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TRANSMISSION FACILITY PLANNING IN TELECOMMUNICATION NETWORKS: A HEURISTIC APPROACH

by

I. Baybars & K.O. Kortanek

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İ. Baybars[(1)] and K.O. Kortanek[(2)]

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(1) Graduate School of Industrial Administration

(2) Department of Mathematics

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ABSTRACT

The transmission system cost functions can be decomposed — approximately — into a fixed charge part and linear cost part. These parts represent the initial investment cost of installing a transmission system on a specific link of the network and the cost of installing circuits of that system, respectively. We present a mixed integer programming (MI?) model to minimize the present value of facility installation costs subject to satisfying linkwise circuit requirements in each period of a fixed planning horizon* The model treats (i) alternate transmission systems with limited supplied, (ii) general circuit requirements and (ill) pre-specified alternate routes, for circuit assignment.

A heuristic procedure is developed for obtaining approximate optimal solutions for a case of empirical interest, where transmission supplies are unlimited *nvi where there is only one alternate route for circuit assignment. Numerical results are presented for moderate size facilities networks over a 3 period planning horizon.

1. INTRODUCTION

1.1 Telecommunications Networks

A telecomminications network is a collection of junctions (or points) some or all of which are joined by direct communication links. A link is a collection of facilities known as transmission equipment which when taken * together comprise a transmission system such as cables, radios, satellites, etc. The main components of a transmission system are the circuits, such as wires, frequencies, channels, etc. One of the traditional transmission facilities is the cable consisting of a large number of wires. Recently, however, the use of radios, satellites, and fiber optics has been rather common.

Traffic, in the form of voice telephone calls, originate at a junction A, such as a city, to be transmitted to another junction B, termed the destination. If there is a direct link in the network which joins point A to point B, the call is transmitted through that link as long as not all the circuits of that link are in use at a given time. If all the lines of the direct A-to-B link are busy, then there are two possibilities: the caller may receive a message requesting that the destination number be re-dialed at a later time, or if there are switching facilities at A, the A-to-B traffic may be switched to some other link, say A-to-C. The call can then be transmitted to B through the links A-to-C and C-to-B or via some other sequence of direct links. In this paper switching facilities shall be distinct from transmission facilities, and emphasis will be on planning models for the latter only.

A telecommunication network can be pictorially represented by a graph whose "vertices" and "edges" correspond to the "points" and the "direct communication links" of the network, respectively. The graph of Figure 1 represents a telecommunications network with 8 points and 15 direct links.

From hereon, "link" shall mean "direct communication link". Furthermore, we will not distinguish between A-to-B and B-to-A traffic. Our graph theoretic terminology is standard; for definitions not given here, see Harary [9]. A link joining point i to point j will be denoted by the doublet (i, j). Occasionally, a link will be represented by e_j, when the point-pair it connects need not be distinguished.

1.2 Transmission Facility Planning in Telecommunications Networks

The process of facilities planning consists of two major steps. Originating demand for a service such as voice transmission is estimated in units of traffic load, typically erlangs, or hundred call seconds (CCS) per hour, where 36 CCS equals one erlang. Actually, to assess the eventual alterations of the network one requires estimated traffic for the peak times of the year ("busy-season, busy-hour"). It is necessary to first translate these demands into transmission channels or trunks which by definition are dimensionless units with a single trunk being needed to carry on a two-way voice communiuation. One could term this first step of the planning process as trunking analysis, and an example of an optimization approach to this task is given in Kortanek, Lee, and Polak [11]. The approach in [11], as well as many others, employs a network hierarchy which permits blocked traffic on a link to be switched through other junctions, eventually reaching the intended destination. The idea of alternate routing appears to have originated in a classic paper of Truitt [17] in 1954.

The output of the trunking analysis is a list of trunks between all point pairs (including 0 trunks between some point pairs). Normally, these requirements for trunks would be computed in the short run, for a given year, say t_o. The trunk requirements get satisfied by actual facilities installation regardless of their level of technology (e.g., overhead cable vs. underground fiber optics). But facilities alterations are definitely long run phenomena, since, once in place, such facilities remain so for possibly 20 to 30 years comprising the planning horizon. Therefore, it is necessary to obtain trunking requirements for the long run also, say for years t_o, $t_o + 1, \ldots, t_o + 29$. One way of doing this is to first estimate customer calling demand in CCS between point pairs for each year of the planning horizon. Then, repeated implementation of a trunking analysis procedure such as the one given in [11] would yield point-to-point trunking requirements for each year of the planning horizon.

At this point of the overall facilities planning process, it is convenient to anoint the so-computed trunks as inputs to a facilities planning model by referring to them formally as circuits. The task then at this stage may be termed the transmission facilities planning problem in telecommunications networks: given point-pair circuit requirements for each year in the planning horizon, find a minimum present value cost facility installation plan by specifying the type of transmission systems and the links themselves on which the systems are to be installed as well as the number of circuits to be installed on each such link in each period of the finite planning horizon. Formally, this combinatoric optimisation problem is a fixed-charge multi-commodity flow synthesis problem and is an enormously difficult one to solve (see, for instance, Lawler [12]. A recent survey by Luss [13] on capacity expansion problems provides an excellent discussion and comparison of similar problems and solution approaches presented in [5], [7], [16], [18] and [19]. Other relevant works include [1], [2] and [3].

Our purpose here is to attack this computationally intractable problem through approximate, tractable means by simplifying the problem as described below.

Basically, an overall model formulation such as Yaged [18] permits imputed circuit demand to be literally "routed" along any sequence of links joining any particular pair of points. Concurrent with this task is the determination of actual facility equipment for any or all of these links. The idea of our formulation is to severely limit the number of choices for routing che circuit requirements of pre-specified links. For example, the simplest model approximation would permit any circuits required for a point-pair (a) to be installed on that direct link or (b) routed along the uniquely determined alternate route stemming from the original, a priori, network hierarchy underlying the trunking analysis which generated the circuit inputs for the planning horizon in the first place* This approximation is designed to be consistent with the given network hierarchy. The model that we present in Section 3 allows more complicated hierarchies in the sense that some point pairs may have many alternate routes.

2. A GRAPH THEORETIC-MATHEMATICAL PROGRAMMING APPROACH

2.1 Network Hierarchy

In most telecommunication networks not all pairs of points are connected by links because of obvious economic reasons. In graph theoretic terms, this means that the corresponding graph is non-complete. However, we assume that the graph is connected and, therefore, given switching facilities, it is possible to reach any point of the network from any other point.

Consider a network represented by the graph of Figure 2. Suppose that the dashed edges of this graph correspond to links which can only carry its own traffic. For instance, the link (3,5) can only carry point 3-to-point 5 traffic. Note that there is no link between points 4 and 6. This means that the point 4-to-point 6 traffic is transmitted along links (4,7) and (7,6). If all the line-circuits of the link (4,7) or (7,6) are busy, then the 4-to-6 traffic is blocked, i.e. lost.

On the other hand, there is a link, namely (5,7), joining point 5 to point 7, and, therefore, the point 5-to-point 7 traffic can be carried along this link. However, if all the circuits of link (5,7) are busy, then the excess 5-to-7 demand can be transmitted along links (5,1), (1,2) and (2,7) subject to idle capacity. The designation of a subset of the links of the telecommunication network in this fashion results in a simple hierarchy: a link (A,B) is termed high-usage if (i) (A,B) can not carry the traffic of pairs of points other than that of A and B, and (ii) it is possible to transmit the A-to-B traffic via an alternate route. All other links are termed final. Thus, by definition, the excess traffic on a final link is lest; the (excess) traffic on a high-usage link can be switched to an alternate route.

This particular type of network hierarchy can be described in graph theoretic terms as follows: the final links are chosen in such a manner that the graph induced by the corresponding edges is a spanning tree, i.e. a connected graph with no cycles. As shown in Figure 3, the solid edges of the graph of Figure 2 induce the spanning tree T. (A graph G_S is a spanning subgraph of the graph G if G_S contains all the vertices of G and no other vertex.) The remaining links of the network are the high-usage links, and they are, in graph theoretic terms, the chords of that spanning tree. Thus, T, together with any chord, contains exactly one cycle. For instance, the graph G_1 of Figure 4 is T together with its chord e_{15} and it contains exactly one cycle: $e_{15}e_2e_1e_3e_7e_{15}$. This implies that there is one and only one path in G_1 joining the end vertices of the edge e_{15}, namely $e_2e_1e_3e_7$. A similar case actually is true for any chord (high-usage link) adjoined to T. Thus, designation of the network hierarchy in this manner offers a unique alternate path for the traffic of each high-usage link. By the length of an alternate path we shall mean the number of final links which it contains.

As an approximation to all conceivable routings, we describe this simplest of all network hierarchies as follows. The circuit requirements of a high-usage link can be met in two ways: either by installing a system -- and therefore, circuits -- on that link or by wholly or partially meeting the demand through routing along the links of its unique alternate route. Even under this restricted routing plan it follows that more than one set of circuits can be installed to meet the requirements of a particular high-usage link. While the fixed cost of installing a system is relatively large, the fixed cost per unit of capacity, as well as the variable cost of circuit installations decreases as system capacity increases. As a consequence, economies of scale may be realized by installing larger systems for which it may be more economical to route the circuit requirements of (some) of the high-usage links than install transmission equipment on a high-usage link itself. This will become apparent when we present numerical examples in Section 3.3.

2.2 Assumptions

A network topology and-network hierarchy of Section 2.1 is given and fixed. There is no provision for the installation of switching equipment or multiplex equipment at the nodes; nor for the installation of new links in the hierarchy. In addition, all input parameters such as network costs, circuit requirements, and system capacities are assumed to be known constants. No monotonicity assumptions, however, are placed on circuit requirements over the planning horizon.

We shall assume that the life of each transmission facility exceeds the length of the given planning horizon and that end-of-planning horizon effects are negligible.

la. the telecommunications field it has been assumed that cost functions associated with installing transmission systems are concave« reflecting economies of scale. These functions may be decomposed — approximately — into a fixed charge and a linear cost part. The fixed charge part represents the initial investment cost of installing a transmission system (e.g., cable) on a link. The linear cost part, on the other hand, represents the cost of installing the circuits (e.g., wires in cables) of that system. Generally, it is assumed that both of these costs depend on the length of the individual links (i.e., the actual distance between the two points joined by that link).

We assume that there are alternative transmission systems such as cables, satellites, microwave radios, and that any of these systems may be available for only specific periods of the planning horizon. Their supply is limited, as well as the supply of the individual circuits of the specific systems.

We shall assume that system reliability will be enhanced if more than one type of transmission facility is present on each link. This would enable the users to maintain direct contact between specific pairs of points, if, for instance, the links are installed with cables as well as satellites facilities, in case the satellite may fail.

3 A GENERAL MIXED-INTEGER PROGRAMMING >K)DEL

We shall use the following notation:

L * {links}; *I* will denote a link and q ^{s} |L{

H » {high-usage links}; h will denote a high-usage link and q^1 s |H|

F » {final links}; f will denote a final link and q" a |FI

$T_{i(h)}$» {final links which constitute the $i(h)^{th}$ alternate route for high-usage link h_f i(h) €lh - {l(h), 2(h),..., $\bar{h}$(h)}

H_f « {high-usage links for which at least one alternate route contains the final link f}

cj_{st} : cost of installing one system s unit on link *I* in period t, where t€T * £l,...,t^f} and s€S * {l,...,s'}

$c?_{ist}$: cost of installing one system s circuit on link *K* in period t

b_{st} : number of system s units available in period t

B_s : number of system s units available throughout the planning horizon

a_s : circuit capacity of one system s unit

a_{sc} : number of system s circuits available for installation in period t

A^s .: number of system s circuits available for installation throughout the planning horizon

r : annual interest rate

w^t « $(1+r)^{"t}$: discount factor for t years

k^s : parity factor

d^{t} : circuit requirement of link *L* in period t

The variables under control are as follows:

> x_{lat} : number of system s units installed on link I in period t
>
> y_{lst} : number of system s circuits Installed on link I in period t
>
> . $U_{i(h),t}$: the number of circuits which will be routed to the $i(h)^{th}$ alternate route of the high-usage link h in period t.

Before presenting the model, we first note that H U F • L and H *D* F • 0 . For the network depicted in <u>Figure 2</u>, $\bar{h}(h)$ • 1 Y h c H and;

$^F l(8)$ * $t^{1,2}$}' $^F l(9)$ * fr.3.5}.$^F l(10)$ -.Cl.3.6}. $F_{1(U)}$ - (2,1,3)

$F_{1(12)}$ = {2,1,4}, $F_{1(13)}$ - {2,1,3,5}, $F_{1(M)}$ - {2,1,3,6}, $F_{1(15)}$.- {2,1,3,7}.

On the other hand, $\bar{h}(h)$ • 2 V h c H in the network of <u>Figure 5</u> :

$^F l(9)$ " Jl,2}, $^F 2(9)$ " $t^{3,4,5,6}$}, $^F l(10)$ " $C^{1,2,3}$}, $F_2(10)$ " {4,5,6}

$F_{1(11)}$ = {4,3,8}, $F_{2(11)}$ = {5,6,1,2,8}, $F_{1(12)}$ - {3,4}, $F_{2(12)}$ - {2,1,6,5}

$F_{1(13)}$ = {2,7}, $^F 2(13)$ " $C^{1,6*5*4,3,7}$]. $*_{1(14)}$ - C2.3.4}, $F_{2(U)}$ - {1,6,5}

$F_{1(15)}$ = {3,4,5^ ^ $^F 2(15)$ " t^2>^{l}>V'

And, finally, $\bar{h}(h)$ - 3 , V h « H in the network of <u>Figure 6</u> ;

$^F l(10)$ " C2,4}, $^F 2(10)$ " $C^{1,9,6,5}$)' $^F 3(10)$ " ^3,7 3

$^F 1(U)$ " *4,5,6}' $^F 2(U)$ " {2,3,7,5,6}, $F_{3(U)}$ - {2,1,9}

$F_{1(12)}$ = {5,6 3, $^F 2(12)$ " ^.2,1,9}, $F_{3(12)}$ - {7,3,1,9}

$F_{1(13)}$ = {5,6,9 }' $^F 2(13)$ " CL2.4J, $F_{3(13)}$ - {1,3,7}

$F_{1(14)}$ = {1,2,4,8}, $F_{2(14)}$ - {9,6,5,8}, $F_{3(U)}$ - {1,3,7,8}

$F_{1(15)}$ = {6,5,7!' $^F 2(15)$ " {3'2.4,5,6}, $F_{3(15)}$ - {3,1,9}

We can now present the model:

<u>Program P</u>

minimize

$$z = \sum_{\ell \in L} \sum_{s \in S} \sum_{t \in T} w_t \left(c'_{\ell st} x_{\ell st} + c''_{\ell st} y_{\ell st} \right) \qquad (1)$$

from among $x_{\ell st}$, $y_{\ell st}$, $u_{i(h),t}$ for each $\ell \in L$, $s \in S$, $t \in T$, $h \in H$, and $i(h) \in I_h$

subject to:

$$\sum_{s \in S} \sum_{t=1}^{\bar{t}} y_{fst} \geq d_{f\bar{t}} + \sum_{h \in H_f} \sum_{i(h) \in I_h} u_{i(h),\bar{t}} \qquad \forall f \in F,\ \forall \bar{t} \in T \qquad (2)$$

$$\sum_{t=1}^{\bar{t}} y_{hst} \geq d_{h\bar{t}} - \sum_{i(h) \in I_h} u_{i(h),\bar{t}} \qquad \forall h \in H,\ \forall \bar{t} \in T \qquad (3)$$

$$\sum_{t=1}^{\bar{t}} y_{\ell st} \leq a_{\ell} \sum_{t=1}^{\bar{t}} x_{\ell st} \qquad \forall \ell \in L,\ \forall s \in S,\ \forall \bar{t} \in T \qquad (4)$$

$$\sum_{\ell \in L} \sum_{t \in T} x_{\ell st} \leq B_s \qquad \forall s \in S \qquad (5)$$

$$\sum_{\ell \in L} x_{\ell st} \leq b_{st} \qquad \forall s \in S,\ \forall t \in T \qquad (6)$$

$$\sum_{\ell \in L} \sum_{t \in T} y_{\ell st} \leq A_s \qquad \forall s \in S \qquad (7)$$

$$\sum_{\ell \in L} y_{\ell st} \leq a_{st} \qquad \forall s \in S,\ \forall t \in T \qquad (8)$$

$$\sum_{t=1}^{\bar{t}} x_{\ell st} \geq \ldots \sum_{t=1}^{\bar{t}} y_{\ell st} \qquad \forall s \in S, \; \forall \ell \in L, \; \forall \bar{t} \in T \qquad (9)$$

$$x_{\ell st} \geq 0 \text{ and integer} \qquad \forall \ell \in L, \; \forall s \in S, \; \forall t \in T \qquad (10)$$

$$y_{\ell st} \geq 0 \qquad \forall \ell \in L, \; \forall s \in S, \; \forall t \in T \qquad (11)$$

$$u_{i(h),t} \geq 0 \qquad \forall i(h) \in I_h, \; \forall t \in T \qquad (12)$$

Description of Program P:

The objective function (1) is the sum of the discounted fixed costs of installing transmission systems and the discounted variable costs of installing circuits on the links of the network throughout the planning horizon.

The first two constraints represent "circuit requirements[11]: The number of circuits on a final link f in period $\bar{t}$ must be greater than or equal to the sum of the circuits required for that link and the circuits to be used to (partially) meet the requirements of the high-usage links whose alternate route(s) contain f. This requirement is expressed by (2). (3), on the other hand, is for high-usage links: the number of circuits on a high-usage link h in period $\bar{t}$ must meet the demand not satisfied by re-routing. (4) represents the capacity of system s units: system s circuits can be installed in link t in period $\bar{t}$ only if system s units have been installed in periods $1,2,\ldots,\bar{t}$ and there is idle capacity. (5) represents the limited supply of a specific transmission system over the planning horizon whereas (6) represents the limited supplies periodvise. Similarly, (7) represents the limited supply of the circuits of a specific transmission system over the planning horizon and (8) represents the limited supplies of the circuits in each period of the planning horizon. (9) is the parity constraint: in order to avoid dependency on a sole system on a link, at least a specified proportion k_s of circuits on every link must be of system s type.

<u>Program ?</u> is a rather large program. Let q^{ffl} be the number of alternate routes In the network, i.e.,

$$q^{"f} - \sum_{h \in H} \bar{h}(h).$$

Then, P has $qt^f(1+2s^f) + 2s^f(1+2t^f)$ constraints other than non-negativity requirements and $t^f(2qs^f 4q^{fff})$ variables, $qs^f t^f$ of which are integer. (We should note here that the number of circuits installed on a link in real-life is sufficiently large enough to enable us to permit fractional values for the variables y_{slt}^{fs}-) For instance, the number of constraints would be 357 if s^f » t^1 * 3 and q^f » 7, q" * 8 and q^{lff} * 8; the number of variables would be 294 with 135 integer variables. Thus, even for relatively small s^f and t^f, <u>Program P</u> remains large.

A reduced problem, which has empirical interest for planning, may be obtained by deleting the supply of equipment constraints (5)-(8-> and the parity constraints (9), and by restricting the number of alternate routes of each high-usage links to be one.

In addition we make a simplifying assumption on costs, namely all costs are independent of link length. In other words unit circuit costs are independent of the subscript £ (corresponding to link 4). This assumption is plausible for networks having links approximately the same length, or at least where the fixed costs are somewhat insensitive to link length.

We shall denote the resulting simplified mixed-integer program as Program &, which in terms of £. consists of (l)-(4), (10), (11) with Ih = {l(h)} for each heH.

Our previous computational experiences in solving Program Q for illustrative problems are reported in [3]. Program Q is sufficiently large and complex that in our attempts to solve it using "flow theoretic methods" [3] and "general purpose mixed 0-1 programming codes" [14] numerical difficulties were encountered. The magnitude of this computation time led us to seek a methodology capable of finding a "good" solution, rather than the optimum, in a "reasonable" CPU time, two terms which are difficult to define in this context. The need for heuristics for problems of this type are also made clear by Luss [13].

4. A HEURISTIC METHOD FOR SOLVING PROGRAM Q

In order to be able to find a "good" solution in "reasonable" CPU time, we adopted a number of simple rules and constructed a seven step heuristic procedure called HTCPQ (See Appendix). The procedure assumes that $|Ih| = 1$, $\forall\ h \in H$, i.e. we consider only __one__ alternate route for each high-usage link. Hence, F_h will represent $F_{1(h)}$. We shall now present a summary of HTCPQ.

Given the network configuration, we first determine the sets F_h and H_f, and all control variables are set equal to zero. At this point, if the circuit requirements of a final link $\bar{f}$ in period 1 exceeds the circuit capacity of the largest transmission system, a decision is made to install as many units of that system as necessary so that the updated circuit requirement of $\bar{f}$ prior to the implementation of __Step 1__ is less than the circuit capacity of the largest system.

In __Step 1__ we determine the collection of high-usage links whose circuit requirements should __not__ be routed. This decision is based on the number of final links in the alternate route for a specific high-usage link $\bar{h}$; for $|F_{\bar{h}}|$ and/or $d_{\bar{h}t}|F_{\bar{h}}|$ may be sufficiently large so that the total variable cost of installing circuits on the links of the alternate route for $\bar{h}$ may exceed the total fixed and variable costs of installing circuits on that link. If such a high-usage links exists, an installation decision is made, and the circuit requirements of that link are updated as well as the sets H_f, $\forall f$ such that $\bar{h} \in H_f$.

Steps 2 and 3 are for initial installations on final links. Since their circuit requirements cannot be routed, the links own requirements are to be met immediately. However, instead of merely satisfying the requirements of a final link $\bar{f}$, we try to exploit the economies of scale by installing large systems so that excess circuit capacity can be used for routing the circuit requirements of the high-usage links. After the installation decisions are completed, the circuit requirements are updated and excess circuit capacities are calculated.

Step 4: Given the installations on the final links made in Steps 2 and 3, we now turn to high-usage links again. In each period, we first consider the high-usage link h with the smallest demand. If the links of the alternate route of h have unused circuit capacity, sufficient number of circuits are installed on those links to meet the requirements of h. Otherwise, we move on to the high-usage link with the next smallest circuit requirements and continue in this fashion until all high-usage links are considered in each period. The high-usage link circuit requirements are updated accordingly.

Obviously, it is possible that $d_{ht} \neq 0$ for some $h \in H$ at this point, and the question becomes whether routing is best for such a high-usage link or not. In Step 5, the high-usage links are ordered in ascending order of circuit requirements in each period. Starting with $t = 1$, the possibility of installing a system on h versus installing an additional system on a link of the alternate route is explored. First, the two final links on the alternate route with the least excess capacity are identified. Of these two finals one (or both) having smallest excess capacity is now "eligible" to have a system 1 unit installed during $\bar{t}$. Without the system added, the limiting factor for the alternate route is the excess capacity of this final. With the system added, the limiting factor is the minimum of the excess capacity of this final plus the capacity of a system 1 unit and the excess capocity of the other of the two. Second, the cheapest way of meeting the circuit requirements for this high-usage is found by considering only the alternatives described above. This least-cost solution will include a system and circuit configuration. However, only the system part of the solution is used here. In this step, as many circuits as possible are installed on the high-usage link; the remainder of the requirements being satisfied via the alternate route. (This is temporary, and <u>Step 6</u> will adjust this solution.) The idea is to keep, for the moment, as much excess capacity on the finals so that the remaining high-usages to be considered will not be restricted more than necessary.

<u>Step 6</u> accepts the system configuration from <u>Step 5</u>, and decides whether it would be cheaper to "trade" some of the circuits on a direct high-usage link for circuits on the alternate route. If it is cheaper, then as many circuits as possible are rerouted along the final.

At the beginning of <u>Step 7</u>» the number of circuits to be installed on each link has already been determined^and thus we now re-evaluate the systems to be installed. We do this by solving a small 0*1 program which identifies the systems needed given the number of circuits to be installed on each link. This is where the discount rate plays a major role. For instance, it is possible that

$$^{2W}t\ s^{*+W}{}_{t+1}\ \%^{*} < V\ i_{*+1}{}^{+W}\ t+X\langle ;\ *-1.$$

that is, it may be cheaper to install two units of system s* in period t and one unit of system s* in period (t + 1) than one unit of system (s* + 1) in period t and one system of (s* - 1) in period (t + 1).

Numerical examples using HTCPQ are presented in the next section.

5. NUMERICAL EXPERIMENTS ON FOUR NETWORKS

In this section we consider four networks N_1, N_2, N_3 and N_4 shown in Figures 2, £, £ and £, respectively. Network N_1 is the one studied in [3] and [11}. The other three networks have been constructed arbitrarily. In each network there exists exactly one alternate route for each high-usage link. Various statistics for these four networks are given in Table 1.

For illustrative purposes, we use a 10-year planning horizon. Period 1, i.e. t » 1, is the base year of this planning horizon, t = 2 corresponds to the fifth year and t « 3 corresponds to the 10th year. The annual interest rate is set at 10%. The linkwise circuit requirements for N_1, N_2, N_3 and N_4 are given in Tables j2, 3^ ft and *1* respectively. The circuit requirements in N_1 are obtained from [3] whereas the circuit requirements in N_2, N_3 and N_4 have been generated randomly except that all circuit requirements in period 1 were restricted to values not exceeding 1000. Furthermore, high-usage link requirements were in general chosen to be smaller than final link requirements. The rationale for the latter is that a typical final link carries the traffic of point-pairs other than its end-points.

Again, for illustrative purposes, we consider three alternate transmission systems. The fixed and variable costs of each transmission system, as well as the circuit capacity of each such system are based on a hypothetical rescaling of the data in Table 1 of Yaged [18] and are given in Table 6. We should re-state here that facility costs are assumed to be independent of the actual length of the links. In Table 7 we present the size of each Program Q corresponding to the four networks considered.

The optimal solution for N_1 is given in Table 8, This solution was obtained by using LINDO (14] on DEC-20 at Carnegie-MelIon University. The solution was found in 37 CPU minutes. We should emphasize that LINDO is a general branch-and-bound procedure and that the magnitude of the CPU time for our problem is not a reflection of the capabilities of LINDO. It is well-known that general procedures for solving integer programs of more than 100 variables do not perform well. (See, for instance, [4]*)

The 'approximate optimal[1] solutions for N_1, N_2, N_3 and N_4 in Tables £, K), ^L and 12, respectively, have been obtained by the heuristic procedure HTCPQ. The entries under 'installations[1] in all those tables are the types of transmission systems to be installed on the respective link in the respective period. For instance, in Table 8. we see that one system 2 unit is to be installed on link 3 in period 2, and, in Table 10, we see that two different systems, namely 1 and 3, are to be installed on link 13 in period 1. A zero entry in any period for any link means that no system is to be installed. The number of circuits installed can be easily computed based on the systems installed.

A natural question that arises is on the 'goodness* of the HTCPQ solution. The best way to measure the heuristic solution is, of course, to compare it with the optimal solution. This we can do for N_1: the optimal total cost is $12,188,683 and the heuristic total cost is $12,195,223, within 0.067* of the optimal. For networks N_2, N_3 and N_4, optimal solutions are not available. We have tried solving a reduced but equivalent Program Q for N_2 using LINDQ, and we had to terminate the computations after 3 CPU hours.

We have also used measures of average circuit costs to test the goodness of the HTCPQ solution. All averages are stated in terms of present values.

- $\bar{C}_{LM}$ [Average Circuit Cost Using Least Cost Circuits on High Usage Links]

  For each high-usage link the required number of circuits are met by System 3 circuits (the least cost ones), while no fixed costs are assigned to it.

- $\bar{C}_O$ [Optimal Solution Average Circuit Cost]
- $\bar{C}_H$ [Heuristic Solution Average Circuit Cost]
- $\bar{C}_{NS}$ [Average Circuit Cost for No Network Hierarchy]

Observe that for both $\bar{C}_{LM}$ and $\bar{C}_{NS}$ there is no routing of circuits and the facility planning problem is solved for each final link individually. For the no network hierarchy case, of course, all links are treated as final links. The relaxations defining $\bar{C}_{LM}$ result in a non-feasible solution to the original transmission planning problem and thus $\bar{C}_{LM}$ is an unattainable lower bound on $\bar{C}_O$. Without the network hierarchy on the other hand, one obtains feasibility and consequently an upper bound on $\bar{C}_O$.

The average circuit costs for each of the four examples are given in Table 13.

Since HTCPQ itself is a heuristic procedure, there are likely to be other rules which improve the solutions possibly leading to an optimal solution. HTCPQ is consistent with the underlying rationale for constructing heuristic procedures, namely, to be able to find a good solution in realistic CPU times. However, for a specific problem a solution found by this procedure can possibly be improved by studying that specific problem.

Finally, we report on numerical experiments using the heuristic under partial restrictions of the network hierarchy defined as follows. Each high-usage link whose alternate path length exceeds a pre-specified number m, is treated as a final link so that no routing of its circuits is permitted* Each of the remaining high-usage links in a partial restriction therefore has a tiny-fumm alternate path length of m.

In Figures 10 and 11 we plot the maximum length of any alternate path (horizontal axis) against the heuristic solution average cost (vertical axis) for networks N_3 and N_4 respectively. Observe that the minimum heuristic average cost is obtained at $m = 6$ for N_3 (Figure 10) and at $m = 5$ for N_4 (Figure 11). These findings support our conjecture that there is a maximum bound on the alternate path length above which it is not economical to route its circuits.

The CPU times for the four sample problems are in Table 14. HTCPQ was run on DEC-20 at Carnegie-Mellon University. Computational complexity of the procedure is $0(n^2)$. The program is written in FORTRAN and has 729 lines.

6. CONCLUSIONS

The process of telecommunications network analysis includes at least three major tasks: (I) Trunking Analysis, (II) Switching Analysis, and (III) Transmission Facilities Planning. In this paper we have set forth a new mixed integer programming model for a class of problems of type (III) and have provided a heuristic procedure for solution. The basic simplifying feature of the model is that circuit requirements between any two nodes have only one way for which they may be alternatively routed. This restriction aids the development of closed form models, which are inter-temporally dynamic in that one period's decisions depend on decisions in the other periods in the fixed finite planning horizon. The model highlights the interaction between large, fixed cost components and the much smaller marginal costs of additional circuit equipment. It facilitates the construction of a solution heuristic which incorporates fixed cost information, departing markedly from heuristics which depend heavily on marginal costs* No dynamic programing is required, and one can reasonably anticipate application to large scale problems. For those cases where it is important to consider a large number of alternate routes for point-pair circuit requirements, then our procedure could provide a good initial start for these more complicated computational methods, see Yaged [18].

Task (III) is certainly tied to Task (I) because circuit requirements for (III) are a result of having first solved problems of type (I), repeatedly, once for each time period within a fixed planning horizon. The output from (I) is actually more extensive, providing in addition, period by period circuit terminations and total switched traffic (in erlangs) at each node. These are some of the inputs required to solve problems of type (II). Using these inputs one could incorporate nodal cost models into the facilities planning models of this paper in order to account for switching costs and also in a related way to account for nodal multiplexing costs.

The heuristic procedure has been illustrated on four numerical examples whose number of nodes and number of links are (8,15), (15,29), and (32,63) and (98,283), respectively. The sizes of the mixed integer prograoming problems of the latter two problems exceeds the capability of known mixed-integer programming algorithms and codes.

ACKNOWLEDGEMENT

We are grateful to Richard Edahl for the computational work and his suggestions to improve the heuristic algorithm.

APPENDIX: Procedure HTCPQ

For the heuristic method that will be given below, we need two new notations: let (i) $y'_{\ell t}$ denote the number of circuits on link ℓ at the end of period t, and (ii) v_{ft} denote the unused transmission system capacity on final link f at the end of period t. That is,

$$y'_{\ell t} = \sum_{s=1}^{s'} \sum_{\bar{t}=1}^{t} y_{\ell s \bar{t}}$$

and

$$v_{ft} = \sum_{s=1}^{s'} \sum_{\bar{t}=1}^{t} a_s x_{fs\bar{t}} - y'_{f\bar{t}}$$

STEP 0: Find $F_h \; \forall h \in H$

Set (i) $x_{\ell st} = y_{\ell st} = 0 \quad \forall \ell \in L,\ s \in S,\ t \in T$

(ii) $v_{ft} = 0 \quad \forall f \in F,\ t \in T$

(iii) $y'_{\ell t} = 0 \quad \forall \ell \in L,\ t \in T$

If for any link ℓ, positive integer n

$$n\, a_{s'} \leq d_{\ell 1} < (n+1)\, a_{s'} ,$$

Set $x_{\ell s'1} = n,\ y_{\ell s'1} = n a_{s'},\ y'_{\ell t} = y_{\ell s'1}$

$d_{\ell t} = d_{\ell t} - y'_{\ell 1} \quad \forall t \in T$

STEP 1: $\forall h \in H$;

(i) set $\bar{s} = 0$

(ii) if (a) $d_{h1} \geq a_2 + a_1$

or (b) $d_{h3} \geq 2a_2$ and $d_{h1} \geq a_2$

then set $\bar{s} = 3$ and go to (iii)

if for $s^* \ni \quad a_{s^*} \leq d_{h1} < a_{(s^*+1)}$,

$c''_{s'1} (|F_h| - 1)\, d_{h1} \geq c_{s^*}$

then set $\bar{s} = s^*$ (if $s^* > 0$) and go to (iii) .

Otherwise, go to (i) for next h.

(iii) set $x_{h\bar{s}1} = x_{h\bar{s}1} + 1,\ y_{h\bar{s}1} = y_{h\bar{s}1} + a_{\bar{s}}$

$y'_{ht} = y'_{ht} + a_{\bar{s}},\ d_{ht} = d_{ht} - a_{\bar{s}} \quad \forall t \in T$

STEP 2: $\forall f \in F$; (i) if $d_{f3} < a_{s'}$ go to (ii)

otherwise: set $x_{f31} = x_{f31} + 1 \quad v_{ft} = v_{ft} + a_{s'} \quad t \in T$

$$\left.\begin{array}{l} y_{f3t} = y_{f3t} + b_t \\ y'_{ft} = y'_{ft} + b_t \\ v_{ft} = v_{ft} - b_t \end{array}\right\} t \in T$$

where $b_t = \min \{d_{ft}, a_{s'}\}$

set $d_{ft} = d_{ft} - a_{s'}$, $t \in T$

(ii) if $d_{f3} > a_2$ or $\sum_{t=1}^{3} (d_{ft} + \sum_{h \in H_f} d_{ht})/3 > a_2$, then set $\bar{s} = 3$.

otherwise, if $d_{f3} > a_1$, set $\bar{s} = 2$

otherwise, set $\bar{s} = 1$.

(iii) Perform operations A-2 and A-3 with $\bar{t} = 1$, $\ell = f$.

STEP 3: If, for any final link f, $d_{f3} > a_2$ and $x_{f31} \geq 2$, then set $\bar{s} = 3$.

Otherwise, compute $\bar{d}_f = {}_h\Sigma\, d_{h2}$

Then, if $d_f - v_{f2} > a_1$, set $\bar{s} = 2$. If, on the other hand,

$d_f - v_{f2} > a_2 + a_1$, set $\bar{s} = 3$. Then if $\bar{s} = 0$, go to the next link.

Otherwise, perform operation A-2 on f with $\bar{s}$ and $\bar{t} = 2$.

STEP 4: $\forall h \in H$, in increasing order of $d_{h\bar{t}} |F_h|$;

If $d_{h\bar{t}} < v_{f\bar{t}}$, $\forall f \in F_h$, perform operations A-3, with $\ell = s$, $\forall f \in H_f$ and set $d_{h\bar{t}} = 0$. Otherwise, go to next link.

STEP 5: $\forall h \in H$, in increasing order of $d_{h\bar{t}}$;

If $d_{h\bar{t}} = 0$, go to next link, otherwise define

(i) $v_1 = \min_{f \in F_h} \{v_{f\bar{t}}\}$

(ii) $v_2 = \min_{\substack{f \in F_h \\ f = \bar{f}}} \{v_{f\bar{t}}\}$ where $\bar{f}$ is the link which gives v_1 in (i)

(iii) $MC = \sum_{\substack{f \in F_h \\ f \notin F}} MC_{f\bar{t}}$ where MC_{ft} is the marginal cost of adding one more circuit to link f in period t.

Then solve the following for $\bar{t}$:

Program P_6

$$\min \quad \sum_{s=1}^{3} c'_{s\bar{t}}\, x_{hs\bar{t}} + c'_{1\bar{t}}\, x^{(1)}_{\bar{f}1\bar{t}} + \sum_{s=1}^{3} c''_{s\bar{t}}\, y^{(1)}_{hs\bar{t}} + (MC+MC_{\bar{f}\bar{t}})\, y^{(2)}_{\bar{f}s\bar{t}} + (MC+c''_{1\bar{t}})\, y^{(3)}_{\bar{f}s\bar{t}}$$

$$\text{s. to:} \quad \sum_{s=1}^{3} y^{(1)}_{hs\bar{t}} + y^{(2)}_{\bar{f}s\bar{t}} + y^{(3)}_{\bar{f}s\bar{t}} \geq d_{h\bar{t}}$$

$$y^{(1)}_{hs\bar{t}} \leq a_s x_{hs\bar{t}}, \quad s = 1,2,3$$

$$y^{(2)}_{\bar{f}s\bar{t}} \leq V_{\bar{f}1}$$

$$y^{(3)}_{\bar{f}s\bar{t}} \leq a_1 x^{(1)}_{\bar{f}1\bar{t}}$$

$$y^{(2)}_{\bar{f}s\bar{t}} + y^{(3)}_{\bar{f}s\bar{t}} \leq V_{\bar{f}2}$$

$$y^{(1)}_{hs\bar{t}},\ y^{(2)}_{\bar{f}s\bar{t}},\ y^{(3)}_{\bar{f}s\bar{t}} \geq 0$$

where (a) $x^{(1)}_{\bar{f}1\bar{t}} = \begin{cases} 1 & \text{if a system 1 unit is added to } \bar{f} \text{ in } \bar{t} \\ 0 & \text{otherwise} \end{cases}$

(b) $y^{(1)}_{hs\bar{t}}$ is the number of system s circuits added to link h in period $\bar{t}$

(c) $y^{(2)}_{\bar{f}s\bar{t}}$ is the number of system s circuits added to each $f \in F_n$ before a new system s unit is added to $\bar{f}$.

(d) $y^{(3)}_{\bar{f}s\bar{t}}$ is the number of system s circuits added to each $f \in F_n$ after a new system s unit is added to $\bar{f}$.

Then, if (i) $x_{hs\bar{t}} \geq 1$, perform operation A-4, and

(ii) $d_{h\bar{t}} > V_{\bar{f}1}$, perform operation A-2.

Then perform operation A-3 $\forall f \in F_n$ with $\bar{f} = h$.

Step 6: $\forall h \in H$, in increasing order of $(y_{ht} - y_{h(t-1)})$;

If $y_{ht} - y_{h(t-1)} = 0$ go to next link. Otherwise,

compute $MC = \sum_{f \in F_h} MC_{ft}$ where MC_{ft} is computed as in Step 5.

Let S(L) be the last system installed on link h. Then solve the following:

Program P_t

$$\min \quad c_{s(L)t}\, y^{(1)}_{hst} + MC\, y^{(3)}_{fst}$$

$$\text{s. to:} \quad y^{(1)}_{hst} + y^{(3)}_{fst} \geq y_{ht}$$

$$y^{(1)}_{hst} \geq y_{h(t-1)}$$

$$y^{(3)}_{fst} \leq \min_{f \in F_h} \{v_{ft}\}$$

$$y^{(1)}_{hst},\ y^{(3)}_{fst} \geq 0$$

If $y^{(3)}_{fst} = 0$, go to next link. Otherwise;

set (i) $y_{ht} = y_{ht} - y^{(3)}_{fst}$

(ii) $v_{ft} = v_{ft} - y^{(3)}_{fst}$ } $\forall f \in F_h$

(iii) $y_{ft} = y_{ft} + y^{(3)}_{fst}$

<u>Step 7</u>; $\forall \ell \in L$, solve the following:

<u>Program P_7</u>

$$\min \sum_{s=1}^{3}\sum_{t=1}^{3} \ldots \sum_{s=1}^{3}\sum_{t=1}^{3} c^*_{st}\, y_{\ell st}$$

subject to: $$\sum_{s=1}^{3}\sum_{t=1}^{\bar t} y_{\ell st} \geq y'_{\ell \bar t}, \quad \bar t = 1,2,3$$

$$\sum_{t=1}^{\bar t} y_{\ell st} \leq a_s \sum_{t=1}^{\bar t} x_{\ell st}, \quad s = 1,2,3$$

$$y_{\ell st} \geq 0$$

$$x_{\ell st} = 0,1$$

<u>Operations</u>:

<u>Operation A-1</u>: Set (i) $x_{hs1} = x_{hs1} + 1$, (ii) $y'_{h1} = y'_{h1} + d_{h1}$,

(iii) $y'_{h2} = y'_{h2} + d_{h2}$, (iv) $y'_{h3} = y'_{h3} + d_{h3}$

and (v) $d_{h1} = d_{h2} = d_{h3} = 0$.

<u>Operation A-2</u>: Set (i) $x_{rts} = x_{rts} + 1$ and (ii) $v_{rt} = v_{rt} + a_s$,

$t = \bar t, \ldots, 3$

<u>Operation A-3</u>: Set (i) $v_{\ell t} = v_{\ell t} - d_{\ell t}$ and (ii) $y'_{\ell t} = y'_{\ell t} + d_{\ell t}$.

<u>Operation A-4</u>; Set (i) $x_{hs\bar t} = x_{hs\bar t} + 1$ and $t^* = \bar t$.

(a) If $t^* > 3$, return. Otherwise;

Define $z = \min\{a_s, d_{ht^*}\}$

Set $y'_{ht^*} = y'_{ht^*} + z$

Set $d_{ht^*} =$

Set $t^* = t^* + 1$

Go to (a) above.

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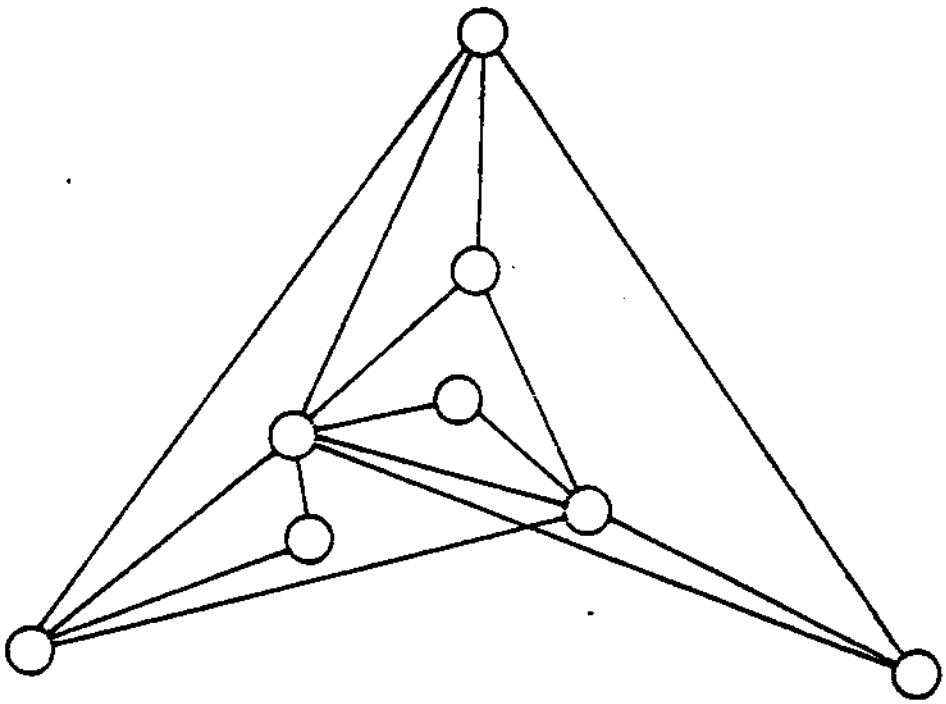


Figure 1 .* A Telecommunications Network

(I. Baybars and K. 0. Kortanek)

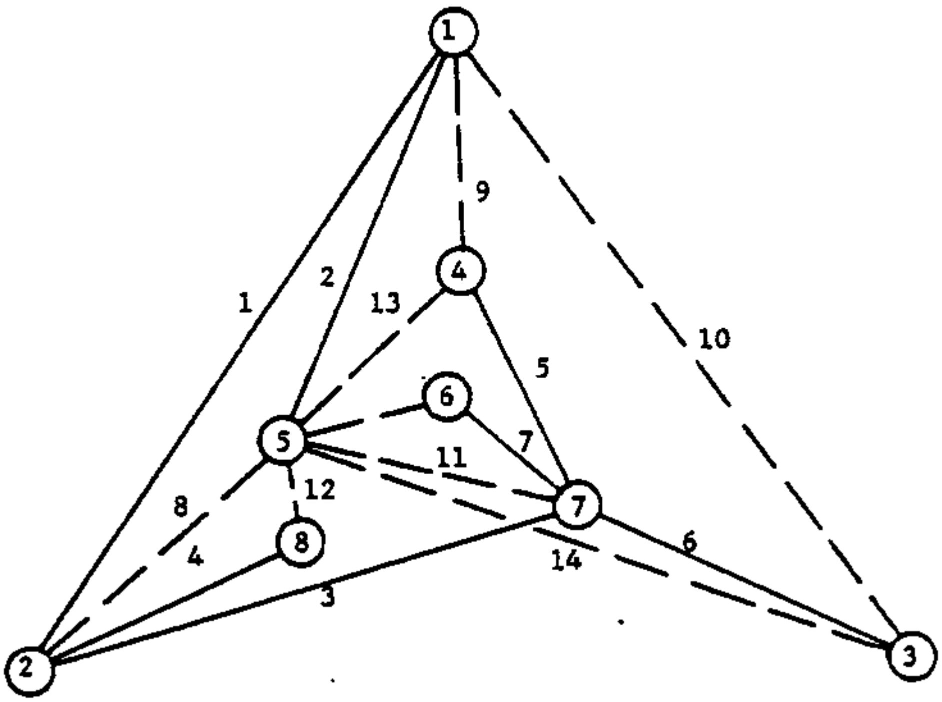


Figure 2: Telecommunications Network N^
(solid lines represent final links; dashed lines
represent high-usage links)

(I. Baybars and K. 0. Kortanek)

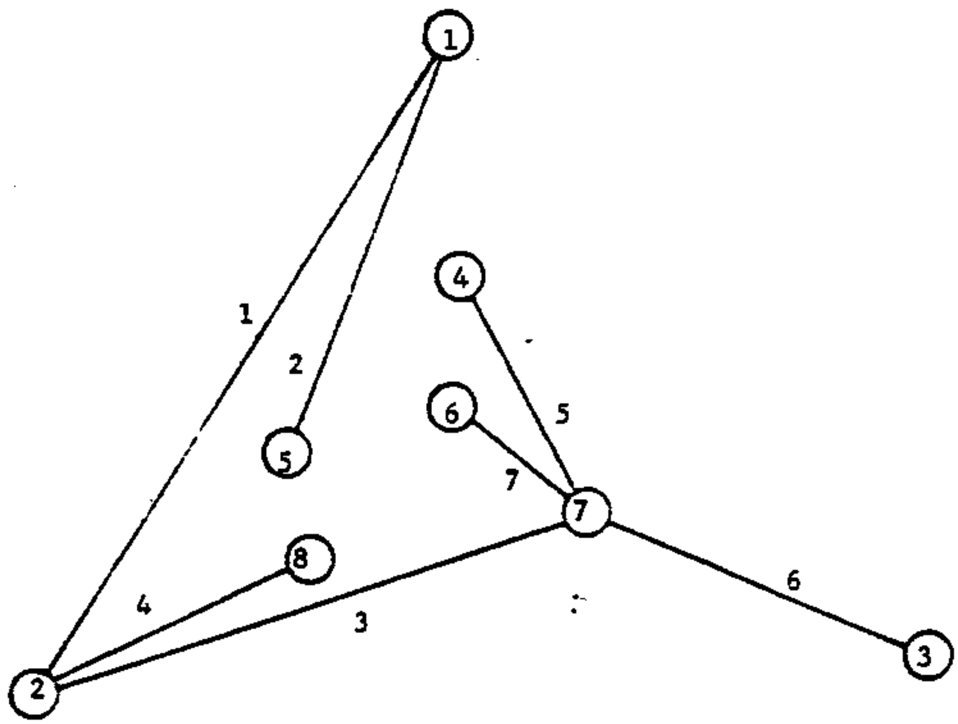
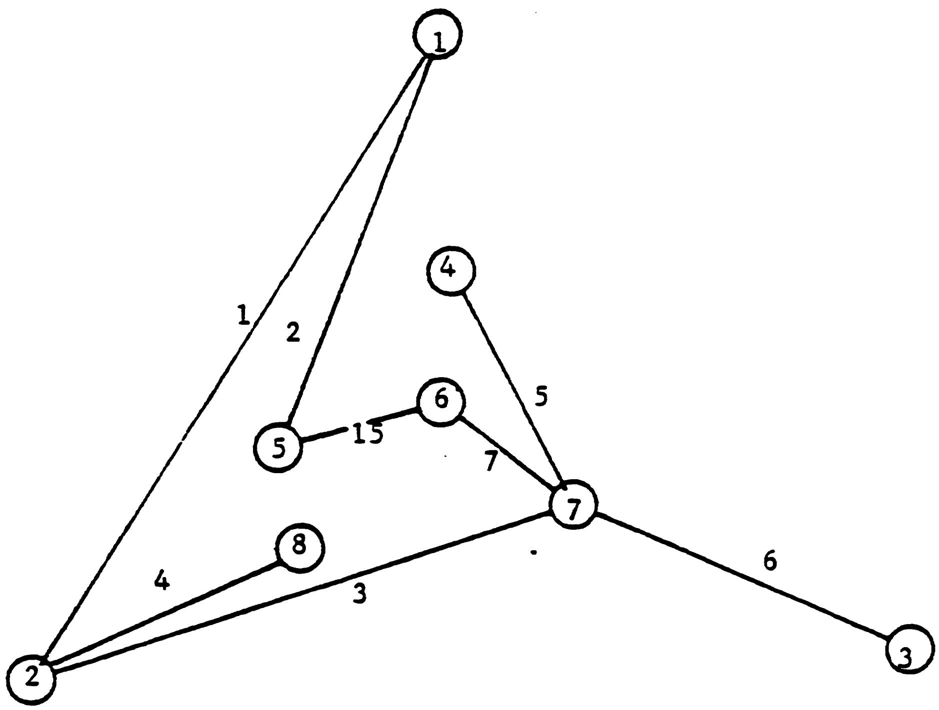


Figure 3: A Spanning Tree T of the Network ^ (Figure 2)

(I. Bayfaars and K. 0. Kortanek)



Firgure 4: The Graph G_1 of N_1 consisting of spanning tree T (Figure 3) and the chord e_{15}

(I. Baybars and K. O. Kortanek)

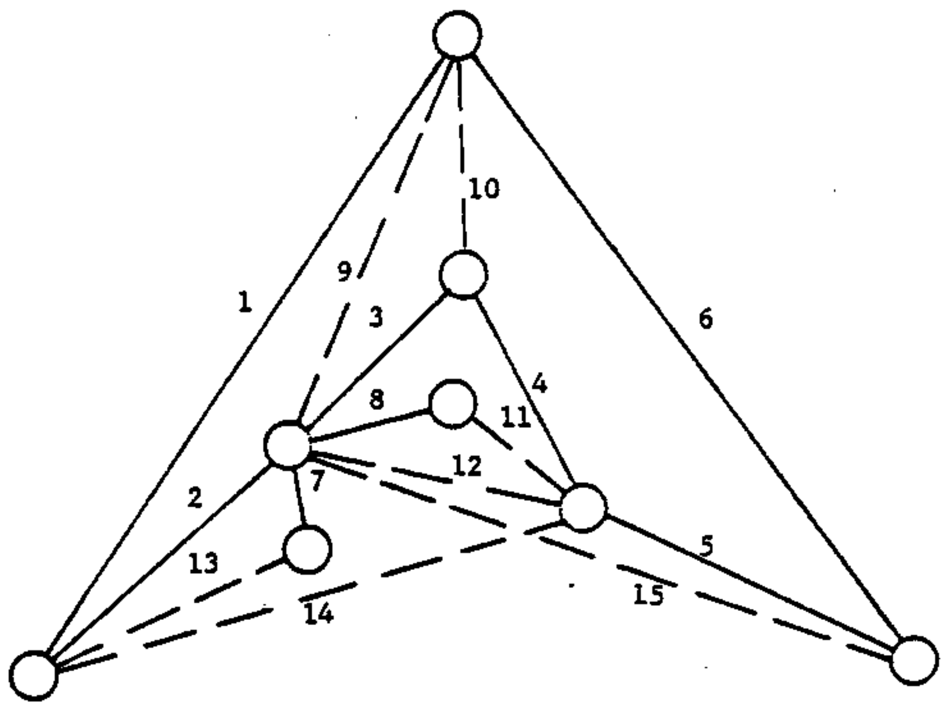


Figure 5: A Telecommunications Network in which each high-usage link has two alternate routes

(I. Bayfaars and K. 0. Kortanek)

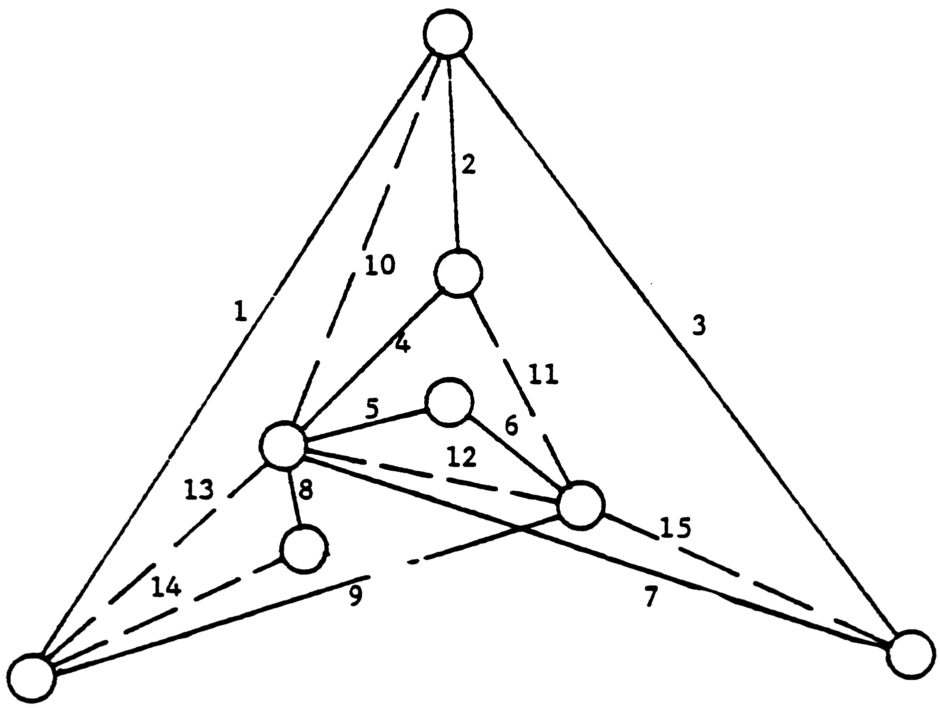


Figure 6: A Telecommunications Network
in which each high-usage link
has three alternate routes

(I. Baybars and K. O. Kortanek)

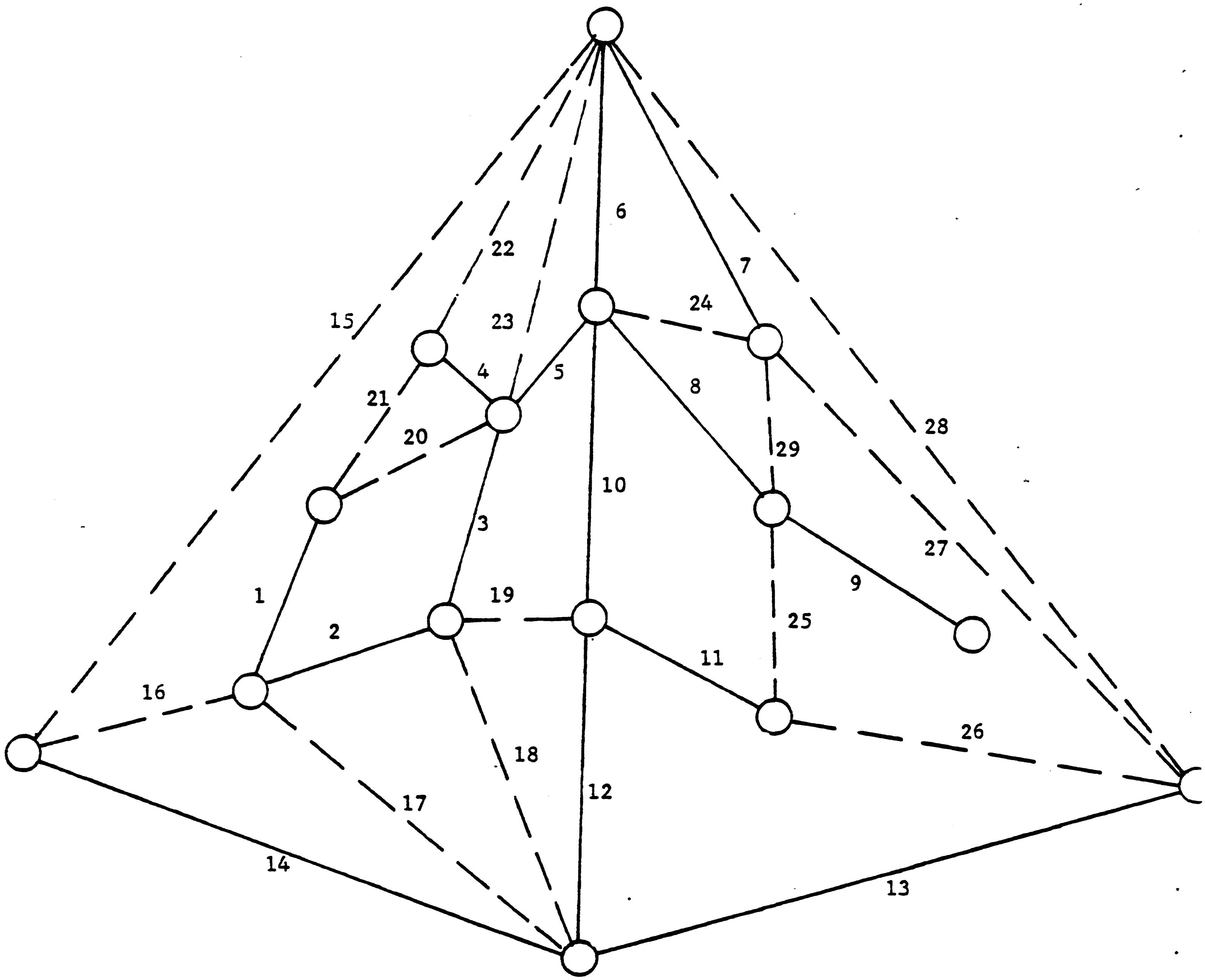


Figure 7: Network N_2

(I. Baybars and K. O. Kortanek)

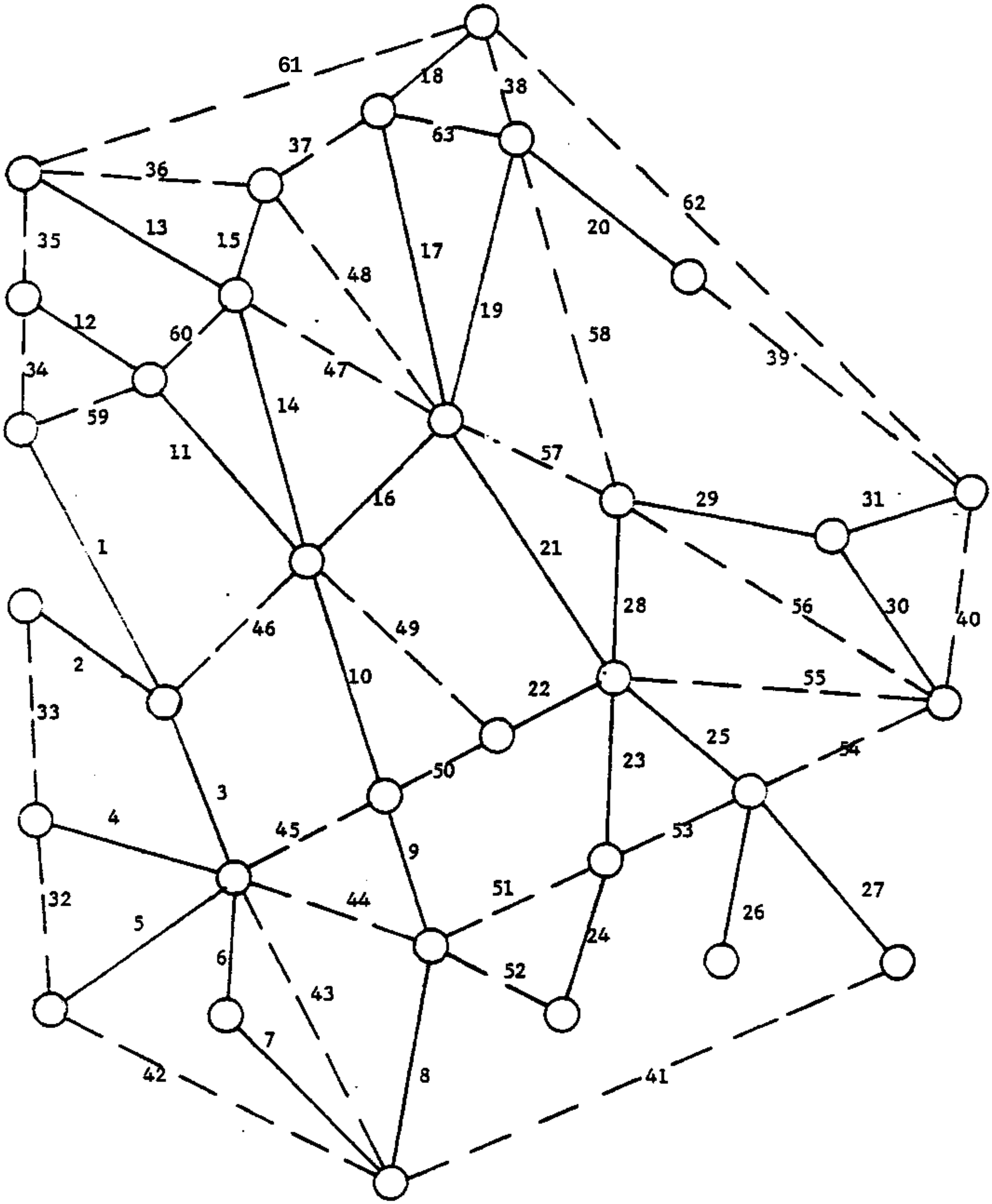


Figure 8: Network N_5

(I. Baybars and K. O. Kortanek)

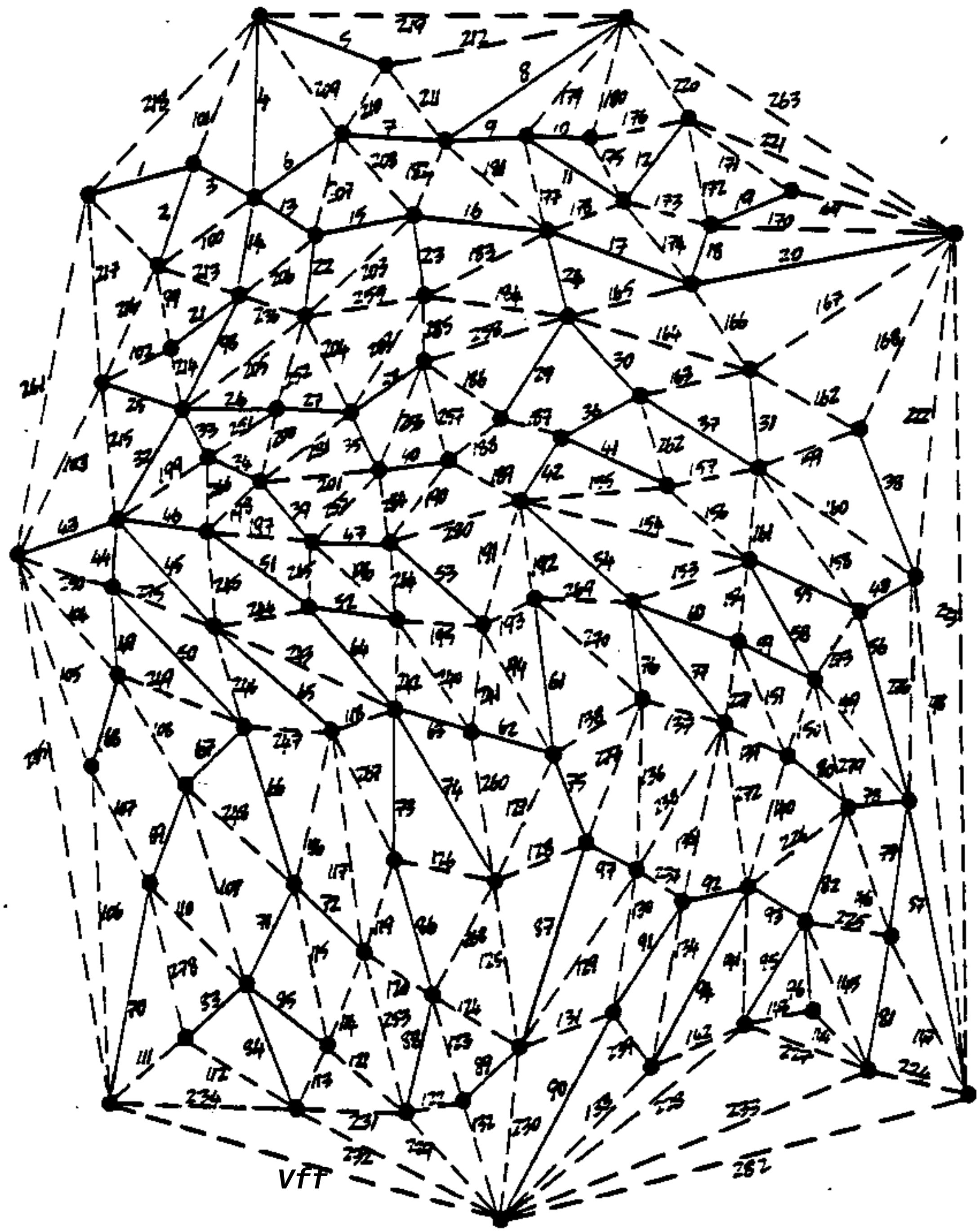


Figure 9: Network N_4

(I. Baybars and K.O. Kortanek)

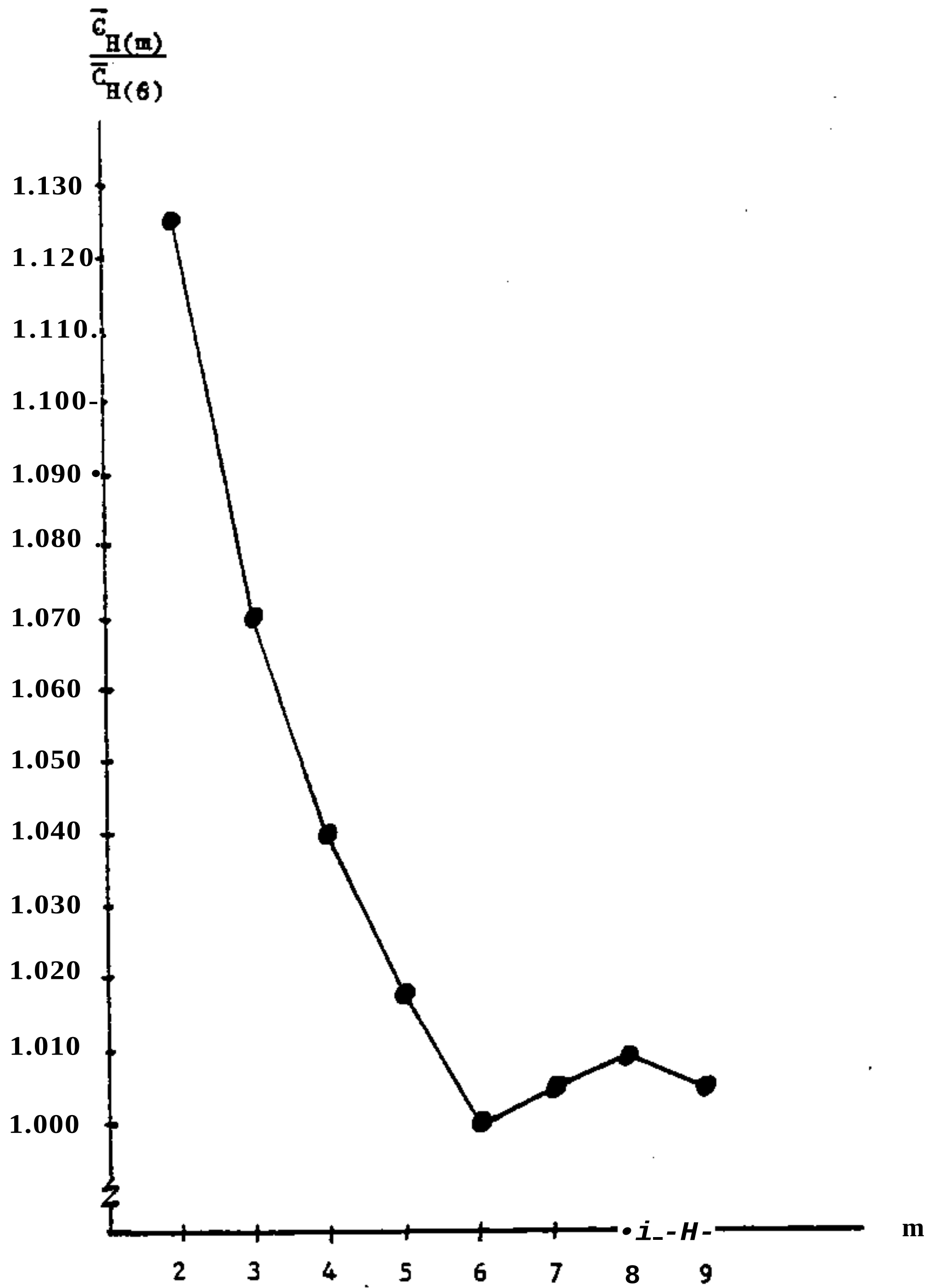


Figure 10 Heuristic Average Cost, $\bar{C}_{H(m)}$
as a Function of M"^*™""
Permitted Length of Alternate Paths, m.

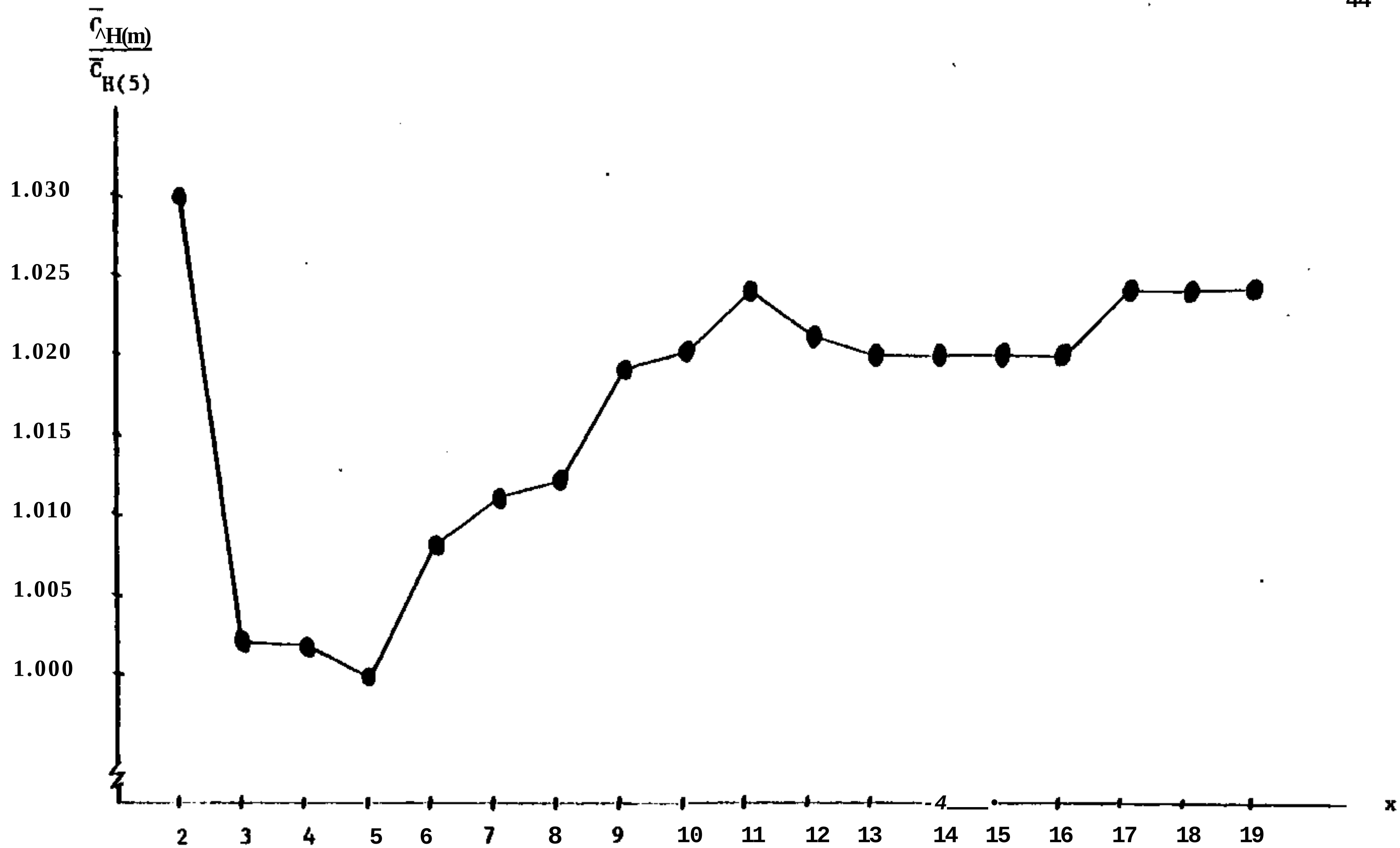


Figure 11 Heuristic Average Cost, $\bar{C}_{H(m)}$
as a Function of Maximum
Permitted Length of Alternate Paths, m.

Network	Nodes	Final Links	High-Usage Links	Mia Alternate Path	Ave Alternate Path	Max Alternate Path
N_1	8	7	8	2	3.25	4
N_2	15	14	15	2	3.60	6
N_3	32	31	32	2	3.81	9"
N_4	99	98	185	2	6.30	30

Table 1: Numerical Characteristics of the Four Networks N_1, N_2, N_3, N_4 of Figures 2,7,8,9.

Link	t-1	t-2	t-3
1	35	60	70
2	21	42	63
3	92	184	184
4	58	99	174
5	47	80	188
6	47	80	177
7	59	100	177
8	2	5	8
9	17	34	68
10	17	39	51
11	7	14	21
12	18	31	72
13	18	31	72
14	18	36	54
15	18	41	72

Table 2: Circuit Requirements for N_1

Link	t=1	t=2	t=3
1	110	130	148
2	201	230	256
3	166	193	193
4	138	178	219
5	173	196	227
6	208	275	299
7	162	169	179
8	233	284	327
9	138	189	231
10	225	252	275
11	146	197	209
12	207	246	259
13	243	321	350
14	237	318	410
15	19	26	33
16	40	51	65
17	29	37	42
18	5	7	8
19	74	97	120
20	50	65	74
21	50	72	91
22	10	15	19
23	50	56	65
24	37	42	45
25	4	5	6
26	32	46	52
27	49	61	79
28	65	69	82
29	61	73	75

Table 3: Circuit Requirements for N_2

Link	t-1	t»2	t-3
1	4	4	4
2	151	152	196
3	157	230	232
4	69	87	111
5	248	409	609
6	32	47	67
7	168	250	367
8	181	320	432
9	271	382	546
10	41	43	45
11	236	261	331
12	168	255	334
13	116	142	147
14	128	238	242
15	196	313	403
16	68	92	110
17	2	2	2
18	276	425	539
19	98	110	151
20	72	85	85
21	111	157	191
22	51	54	70
23	60	95	104
24	157	262	343
25	144	192	255
26	24	45	* 47
27	71	105	119
28	114	165	217
29	24	31	34
30	130	201	293
31	174	309	333
32	24	26	30
33	25	42	55
34	30	49	68
35	22	22	26
36	21	22	29
37	16	23	30
38	25	25	34
39	15	21	27
40	7	13	16
41	17	29	31
42	9	16	22
43	18	24	33
44	11	15	15
45	25	34	38
46	5	9	12
47	13	18	27
48	21	23	31
49	52	78	81
50	9	15	15
51	19	19	25
52	56	81	98
53	15	15	20
54	42	53	73
55	66	72	90
56	17	17	18
57	11	16	18
58	15	15	18
59	14	15	21
60	22	30	44
61	15	25	30
62	16	29	40
63	11	21	31

Table 4: Circuit Requirements for N_3

Link	t=1	t=2	t=3	Link	t=1	t=2	t=3
1	489	537	563	45	112	183	195
2	167	260	343	46	404	472	641
3	125	245	257	47	238	276	369
4	276	380	497	48	240	472	646
5	162	241	281	49	314	329	454
6	422	481	596	50	297	412	412
7	315	365	390	51	459	610	671
8	332	590	590	52	218	398	577
9	288	313	444	53	478	497	591
10	181	260	293	54	408	518	574
11	256	471	546	55	347	440	484
12	234	425	616	56	430	756	816
13	262	393	471	57	363	468	496
14	378	495	712	58	237	367	502
15	351	491	500	59	484	566	616
16	283	390	522	60	498	692	851
17	497	656	728	61	397	432	622
18	160	212	226	62	410	541	589
19	311	466	484	63	316	436	518
20	342	530	535	64	312	355	468
21	373	484	687	65	370	388	442
22	274	405	510	66	278	428	440
23	299	502	687	67	339	454	526
24	169	309	321	68	479	507	507
25	112	118	129	69	347	388	527
26	352	468	547	70	360	511	526
27	382	542	601	71	186	347	464
28	319	424	614	72	481	658	684
29	333	502	672	73	276	438	477
30	149	260	291	74	369	542	617
31	340	452	519	75	338	385	504
32	204	401	489	76	276	347	385
33	321	369	439	77	496	639	677
34	344	509	519	78	356	519	685
35	269	425	437	79	472	585	672
36	146	273	360	80	267	365	503
37	372	531	584	81	393	495	613
38	424	466	615	82	342	533	575
39	331	377	539	83	324	411	567
40	343	445	614	84	462	535	663
41	478	683	730	85	489	523	523
42	277	426	621	86	324	450	670
43	217	269	392	87	199	250	372
44	163	317	418	88	351	526	662

Table 5: Circuit Requirements for N_4

Link	t=1	t»2	t»3	Link	t=1	t»2	t»3
89	121	145	159	134	7	7	13
90	373	559	721	135	68	97	109
91	227	442	574	136	26	30	51
92	324	612	862	137	14	15	19
93	137	178	299	138	74	85	118
94	329	365	386	139	42	47	84
95	434	546	627	140	62	95	141
96	296	390	436	141	66	66	88
97	471	480	590	142	68	76	115
98	34	46	67	143	95	96	166
99	3	5	8	144	27	44	75
100	97	106	186	145	21	37	52
101	16	20	32	146	16	20	24
102	12	21	37	147	11	16	19
103	55	84	107	148	8	13	24
104	16	16	27	149	80	157	243
105	84	147	224	150	23	34	66
106	63	87	113	151	24	41	56
107	33	59	112	152	62	120	180
108	57	64	115	153	59	108	136
109	18	28	42	154	91	91	158
110	25	47	58	155	21	28	54
111	23	44	57	156	95	165	173
112	52	71	140	157	81	143	177
113	37	48	77	158	69	115	159
114	70	116	208	159	43	47	70
115	56	89	115	160	72	141	177
116	99	131	227	161	23	42	52
117	16	24	35	162	96	184	255
118	31	60	85	163	49	81	82
119	68	74	139	164	8	9	17
120	74	84	152	165	79	146	233
121	27	35	66	166	41	75	130
122	29	56	77	167	31	54	95
123	16	28	40	168	62	78	131
124	11	11	20	169	74	79	121
125	35	38	39	170	8	12	12
126	38	56	66	171	55	81	82
127	31	44	53	172	67	97	169
128	66	128	142	173	95	143	221
129	14	14	27	174	69	80	121
130	68	70	116	175	71	97	189
131	20	30	30	176	18	30	57
132	64	83	161	177	96	100	156
133	5	5	9	178	55	79	101

(Table 5 continued)

Link	t=1	t=2	t=3
179	73	116	143
180	67	125	235
181	27	33	60
182	99	181	260
183	35	69	74
184	94	145	159
185	53	102	189
186	78	105	112
187	34	66	128
188	32	63	86
189	92	154	177
190	97	191	212
191	64	89	90
192	19	26	26
193	35	51	64
194	12	18	19
195	37	55	80
196	22	22	33
197	4	6	7
198	32	62	71
199	13	18	27
200	65	103	124
201	86	131	187
202	29	36	38
203	94	133	168
204	34	37	53
205	77	132	201
206	67	132	137
207	25	38	68
208	55	79	113
209	62	94	165
210	44	63	79
211	30	38	63
212	32	59	68
213	76	123	227
214	81	153	165
215	56	106	144
216	34	55	84
217	23	44	51
218	78	135	202
219	2	3	3
220	66	104	118
221	94	124	243
222	60	90	142
223	11	12	21
224	83	85	159
225	45	48	73
226	74	138	155
227	3	3	4
228	99	132	211
229	69	75	93
230	12	17	17
231	52	81	104
232	82	106	165
233	19	36	53
234	66	90	109
235	49	70	104
236	80	151	160
237	4	4	6
238	41	75	75
239	52	60	81
240	79	109	119
241	87	125	150
242	69	91	157
243	46	46	84
244	43	64	118
245	24	46	46
246	62	65	89
247	85	105	123
248	56	59	97
249	77	112	132
250	17	32	52
251	63	75	90
252	12	22	33
253	86	149	213
254	7	8	8
255	38	69	* 80
256	40	71	126
257	28	36	69
258	94	145	184
259	17	21	22
260	18	27	30
261	55	64	86
262	15	26	48
263	93	140	212
264	30	52	59
265	25	49	59
266	37	52	91
267	68	90	128
268	98	168	310
269	82	109	172
270	94	137	198
271	10	12	17
272	16	27	51
273	25	25	38
274	40	73	144
275	84	111	170
276	51	90	140
277	78	117	228
278	58	67	72
279	93	123	156
280	10	11	17
281	86	91	170
282	61	73	81
283	52	59	112

(Table 5 continued)

*SfMtm** (»)	ttaad Cost CcM (dollars) *	Variable Cost. (c^a) (dollars) *	Capacity (a) (uni^s) *
1	530,000	3,100	30
2	•70,000	1,070	90
3	1,400,000	277	270

Table 6: Costs and Capacities of the Transmission Systems and Cost of the Circuits

	Program Q		
Network	Constraints	Integer Variables	Other Variables
N_1	180	135	159
N_2	348	261	306
N_3	756	567	663
N_4	3396	2547	3102

Table 7: Numerical Characteristics of the Associated Optimization Problems of the Four Networks

Link	Installations		
	Si	t»2	t»3
1	3	0	0
2	3	0	0
3	3	2	0
4	3	0	0
5	3	0	0
6	3	0	0
7	3	0	0
8	0	0	0
9	0	0	0
10	0	0	0
11	0	0	0
12	0	0	2
13	0	0	2
14	0	1	0
15	0	0	2

Table 8: Optimal Solution for N_1

(Total Cost: $ 12188683)

Link	Installations t-1	t-2	t-3
1	3	0	0
2	3	0	0
3	3	2	0
4	3	0	0
5	3	0	0
6	3	0	0
7	3	0	0
8	0	0	0
9	0	0	2
10	0	0	0
11	0	0	0
12	0	0	0
13	0	0	2
14	0	0	2
15	0	1	0

Table 9: Heuristic Solution for N_1
(Total Cost: $ 12195223)

Link	Installations		
	t=1	t=2	t=3
1	3	0	0
2	3	2	0
3	3	1	1
4	3	0	0
5	3	2	0
6	6	a	0
7	3	0	0
8	5	0	0
9	3	0	0
10	3	2	0
11	3	0	0
12	3	2	0
13	4	2	[illegible]
14	3	3	0
15	0	0	0
16	2	0	0
17	1	1	0
18	0	0	0
19	2	0	0
20	0	0	0
21	2	0	0
22	0	0	0
23	0	0	0
24	0	0	0
25	0	0	0
26	0	0	C
27	2	0	C
28	2	0	0
29	C	1	1

Table 10: Heuristic Solution for N_2
(Total Cost: $ 34,411,299)

Link	Installations t=1	t=2	t=3
1	1	0	0
2	3	0	0
3	3	0	0
4	3	0	0
5	4	3	3
6	3	0	0
7	3	3	0
8	3	3	0
9	6	0	1
10	2	0	0
11	5	0	0
12	3	2	0
13	3	0	0
14	3	2	0
15	3	3	0
16	3	0	0
17	2	0	0
18	6	0	0
19	3	0	0
20	2	0	0
21	3	0	0
22	3	0	0
23	3	0	0
24	3	0	2
25	3	0	0
26	1	1	0
27	2	1	0
28	3	2	0
29	3	0	0
30	3	3	0
31	3	2	0
32	0	0	0
33	0	1	0
34	2	0	0
35	0	0	0
36	0	0	0
37	0	0	0
38	0	0	2
39	1	0	0
40	0	0	0
41	1	0	1
42	0	0	0
43	0	0	0
44	0	0	0
45	0	0	1
46	1	0	0
47	0	0	0
48	0	0	0
49	0	1	1
50	0	0	0
51	0	0	1
52	2	0	1
53	0	0	1
54	0	0	2
55	0	0	0
56	0	0	0
57	0	0	0
58	0	0	0
59	1	0	0
60	0	0	2
61	0	1	0
62	1	0	1
63	0	0	0

Table 11: Heuristic Solution for N_3
(Total Cost: $ 63,143,533)

Link	Installations t=1	t=2	t=3
1	[illegible]	[illegible]	0
2	3	3	0
3	3	3	[illegible]
4	6	3	[illegible]
5	3	2	1
6	3	2	[illegible]
7	6	0	2
8	[illegible]	[illegible]	[illegible]
9	[illegible]	0	[illegible]
10	[illegible]	0	[illegible]
11	6	3	0
12	6	0	2
13	6	3	0
14	8	3	0
15	6	2	0
16	6	2	0
17	6	3	0
18	3	2	0
19	6	0	0
20	6	2	0
21	6	3	0
22	6	2	0
23	6	3	0
24	3	3	0
25	3	0	0
26	5	3	0
27	6	2	0
28	6	0	2
29	6	1	3
30	3	3	0
31	6	2	0
32	6	3	0
33	[illegible]	3	0
34	[illegible]	3	0
35	4	3	0
36	3	3	0
37	[illegible]	3	0
38	[illegible]	0	2
39	[illegible]	0	0
40	[illegible]	3	2

Link	Installations t=1	t=2	t=3
41	9	3	0
42	6	2	0
43	6	1	0
44	6	3	0
45	3	3	0
46	9	0	0
47	3	1	2
48	6	3	0
49	6	2	0
50	6	0	1
51	6	3	0
52	6	1	3
53	6	0	2
54	6	2	0
55	6	0	1
56	9	3	0
57	6	3	0
58	6	3	0
59	9	0	0
60	9	3	0
61	6	0	2
62	6	3	0
63	6	3	0
64	6	2	0
65	6	0	0
65	6	3	0
67	6	3	0
66	6	1	0
69	6	0	0
70	[illegible]	0	0
71	3	3	0
72	6	3	0
73	6	2	0
74	8	3	0
75	6	0	0
76	6	0	0
77	9	0	0
78	6	3	0
79	9	0	0
80	6	0	0

Table 12: Heuristic Solution for N_4
(Total Cost: $ 473,501,920)

Link	Installations t=1	t=2	t=3
81	6	2	0
82	6	3	0
83	6	0	1
84	7	3	0
85	6	3	0
86	6	0	3
87	6	0	0
88	6	0	3
89	3	0	0
90	6	1	3
91	5	3	2
92	6	4	3
93	6	0	0
94	6	3	0
95	9	0	0
96	6	0	0
97	6	2	0
98	3	2	0
99	0	0	1
100	3	0	0
101	0	0	0
102	0	0	0
103	0	1	2
104	0	0	0
105	0	0	3
106	1	0	1
107	2	0	1
108	2	0	1
109	0	0	0
110	0	0	1
111	0	0	2
112	2	0	2
113	0	0	2
114	1	0	2
115	2	0	0
116	3	0	0
117	1	0	1
118	2	0	0
119	2	0	2
120	2	0	2

Link	Installations t=1	t=2	t=3
121	2	0	0
122	2	0	0
123	1	0	1
124	1	0	0
125	2	0	0
126	2	0	0
127	0	0	0
128	0	1	0
129	0	0	0
130	0	0	2
131	1	0	0
132	2	0	2
133	1	0	0
134	0	0	0
135	0	0	0
136	1	0	1
137	1	0	0
138	0	0	2
139	2	0	0
140	3	0	0
141	0	0	2
142	0	0	2
143	2	0	0
144	0	0	0
145	1	1	0
146	0	0	1
147	0	0	0
148	0	0	0
149	0	0	3
150	0	0	1
151	1	1	0
152	3	0	0
153	0	0	0
154	0	0	3
155	0	0	0
156	0	0	3
157	3	0	0
158	0	0	2
159	2	0	0
160	3	0	0

(Table 12 continued)

I.Ink	Installations t-l	.5=1	t=3
161	1	1	0
162	3	0	0
163	2	0	0
164	0	0	0
165	0	0	3
166	0	0	2
167	2	0	1
168	2	0	2
169	2	0	2
170	0	0	0
171	0	0	0
172	3	0	0
173	3	0	0
174	2	0	2
175	3	0	0
176	0	•O	0
177	0	0	^
178	2	0	1
179	3	0	0
180	0	0	3
181	0	0	0
182	3	0	0
163	2	v	0
184	0	0	2
ies	0	1	3
186	2	1	C
187	2	0	2
188	0	0	0
189	3	0	0
19C	t	C	0
191	2	0	0
192	1	0	0
193	2	0	0
194	1	0	0
195	2	0	0
196	1	0	1
197	1	0	0
198	2	0	0
199	0	0	C
200	0	0	0
201	2	0	0
202	1	1	0

I•Ink	installations f l	£=2	f 3
203	3	0	0
204	0	0	0
205	3	0	0
206	0	0	3
207	0	0	0
208	0	0	0
209	0	1	3
210	0	0	0
211	0	0	0
212	0	0	1
213	3	0	0
214	0	0	2
215	0	0	1
216	0	0	0
217	1	1	0
218	3	0	0
219	0	0	0
22C	0	0	2
221	3	0	0
222	2	0	2
223	1	0	0
224	2	0	2
225	0	0	2
226	0	2	P
227	0	0	0
228	3	0	0
229	2	0	1
230	1	0	0
231	&%	0	1
232	3	0	0
233	1	1	0
234	2	0	1
235	2	0	1
236	3	0	0
237	0	0	0
238	2	0	0
239	2	0	0
240	0	0	1
241	0	0	2
242	3	0	0
243	0	0	0
244	1	0	•?

(Table 12 continued)

Link	Xnstall*tions t=1	t=2	t=3
245	0	0	0
246	0	0	0
247	2	0	C
248	2	C	C
249	C	0	C
250	C	0	C
251	0	0	0
252	0	0	0
253	3	0	0
254	1	0	0
255	2	0	0
256	2	0	2
257	0	0	2
258	3	0	0
259	1	0	0
260	0	0	0
261	0	0	0
262	1	0	1
263	3	0	0
264	1	1	0
265	1	2^	0
266	2	0	1
267	0	0	3
268	3	0	2
269	0	0	3
270	3	0	0
271	1	0	0
272	1	0	1
273	0	0	0
274	0	1	2
275	3	0	0
276	0	0	0
277	0	0	2
278	0	0	0
279	3	0	0
280	0	0	0
281	3	0	0
282	0	0	2
283	2	0	1

(Table 12 continued)

Network	$\bar{C}_M$ Minimum	$\bar{C}_O$ Optimal	$\bar{C}_H$ Heuristic	Maxizsum
N_1	6228	8400	8405	10706
N_2	5265	N.A.	7754	8217
N_3	5933	N.A.	7791	8465
N_4	4575	N.A.	6706	6906

Table 13: Average Circuit Costs:
$\bar{C}_M$ (Average Using Least Cost Circuits on High Usage Links); $\bar{C}_O$ (Average Optimal); $\bar{C}_H$ (Average Heuristic); C_{NS} (Average without Network Hierarchy

Network	CPU Seconds
N_1	0.36
N_2	0.41
N_3	0.57
N_4	3.98

Table 14: CPU Times