COMPARISON OF TWO METHODS FOR DESIGN CENTERING

by

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ABSTRACT

Two methods recently proposed to solve the design cantering problem [2,9] are compared. Although the methods are formulated differently, they are shown, under general assumptions, to yield the same solutions. Simplifications required to make the methods efficiently implementable introduce, however, significant differences from an utilization point of view.

1. Introduction

Finding the best nominal design in order to maximize the yield is an important problem in tC design. Because of the unavoidable fluctuations in the manufacturing process, the actual values of the circuit parameters, denoted by the vector p, is characterized by the known joint density distribution <jrtp-p°). where p° represents the nominal values. Oesign centering methods [1.5,31 try to imbed in the so-called region of acceptability R_{Jf} the largest convex domain $B_r(p)$ related to <pip-p°). as shown below. The region of acceptability can for our purposes be defined as the i

$$R_a = \{p \mid f_i(p) \leq f_{max},\ i = 1,\dots,m\} \tag{1}$$

where the f_i represent the performance functions which characterize the circuit behavior. R_t is assumed to be simply connected. The contours of equal probability of ?<p) can be associated for all the distribution of interest with a norm n(p). See for example [3]. $B_r(p)$, often referred to as a norm body, is defined relative to n(p)

$$B_r(p°) = \{p \mid n(p-p°) \leq r\} \tag{2}$$

and represents a body centered at p° whose size is proportional to r. The first method we will look into is the approach referred to as (VTP) in [2], and can be formulated as

$$(VTP):\ \underset{p^0}{\text{maximize}}\ r$$

$$\text{such that}\ \max_i \max_{y \in B_r} f_i(y) \leq f_{max} \tag{3}$$

In this approach a maximally sized body is to be found, such that inside the body none of the performance functions wiH exceed their maximum allowable value. We note from the outset that the main difficulty in solving (3) derives from the maximization subproblem

$$\underset{y \in B_r(p^0)}{\text{maximize}}\ f_i(y) \tag{4}$$

as it must be solved for all constraints i * 1.....m and is likely to have local maxima, making gradient based methods unreliable. That (4) is iikely to have local maxima can be inferred from the fact that quite often the $f_i(y)$ are convex functions and $B_r(p°)$ is a parallelepiped.

The second method, referred to as the *{HP)* method, is formulated as:

$$\{UP):\ \max_r\ \min_i\ \min_{y \in R_a} rtfyp^0\}) \tag{5}$$

In this method the points on the boundary of the region of acceptability which are closest to the nominal design are located and then this distance is maximized. Searching for these near points is based uoon the fact that they limit further expansion of the body $B_r(p)$. It is possible to prove that if the performance functions are either quastconvex or quasiconcave, their accumulation points are the points where the largest body touches the boundary of the region of acceptability.

The domain of the innermost minimizatio n is the intersection of the region of acceptability with the surface f.,(y) * f ^, i.e. in the boundary of R_a, we will assume this domain to be nonempty. Otherwise the constraint ty is superfluous in the sense that if dropped from the set of constraints the region of acceptability remains unchanged.An algorithm can easily detect this situation by verifying that for the constraint i there is no feasible solution to the minimization problem. The main difficulty that arises in solving (5) isthesubproblem

$$\begin{aligned} &\underset{y}{\text{minimize}} && n(y-p°) \\ &\text{such that} && f_j(y) \leq f_{max},\ j = 1,\dots,m \\ & && f_i(y) \geq f_{max} \end{aligned} \tag{6}$$

The difficulty here is that (6) is a constrained minimization problem which is computationally intensive in itself, and one which must be solved repeatedly.

2. Equivalence of Methods

It is ilustrative to recognize that if R_a is simply connected, the performance functions are differentiate and their gradients do no vanish at the boundary of R_a, bd(R_a). then a locally optimal solution to (3) is also locally optimal to (5).

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For convenience we define

$$bd_i(R_j\text{-}\{p\in R_i | f_i(r t\text{-}f,,,.,\}$$ (7)

andthesats

$$M_i(p,r) = 01\ |f_i(0)\ .max\quad f_i(p)>$$ (8)

VTncenteUi proved [10] that a solution 0°, ?) to problem (3)* locally optimal if

$$f_i(M_i(\beta^{\circ}, r^{*})) \leq f_{max}$$ (9)

and

$$0 \in CO^{m}y_t\ \{Vfjtp) \mid pcM_i.0^{\circ}/).fj(p) = f_{max}\}$$ (10)

This last expression can oe mierprataa geometrically to mean that the convex hull defined by gradients of the cunäuaima at the points where the body touches the boundary of R_t, contains the origin, intuitively, if this situation occurs, there is no direction in which the center could be moved such that the radius of the body couwincrease.

To establish the equivalence between the two problems* it is enough to show that any point 0 €M_i(fl °, f), exists in bd_i(Ra) and is at a nẁtimai distance from &°. Assume ttwe is a point p"ebd$_i$(Ra) such that n<p"-S °) < n{fi^ °) then p* exists in the interior of B_r. (fl °) and there is an «>0 such that B_ϵ(p) C B? « °). By assumption Vf_i(p") m 0 and there ta therefore a point P* € B_ϵ(p*) such that ftp") > Kp"). But this contradicts the hypothesis that (0 V) is optimal as for p* we would have p* € B_r (β °), f(p) > f_{max}.

3. Relation Between Subproblems (4) and (6)

The relation between problems (4) and (8) can be made dearer by dropping from (6) the requirement that $f\text{-}fi)^{\wedge}_{nm}$. i«i.~«i-i,i + i,...,m. if the constraint i is not superfluous, i.^se constraints for most cases are not active and the solution to (6) would remain unchanged. We rewrite therefore (6) in the following form

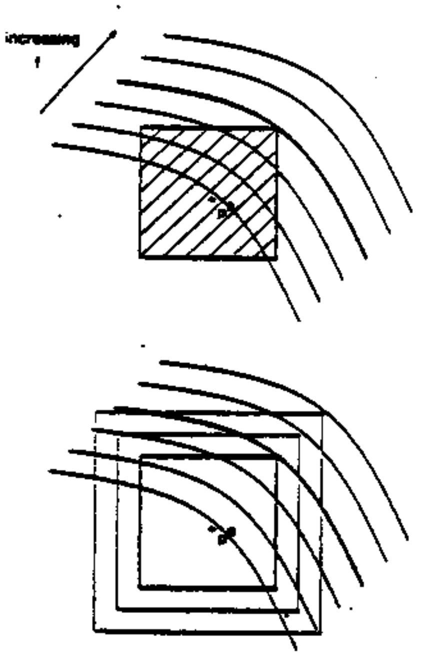
minimize $n(y-p^{o})$ (11)
such that $f_i(p) \geq f_{max}$

On the other hand, using (2) subproblem (4) can also be rewritten as

maximize $f_i(y)$
such that $n(y-p^{o}) \leq r$ (12)

for p° and r constants.

As illustrated in Rg. 1 for the norm $n(x) = \|x\| = \max |x_i|\ 1\text{^-}1,...\text{^}$, in subproblem (10) for a constant sized paraHeHpiped we search for the



Rgurei:

point where f_i is maximized. In (11) the opposite problem is solved: given a fixed value of the function f_i, the minimum value of r is found such that the resulting norm body will have at least a common point with the surfac»f_i(p) - $t_{(xmx}$.

It is interesting to rfote that (11) and (12) are in some sense duals of each other since they have the objective function *and* the constraint interchanged. We saw before that the solution to these two subproblems for *{$.?)* was the same. We further notice that a local solution to (11), assuming that n(x) is locally Lipschitz, is given by [4].

$$0 \in r_o 3rKy\text{-}p^{o}) - r_1 Vf_i(y)$$
$$r_o, r_1 \geq 0$$ (13)
$$r_1\,[f_i(y) - f_{max}] = 0$$

where $3n(y-p^{o})$ represents the generalized gradient of $n(y-p^{o})$. Similarly, for (12)

$$0 \in -r_0\,\nabla f_i(y) + r_1\,\partial n(y-p^{o})$$
$$r_0, r_1 \geq 0$$ (14)
$$r_1[n(y-p^{o})-r] = 0$$

The similarity of these problems is enchanced if we nouca that at the solution of (3) and (5) we must have r ^.

4. Implementation of the Methods

The implementation of the two methods is similar in the sense that the subproblems (4) and (6) are solved at each iteration and the function and gradient information gathered during this process is used by the outer maximization. In the first case the problem can be reduced to a constrained nonlinear problem which can be solved by some constrained variable metric methods. Specifically, a variation of Poweffs algorithm [7] is used, in the second method, the equivalent information is used to generate a second order approximation to the constraining surfaces and the largest normbody is inscribed in that simplified approximation to the region of acceptability.

Serving subprotxeni (4) at each iteration would result in a computationally sxpansnw agofiuvn. nwrarora nw nonnoaay $B_r(p)$ is imricad to ttw cas* wtwr* ft* corresponding norm is $\|\cdot\|_\infty$, i.e.

$$B_r(p) = \{y \mid |y_i - p_i| \leq r\} \quad (15)$$

The basis ror irns resmcoon is xnac m most practical cases the maximum of (4) will occur at a vertex of $B_r(p)$, therefore reducing the set where the search is to be done to a finite set This set can however stiii be very large and in [2] a scheme referred to as splitting is introduced. TWa technique, which is baaed on previous information, predicts where the local maxima of (4) are Hkafy to occur. Reducwgmeiearchfbramaxirnumtothesatofverticaeof $B_r(p)$ can however introduce significant errors in the case where the region of acceptability is not one (finenakraly convex [1], or the region of acceptability has hoiee in its interior, as iHuatratad in Fig. 2.

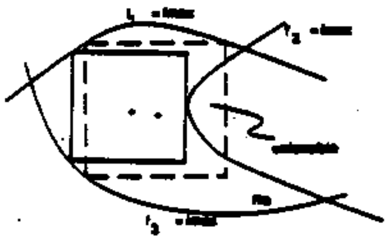


Figure 2:

The (NP) method doea not have thia limitation*

Although methods exist tar immmnad problems when the obiective function is imodifterentiaMe. ag> [6], and thia is presentfy an area of active raae*ch, all algorithms for thia general problem tend to require a very large number of function and gradient (generalized gradients) evaluations making them unsuitable to be used in subproblem (6) where repealed solutions are required. In our HiHxemeiHation we limited the algorithm (NP) to the casee where the norm is diffeientfable, and used alao Powers constrained variable metric algorithm to solve subproblems (6).

From the designer's point of view, the (NP) method yields interesting information on howweilthe constraints are formulated. At early stages of a design, it is very often the case that a large number of cmisuaintt *m* tentatively specified, dearly if the solution to (6) corresponds to a value of t M ^ the constraints can be modified. Further, the relative distances to the final nominal?[0] give an indication on how strongly the* constraint affects the circuits yield.

A limitation of the NP method which could be serious is 9 the initial design is very far from the final solution, the quadratic approximations might have to be updated, thereby significantly increasaig xnecoai of me atgoninni.

5. Examples

We tried both methods on a group of examples to compare the behavior of both methods. For the (VTP) we used the infinite norm, while for the (NP) we used the L_2-norm, this accounts for the different solutions obtained with both methods. (Note also the theoretical advantage of the (VTP) methods of not requiring a feasible starting point).

Example 1 In this example we have a single aonve. Quadratic performance function

$$f_1(p) = 0505(p_1^2 + p_2^2) - 0.99p_1p_2$$

constrained to be smaller than a1.

	Starting Point	Rn * Point	Number of F.E.	Number of OE.
(VTP)	(2.O40J)	(OOOO)	8	8
(NP)	(05,06)	(OOOO)	6	6

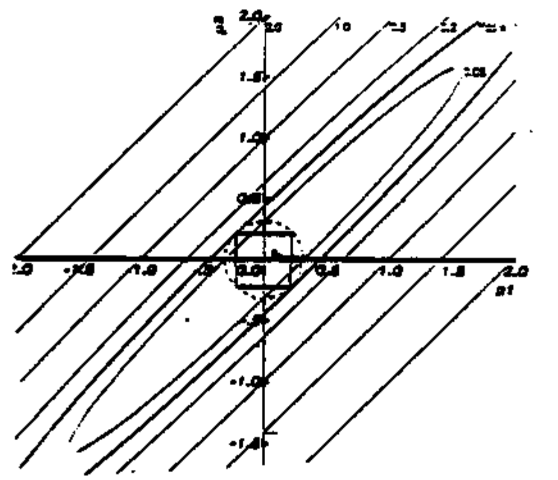


Figure 3:

Example 2 Thia example is taken from (8)

$$f_1(p) = e^{-p_1+1}((p_2-1)^2+1)$$
$$f_2(p) = e^{p_1-2p_2+1}$$
$$f_3(p) = p_1^2 + p_2^2 - 1$$

The constraints are $f_i \leq 1.5$, $i = 1,2,3$

	Starting Point	FinalPoint	Number of F.E.	Number of G.E.
(VTP)	(2.0,2.0)	(03a1.01)	18	20
(NP)	(O7,O9)	(035,1.01)	15	46

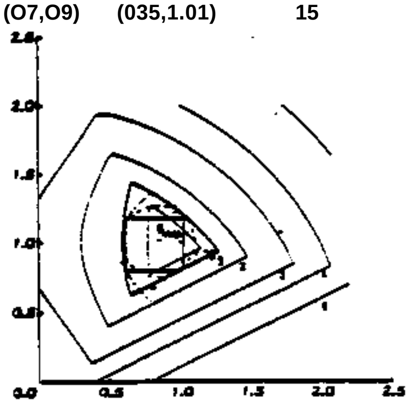


Figure 4:

Example 3 This example was taken from [8]

$f_1(p) = 1.5 - p_1(1-p_2)$ $\quad f_4 = -f_1$

$f_2(p) = 2.25 - p_1(1-p_2^2)$ $\quad f_5 = -f_2$

$f_3 = 2.625 - p_1(1-p_2^3)$ $\quad f_6 = -f_3$

constrained to $f_i \leq 1.5, i = 1,\ldots,6$

	Starting Point	Final Point	Number of F.E.	Number of G.E.
(VTP)	(2.0,2.0)	(1.851,0.134)	33	87
(NP)	(1.3,0.3)	(1.832,0.142)	24	20

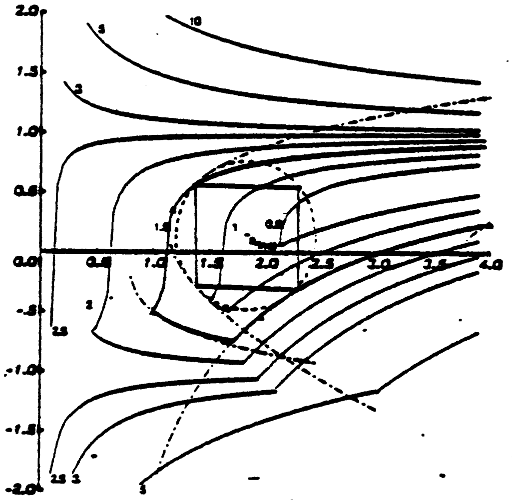


Figure 5:

Example 4 Also taken from [8] has two performance function, one convex and the other concave, both constrained to be smaller than 1.15

$f_1(p) = 0.96 + (p_1-1)^2 + (p_2-0.8)_2$

$f_2(p) = 1.16 - (p_1-1)^2 - (p_2-0.6)^2$

	Starting Point	Final Point	Number of F.E.	Number of G.E.
(VTP)	(2.0,2.0)	(1.000,1.004)	31	32
(NP)	(0.8,0.9)	(1.000,1.016)	11	22

For this example and for function f_2 the solution to subproblem (4) is not in a vertex of the parallelipiped and therefore a significant part of the final tolerance box is infeasible.

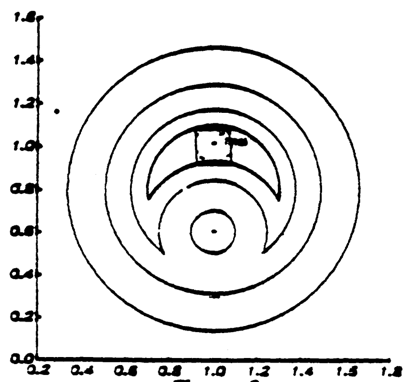


Figure 6:

Conclusion

We have shown that two apparently very different problems can be seen as different formulations of the same body imbedding problem, yielding under general conditions an identical result. With reasonable assumptions the problems lend themselves to very different implementations : one (VTP) suitable for the worst case problem, the other (NP) for a general body center problem when the norm is differentiable. It is interesting to note further that when applied to similar problems they seem to have computational requirements of the same order, when measured in terms of the number of function and gradient evaluations.

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