YIELD MAXIMIZATION FOR USE IN MULTIPLE CRITERION OPTIMIZATION OF ELECTRONIC CIRCUITS

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ABSTRACT

In this paper we examine the problem of designing electronic circuits using Multiple Criteria Optimization where one of the competing criteria is circuit yield. The yield and gradient of vield are estimated using a method based upon Simplicial Approximation which is used to form a piecewise linear approximation to the probability density function of the designable parameters. An example illustrates that It may be possible to significantly alter the values of various circuit criteria, over their value at the maximum yield point, with very little change in yield.

I. INTRODUCTION

Historically circuit design can be viewed as consisting of two broad methodologies: performance design and statistical design. In performance design the circuit designer chooses a circuit configuration, adjusts parameters to attain a desired performance and then tests the circuit yield. If the yield is too small the parameters are re-adjusted. Statistical design arose mainly in response to integrated circuit design problems. In statistical design a circuit configuration is chosen and then the parameters are adjusted to achieve maximum circuit yield (worst case design being the extreme of 100% yield).

These two methodologies can be unified by considering circuit design as a Multiple Criteria Optimization (MCO) problem with yield as one of the competing objectives. In this way we can investigate the possible tradeoffs. However, it seems natural that there might exist a trade off between the performance characteristics and the statistical characteristics of a circuit. A heuristic argument for this trade-off can be made by noting that extremes of circuit behavior, e.g., low power dissipation or fast switching times, usually occur at the extreme limits of some parameter values which makes the design very sensitive to parameter variations - just what statistical design tries to avoid. In this paper we will consider yield maximization and develop a technique, which is an extension of the Simplicial Approximation technique [1,2], that can be used when yield and performance are to be traded-off in an optimization procedure. (Note: the techniques of Multiple Criterion Optimization (MCO) [3,4] will be used to generate the trade-offs between yield and other competing design objectives.)

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II. REVIEW OF EXISTING YIELD MAXIMIZATION PROCEDURES

Assume that the circuit under consideration has n statistical parameters, denoted by x. The specified value of these parameters, x_0, is called the nominal value and it is a point in the n dimensional parameter space. A circuit is considered to have acceptable performance if the following constraints are satisfied:

$$g(x_0) \leq 0$$
$$h(x_0) = 0. \qquad (1)$$

The feasible or acceptable region 3 is the set of all parameters that satisfy the constraints,

$$\Omega = \{ x_0 \mid g(x_0) \leq 0,\ h(x_0) = 0 \} \qquad (2)$$

Given that the parameters have a joint probability density function (p.d.f.), $F(x, x_0)$, the yield associated with a circuit whose nominal point is x_0 is given by

$$y(x_0) = \int_\Omega F(x_0, x)\,dx \quad . \qquad (3)$$

Note that since ?. is implicitly defined in terms of x_0 direct evaluation of (3) is impossible. This fact In turn makes difficult the maximization of y liv varying jc^, oxpee L.illv within the context of a multiple criterion optimization problem. (Since we have limited our interest here to the design of integrated circuits only, we do not consider tolerances as designable.)

Since our primary concern is the evaluation and ultimate maximization of ^/(x^), we consider some of the techniques which have been developed for these purposes, and indicate -./here their weaknesses lie. We begin with the Simplicial

Approximation method [1]. This technique is an iterative procedure for creating a piecewise linear inner approximation, SA, of the feasible region, Ω. The feasible region is assumed convex. Thus we have

$$SA = \{ \underset{\sim}{x} \mid \underset{\sim}{n}_i^T \underset{\sim}{x} \le b_i,\ i = 1,2,\ldots,n_F\} \qquad (4)$$

and

$$SA \subseteq \Omega \qquad (5)$$

where $\underset{\sim}{n}_i, b_i$, describe the hyperplanes that bound the approximation. Assuming that at some stage, SA is a good approximation to Ω, Director and Hachtel propose a yield maximization procedure which is based upon inscribing a convex body into the SA. The convex body (yield body or norm body) to be inscribed has the same shape as a level contour of the p.d.f. This inscription is done using a linear program and the center of the largest inscribed body is taken as the nominal point which maximizes yield.

We now carefully examine the assumptions of this procedure to see if it might be useful when trading-off between yield and performance. An underlying assumption of the Simplicial Approximation design centering scheme is that inscribing the largest norm body in the approximation, SA, is equivalent to maximizing yield over SA. (Note: for the rest of this discussion the simplicial approximation will be taken as an adequate approximation of the feasible region, Ω, and the assumption will be made that we must maximize yield over SA). In order to show that the assumption used in Simplicial Approximation design centering scheme is in general invalid, we will assume that $0 \le F(\underset{\sim}{x},\underset{\sim}{x}_0) \le M < \infty$ for all $\underset{\sim}{x}$, $\underset{\sim}{x}_0$, where $(F(\underset{\sim}{x},\underset{\sim}{x}_0)$ is the p.d.f. and $\underset{\sim}{x}_0$ the nominal point. Further, we assume that each level set of $F(\underset{\sim}{x},\underset{\sim}{x}_0)$

$$K_{\underset{\sim}{x}_0}(u) = \{\underset{\sim}{x} \mid F(\underset{\sim}{x},\underset{\sim}{x}_0) \ge u\},\ 0 \le u \le M\} \qquad (6)$$

is a convex, connected set. Recall that the yield, at a given nominal, over the simplicial approximation, SA, is

$$Y(\underset{\sim}{x}_0) = \int_{SA} F(\underset{\sim}{x},\underset{\sim}{x}_0)\, d\underset{\sim}{x}. \qquad (7)$$

Let $V\{K_{\underset{\sim}{x}_0}(u) \cap SA\}$ represent the Euclidean volume in input $\underset{\sim}{x}_0$ space of the intersection of the level set $K_{\underset{\sim}{x}_0}(u)$ with the approximation, SA. We now write (7) as a Lebesgue-Stieltjes integral [5,6,7],

$$\int_{SA} F(\underset{\sim}{x},\underset{\sim}{x}_0)d\underset{\sim}{x} = \int_0^M V\{K_{\underset{\sim}{x}_0}(u) \cap SA\}\ du\ . \qquad (8)$$

In order to relate (8) to yield body inscription, we need the notion of the largest yield body-level set- that can be inscribed in SA at a given nominal point $\underset{\sim}{x}_0$. We define the level set of this body as

$$u_I(\underset{\sim}{x}_0) = \min\{u \mid K_{\underset{\sim}{x}_0}(u) \cap SA = K_{\underset{\sim}{x}_0}(u)\}\ . \qquad (9)$$

The complementary idea of the smallest yield body, at a given nominal, that just contains SA is given by

$$u_0(\underset{\sim}{x}_0) = \max\{u \mid K_{\underset{\sim}{x}_0}(u) \cap SA = SA\}. \qquad (10)$$

The fact that a small u in (6) corresponds to a large yield body, and vice versa, accounts for the min and max in (9) and (10).

Now using (9) and (10), we rewrite (8) as

$$\int_0^M V\{K_{\underset{\sim}{x}_0}(u) \cap SA\}\ du = u_0(\underset{\sim}{x}_0)V\{SA\} \qquad (11a)$$

$$\int_{u_0(\underset{\sim}{x}_0)}^{u_I(\underset{\sim}{x}_0)} V\{K_{\underset{\sim}{x}_0}(u) \cap SA\}du \qquad (11b)$$

$$\int_{u_I(\underset{\sim}{x}_0)}^{M} V\{K_{\underset{\sim}{x}_0}(u)\}du \qquad (11c)$$

Examining (11c) we recognize that inscribing the largest yield body into SA is equivalent to minimizing $u_I(\underset{\sim}{x}_0)$ and thus maximizing (11c). However in order to maximize (11a) we must maximize $u_0(\underset{\sim}{x}_0)$ which implies finding the smallest yield body that contains SA. If the nominal point which maximizes (11a) coincides with the maximizer of (11c) then we have a maximum yield. Anderson [7] has shown that if the level sets of $F(\underset{\sim}{x},\underset{\sim}{x}_0)$ are symmetric about the nominal and if the SA is symmetric about some point $\underset{\sim}{x}^*$, then maximum yield occurs when $\underset{\sim}{x}_0 = \underset{\sim}{x}^*$. However, in the general case, (11b) will determine the maximum yield as a trade-off between (11a) and (11c). In fact, direct differentiation of (11) shows

$$\frac{\partial}{\partial x_0} Y(\underset{\sim}{x}_0) - \int_{u_0(\underset{\sim}{x}_0)}^{u_I(\underset{\sim}{x}_0)} V\{K_{\underset{\sim}{x}_0}(u) \cap SA\}du \qquad (12)$$

Thus we see that the Simplicial Approximation design centering scheme of inscribing the largest yield body into SA only maximizes (11c) and in general does not maximize the yield. Further, it seems likely that this method of approximating the maximization of yield becomes less accurate as we move away from the center of the largest inscribed yield body and therefore will not be useful for generating trade-offs between yield and performance. But the concept of Simplicial Approximation does form the basis of the technique to be described below.

Other yield estimation procedures have been proposed by Tahim and Spence [8] and Bandler and Abdel-Malek [9-11], amongst others. Space limitations preclude discussion of these techniques here. Suffice it to say that the heuristic nature of [8] and the computational complexity of [9-11] limit the usefulness of these methods for MCO problems [3].

III. A NEW YIELD MAXIMIZATION PROCEDURE

We will describe a new yield maximization procedure based upon the Simplicial Approximation, SA, to the feasible region. Using this approximation we can replace the original nonlinear constraints of the optimization with the linear constraints of the simplicial approximation, thus reducing the work involved in solving the optimization problem.

By construction, each face of the approximation, SA, is an (n-1) dimensional simplex. Thus, as shown in Fig. 1, each nominal point x_0, interior to SA, induces a unique interior simplicial decoraposition of SA. Our method estimates the yield by first dividing each interior simplex into a number of segments (Fig. 2). A piecewise linear approximation of the p.d.f. is then made over each segment and the total yield is found by estimating the yield integral using these piecewise linear approximations to the p.d.f.

Specifically, the Integral over each segment is found as follows: first, using Fig. 2, with

$$\xi_{imk} = x_0 + \frac{k}{\ell}\left(x_{im} - x_0\right) \quad \begin{matrix} m=1,\dots,n \\ k=1,\dots,\ell \end{matrix} \tag{13}$$

where ℓ is the number of segments desired, x_{im} the vertices of SA_i, we estimate the yield over the i^{th} simplex as

$$Y_i(x_0) = \frac{1}{\ell^n} V\left\{SA_i(x_0)\right\} \frac{1}{n+1}\left\{F(x_0,x_0) + \sum_{k=1}^{n} F(\xi_{ik\ell},x_0)\right\} + \sum_{j=2}^{\ell} \frac{j^n-(j-1)^n}{\ell^n} V\left\{SA(x)\right\}\frac{1}{2n}\left\{\sum_{k=j-1}^{j} \sum_{m=1}^{n} F(\xi_{imk}, x_0)\right\} \tag{14}$$

The gradient of the yield over the i^{th} interior simplex is estimated by taking the gradient of (14) (similar to the Bandler Abdel-Malek approach) i.e.,

$$\nabla_{x_0} Y_i(x_0) = \frac{1}{\ell^n} V\left\{SA_i(x_0)\right\}\left(\frac{1}{n+1}\right)\left\{\nabla_{x_0} F(x_0,x_0) + \sum_{k=1}^{n} \frac{\partial F(\xi_{ik\ell},x_0)}{\partial x_0}\right\} + \frac{1}{\ell^n}\nabla_{x_0} V\left\{SA_i(x_0)\right\}\frac{1}{n+1}\left\{F(x_0,x_0) + \sum_{k=1}^{n} F(\xi_{ik\ell},x_0)\right\} + \sum_{j=2}^{\ell}\frac{j^n(j-1)^n}{\ell^n} V\left\{SA_i(x_0)\right\}\frac{1}{2n}\left\{\sum_{k=j-1}^{j}\left[\sum_{m=1}^{n} \frac{\partial F(\xi_{imk},x_0)}{\partial x_0}\right]\right\} + \sum_{j=2}^{\ell}\frac{j^n-(j-1)^n}{\ell^n} V\left\{SA_i(x_0)\right\}\frac{1}{2n}\left\{\sum_{k=j-1}^{j}\left[\sum_{m=1}^{n} F(\xi_{imk},x_0)\right]\right\}. \tag{15}$$

Note that (15) is the exact gradient of (14). Where $V\{SA_i(x_0)\}$ represents the Euclidean volume of the i^{th} interior simplex. Notice that to calculate (14) and (15) we need the volume of the i^{th} simplex and its gradient. It can be shown [3,12] that

$$V\{SA_i(x_0)\} = \frac{1}{n!}\left|\det(X_i)\,\{1-x_0^T X_i^{-1} e\}\right| \tag{16}$$

$$\nabla V\{SA_i(x_0)\} = \frac{1}{n!}\, \mathrm{Sgn}[\det(X_i)\{1-x_0^T X_i^{-1} e\}](-X_i^{-1} e) \tag{17}$$

where X_i is a matrix whose columns are the coordinates of the vertices of the i^{th} interior simplex excluding the nominal and e is a vector of all ones. Notice that $X_i^{-1}e$ and $\det(X_i)$ can be calculated once, independent of the nominal point, and then the evaluation of (16) and (17) and therefore (14) and (15) becomes straightforward. The total yield and the gradient of yield are found by summing (14) and (15) over all interior simplices.

IV EXAMPLE

As an example of uaing the vieid estimation procedure lit a Multiple Criteria Optimization to trade-off between yield and two other objectives we consider the following problem:

$$\text{MIN} \quad f_1 = (x_1 - 1.5)^2 + (x_2-3)^2 \tag{18}$$
$$f_2 = (x_1 - 7)^2 + (x_2 - .5)^2$$
$$1 - Y(x_0)$$

subject to

$$x_1, x_2 \in \Phi \left\{\left(\frac{x_1-6}{5.5}\right)^2 + \left(\frac{x_2-6}{2}\right)^2 \le 1\right\} \tag{19}$$

where $Y(x_0)$ is the yield and x_1 and x_2 are independent Gaussian variables with equal variances. Φ indicates that we will rotate the ellipse defined by the inequality 45° to define the feasible region. Note that minimizing $1-Y(x)$ is equivalent to maximizing $Y(x)$.

The results of this example are presented in Figs. 3 and 4. In each figure we have shown the piecewise linear approximation to the feasible region, the points 1,2, and 3 which are the minimum of f_1, f_2 and maximum of yield respectively and a dashed line which consists of the trade-off solutions for f_1 and f_2 independent of yield. Notice that if we considered only f_1 and f_2, any trade-off solution we found would be far from the maximum yield point. Points 4 and 5 in each figure are trade-offs between yield, f_1, and f_2 which were generated using the techniques of Multiple Criterion Optimization [3], These trade-off points indicate that if we allow yield to decrease slightly we can achieve a marked difference in the values of the other objectives. This indicates the usefulness of considering trade-offs among all the objectives in a design, including yield, at the same time.

V CONCLUSIONS

We have presented a new method of estimating yield based upon the Simplicial Approximation to the feasible region. This method is suitable for use in a Multiple Criterion Optimization design where yield and other design objectives are trade-off. An idealized example was used to illustrate the important point that it may be possible to achieve significant improvement in the value of some objectives, over their value at the maximum yield point, with only a slight degradation in yield.

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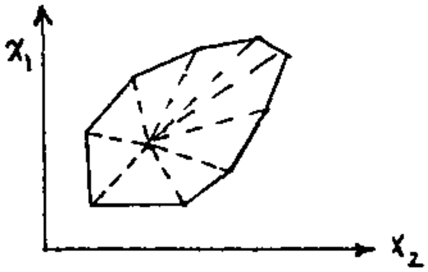

Fig. 1 An interior simplicial decomposition of the simplicial approximation induced by a nominal point.

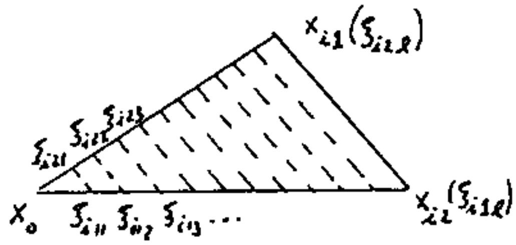

Fig. 2 Division of the i^{th} interior simplicial

NONINFERIOR POINTS

PT	Fl	F2	YIELD
1	.3195	24.58	37.68
2	' 27.26	.2935	46.78
3	29.25	7.25	96.44
lx	16.39	6.57	93.23
5	12.72	7.29	90.24

NONINFERIOR POINTS

PT	Fl	F2	YIELD
1	.3195	24.58	**26.97**
2	27.26	.2935	42.9
3	29.25	7.25	62.47
4	25.28	2.41	56.61
5	13.36	6.53	60.62

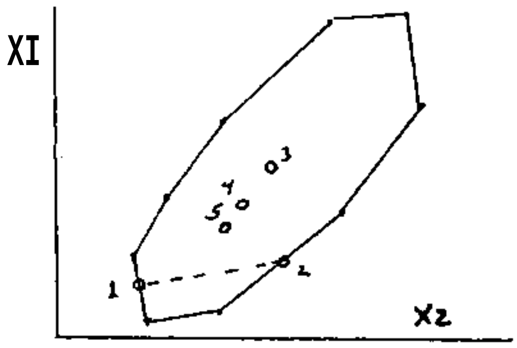

Fig. 3 Noninferior points for example including yield with standard deviation of 1.

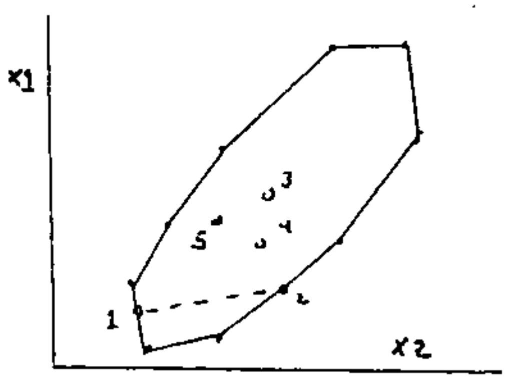

Fig. 4 Noninferior points for example including yield with standard deviation of 2.