the less than economical distribution of driving forces. In a problem like 10SP1 where full heat recovery is easy, the conservative use of driving forces that is characteristic of the synthesis algorithm leads to some matches with small driving forces (C_4/H_{10}, C_3/H_9) being chosen near the beginning of the synthesis and others with large driving forces (H_9/C_1, H_9/C_3) being chosen near the end. The disparity in driving forces increases surface area, and hence, required investment but is easy to remedy.

The coolers on H_9 and H_{10} comprise three vertices of a rectangle. A match between H_9 and C_4 would complete the rectangular cycle. As heatloads are shifted around the cycle, the distribution of driving forces is enhanced by the elimination of the "greedy" cooler on H_9. The resulting network can be further improved by reordering the matches involving H_9 to give the network shown in Figure 18.

Improvement is still possible. The match between H_9 and C_3 remains as in the first network. This match, H_9/C_1 and H_6/C_1 are three vertices of a rectangle that can be completed by introducing a match between R_6 and C_3. Shifting heatloads leads to the network shown in Figure 19.

This network took little work and less ingenuity to discover. Yet it is less expensive than any network that has been offered in the literature other than on the basis of exhaustive search. It is not, however, even a local optimum. It has an immediate neighbor that is less expensive still.

By deleting H_6/C_1 in favor of a cooler on H_6, the network in Figure 20 is obtained. The low cost given for that network presupposes that the cooling water has been split into two rather than three branches. One branch services H_6 while the other is matched with H_{10} and then H_9.

This is the least expensive network without split streams to have appeared in the literature. It is slightly less expensive than that offered by Boland and Linnhoff (1978) on the basis of an exhaustive search that neglected the option of having the cooling water meet two streams in series rather than parallel.

Greenkorn et al (1978) have carried out an even more exhaustive search than Boland and Linnhoff. Using a notion essentially the same as that of a task-reducing match, they have searched the networks for 10SP1 that feature full heat recovery and the minimum number of units without any limit on the number of times streams may be split. The only constraint that they seem to impose is that all mixing be isothermal. They present a network that features split streams and is slightly less expensive than that in Figure 20 (Greenkorn et al, 1978). Because they do not curtail unfruitful search with repeated utility analyses, their method uses prohibitive amounts of computer time. Exhaustive search based on task-reducing matches and repeated utility analyses would, however, surely be a most fruitful approach. Their optimum is, as far as the authors know, the global optimum. It has not been improved upon by the present methods. Their network can, however, easily be generated by the present evolutionary method (Grimes, 1980).

In addition to the four cases discussed in this work, the present methods have been applied to eight other test cases. In five the best networks presented by others have been generated: the two test cases of Linnhoff and Flower (1978), the unconstrained 5SP1, 6SP1 and 7SP2. Two new optima are presented for 4SP2. The network in Figure 21 is the best network to appear in the literature that splits C1 into only two branches. Figure 22 displays what is, as far as the authors know, the global optimum for 4SP2. Figure 23 presents a new optimum for 6SP2, a problem that features non-thermodynamic constraints. 6SP2 was first presented by Grossmann and Sargent (1979), In the solutions involving split streams, split fractions have not been optimized. Optimality claims apply only to configurations.

CONCLUSIONS AND SIGNIFICANCE

A high degree of heat recovery and the attendant low operating costs are desirable features of a network of heat exchange, but the capital costs should also be kept low. One way to minimize the capital costs that might appear attractive would be to minimize the overall heat transfer area of the network. A more effective way, however, is to minimize the number of exchangers required. Hohmann (1970) presented an algorithm for minimizing both utility consumption and surface area, but concluded that minimizing the number of units is more important than minimizing surface area even though he had no special design technique for doing so.

Later authors presented algorithmic/evolutionary methods for designing thermally integrated minimum unit networks but usually emphasized heat recovery to the exclusion of minimizing the number of units during the preliminary synthesis. The tedius task of reducing the large number of matches that were introduced to achieve full heat recovery systematically was left to the ensuing evolutionary development and became one of the principal objectives of that development. Linnhoff and Flower (1978) and Su (1979) present the best of these methods. Another significant contribution is that of Nishida, et al. (1971, 1977) who also minimize surface area in the preliminary synthesis algorithm although minimum surface area is then sacrificed to reduce the number of units in the evolutionary phase as economic optimality dictates.

We have attempted to restore a balance between network complexity (as seen in the number of matches) and heat recovery to the preliminary synthesis. The evolutionary development can then tidy up any shortcomings in terms of heat recovery that might on occasion remain but is primarily intended to allow the convenient identification of alternative networks that feature both a high degree of heat recovery and network simplicity so that these alternatives can be evaluated in terms of whatever standards might be most relevant to the problem at hand.

A synthesis algorithm that strives for the minimum number of units requires a good preliminary estimate of what that number will be. Accordingly, we have loosened the traditional definition to allow for the extra units required by the presence of pinch points. The failure of the traditional definition has been noted before in individual cases involving pinch points (Linnhoff, 1979), but neither the reason for failure nor an alternate criterion have been offered. As the discussion of 6SP3 indicates, however, no such estimate will be infallible. It is therefore important that the evolutionary method offered, while emphasizing the conservation of the number of matches, allows the introduction of extra matches when, as in the case of 6SP3, that is desirable.

Some of the elements of this method have been noted by others. Greenkorn, et al. (1978) exploit task-reducing matches but not in the context of a convenient initial synthesis algorithm. Nor do they demonstrate that it is possible in every case to see a minimum unit network in terms of a sequence of task-reducing matches. The interdependent variability of the heatloads of the matches in a cycle can be discerned in the work of both Hohmann (1971) and Linnhoff and Flower (1978). The possibility of removing cycles by shifting heatloads until one goes to zero has been exploited by Su (1979) to reduce the number of matches resulting from the use of the Temperature Interval method of Linnhoff and Flower.

The power of the evolutionary technique of introducing and deleting cycles is supported by proof. A network of heat exchange can be viewed on two levels, a set of matches between streams and the order in which the streams encounter the matches that involve them. Any two coherent set of matches, sets having the correct number of matches, meeting a condition of topological coherence (connectedness) and including no matches with negative heatloads, are joined by a sequence of coherent sets each of which differs from each of its immediate neighbors by the presence (and absence) of exactly one match. It is possible, therefore, to move from any-coherent set to any other by passing through these intermediate sets, introducing and deleting one match at each step. In a similar way, it is possible to move from any thermodynamically feasible ordering of a coherent set of matches to any other by passing through intermediate thermodynamically feasible orderings by interchanging the order of two matches contiguous on a common stream at each step. Rigorous proof of these claims is given in Grimes, 1980.

These methods were developed as part of a production-rule system written in the experimental artificial intelligence language OPS3RX. This approach derives from current work in the area of developing computer systems that capture the expertise of engineers, doctors and other professionals, in the form of rule-based, knowledge-intensive programs (Rychener, 1981). The use of 0PS3RX was motivated by the desire to test the use of a production-rule system for allowing one to readily add and test various evolutionary strategies. It was hoped that the program would gradually do more of the intelligent processing without the need for human guidance. The flexibility of production systems seemed to lend itself to the problem of synthesizing networks of heat exchange, but the slow response of the system developed was prohibitive for engineering purposes. It should be noted that future production systems promise to overcome this difficulty. In an interactive context, but couched in a more traditional procedural language, these methods could be of considerable assistance in helping the design engineer to identify the options available to him or her quickly.

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CAPTIONS FOR FIGURES

	Tm	Tout	C_p	Q
C1	140	320	1.4450	260.10
H2	320	200	1.66668	200.00
C3	240	500	1.1530	299.78
H4	480	280	2.0000	400.00

Figure 1. Stream Data for 4SP1

Stream	T_{in}	Tout	C_p	Q
H2	320	200	1.66668	200.00
H4	480	280	2.00000	400.00
C1	140	320	1.44500	260.10
C3	240	460	1.15300	253.66
C	100	180	1.07800	86.24

Figure 2. Below the Pinch Point

Stream	T_{in}	Tout	C_p	Q
H2	320	200	1.66668	200.00
H4	353.2	280	2.00000	146.34
C1	140	320	1.44500	260.10
C	100	180	1.07800	86.24

Figure 3. 4SP1 Reduced Once

Stream	T_{in}	T_{out}	Cp	Q -
H2	320	200	1.66668	200.00
C1	140	218.7	1.44500	113.76
C	100	180	.1.07800	86.24

Figure 4. 4SP1 Reduced Twice

```
             C3          Cl
              |500
            ( H )
            (  46)
              |460          |320
H4    480-(  1  )-353-(  2  )-280
            (254)        (146)
              |240          |219
H2                 320-( 3 )-252-( C )-200
                       (114)       ( 86)
                         1140
```

Figure 5: Global Optimum for 4SP1

	Cl	C3
H2	E.102	-
H4	*	E.133

Figure 6: Search Matrix for 4SP1

```
           C3          Cl
            1500
          ( H )
          (46)
            |460        |320
H4   480-( 3 )-353-( 2 )-323-( C )-280
          (254)      ( 60)      ( 86)
            |240        |278
H2               320-( 1 )-200
                      (200)
                       1140
```

Figure 7: Another Network for 4SP1

```
           C3          Cl
            |500
          ( H )
          ( 46)
            1            1320            1180
H4   480-( • )-      "( 3 )-    0   -( 4 )-280
          (254)      (146            (0-->
            1240      — >60)           86)
H2               320-( 2 )-         -( 1 )-200
                     (114             (86
                     —>200)           —>0)
                       1140            1100
```

Figure B: A Cycle

```
          C5               C3          Cl
                           |360         1400
H2                   480-( 5 )-456-( 3 )-250
                         ( 77)     (648)
            |400           |329         1100
          ( H )          ( H )
          ( 49)          (307)
            |380           |204
H4 400-( 4 )-       234-( 1 )-172              ( C )-150
          (445)         (131)                  ( 54)
            |200           |150
```

Figure 9. Linnhoff and Flower ($50,341/yr)

```
        C5            C3           Cl
         |400                       |400
H2  480-( 2  )-323              -( 3  )-250
       ( 77)                     (648)
                       |360         |100
                     ( H )
                     (357)
         |369          1214
H4  400-( 4 )-  234-( 1 )-172            ( C  )-150
       (417)        (158)                ( 55)
         |200          |150
```

Figure 10. A Minimum Unit Solution ($49,834/yr)

```
           C5            C3          Cl
            |400                      |400
H2   480-(  1  )-435                -(5  )-250
          (141)                     (583)
                           |360       |130
                         ( H )
                         (302)
            |343           |237
H4   400-(  3  )-260   -(  4 -)-150
          (353)          (277)
            |200           |124
```

Figure 11. After Cooling C_3 with (^ ($39,706/yr)

	T*	Tout	C_p	Q
C1	100	400	1.6000	528.00
H2	397	150	2.8000	691.60
H3	520	300	2.3800	523.60
C4	180	350	3.2760	556.92
C5	200	400	2.6350	527.00
H6	390	150	3.3600	806.40

Figure 12. Stream Data for 6SP3

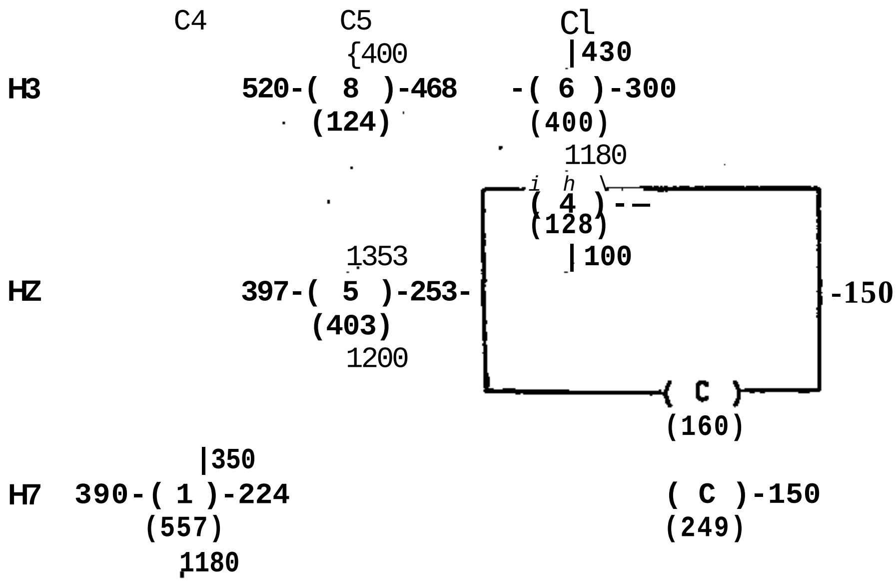

Figure 13. Full Heat Recovery ($29,678/yr.)

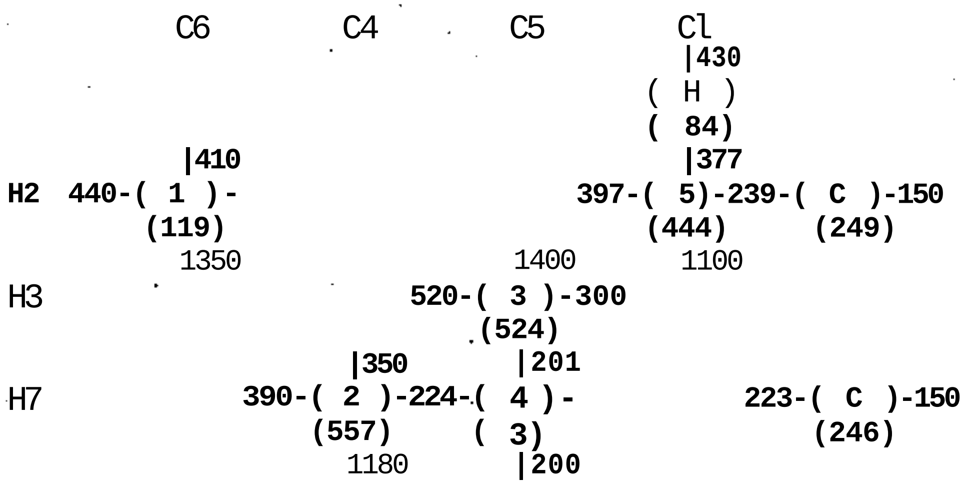

Figure 14. Algorithmic Network

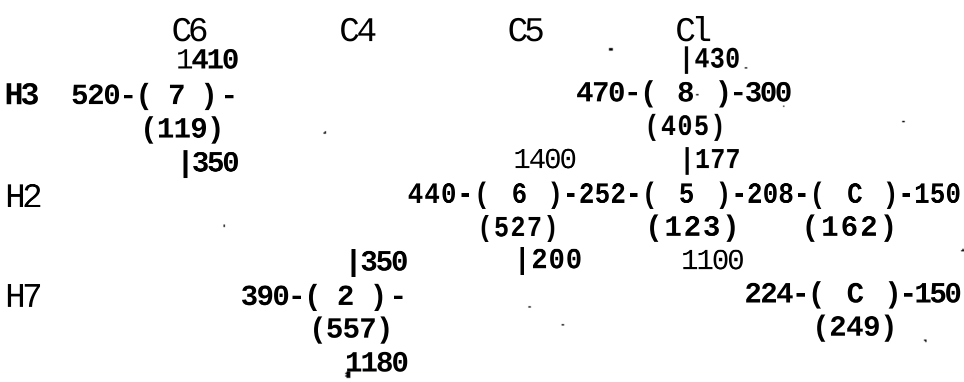

Figure 15. Final Result ($83,356)

```
          C6          C4          C5          C1
                                  |400
K2                          440-( 6 )-              252-( C )-150
                              (527)                      (285)
          1410                    1200        1430
H3   520-( 7 )-                         470-( 8 )-300
       (119)                                (405)
          |350        1350                    1177
H7                390-( 2 )-            224-( 9 )-188-( C )-150
                       (557)                (123)       (129)
                      1180                    1100
```

Figure 16. An Improvement (?) ($82,522)

```
          C2        CI        C4        C5                    C3
                                                              1430
H7                                                      480-( 37J-280
                                                           (400)
                                        |400
H8                                440-( 38)-252-( C )-150
                                     (527)      (285)
       |431      |320                   |200                  |180
H9   520-( 40)-427-( 41)-                    402-( C )-354-( 50)-300
        (220)     ( 60)                         (115)      (128)
       |240      |278                                         |100
H6            320-( 46)-200.
                 (200)
                 1140        1350
H10                   390-( 42)-                224-( C )-150
                          (557)                     (249)
                          1180
```

Figure 17. Algorithmic Output ($99,496)

```
          C4        C2        C1        C5                  C3
                                                            |430
H7                                                    480-( 37)-280
                                                          (400)
                                        |400
H8                                440-( 38)-252-( C )-150
                                      (527)     (285)
          |350      |431      |320      |200                |180
H9  520-( 39)-472-( 40)-379-( 41)-                    354-( 50)-300
        (115)     (220)     ( 60)                         (128)
                    |240       |278                         |100
H6                      320-( 46)-200
                            (200)
          |315                |140
H10 390-( 42)-                              259-( C )-150
        (442)                                   (365)
          |180
```

Figure 18. Reordering the Matches ($95,981)

```
          C4        C2        C5                  C1        C3
                                                            |430
H7                                                    480-( 43)-280
                                                          (400)
                                                  |190      |180
H6                                          320-( 52)-277-( 42)-200
                                                ( 72)     (128)
                              |400                |140      |100
H8                      440-( 44)-252-( 49)-150
                            (527)     (285)
          |350      |431      |200                |320
H9  520-( 45)-472-( 46)-                    379-( 47)-300
        (115)     (220)                         (188)
          |315      |240
H10 390-( 48)-                    259-( 50)-150
        (442)                         (365)
          |180                          |100
```

Figure 19. A Second Evolution ($93,724)

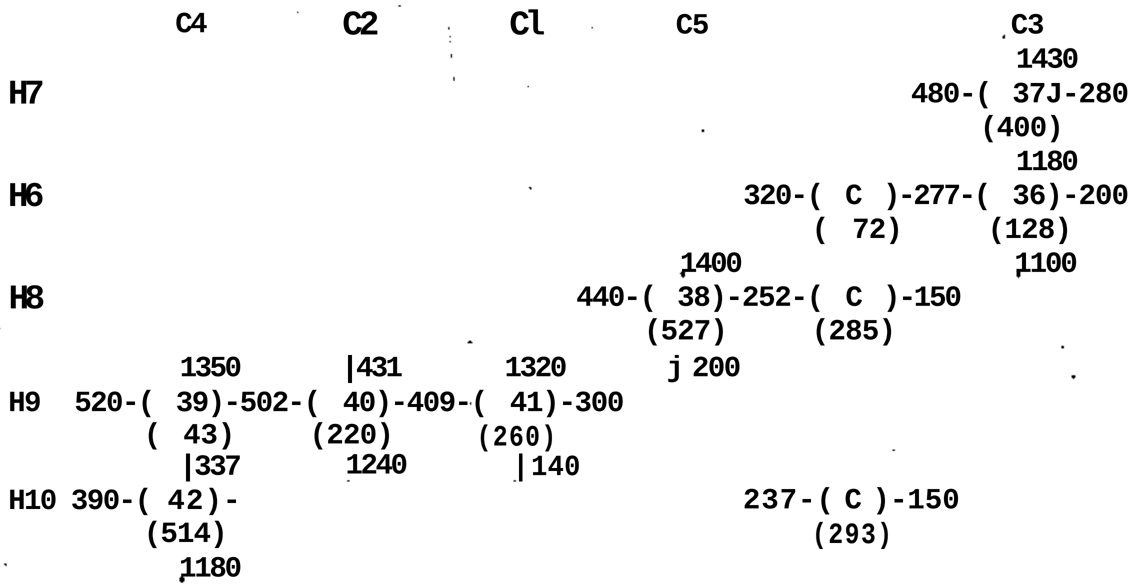

Figure 20. A Third Evolution ($93,352)

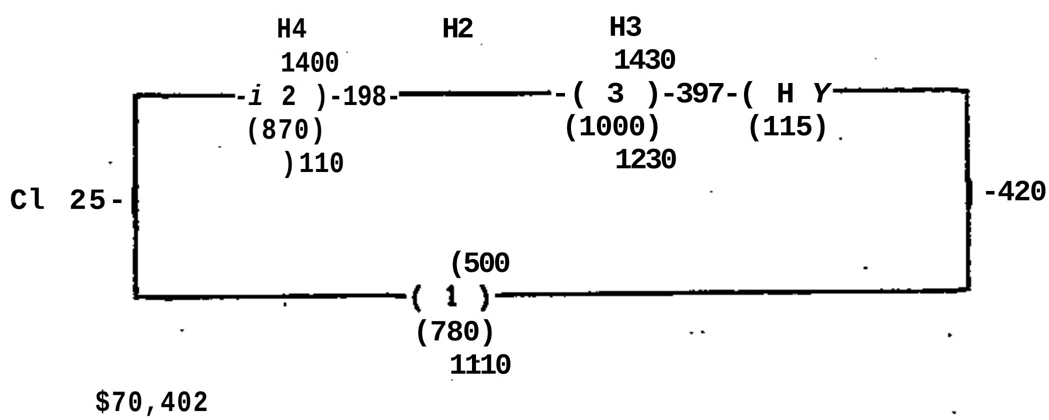

Figure 21. 4SP2: Search matrix without utilities (Optimum)

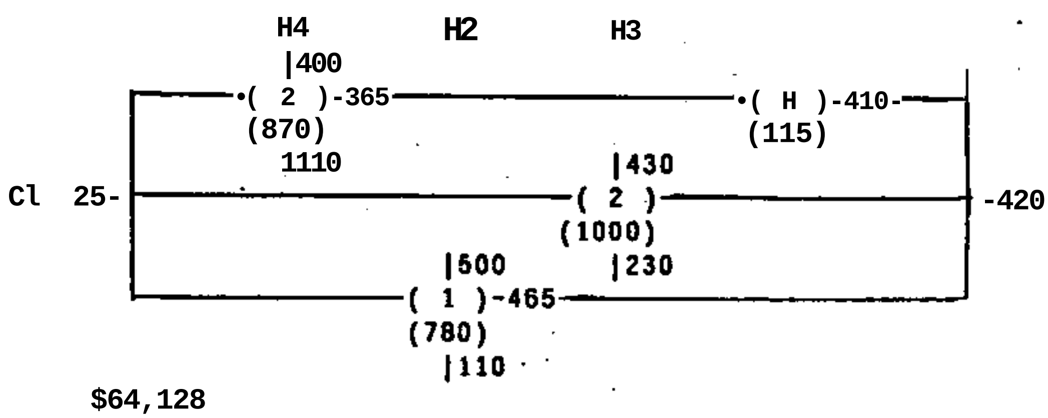

Figure 22. 4SP2: Search without utilities, non-isothermal mixing (Optimum)

```
          C2          Cl          C3
          |201        |300        |250
H4   500-( 1 )-400-( 2 )-388-( 3 )-318-( C )-300
        (2000)      (250)      (1400)      (350)
          |200       |283   .   |180
H5      400-( 4 )-399
              . (2000)
                 1150
H6                             350-( C )-250
                                    (1000)
```

$89,511 ($80.670/yr.)

Figure 23. A new optimum for 6SP21