is TP in (a,b) if and only if the following two conditions hold:

(a) $a_{ij}(t) = 0$, $|i - j| \geq 2$, $i, j = 1, \ldots, n$, $a < t < b$.

(b) $a_{i,i+1}(t) \geq 0$, $a_{i+1,i}(t) \geq 0$, $i = 1, \ldots, n-1$, $a < t < b$.

Proof. As total positivity of the system (1.2) implies its positivity, it follows from Lemma 1 that all off-diagonal elements $a_{ij}(t)$, $i \neq j$, of $A(t)$ have to be non-negative in (a,b). If an element $a_{ij}(t)$, $|i - j| \geq 2$, were to be positive for some t part (ii) of Corollary 1 would imply that the matrix $B^{(2)}(t)$ of the second compound system has an off-diagonal element which is somewhere negative, and Lemma 1, applied to this second compound system, then shows that (1.2) is not TP. Conditions (a) and (b) are thus necessary. Their sufficiency follows from part (i) of Corollary 1 and the sufficiency part of Lemma 1, applied to all compound systems (1.3). (We remark that we also use that the pth compound of the unit matrix $I = (\delta_{ij})_1^n$ is again $I = (\delta_{ij})_1^N$. Hence if $Y(t) = Y(t,r)$ is the solution of (1.2) which satisfies (1.5), then its compound also satisfies $c_p(Y(r)) = I$.)

Theorem 3. Let the n^2 real functions $a_{ij}(t)$, $i,j = 1, \ldots, n$, be continuous in (a,b), $-\infty \leq a < b \leq \infty$, and set $A(t) = (a_{ij}(t))_1^n$. The differential system

$$(1.2) \qquad Y'(t) = A(t)Y(t),$$