MULTIPLE CRITERION OPTIMIZATION OF ELECTRONIC CIRCUITS

by

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MULTIPLE CRITERION OPTIMIZATION OF ELECTRONIC CIRCUITS[1]

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ABSTRACT

In this paper we examine the problem of designing electronic circuits using Multiple Criteria Optimization. A new technique for generating solutions to the MCO problem based upon a family of weighted p-norms is presented. We concentrate on the max norm member of this family (this gives a minimax problem) and propose a method of solution based upon a new constrained optimization method due to Powell. An example illustrates this procedure.

I. INTRODUCTION

Over the past decade many papers have appeared which employed optimization techniques for the choice of the parameter values of a design (see for example fl-Jj amontf sellers). In spite of this work, and In spLte of wiJe industrial use of other computer aids such as circuit simulation packages, industrial designers have been most reticent in accepting optimization as a useful design aid. The basic reason for the lack of enthusiasm for optimization is that when applied to many realistic design problems, these computationally expensive algorithms have yielded little improvement over the engineer's initial design. We feel that Multiple Criterion Optimization (MCO) may overcome this problem.

To see why the MCO approach may be better consider the typical situation in which a designer wants to choose the circuit parameter values, such as resistor and capacitor values, device geometries, and processing parameters, in order to achieve certain design objectives. These design objectives could be, for example, the minimization of power, area, and propagation delay, maximization of gain, and the requirement of maintaining a certain noise margin. The main characteristic of these design objectives is that they cannot be simultaneously achieved. That is, the design objectives are competing and the final design will be a trade-off among the competing design objectives. To formalize this situation assume that the soal is to minimize the objectives $f_1(x)$, $f_2(x)$, ..., $f_m(x)$ simultaneously, where $x = (x_1, x_2, \ldots, x_n)$ are the Jesignable parameters. A typical approach is to choose weights w_1, w_2, ..., w_m and fora

$$p(x) = \sum w_i f_i(x) \qquad (1)$$

Now $p(x)$ is minimized by a standard gradient optimization technique such as that described by Fletcher-Powell [4]. Based upon the results of minimizing $p(x)$, a new set of weights could be chosen and the minimization repeated. The weights have been chosen in an ad hoc fashion such as making w_i 10 tiaes greater than other w's because the designer felt strongly about f_i as an objective. However, the choice of weights Is critical and, more often than not, in real examples satisfactory trade-offs between design objectives could not be generated using this technique. Thus the ill-repute of optimization with many designers.

The class of optimization techniques, known as Multiple Criterion Optimization (MCO) [5,6] has been developed specifically to deal with the problem of optimizing competing objective functions. These techniques have received little attention in the electrical engineering community. Lin [5] presented the first application of MCO ideas to a circuit design problem, but his example was solved by hand and did not demonstrate the power of MCO methods. Fraser [7] used the theory of MCO to systematically choose the weights for (1) when designing several logic gates. However, Fraser did not explore the use of more powerful MCO techniques for the design of electronic circuits. In this paper we will explore the use of the full power of MCO techniques for the design of electronic circuits.

II. MULTIPLE CRITERION OPTIMIZATION

In discussing the ideas of MCO we will use the following notation. The n designuble parameters, x_i, i=1,2, ...,n, in the multiple criterion optimization problem will be denoted by the n-vector $x = (x_1, x_2, \ldots, x_n)$. It is convenient to view x as a point in the n-dimensional input space I. The m design objectives, f_j, j=1,2,...,m, will be denoted by the m-vector $f = f(x) = (f_1(x), f_2(x), \ldots, f_m(x))$. It is convenient to view f as a point in the m-dimensional output space O.

In general the optimization problems under consideration will be subject to certain constraints which will be expressed as $g(x) \le 0$ and $h(x) = 0$. In general these constraints will be nonlinear functions of x.

The MCO problem can now be stated as

$$\min f(x), \text{subject to } g(x) \le 0 \text{ and } h(x) = 0. \qquad (2)$$

As previously mentioned, if some (usually all) of the components of f are competing there will be no point x that simultaneously minimizes of all the components of f.

Instead of optimality, the concept of noninferiority [8] is used to characterize a solution to the MCO problem. In order to concisely define noninferiority we introduce the following definition.

Definition. The feasible region in input space, Ω, is the set of all designable parameters that satisfy the constraints, i.e. $\Omega = \{x \mid h(x) = 0, g(x) \le 0\}$ (3)

Definition. The feasible region in output space. Λ, is the image by f of the feasible region in input space, i.e. $\Lambda = \{f \mid f = f(x), x \in \Omega\}$ (4)

Definition. A point x^* is a noninferior point if and only if there does not exist an $x \in \Omega$ such that

$$f_i(x) \le f_i(x^*) \quad i=1, \ldots, m$$
$$f_j(x) < f_j(x^*) \quad \text{for soae } j. \qquad (5)$$

The image of a noninferior point is a noninferior solution.

In general there are an infinite number of noninferior points for a given MCO problem. The collection of noninferior points is the noninferior set. The Image

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of the noninferior set by $\underset{\sim}{f}$ is called the noninferior solution set, the noninferior surface, or the trade-off surface. As with scalar optimization procedures, procedures for solving MCO problems in fact can only generate local noninferior points. However, in the sequel whenever we discuss a noninferior solution we will mean a global noninferior solution.

In solving an MCO problem we desire to find various noninferior solutions. The designer will examine the known noninferior solutions, choose one as a final design or request that more noninferior solutions be generated. We will discuss a method of generating noninferior solutions based upon minimizing weighted p-norms [9,10], i.e. we will consider the following single objective optimization:

$$\min_{\underset{\sim}{x}} \|W\underset{\sim}{f}\|_p \quad \text{subject to } x\in\Omega \tag{6}$$

where

$W = \text{diag}(W_1 W_2, \ldots, W_m), W_i \geq 0$ and

$$\|W\underset{\sim}{f}\|_p = \left(\sum_{i=1}^{m} \left(W_{ii} f_i\right)^p\right)^{1/p} \tag{7}$$

(Note: we assume, without loss of generality, that $\Lambda>0$, i.e. $f \in \Lambda \Rightarrow f_i > 0$ for all i).

It can be shown [11] that for each noninferior solution, $\bar{f}$, there exists a p and a W such that $\bar{f}$ is the solution to (6). Notice that for p=1 (7) is equivalent to minimizing a weighted sum and for $p=\infty$ (6) is a minimax optimization problem.

In order to understand the importance of the choice of p let us consider the level sets for the weighted p-norm:

$$L_p^W(\alpha) = \{f \,|\, \|Wf\|_p \leq \alpha\}$$

The level sets for a 1-,2-, and max-norm, with $\underset{\sim}{W}=\underset{\sim}{I}$ are shown in Fig. 1a. The same level sets but for a different weight are shown in Fig. 1b. Minimizing the weighted p-norm shrinks the level set about the origin as much as possible while still remaining in contact with Λ. In Fig. 2 we show the point found by minimizing a 1-norm with weight W_1. The problem inherent in the 1-norm method is indicated in Fig. 3 using weight W_2, here because of the nonconvexity of the noninferior surface there are many noninferior points that cannot be found using the 1-norm. However, the noninferior solutions on nonconvex portions of the noninferior surface can be found using higher p's. In particular Fig. 4 shows the use of the max-norm.

Clearly both the choice of p and W are important in searching for various noninferior solutions. An important characterization of the weights can be found as follows: let $\underset{\sim}{n}^*$ be the inward pointing normal (assumed to exist uniquely) to the noninferior surface at $\underset{\sim}{f}^*$ such that

$$\underset{\sim}{n}^{*T} \underset{\sim}{f}^* = 1$$

Further we require that for the p-norm the value of $\|W(p)f^*\|_p$ be one. Then the canonic weight used to find $\underset{\sim}{f}^*$ by minimizing $\|W(p)\underset{\sim}{f}\|_p$ (assuming this can be done) is defined as

$$W_{ii}(p) = \frac{n_i^{*\frac{1}{p}}}{f_i^{*\frac{p-1}{p}}} \tag{8}$$

Notice for p=1,

$$W_{ii}(1) = n_i^*$$

and for $p=\infty$

$$W_{ii}(\infty) = \frac{1}{f_i^*}$$

Also note that any positive multiple of the canonic weight can be used to find the same noninferior solution.

Based upon our understanding of the MCO problem and the interpretation of the canonic weight, we can now discuss heuristics for weight selection. The first step in an MCO should be the minimization of each function separately. These minimizations give the designer an idea of the possible range of values of the objectives. The next step would be to generate a weight that would provide a trade-off between these extremes. Using the 1-norm we might consider finding the plane through the m extreme or boundary points. If $\underset{\sim}{f}^i$ is the noninferior solution found by minimizing the ith objective function then the normal to the plane through the boundary points is found by solving

$$\begin{bmatrix} ----- \underset{\sim}{f}^1 ----- \\ ----- \underset{\sim}{f}^2 ----- \\ \cdot \\ \cdot \\ \cdot \\ ----- \underset{\sim}{f}^m ----- \end{bmatrix} W = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \tag{9}$$

The W that solves (9) is used as the weight in the next 1-norm minimization. This sequence is illustrated in Fig. 5. This process can be continued as follows: if the $(m+1)^{st}$ noninferior solution was unsatisfactory because the jth component was too large, then $\underset{\sim}{f}^{m+1} \leftarrow \underset{\sim}{f}^{j*}$ (minimum of jth component) and m-2 further noninferior solutions could be chosen (perhaps deleting the two noninferior solutions with highest solutions of f_j) and a weight normal to this plane could be found using (9). A minimization using this weight should decrease the jth component of the objective function.

An alternate procedure for weight selection can be generated for use with the max-norm. As was discussed previously, the canonic weight associated with a noninferior solution $\underset{\sim}{f}^*$, for ℓ_∞ is

$$W_{ii} = \frac{1}{f_i^*} \qquad i = 1, \ldots, m.$$

Clearly any positive multiple of W* is also a valid weight. Thus we can take any set of m previously found noninferior solutions (initially the points found by the boundary search) and ask the user to assign a weight, $\alpha_i > 0$, to each point indicating how much he favors that particular solution. The weights α_i can be normalized so that $\sum\alpha_i = 1$ and then we form

$$\hat{\underset{\sim}{f}} = \sum\alpha_i \hat{\underset{\sim}{f}}.$$

$\hat{f}$ will be in the plane specified by the m points. We now form the weight

$$\hat{W}_{ii} = \frac{1}{\hat{f}_i} \qquad i = 1, \ldots, m.$$

If there is a noninferior solution along the ray extending through $\hat{\underset{\sim}{f}}$, the weighted ℓ_∞ method using $\hat{W}$ will find it. If $\hat{W}$ is not a scaled canonic weight for any noninferior solution, a noninferior solution will still be found (unless the noninferior surface has pathological irregularities) and we have the valuable information that a noninferior solution does not exist in the direction specified. Notice that this method of weight selection is much more direct than the 1-norm method because we are not specifying a normal to the support plane at the desired solution, but an actual value, $\hat{\underset{\sim}{f}}$, for the solution which, if the weight is a scaled canonic weight, will be a scaled version of the solution.

A number of variations on this method are clearly possible. For instance, if the user examined the existing noninferior solutions he could simply specify a new desired solution $\hat{\underset{\sim}{f}}$, then a weight $\hat{W}_{ii} = 1/\hat{f}_i$ is immediately generated.

III. EXAMPLE

In this section we apply the ideas developed in Section 2 to the optimization of an MOSFET HAND gate circuit [7]. The designable parameters will be the width, W^, of transistor Tl, the width of the bottom devices, W23, (constrained to be the same) and the flat band voltage Vp«. The objectives in our design will be: to minimize the area used by the transistors, to minimize the switching time of the gate, and to require the ON voltage V_Q to be as close to zero as possible. The switching tine is dominated by the turn OFF time which is approximated by a first order approximation (7). Thus to evaluate the objective functions we only need to analyze the gate in the ON state.

The MCO we want to solve Is

$$\min_{x = (W_1, W_{23}, V_{FB})} \quad \phi_1 = \text{propagation delay}, \quad \phi_2 = \text{area}, \quad \phi_3 = -V_o \qquad (10)$$

subject to

$$\phi_1 \leq 110 \text{ nsec}$$
$$\phi_2 \leq 2500 \text{ mils}^2$$
$$\phi_3 \leq .7 \text{ volts}$$
$$-2 < V_{FB} < -1 \text{ volts}$$
$$5 \leq W_1 \leq 50 \text{ microns}$$
$$50 \leq W_{23} \leq 50 \text{ microns}$$

In order to reduce coramutational effort we employed the Simplicial Approximation algorithm [12] to approximate the constraints of (10) by the linear constraints:

$$a_i^T x \leq b_i \qquad i = 1,2, \ldots, 38 \qquad (11)$$

where a_i and b_i define the bounding hyperplanes of the approximation of the feasible region.

We will generate noninferior solutions to (11) using the weighted ... and weighted I.* techniques. Thus we solve two problems:

Problem 1

$$\min_x \; (W_1 \phi_1 + W_2 \phi_2 + W_3 \phi_3) \qquad (12)$$

subject to

$$a_i^T x < b_i \quad i = 1, \ldots, 38$$

Problem 2

$$\min_x \; y$$

subject to

$$w_1 \phi_1 \leq y$$
$$w_2 \phi_2 \leq y$$
$$w_3 \phi_3 \leq y$$
$$a_i^T x \leq b_i \quad i = 1, \ldots, 38$$

Both of these problems will be solved using Powell's constrained variable metric method [4]. (The program of problem 2 is one method of solving the minimax optimization problem.)

III.2 Problem 1

Our first three optimizations were to find the minimum of ϕ_1, ϕ_2, ϕ_3 separately. The weight for the next optimization was the normal of the plane passing through the solutions of these minimizations (in output space). Our starting point for these four runs was the de3ign center given by the Simplicial Approximation algorithm;

$$W_1 = 10.6$$
$$W_{23} = 209$$
$$V_{FB} = -1.47$$

The objective function values at the nominal were;

$$\phi_1 = 86.23$$
$$\phi_2 = 2258.06$$
$$\phi_3 = .56423$$

The results of the first four optimizations are presented in Table 1. Notice that by choosing the weights normal to the plane defined by the boundary search point, the minimization gave an interesting solution showing a reasonable trade-off between all objectives. Next we decided to try and reduce the propagation delay by choosing, as the weight, the normal to the plane through the first, second and fourth noninferior solutions (see Table 1). The result of this minimization did reduce the propagation delay, at a high cost in area and many function evaluations. In an effort to reduce the computational expense of finding this solution, our sixth optimization used the same weights. However, we now started at W_1 « 15.46, W_{23} - 226.74, and V_{FB} » -1, which was a point on the plane used to determine the weights for this run. The same final solution was found using considerably fewer function evaluations.

We next decided to form a weight based upon noninferior solutions, two, four and five. Using this weight and a starting point equal to the fourth noninferior point, we achieved the results shown. However, the algorithm appeared to have difficulty in choosing the correct final step size. Using the same weight, but starting at W_1 • 8.5, W_{23} * 165, V_{FB} - -1, we converged, albeit with the same difficulty, to a different solution. Finally we used the endpoint of the last (eighth) optimization as the initial point for the ninth optimization (see Table 1). The conclusion we draw from these results is that we have found indications of a nonconvexity on the noninferior surface. Thus there are many (possibly infinite solutions to the l_1 minimization with this weight and that this ambiguity of solutions in a small region of the noninferior surface was the cause of the problems in the optimizations. Therefore we have indication of the need to use more powerful MCO techniques on this problem.

III.3 Problem 2

We now find noninferior solutions to (11) using the minimax approach in [13]. The results of the boundary search are presented in Table 2. The starting point was again the design center predicted by the Simplicial Approximation algorithm, with the addition of *y* which was assumed initially zero. The interesting point, in comparing the boundary searches of Problem 1 and Problem 2, is to notice the difference in computational effort. Experience indicates that whether the weighted sum method or minimax method Is more efficient is problem dependent.

The fourth noninferior solution to Problem 2 was based upon an equal weighting of the boundary solutions and uses the minima* weight description cf Section II. Notice how closely the results of run four match In Problems 1 and 2. This indicates a convex region of the noninferior surface and that the solution to this minimization lies near the centroid of the three points used to define the weights. The next minimization was performed with weights which are equal combinations of the first, second and fourth noninferior solutions.

Tn order to test the effect of starting point on the minim.ix method, we used tin; weights of run five with the starting point W^ - 9.84, W23 - 134.2, V_{fB} • -1, che noninferior point corresponding to the fourth noninferior solution. Again, a significant reduction In computational expense was achieved supporting the idea of choosing a starting point somewhere in the plane specified by the noninferior solutions used to calculate weights.

A number of runs were made usin-g minimax method, all of them worked well; in fact, better than the weighted sura method. The last experiment we will report was suggested by the possible: nonconvex region detected in Problem 1. We found the centroid of the noninferior solutions in runs seven, eight and nine of Table 2. This point could possibly be in nonconvex portions of the noninferior surface. This point is tp_n * 93.3, AREA » 1791, $-V_o$ = .55. The results of this run are in Table 3 and the final function values were incredibly close to che desired values.

Based upon the comparison of Table I and Table 2, as well as our subjective evaluation of using both veighted sum and minimax methods on the same MCO problem, ve feel that the miniraax methods were far superior (at least on this problem).

IV. SUMMARY AMD CONCLUSIONS

The ideas of Multiple Criterion Optimization have been introduced and shown to be a natural way of starting the problem of electronic circuit design. The weighted p-norm method for finding solutions to the MCO problem has been presented. The canonic weight for the p-nonn and heuristics based on the canonic weight have been discussed. Finally the techniques of MCO have been used for the design *of* an MOSFET NAM) gate.

We feel that the use of MCO ideas for circuit design has the potential for making optimization a powerful and useful design* aid. Further, the interpretation of least pth methods in light of MCO methodology should revive interest and promote the proper use of these classical and much abused methods. Finally, it should be possible ro consider an extended MCO problem where one of the objectives is to maximize the yield of the design [13]. This extended MCO problem would truly capture the major trase-offs faced by the designer of an electronic circuit.

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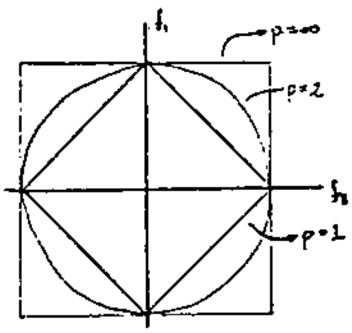

Fig. la

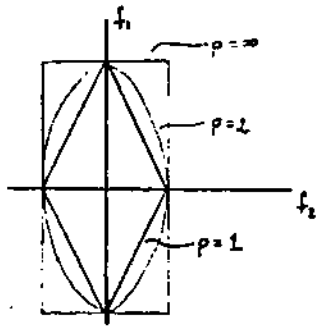

Fig. lb

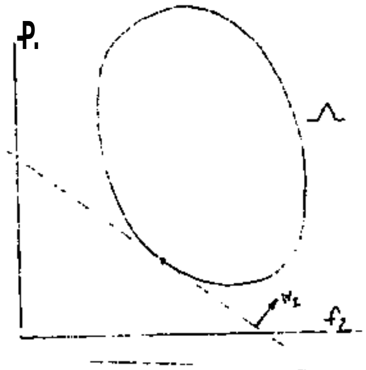

Fig. 2

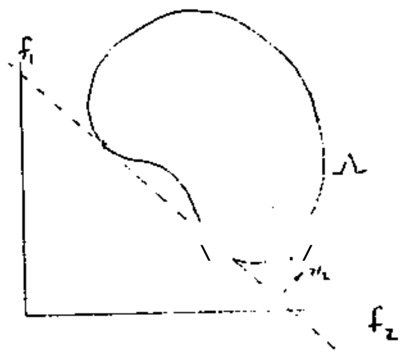

Fig. 3

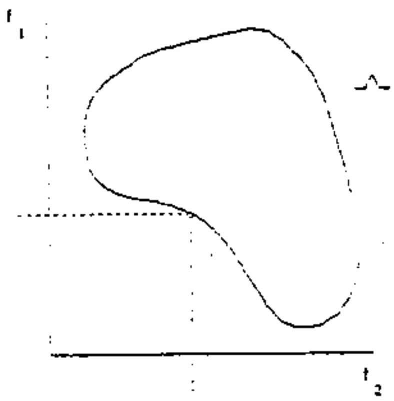

Figure 4.

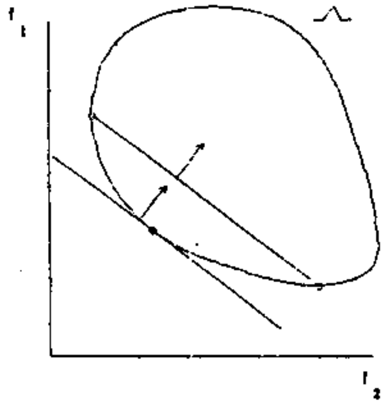

Figure 5.

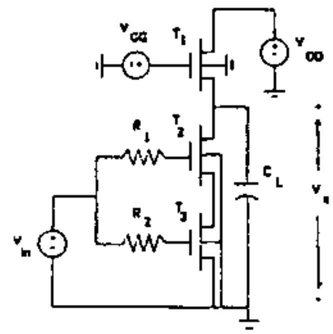

Figure 6.

RUN	WEIGHT			FITJAL PARAMETERS W_1	W_{23}	V_{FB}	FINAL FUNCTION t_{PD}	AREA	$\lvert V_0 \rvert$	NITER	NFUNC	N'CRAD
1	1	0	0	15.66	226.76	-1	56.2	2500	.7	2	3	3
2	0	1	0	7.896	115.8	-1	110	2500	.7	1	2	2
3	0	0	1	7.32	236 .23	-1	110	2500	.35	15	16	16
L	.0109	.0005	1.71	9.85	186 .2	-1	87.9	1997	.557	17	18	18
5	.0118	.0005	1.53	10.52	171 .25	-1	82.4	1373.5	.635	17	18	13
6	.0113	.0005	1.53	10.52	171 .25	-I	82.6	1873.5	.635	9	10	10
7	.0102	.0005	1.81	8.788	165 .6	-1	98.66	1793.97	.554	19	21	20

TABLE 1. Results of minimax optimization of NAND gate.

RUN i	WEIGHTS			FINAL PARAMF.TF.RS	W_{23}	V_{FB}	FINAL FUNCTintIS t_{P0}	AREA	$\lvert V_0 \rvert$	NITER	MFUIIC	NCRAD
I	1	0	0	15.66	220.76	-1	56.2	2500	.7	5	6	6
2	0	1	0	7.896	115.8	-1	110	2500	.7	3	6	6
3	0	0	1	7.R2	2V..2B	-1	110	2500	.35	13	i*	16
6	.0036	.0Q016	.5651	9.86	186.62	-1	87.9	1999	.557	23	24	26
5	.0067	8.96X10"5	.731	11	2J2.3	-1	78.5	2500	.5	27	,7	28
6	.0067	$8.96X10^{-5}$	.731	11	2J2.3	-1	78.5	2500	.5	21	22	22
7	.0032	.0002	.552	9.167	186.3	-I	96.31	19H?	.52	23	28	23
8	.0032	.0002	.552	3.76	165	-1	98.77	1787	.55	7	. U	7
9	.0032	.002	.552	fl.33	167	-I	103.99	1600	.539	6	18	6

TABLE 2. Results of weighted sum optimization of MAND gate.