Dynamically Stable Legged Locomotion

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Abstract

Though vehicles that use legs for locomotion promise superior mobility and versatility, very little is known about their design and control. Balance, resonance, and dynamic control are key issues underlying high performance legged systems, both man-made and biological, yet understanding in these areas is particularly lacking. We focus attention on these important problems by studying hopping systems that have only one leg. A one-legged system must hop to locomote, must balance to hop, and must be dynamically controlled at all times to balance.

An ideal one-legged planar hopping machine is presented with its equations of motion. Control is decomposed into a vertical hopping part and a horizontal balance part. A total vertical energy measure is used to control uniformity of hopping height when there are mechanical losses and irregular terrains. Balance and control of horizontal translation are explored through implementation of three controllers: a linear feedback controller, a stance controller, and a new table look-up controller.

The design and operation of a physical planar hopping machine is discussed, along with preliminary experimental results for vertical control. Three new designs for experimental vehicles that operate in 3-space are presented. Two are mechanically simple designs with functional symmetry; the third is a preliminary concept for a multi-legged balancing vehicle that is optimized for forward motion.

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1. Introduction and Summary

One need only watch a few slow motion instant replays on Saturday afternoon sports television to be amazed by the variety and complexity of ways a human can carry, swing, toss, glide, and otherwise propel his body through space. Orientation, balance, and control are maintained at all times without apparent effort, while the ball is dunked, the bar is jumped, or the base is stolen. Such spectacular performance is not confined to the sports arena -- behavior observable at any school playground is equally impressive from a mechanical engineering, sensory motor integration, or computer science point of view. The final wonder comes when we observe the one year old infant's first wobbly steps with the knowledge that running and jumping will soon be learned and added to the repertoire. Animals also demonstrate feats of agility that make them able to move quickly and reliably over flat, hilly, and mountainous terrain, through forest, swamp, marsh and jungle, and some move very quickly in the open or from tree to tree.

Despite excellence in using his own legs to locomote, man is still at a primitive stage in the development and construction of legged vehicles. So far his designs do not exhibit the mobility, agility, versatility, and energy advantages of biological legged systems. They are slow, ponderous, and rarely leave the laboratory. We have neither theory nor understanding of legged locomotion to explain the behavior we see in natural systems, nor to build machines with similar properties. As a result our mobility is generally limited by dependence on roads, runways, the beasts of burden, and our own two feet.

The work reported here describes modelling, simulation, design, and physical experiments with new types of legged locomotion systems. Our long term goal is to develop a systematic understanding of the legged locomotion problem in biology and in robotics. This will help to determine the potential benefits for domestic and military exploitation, to provide the scientific foundation upon which useful transportation systems may be constructed, and to help explain the behavior we observe in naturally occurring legged systems. No such understanding is available today and its lack is sorely felt.

Our shorter term objective is to do experiments that let us explore the basic issues underlying balance, dynamic control, and resonant oscillatory behavior as they affect legged systems. Our choice of these issues was triggered by two observations:

- Most biological walkers larger than insects depend on active balance and dynamic stability as basic operating principles. Existing man-made walking machines rely solely on static techniques for support.
- Biological systems have elastic muscles and tendons that participate in resonant mechanical oscillations during locomotion [4,186], whereas man-made walkers usually have rigid legs with no suspension.

We have taken these observations to heart in formulating our studies of legged locomotion. The 'main issues then are balance and resonant oscillation. Our method of focussing on these issues is to

experiment with hopping systems having only a single leg -- neither balance nor oscillation may be ignored. Because only one gait is possible for a one legged device [169], the problem of gait selection is particularly simple and does not distract us from the central issues of this research.

During walking, running, and hopping, legs do two things. They store energy in springy muscle and tendon when the leg shortens, returning some of it when they lengthen, and they move back and forth to propel and to provide balance. We have modelled this behavior in an experimental vehicle with a single leg that changes length by means of a large spring, and a simple rotary hip that moves the leg back and forth. We call this model the *Ideal Planar Hopper*. (See Fig. 2-1.)

We decompose control of the planar hopper into two parts, a vertical hopping part and a horizontal balance part. The primary role of the vertical controller is to establish a regular framework within which alternating stance phases and transfer phases occur. The basic timing and rhythm of locomotion is governed by this vertical oscillatory system. The horizontal part is responsible for balancing the system and causing it to translate from place to place. All the actions of the horizontal part are synchronized by key events in the vertical hopping cycle.

It is the purpose of vertical control to achieve stable, energy efficient hopping that provides a framework within which locomotion can take place. We developed an asynchronous hopping cycle in which each control action is keyed to one of four events: LIFT-OFF - the foot leaves the ground, PEAK - the vehicle is momentarily suspended in air at the top of a hop, TOUCH-DOWN - the foot touches the ground during landing, and BOTTOM - the leg is shortened at the bottom of a hop. We found that a measure of total vertical energy in the locomotion system is useful in predicting and controlling hopping altitude, despite mechanical losses and irregular terrains.

There are two ways to control horizontal behavior of the hopper. One is to manipulate placement of the foot just before TOUCH-DOWN -- placement will determine the direction of falling during the subsequent stance period. The other way is to apply torque to the *hip* during stance, thereby changing the angular momentum of the body. We have experimented with three methods for balance, two that only use foot placement, and a third that combines foot placement with hip torque.

Linear Foot Placement

This first method determines foot placement as a linear function of errors in the three important state variables: body angle θ_2, body angular rate $\dot{\theta}_2$, and horizontal body rate $\dot{x}_2$ as defined in Figure 2-1. The leg is positioned just before TOUCH-DOWN so that the horizontal distance between the foot and the projection of the vehicle's center of gravity is specified by this linear function. In simulation we have found that this simple approach is very good at maintaining balance. A wide variety of initial conditions and disturbances are permissible without causing crashes. However, we also found that limit cycles in horizontal velocity are hard to avoid.

Predictive State Space Method

As a follow-up to Raibert's earlier work on robot manipulator dynamics [227, 228], we explored a control method that uses a large table - about 10,000 numbers - to control the simulated hopper. The *Predictive State Space Method* is able to make a very useful prediction just before each TOUCH-DOWN -- for each possible foot placement it predicts the vehicle's state at the time of next LIFT-OFF. These predictions are used to select the *best* foot placement at TOUCH-DOWN during each hopping cycle. This table look-up method provides a *tabular* solution to the vehicle's nonlinear equations of motion, where no closed form solution is known.

The performance of the predictive state space controller is very good. Not only can it balance and perform well controlled translations, but in contrast to the linear method, horizontal velocity is controlled with great precision.

Sweep Control

Sweep control grew from a simple observation: During the stance part of constant velocity horizontal locomotion, a legged vehicle's center of gravity translates by a predictable amount at a fixed rate. Therefore, it is possible to plan a sweeping motion of the leg during stance such that: a) the constant horizontal velocity of the body is not impeded, and b) balance is maintained. This form of control combines foot placement with hip torque. For the special case of zero horizontal velocity, the method simplifies to the linear controller described above.

Simulations of sweep control have provided our most exciting results to date. Very high horizontal velocities are achievable, stability is very good, and the resulting locomotion begins to look very like the behavior of biological systems. Figure 1-1 shows a sequence of vehicle positions obtained in a simulation of sweep control.

LIFT-OFF   TOP   TOUCH-DOWN   BOTTOM
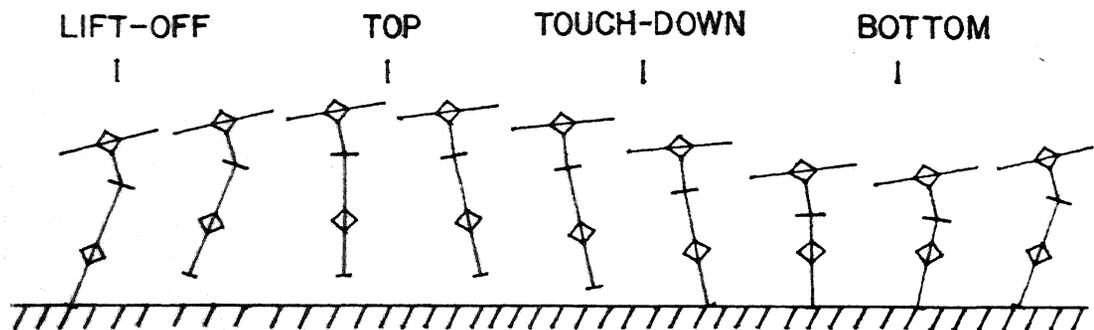
Figure 1-1: Sequence taken from simulation of sweep control. One frame every .1 sec. The x-axis has been stretched to make the figure clearer. Actual translation during flight is about equal to the width of the vehicle.

We have designed and fabricated a physical hopping machine that serves as a testbed for the ideas developed through analysis and simulation. Figure 1-2 is a photograph of the planar hopping machine in its present state. Two pneumatic cylinders act together under computer control to excite vertical oscillations, and to balance through horizontal leg motion. Presently, all sensory information is provided by an electro-optical sensor, Selspot, that watches the hopper's behavior from a nearby tripod. Other sensors are in various stages of development: linear encoder, rate gyro, down looking sonar, and more sophisticated use of the Selspot.

We have implemented a very simple vertical control that excites and maintains resonant hopping oscillations in the physical hopper. Our design goal of three inch ground clearance on each hop is now achieved consistently since improving the mechanical design of the system's air bearings and the leg's linear bearing. We have begun work on controlling balance, but have no results at the time of this writing. Our progress in achieving balance has been delayed because of a bad design decision made early in the project: Bang-bang pneumatic control of coupled actuators is a tricky business that is best avoided. We discuss plans to overcome difficulties imposed by this problem.

We have begun to explore locomotion in three dimensions. Three preliminary mechanical designs are presented for one-legged experimental vehicles that will locomote in 3-space without mechanical support. Two are candidates for experiments in the coming months - the third is a design that looks to the future when multi-legged vehicles will incorporate balancing control systems. As with our planar work, the major emphasis is on balance and resonance. Elastic elements play a central role in all three vehicles providing energy conservative vertical motion and high performance balance motion. These new designs incorporate improvements based on what we are learning from the planar hopper.
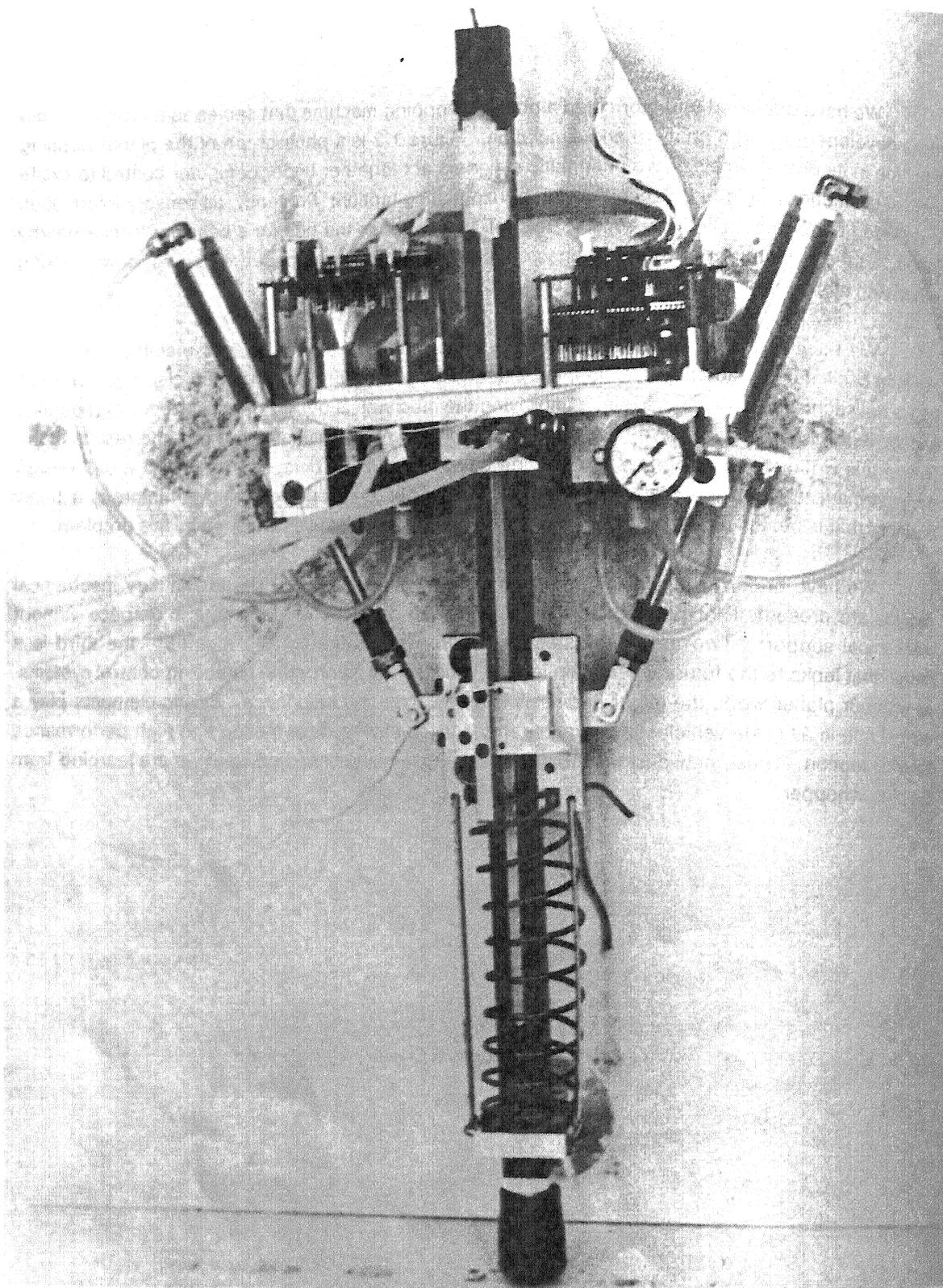
Figure 1-2: Photograph of planar hopper. (Details in Fig. 3-1).

2. Analysis and Simulation of Planar Hopping

2.1 A Simple Model

A seed that lead to these studies was planted when we noticed that during locomotion biological legs do two things: they change length and they change angle. It usually doesn't matter, whether the animal is walking, running or hopping, or whether the leg is part of a quadruped, biped, or monoped (eg. hopping kangaroo).

Legs change length to propel the body forward and upward, and to reduce their own moments of inertia when they swing forward. They absorb energy in springy muscle and tendon during landing, and return it during take-off. Legs also swing back and forth. This motion permits feet to be precisely positioned with respect to an animal's center of gravity, and permits torques to be applied to change angular momentum.
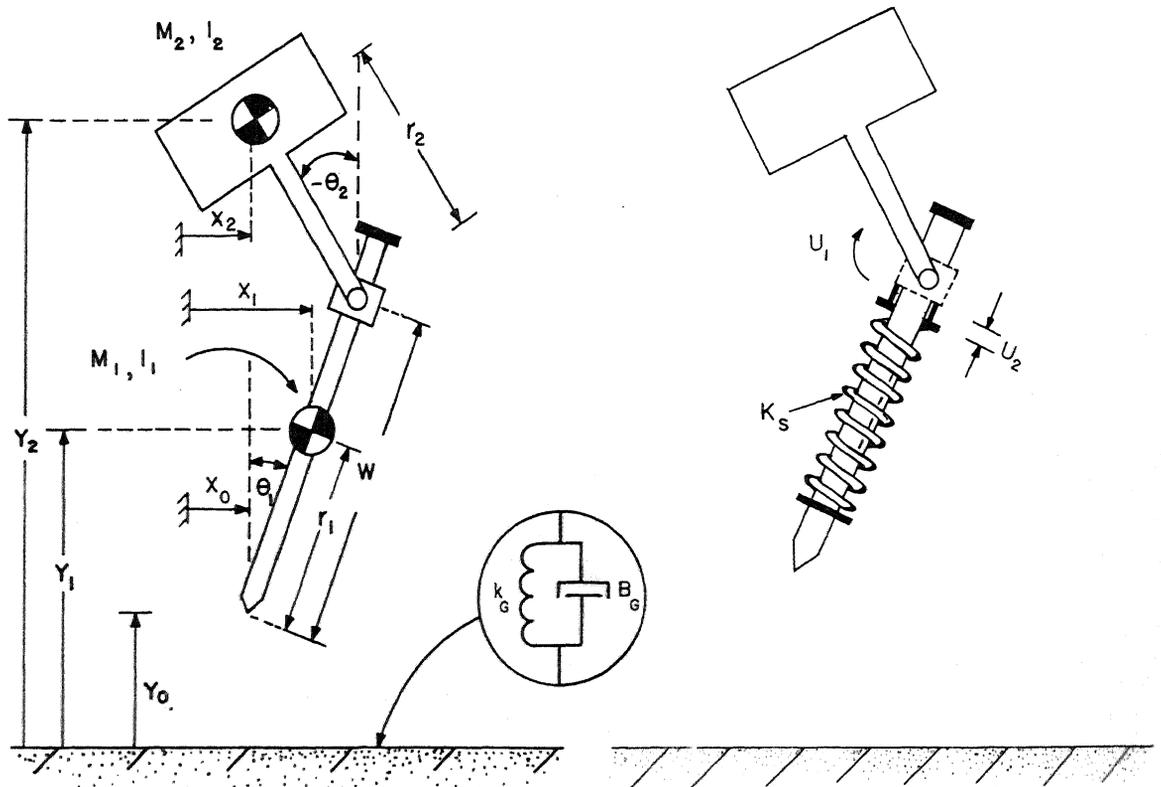
Figure 2-1: Diagram of ideal one legged hopping mechanism, *Ideal Planar Hopper*.

We have modelled this behavior in a vehicle with a single leg that changes length under the influence of a large spring, and that rotates about a simple rotary hip. This very simple model captures what is important in dynamic locomotion while avoiding the complications implicit in systems with many legs. Figure 2-1 shows the idealized mechanical system under study. A leg of mass M_r moment of inertia I_t is connected to a body of mass M_2 moment of inertia I_2. A joint between these links permits linear sliding motion and rotation. Control torque u^t) is applied to a rotary joint at the hip. Linear motion of the body sliding on the leg is controlled by a spring of stiffness K_s and length w in series with a linear position source of length $u_2(t)$. The linear actuator adjusts the spacing between the body and spring to control forces acting between the leg and body. A mechanical stop, modelled as a very stiff region of the main spring, limits extension of the main spring beyond its rest length. For $w > w_n + u_o$ the spring becomes stiff, $K = K_.$, and damped, $B = B_o$.

The ground is modelled as a two dimensional spring and damper (K_G, B_Q). One spring acts vertically, the other horizontally, with no interaction. This spring and damper influences the hopper only when the foot is in contact with the ground, $y_Q < 0$. Each time the foot touches the ground the rest position of the horizontal ground spring is reset.

Equations of motion for the entire mechanical system are found in Appendix I. Since we have no closed form solution to these differential equations, a numerical algorithm is used to determine the system's behavior as a function of time. All the simulations discussed below are accomplished by using such numerical solutions in conjunction with the controllers being studied. The numerical constants used throughout the paper are given in Appendix II.

2.2 Control

There are two sets of behaviors that are central to our study of planar hopping: vertical oscillation and horizontal balance. The vertical part involves generating a stable series of resonant oscillations that take a system off the ground and control the height of each jump. The horizontal part must maintain balance during hopping, and achieve the correct lateral position and translation over a support surface.

2.2.1 Vertical Control - Hopping

Unlike a wheel which changes its point of support continuously and gradually while bearing weight, a leg changes its point of support all at once and must be unloaded to do so. Therefore, in order for a legged system to progress there must be a framework of activity for each teg in which there is a time when the leg bears weight and the foot cannot be moved, and another time when, the leg is unloaded and the foot free to move. Such an alternation between a loaded phase, called *stance,* and an unloaded phase, *transfer,* is found in every form of legged locomotion. For systems with a single leg or a set of legs that act together, the transfer phase is also the *flight* phase, and we call the alternation cycle *hopping.* To first order a vertical controller is responsible for providing such an alternation framework.

We have designed a vertical controller that:

- Initiates hopping
- Controls the height of hopping
- Maintains stable hopping during horizontal control
- Terminates hopping

The basic mechanism through which each of these actions is accomplished is controlled excitation of a spring/mass gravity/mass oscillator. Hopping is initiated by exciting the spring/mass oscillator with a linear actuator until *escape velocity* is reached -- when inertial forces are sufficient to overcome gravity the foot leaves the ground and hopping begins. At this point the system becomes a spring/mass-gravity/mass oscillator. Four events in the oscillation cycle are of particular interest:

- LIFT-OFF - the moment the foot leaves the ground
- PEAK - the point in flight when vertical velocity changes from positive to negative
- TOUCH-DOWN - when foot first touches the ground
- BOTTOM - when vertical velocity changes from negative to positive

To understand regulation of a hopping system's vertical behavior it is useful to think about energy, its storage and its dissipation. Consider the simplified case in which $\$_V$ $\dot{9}_V$ 8_T and 0^*_2 are all small and neglected -- motion is primarily vertical. For the present model there are six ways energy can be stored:

1. Kinetic energy of body
2. Kinetic energy of leg
3. Gravitational potential of body
4. Gravitational potential of leg
5. Elastic potential of the main spring (including mechanical stop)
6. Elastic potential of the support surface, the ground

Therefore, the total vertical energy during stance is:

$$E_{STANCE} = PE_g(M_1) + PE_g(M_2) + KE(M_1) + KE(M_2) + PE\wedge M_1) + PE_e(M_2) \tag{2.1}$$

$$= M_1 g y_1 + M_2 g y_2 + .5 M_1 \dot{y}_1^2 + .5 M_2 \dot{y}_2^2 + .5 K_s (w_0 - w + u_2)^2 + .5 K_Q y \wedge$$

where
    PE_g - gravitational potential energy
    PE_e • elastic .potential energy
    KE - kinetic energy
    g »acceleration of gravity
    w_0 is the rest length of the spring.

Notice that we have chosen expressions for potential energy that are zero when the hopper is

standing vertically with the main spring extended and the foot just touching the ground. This will make the analysis simpler later. Ignoring windage which is typically less than 5% of the total, there are two ways energy can be lost:

1. Damping in main spring (including mechanical stop)
2. Damping in ground spring.

There are two events in a hopping cycle during which most energy loss occurs. At TOUCH-DOWN most of the leg's kinetic energy is dissipated by ground damping[1]:

$$E_{TD\text{-}LOSS} = KE(M_1) = .5\,M_1\dot{y}^2_{1,TD-} \tag{2.2}$$

where
$\dot{y}^2_{1,TD-}$ is the velocity just before TOUCH-DOWN.
Subscript TD– means just before TOUCH-DOWN.

At LIFT-OFF damping in the main spring's mechanical stop, the stiff region, dissipates a fraction of the system's kinetic energy:

$$E_{LO\text{-}LOSS} = \frac{M_1}{M_1+M_2}KE_{LO-}(M_2) = \frac{M_1 M_2}{2(M_1+M_2)}\dot{y}^2_{2,LO-} \tag{2.3}$$

where
$KE_{LO-}(M_2)$ is the total kinetic energy
$\dot{y}^2_{2,LO-}$ is the vertical velocity of the body just before LIFT-OFF
Subscript LO– means just before LIFT-OFF.

The fraction, $M_1/(M_1+M_2)$ was found by equating linear momentum before and after LIFT-OFF. It's interesting that this loss is minimized when the leg/body mass ratio is minimized, ie. when unsprung mass is minimized.

In addition to these energy losses there is an energy source, the linear actuator of length u_2. Since this actuator is in series with a spring it does work on the system when it changes length. If u_2 instantaneously changes from $u_{2,i}$ to $u_{2,i}+\Delta u_2$, then there is an energy change:

$$\Delta E_{u_2} = K_s[\,.5\,\Delta u_2^2 + \Delta u_2\,(w_0-w+u_{2,i})\,] \tag{2.4}$$

Actually this actuator can also be an energy sink -- when Δu_2 is negative total energy is reduced. Also note that the magnitude of ΔE depends on w -- more work is done when the main spring is compressed than when it is relaxed. Therefore, lengthening the actuator at BOTTOM and shortening during flight causes the total energy in the system to increase on each hop. On the other hand, shortening the actuator at BOTTOM and lengthening during flight causes the total energy in the system to decrease, eventually to zero.

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[1]There is actually ground damping throughout stance, but such losses are generally small.

Now we are ready to control hopping height, H. If the main spring is at equilibrium during flight, all energy takes the form of gravitational potential at PEAK where $\dot{y}_2$ = 0 by definition. Therefore, for the case when the vertical actuator is inactive we can predict the height of the next hop at any time during stance. We assume complete state information to be available and neglect ground losses after TOUCH-DOWN. At LIFT-OFF the total vertical energy takes the form of kinetic energy:

$$E_{LO-} = KE_{LO-} = E_{STANCE} \tag{2.5}$$

We know from (2.3) that there is a fractional loss of kinetic energy at LIFT-OFF. Combining (2.3) and (2.1) we get the total energy during flight:

$$E_{FLIGHT} = KE_{LO+} \tag{2.6}$$

$$= \frac{M_2}{M_1 + M_2} KE_{LO-}$$

$$= \frac{M_2}{M_1 + M_2} [M_1 g y_1 + M_2 g y_2 + .5 M_1 \dot{y}_1^2 + .5 M_2 \dot{y}_2^2 + .5 K_s (w_0 - w + u_2)^2 + .5 K_G y_0^2]$$

Now this result can be used to control hopping height. For the body to hop to height H the total vertical energy must be:

$$E_H = M_1 g [H - r_2 \cos(\theta_2) - (w_0 - r_1)\sin(\theta_1)] + M_2 g H \tag{2.7}$$

So at all times during stance the energy that must be supplied or removed by the vertical actuator to produce a hop of height H is:

$$\Delta E_H = E_H - E_{LO+} \tag{2.8}$$

From (2.4) and (2.8) the linear actuator must extend by:

$$\Delta u_2 = -(u_2 - w + w_0) + \sqrt{(u_2 - w_0 + w)^2 + \frac{2 \Delta E_H}{K_s}} \tag{2.9}$$

Actually, if (2.6) can be evaluated rapidly enough, it is not necessary to use (2.9). On each control cycle the sign of ΔE can be used to determine if u_2 should be lengthened or shortened.

This method was used to control the height of hopping in simulation. At BOTTOM, indicated by y_0 changing sign from negative to positive, (2.1) and (2.6) are used to predict hopping height. u_2 is then lengthened or shortened accordingly. Figure 2-2 plots the vertical position of the foot and the body during a period of increasing hopping height, and during stable hopping. Starting at rest, the system executes a positive work cycle on each hop until the vertical energy increases to the correct value. Then it maintains this level. The finite response time of a physical actuator is simulated by making u_2 increase or decrease with a quadratic trajectory. Since the range of the linear actuator stroke is finite the energy that can be injected on a single cycle is limited. Therefore, a number of cycles may be required to achieve a desired energy and height.

In these simulations shortening occurs during TOUCH-DOWN by letting the foot just touch the
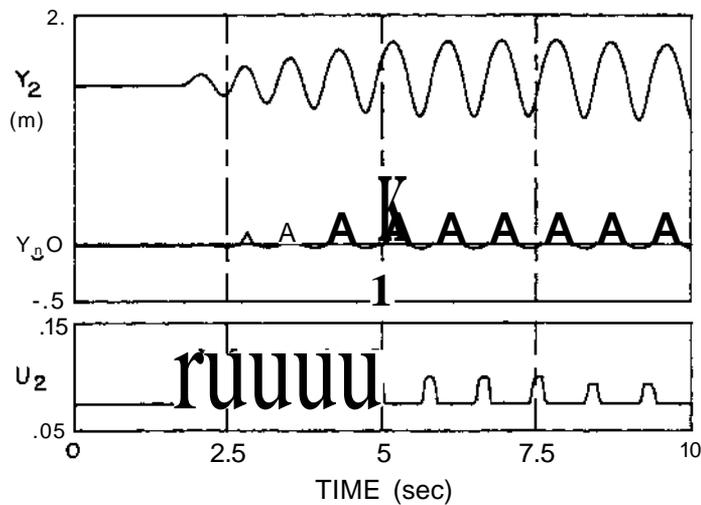
Figure 2-2: Vertical hopping. Starting from rest, total energy is increased until desired hopping height is attained. Hopping height is regulated through control of vertical actuator u^.

ground until the actuator is fully shortened. If there were no losses in the system it would not be necessary to activate u_2 once a desired height of hopping was achieved. However, since there are losses it is necessary to replace the energy lost on each hop to get level hopping. The vertical controller injects or extracts the right amount of energy to achieve a desired hopping height based on of total vertical energy at BOTTOM.

It is sometimes useful to describe the vertical control cycle in the phase plane. If we plot body velocity on the abscissa and body altitude with respect to its resting altitude on the ordinate we obtain a phase plot. (See Figure 2-3.) Note the parabolic trajectory during flight caused by the constant gravitational acceleration, and the circular trajectory during stance due to the spring. The four events mentioned earlier, LIFT-OFF, PEAK, TOUCH-DOWN, and BOTTOM, were chosen as control points to synchronize actions of the controller with ongoing behavior of the hopping system. Use of these events as control switching points allows a simple and reliable asynchronous implementation.

Figure 2-4 plots vertical energy during two cycles of fixed height hopping (ie. from time 7.5 to 9.3 in Figure 2-2). A lossless system would produce a perfectly flat total energy line. It can be seen that the primary losses occur when the foot strikes the ground, and when it leaves the ground. Energy increases during the latter part of stance, when the actuator lengthens. There is also a small energy increase when the actuator shortens, however this energy is dissipated by damping in the mechanical stop. These data are replotted in polar form in Figure 2-5a. In this figure radius is the magnitude of total energy, and angle is $\text{Arctan}(y_2/\dot{y}_2)$, A lossless system would produce a perfectly circular total energy line. In this plot the curve crosses the axes at the same synchronization points LIFT-OFF, PEAK, TOUCH-DOWN, and BOTTOM.
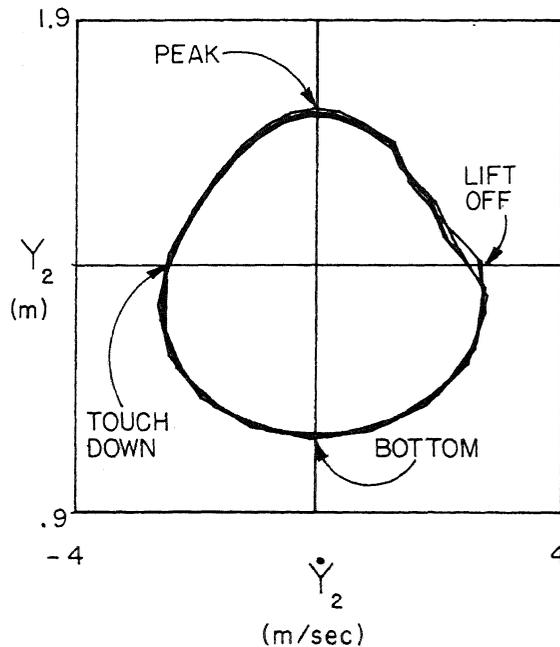
Figure 2-3: Phase plot for vertical hopping. Four control events are indicated where curve crosses axes. Data from stable part of Figure 2-2.

Figure 2-5b plots energy for the initial section of Figure 2-2. The spiral depicts the start-up period during which the system energy is *pumped up*. It takes a few cycles to achieve desired hopping height. Figure 2-6 shows an 80 second sequence of vertical hopping in which desired height is adjusted up and down. The ability to regulate height during vertical hopping is clearly demonstrated here. Notice the difference between the speed of the actively controlled descent beginning at $t = 45$ and the passive descent beginning at $t = 65$.

It is interesting that the time at which the leg is shortened during a steady state hop cycle can be manipulated to optimize a variety of criteria:

- When the leg is shortened at LIFT-OFF, ground clearance of the foot during flight is optimized. This is important when terrain is uneven or when large horizontal sweeps of the leg will occur during flight, as when translating at high speed. If the leg is not short during these sweeps stubbing the toe is hard to avoid. Shortening at LIFT-OFF also minimizes the leg's moment of inertia during flight, so the leg can be swung forward faster and with less angular effect on the body.

- When shortening occurs at PEAK the time between vertical actuations is maximized. This strategy could permit use of an actuator of lower bandwidth.

- If shortening occurs upon TOUCH-DOWN, then the ground impact forces on the foot are minimized. This strategy is normally used by humans when they are asked to hop in place on a flat floor.
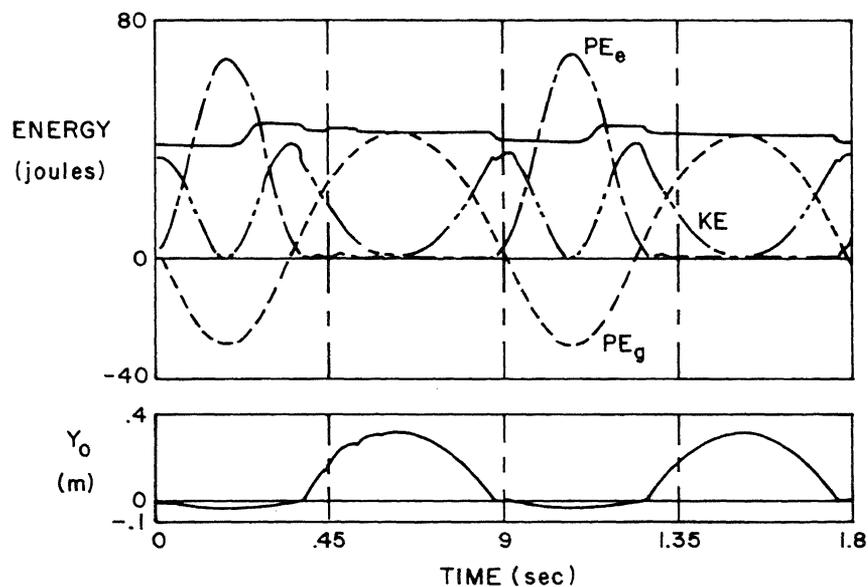
Figure 2-4: Vertical energy for two hopping cycles at constant height. Total energy, kinetic energy, gravitational potential, and elastic potential are shown. Primary losses of energy are at TOUCH-DOWN and LIFT-OFF. Data from Fig. 2-2.
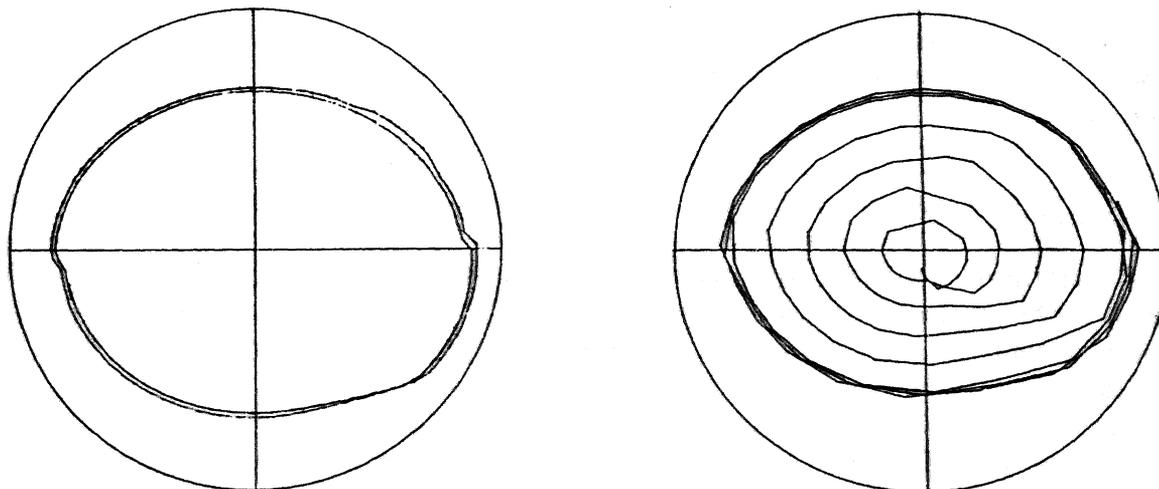
Figure 2-5: Total energy plotted in polar form. Radius is magnitude of energy, angle is Arctan$(y_2/\dot{y}_2)$. Left) Data of Figure 2-4 replotted. Right) Period during which hopping height increases. (Circles indicates 50 joules.)
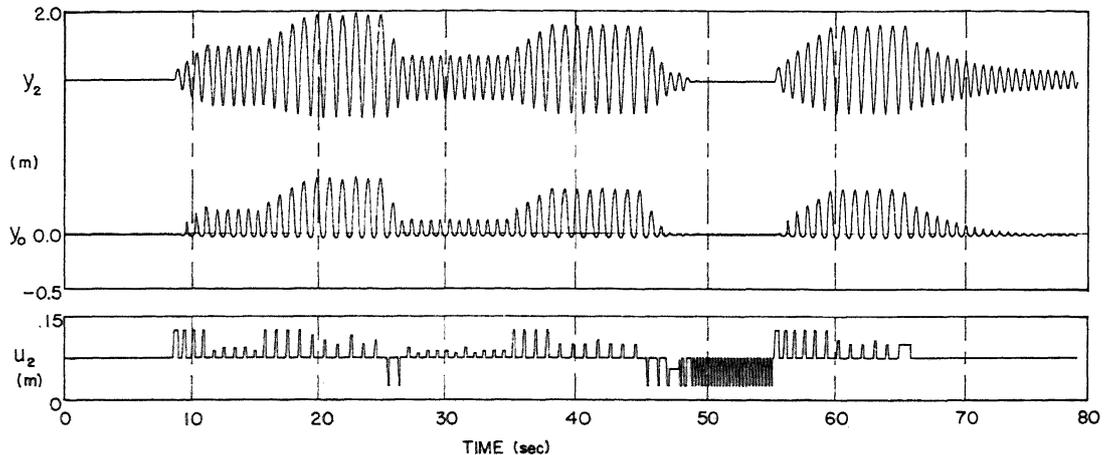
Figure 2-6: Vertical hopping sequence. At times t = 9, 15, 25, 35, 45, 55, 65 desired hopping height H = 1.7, 2.0, 1.6, 1.9, 1.4, 1.9, 1.4. The descent beginning at t = 65 is passive.

It is also possible to shorten the leg at LIFT-OFF, lengthen it again just before the next TOUCH-DOWN, and let it shorten during the landing. This strategy, apparently used by humans when running, maximizes ground clearance <u>and</u> minimizes impact forces on the foot. However this is accomplished at the expense of additional actuator bandwidth. Although more energy is required for these extra lengthening and shortening motions, the leg can be swung forward more efficiently with its lower moment of inertia.

2.2.2 Horizontal Control -- Balance

A horizontal controller maintains balance and controls lateral motion. There are two parts to the control of horizontal and angular behavior of a legged hopping system: a part that works while the vehicle is in flight, and a part that works while the vehicle is in stance. During flight the leg can be moved to change the horizontal distance between the foot and the center of gravity. This distance will affect the direction of falling during the subsequent stance period. During stance torques applied to the hip directly modify the body's angular momentum.

Here we describe three methods for balance, two that only position the foot during flight, and a third that combines foot placement with hip torque. Biological systems usually employ both foot placement and hip torque for balance. However, the use of simpler algorithms in our experiments permits the foot placement component of balance to be examined more clearly in isolation. The third algorithm is more practical, demonstrating high performance.

All horizontal control techniques we've explored operate within a framework. One part of the framework is the regular cyclic activity of vertical hopping that synchronizes the flow of balancing activity. The four events of hopping, LIFT-OFF, PEAK, TOUCH-DOWN, and BOTTOM play an important role here. For instance, all of our methods start positioning the foot at PEAK and finish doing so just before TOUCH-DOWN. So in this case vertical hopping events regulate the flow of horizontal control events.

In addition to this timing part of the framework there is a second hierarchical part. In the context of the one legged model, we analyze horizontal control as a three level process:

- At the highest level desired trajectories of horizontal motion, $x_2(t)$ are controlled by manipulating the vehicle's state. It generally takes many hops to substantially modify horizontal position, so this is a low-bandwidth process. The input to this *task* level are user specified trajectories, and the output is a set of state variables: $(\theta_2, \dot{\theta}_2, \dot{x}_2)$ = $F[x_{2d}(t)]$.

- The middle level of control is where we focus most of our attention. At this level vehicle state variables are controlled by manipulating the leg, either its angular position at TOUCH-DOWN or its motion during stance. This process acts once each hop, so it generally takes a few hop cycles to achieve a desired state. The inputs to this *state* level are state trajectories generated by the task level. The output selects leg angle or foot placement: $\theta_{1,TD} = F[\theta_{2d}, \dot{\theta}_{2d}, \dot{x}_{2d}]$.

- At the lowest level[2] desired leg angles are controlled by manipulating torque at the hip. This process runs at the servo rate of the hardware controller, executing many measurement/control cycles per hop -- a 10 msec control cycle is typical. The inputs to this *servo* level are leg angles specified by the state controller, and the output is an actuator control signal: $u_2(t) = F[\theta_{1d}]$. For all the experiments reported here the following PD controller is used to position the leg:

$$u_1(t) = K_p(\theta_1 - \theta_{1d}) + K_v(\dot{\theta}_1 - \dot{\theta}_{1d})$$ (2.10)

For the present work the task control level is provided by an operator who runs the simulation or experiment. The servo level is fixed for all experiments as given in (2.10), though the gains may vary from experiment to experiment. The main focus of our work is on the state level. At this level the leg is positioned before each TOUCH-DOWN so that the state at next LIFT-OFF is optimized and the behavior of the leg during stance is specified. The following sections describe a number of approaches to state control.

2.2.2.1 Foot Placement Control

The simplest form of horizontal control we explored uses information about the vehicle's center of gravity plus linear feedback to specify foot placement during each TOUCH-DOWN. Our intention was to study a pure foot placement controller, in which no attempt is made to control balance during stance. Actually, during stance the controller maintains a constant hip angle between body and leg, so the vehicle behaves like a rigid inverted pendulum during this period.

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[2] Within the actuator and its support system there are likely to be additional low levels of control.

Two factors determine where the foot is placed: projection of center of gravity and errors in state. A linear function of errors in state generates a displacement, x_{ERR} which is added to the projection of the center of gravity, x_{CQ}. A leg angle is calculated that will place the foot at this point. Since the leg has mass, movement of the leg changes the projection of the center of gravity - simultaneous equations must be solved. The following analysis is done in a moving coordinate system fixed to the vehicle's hip:

First we find horizontal position of center of gravity:

$$x_{\ll} \stackrel{\wedge}{-} \frac{(w-r_1)M_1 \operatorname{Sin}(\theta_1) + r_2 M_2 \operatorname{Sin}(\theta_2)}{M_1 + M_2} \tag{2.11}$$

The following linear combination of state errors provides corrective feedback:

$$x_{ERfl} - \rtimes\!\prec\!(W + W_a) + K_3(\dot{x}_2 - \dot{x}_{2d}) \tag{2.12}$$

Placing foot at desired spot requires that:

$$w \operatorname{Sin}(\theta_i) = x_{CG} + x_{E_RR} \tag{2.13}$$

Substituting (2.11) into (2.13) and solving for foot placement with respect to hip and leg TOUCH-DOWN angle:

$$x_{TD} = w \frac{r_2 M_2 \operatorname{Sin}(\theta_2) - (M_1 + M_2) x_{ERR}}{r_1 M_1 + wM_2} \tag{2.14}$$

$$\#_1 = \operatorname{Arcsin} \frac{r_2 M_2 \operatorname{Sin}(\theta_2) - (M_1 + M_2) x_m}{r_1 M_1 + wM_2} \tag{2.15}$$

Figure 2-7 plots $\theta_2(t)$, $\dot{\theta}_2(t)$, and $\dot{x}_2(t)$ for a step in horizontal position achieved with this linear foot placement method. Simple rate control is used at the top level to control horizontal position. The lowest level servo controller is given by (2.10) with parameters specified in Appendix II. Balance using this method is remarkably stable -- translations at a variety of rates and with different body attitudes are achieved while maintaining the upright posture.

A limitation of the method is the maximum achievable rate of translation. This is limited to about 0.5 m/sec by the relatively restricted motion of the leg while in stance. Also, limit cycte oscillations in horizontal velocity, observable in Figure 2-7, are difficult to avoid. Some of this may be due to the ad hoc procedure used to select the feedback gains, $K^\wedge$ K_y and K_3 « they were adjusted by hand based on observed performance,

The above work focysed solely on the role of foot placement in horizontal control so tiiat we could set a clear picture of its contribution. It was found that balance and translation can be achieved, though transnational progress is very slow and an unnatural, *hunched* posture is required. The back and forth sweeping motions normally made by bioiogical legs when walking, running, or hopping are absent. We now explore the crucial role played by hip torque during stance, awl *9m* how this additional mechanism enriches behavior.
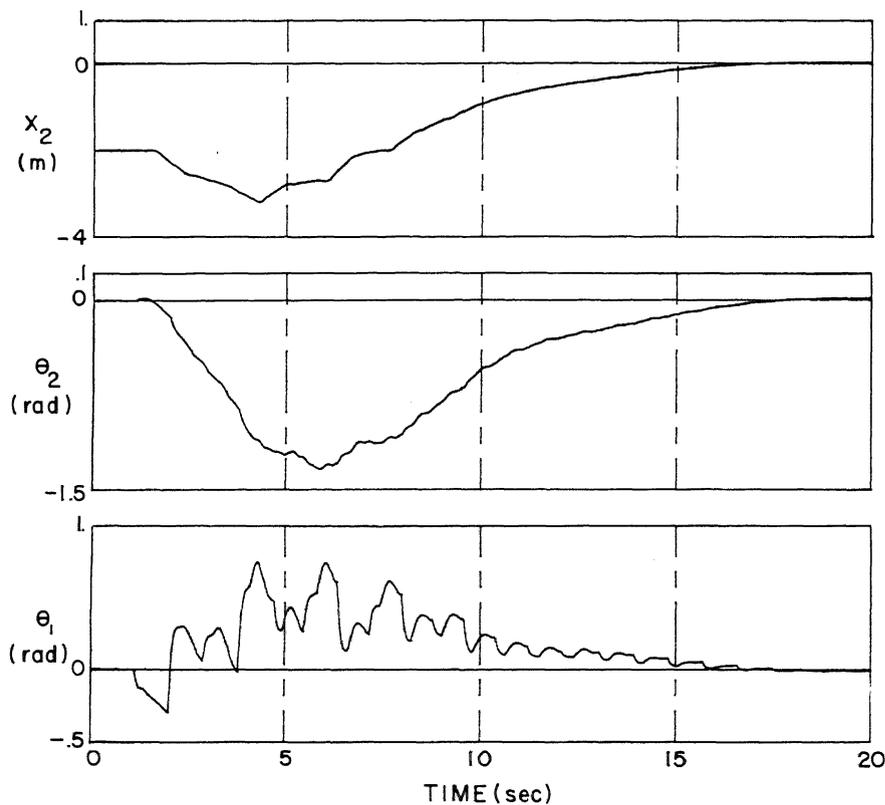
Figure 2-7: Horizontal position step controlled by linear method.

2.2.2.2 Stance Control

As we have said, there are two mechanisms that can contribute to balance: foot placement at TOUCH-DOWN, and leg motion during stance. In this section we describe a very simple approach to horizontal control that employs foot placement and hip torque during stance. The method examined is actually a generalization of the previous work on foot placement, as you will see.

What should a hip actuator do during stance? *During ideal steady state translation a vehicle undergoes no horizontal acceleration.* Therefore, there is a constraint on relative horizontal motion between foot and body for nominal translation at constant velocity. It can be satisfied throughout stance by choosing appropriate leg angles for vertical positions of the body. Satisfying this constraint causes the leg to sweep a range of angles while the foot is touching the ground.

For each stance period there is a locus of points over which the vehicle's center of gravity passes. We call such loci *CG-prints*[3]. Balance of the vehicle is least disturbed when the center of

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[3]Analogous to *footprints*.

gravity spends equal time in front of.the foot, and equal time behind it. Therefore, under nominal, steady state conditions balance is accomplished by placing the foot in the center of the next CG-print, a point that can be predicted from x_2 and $\dot{x}_2$. When state variables deviate from their desired values, foot placement must deviate from its nominal position -- the foot is placed toward the front or rear of the CG-print in order to correct state errors. Figure 2-8 illustrates the three cases possible.
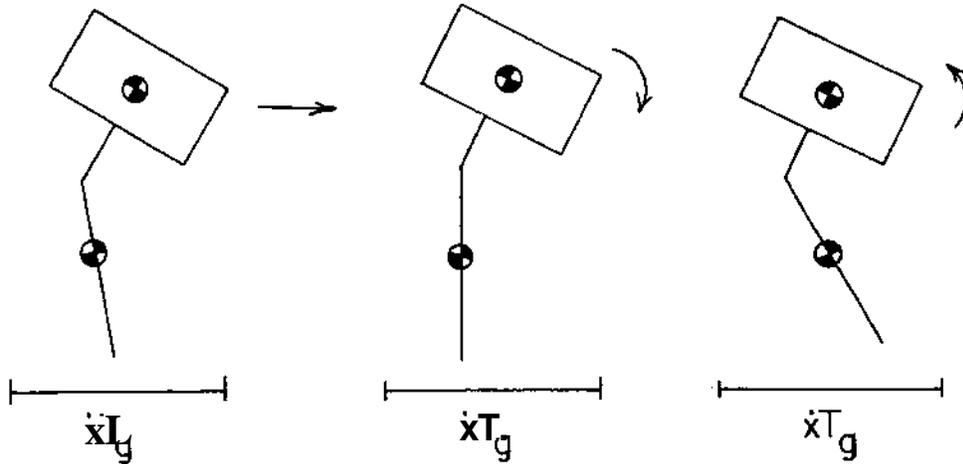
Figure 2-8: Three possible cases during stance period: a) Foot fe placed *in* center of region swept by projection of center of gravity, the CG-*print,* b) Foot Is placed toward rear of   CG-print, causing body to fall forward during stance interval, c) Foot is place toward front of CG-print, causing body to fall rearward during stance interval.

If we know in advance the duration of stance, the horizontal velocity of the body, and the geometry of the vehicle, then we can calculate an appropriate teg angle for TOUCH-DOWN and a sweeping function. To first order the time spent in stance Is determined by the main spring and body mass. The natural frequency of this oscillating system is:

$$\omega_n = \sqrt{\frac{K_s}{M_2}} \qquad (2.16)$$

The vehicle spends about a half cycle on the ground, so the duration of stance is:

$$\text{STANCE} \quad \frac{\pi}{\omega_n} \quad \sqrt{\frac{M_2}{K_s}} \qquad (2.17)$$

For transiationat velocity $\dot{x}$ the horizontal distance traversed by the body during stance, the length of the CG-print is:

$$\text{Ax}_{\text{STANCE}} \quad m \quad \dot{x} T_{\text{STANCE}} \qquad (2.18)$$

The center of the CG-print is calculated In the same way that $x_{|0}$ is calculated for the linear foot placement method. We repeat (2.11), (2,12). (2.13), (2.14) and (2.15) here for convenience. First fie horizontal position of center of gravity:

$$x_{CG} = \frac{(w-r_1)M_1 \sin(\theta_1) + r_2 M_2 \sin(\theta_2)}{M_1 + M_2} \tag{2.19}$$

Linear error function:

$$x_{ERR} = K_1(\theta_2 - \theta_{2d}) + K_2(\dot{\theta}_2 - \dot{\theta}_{2d}) + K_3(\dot{x}_2 - \dot{x}_{2d>}) \tag{2.20}$$

Factor in leg mass:

$$w \sin(\wedge) = x_{CG} + x_{ERR} \tag{2.21}$$

Solving for the center of the foot's sweep during stance with respect to hip:

$$x_{CENTER} = w \frac{r_2 M_2 \sin(\theta_2) - (M_1 + M_2) x_{ERR}}{r_{iMi} + wM_2} \tag{2.22}$$

Foot placement at touchdown, then is:

$$x_{TD} = x_{CENTER} + \frac{\Delta x_{STANCE}}{\hat{2}} \tag{2.23}$$

For vertical hopping in place, where $\dot{x}_{2TD} = 0$, (2.23) reduces to (2.14) of the linear foot placement method. Now the linear sweeping rule. At time t during stance:

$$x(t) = x_{TD} - \Delta x_{STANCE} \frac{(t_{TD})}{t_{STANCE}} \tag{2.24}$$

$$\theta(t) = \text{Arcsin} \left(\frac{\hat{x}}{w}\right) \tag{2.25}$$

We have implemented simulations of linear sweep control that show dramatic improvements in the quality and speed of horizontal locomotion over the linear position method. Figure 2-9 shows a .75 m/sec translation. In this plot the horizontal velocity is well controlled. Body attitude is stable in an upright position. The cartoon of Figure 2-10 lets one more easily visualize the results of this simulation.

A variation of the above stance control method is used when desired horizontal velocity is not constant as originally assumed for the derivation. During horizontal acceleration instead of determining foot placement and sweep magnitude solely on the basis of $\dot{X}_{2TD}'$ as done in (2.18) and (2.23), a compromise between actual and desired velocity is used:

$$\Delta x_{STANCE} = [\max(\min(\dot{x}_{2,d} - \dot{x}_2, \Delta\dot{x}_{MAX}), -\Delta\dot{x}_{MAX}) + \dot{x}_2] T_{STANCE} \tag{2.26}$$

where
   $\Delta\dot{X}_{MAX}$ is the maximum allowed acceleration
   $\dot{x}_{2,d}$ is the desired horizontal velocity.

This algorithm stretches the leg forward and lengthens the stride in order to accelerate. Velocity is automatically ramped up or down with roughly constant acceleration. Figure 2-11 shows data plotted for the case where the vehicle starts at rest with desired velocity of 4 m/sec. Figure 2-12 replots the data of 2-11 in cartoon form where the sequence of positions can be visualized.
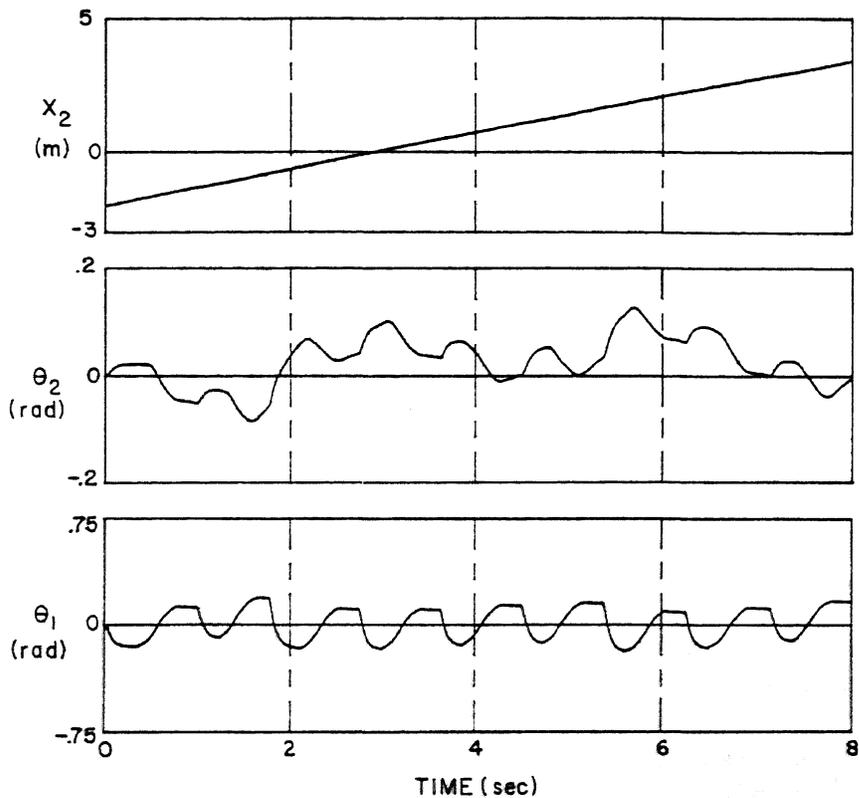
Figure 2-9: Sweep control with constant horizontal velocity of .75 m/sec.
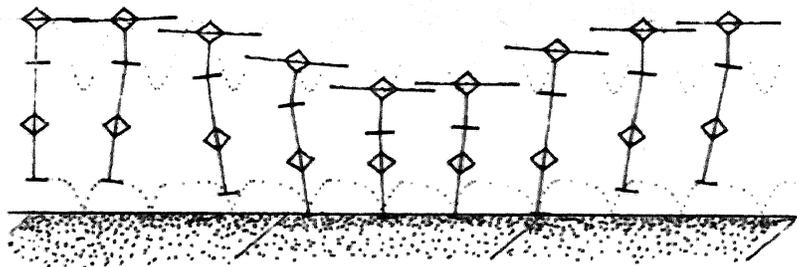
Figure 2-10: The data of Figure 2-9 replotted in cartoon form. Dotted lines indicate path of body and foot. 40 msec/dot, 1 sec/stick figure.

The data shown here are by far the best we have produced for any form of horizontal control. However, there are still some problems. It turns out that at high speeds stubbing the toe is difficult to avoid with a leg of the present design. For this reason an articulated leg that can substantially retract is important for achieving very high velocity translation.
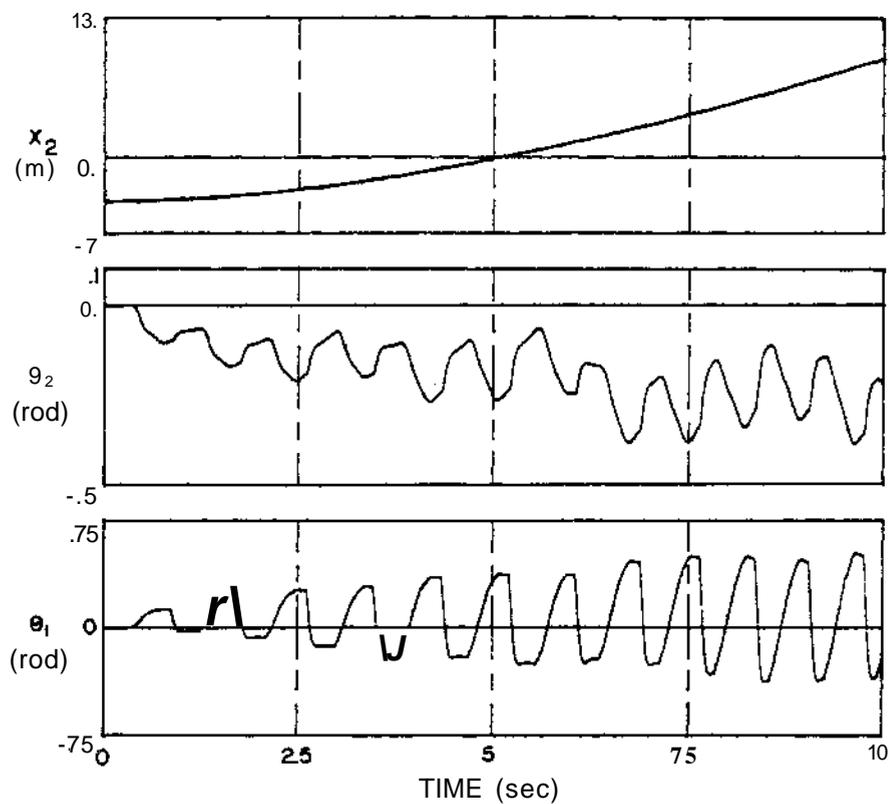
Figure 2-11: Sweep control with horizontal acceleration.
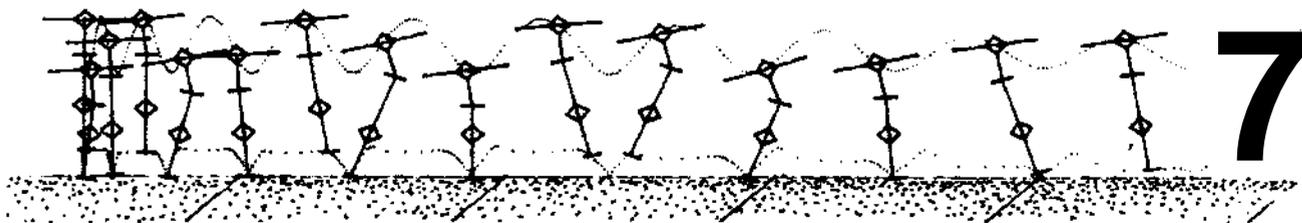
Figu re 2-12: Cartoon of data from Fig. 2-11. 20 msec/dot, .6 sec/stick figure.

2.2.2.3 Predictive State Space Method

Another set of experiments was conducted using an entirely new method of control that extends Raibert's State Space Memory Method [227]. The Predictive State Space Method is a means of solving the complete nonlinear dynamics for a hopping system without having to do lengthy calculations or numerical integrations in real time. Instead, a large table is maintained in which are stored future state data. Each address into the table is a function of the current state and a control vector. At each such address is stored the corresponding future state.

The predictive state space method is a *dynamic inverse* method, in that knowledge of the relationship between desired system state and available control actions is used to choose an appropriate control. We motivate this approach by casting the balance problem for a hopping mechanism in the following terms:

> Given complete state information at the time of TOUCH-DOWN, what leg-body angle maintained throughout stance will move the system to minimize the state error at the time of next LIFT-OFF?

The predictive state space method is a way of predicting state at a future time, t_f, based on augmented state at the present time, t_p. We define a multi-dimensional vector space Γ such that each dimension of Γ corresponds to one element of an augmented system state vector, X. The components of X are n state variables and i control variables. For each point in Γ there is a *future state vector* that predicts the state of the system at a specific time in the future, t_2 -- the present augmented state vector points to a future state vector.

If such a vector field is accessed at TOUCH-DOWN, and if the future state vectors correspond to the time of next LIFT-OFF, and if the i control signals are completely determined by their values at LIFT-OFF, then control is achieved as follows: At each TOUCH-DOWN the current state vector exclusive of the i control elements determines an i-subfield of Γ. By minimizing a performance index over this subfield a best future state vector is found. The location of the best future state vector in the subfield indicates best values for the i control variables.

When each dimension of Γ is quantized, this approach can be implemented with a finite memory. For n state variables and i control variables, each quantized to M values, there are M^{n+i} hyper-regions, each storing n values. A *Nyquist* like sampling rate applies to the selection of M -- the table must be quantized at a fine enough level to capture the bandwidth present in the variations of future state.

At one event, t_p the predictive state space method makes predictions concerning another event, t_f. During the interval that separates these events the system control variables must behave in a restricted way -- they must vary over the interval with at most i degrees of freedom. That is, for each augmented state at t_p, there is exactly one control trajectory during the interval. The question of finding suitable control functions of augmented state is an open problem.

What will the state be at t_F for a given augmented state at t_P? Such data may be obtained by simulation, by evaluating closed form solutions, or by making measurements on a behaving system. Though it has not yet been attempted, data could be obtained by taking measurements from a physical, behaving system. Such a learning controller would benefit from experience gained in practice, and could adapt to changes in the mechanical characteristics of the system [227].

What has been described so far has no specific connection to legged locomotion. But let us ask, what happens if some parts of the mechanical system we are trying to control engage in very regular repeatable activity. Does that make the problem easier or reduce its dimensionality? Normally to control a system, we would have to include all state and control variables in the augmented state vector. However, for a locomotion system engaging in regular rhythmic vertical hopping, to first order the vertical state variables (eg. w, $\dot{w}$, y_2, $\dot{y}_2$, u_2) always have the same value at TOUCH-DOWN and follow nearly the same trajectory throughout stance. Therefore these variables can each be quantized to one level rather than M, resulting in dramatic reductions in table size.

We have used the predictive state space method to control the planar hopper. Body angle θ_2, body angular rate $\dot{\theta}_2$, and horizontal velocity $\dot{x}_2$ are state variables that access a table, $n = 3$. Leg angle θ_1 is a control variable, $i = 1$. These variables index a 4 dimensional space. Each dimension of the memory is quantized to nine levels, $M = 9$, requiring that $nM^{i+n} = 19,683$ values be stored. To compensate for such a course quantization, linear interpolation is performed when accessing the memory.

We obtained tabular data for these experiments by simulating a large set of TOUCH-DOWN - stance - LIFT-OFF cycles with systematically varied initial conditions. As in the earlier work on linear feedback, the angle between leg and hip, $\theta_1 - \theta_2$, is held constant during stance. Therefore, the controller determines a value for control variable θ_1 at TOUCH-DOWN, and its behavior until LIFT-OFF is determined by the augmented state vector, in this case a subvector, $[\theta_1, \theta_2]$.

Use of the table requires that a search be performed for the hyper-region containing the state vector that minimizes a performance index. We employed a quadratic index:

$$PI = Q_1(\theta_2 - \theta_{2d})^2 + Q_2(\dot{\theta}_2 - \dot{\theta}_{2d})^2 + Q_3(\dot{x}_2 - \dot{x}_{2d})^2 \qquad (2.27)$$

where:
Q_1, Q_2, Q_3 are weights chosen by the user

$\theta_{2d}, \dot{\theta}_{2d}, \dot{x}_{2d}$ are desired values.

Figure 2-13 plots the body angle and horizontal position of the hopper for a *drop balance test*. In this test the hopper is dropped from a height of 0.3 m with an initial body angle of 0.8 radians. The task level set points are $\theta_{2d} = 0$, $\dot{\theta}_{2d} = 0$, $\dot{x}_{2d} = 0$. Under control of the predictive state space method desired foot placement for the impending landing is repeatedly calculated at 10 msec. intervals from PEAK until TOUCH-DOWN. After TOUCH-DOWN the desired leg-body angle is no longer
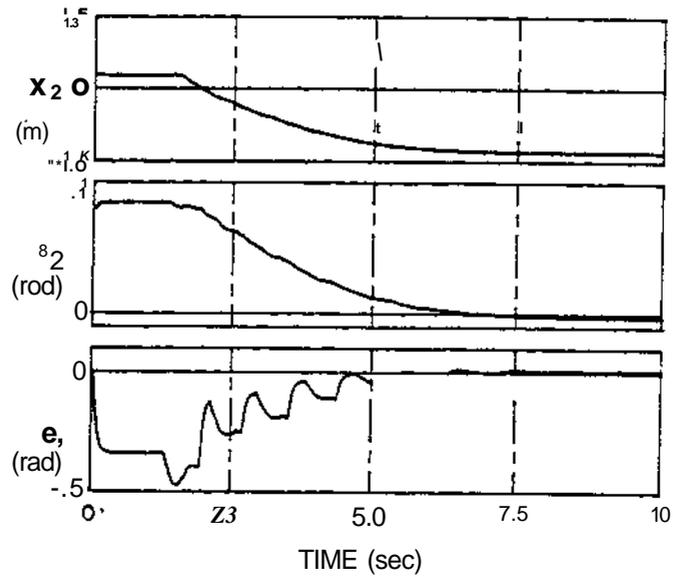
Figu re 2-13: Drop balance test using predictive state space method.

adjusted. Sn this test no attempt is made to control horizontal position, x_2. A vertical posture with no horizontal motion is attained in about 6 sec.
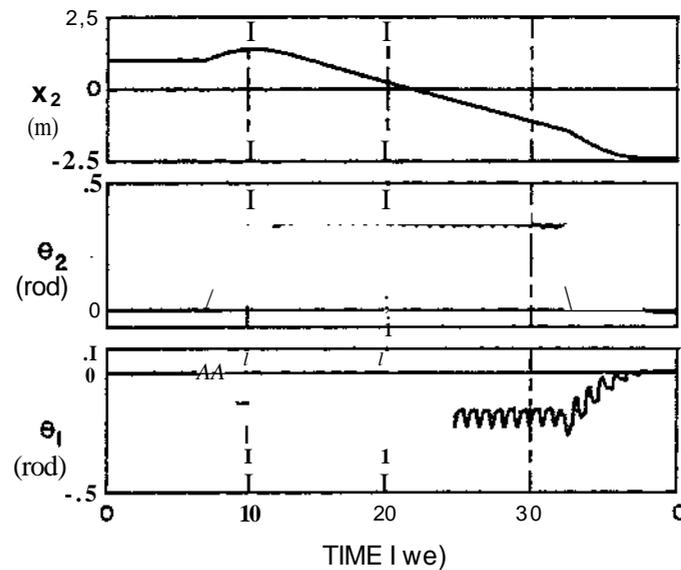
Figo *m* 2-14: Lateral step controlled[1] by predictive state space method.

The predictive state space method was used to control a lateral step in position. Figure 2-14 shows a translation In which x_2 was controlled indirectly through $\dot{x}_2$. Unlike results from the linear method, here horizontal velocity is precisely controlled. Compare Figure 2-14 to Figure 2-7.

The computational costs of using the predictive state space method center on search for the state that minimizes the performance index, and interpolation once the correct vector is found. No time is spent evaluating equations of motion directly. As we said earlier, it is not necessary that the control variables be constant during stance, only that they do not vary throughout state space with more degrees of freedom than are represented in the table. This means that control signals are perfectly acceptable provided control during stance is completely determined by state at TOUCH-DOWN.[4]

We are exploring a form of control that combines our work on the predictive state space method with our work on stance control. As before, foot placement at TOUCH-DOWN is determined from the predictive state space table. Rather than hold the hip angle constant during stance, however, the leg is swept through a region determined by a function of horizontal velocity at TOUCH-DOWN.

So far, we have not gotten good behavior from this type of control. Our diagnosis is that good stance control requires an additional degree of freedom in control that reflects the relationship between desired and actual horizontal velocities. The best linear stance control technique permits the difference between actual and desired horizontal velocity to affect the sweeping rate.

2.2.2.4 Polynomial Approximation to Predictive State Space Table

We have shown the predictive state space method to be an effective approach to control of a non-linear dynamic system with few state and control variables. However, the memory requirements for this approach become severe in larger applications. With this in mind, we have asked the questions, "Can the data in the predictive state space tables be approximated adequately by analytic functions of the state variables? If so, with many fewer coefficients than are in the original tables?" We have undertaken to evaluate the use of polynomial surfaces that approximate the tabular data.

Given $N = n + i$ state and control variables, polynomials are constructed that map points in the augmented state space to *estimated* future state vectors. Each of n polynomials minimizes the total square error for the variable it approximates across all data points in the predictive state space memory.

Let A be a matrix in which each row contains values of the N state and control variables; let B be the matrix that contains future values of the state variables in corresponding rows. The matrices A and B then form a data structure for the predictive state space memory. Given a sequence of distinct terms of the form:

$$\langle x_1^{i_{11}} x_2^{i_{12}} x_3^{i_{13}} x_4^{i_{14}}, \ldots, x_1^{i_{M1}} x_2^{i_{M2}} x_3^{i_{M3}} x_4^{i_{M4}} \rangle \tag{2.28}$$

---

[4] Actually, there is another practical limitation. Since interpolation is necessary to accomplish good control with tables of finite size, it is also required that small variations in state result in small changes in control. This is part of a more general consideration that governs the coarseness with which the predictive state space memory can be divided.

determine a row of the M-column matrix C by evaluating these terms at the values defined by the same row of A. Then

$$(C^T C X = C^T B) \tag{2.29}$$

is a linear system whose solution X contains, in each column, the coefficients of a least squares polynomial that estimates the values in the corresponding column of B [249]. The polynomial is, of course, determined by the choice of the exponents in the above sequence.

We have tested the use of least squares polynomials in control of the simulated one legged hopping machine. In this case 6_v 6_2, $\#_2$, and $\dot{x}_2$ are the independent variables and future values of $\$_r$ 6_2, and $\dot{x}_2$ are the variables to be approximated.
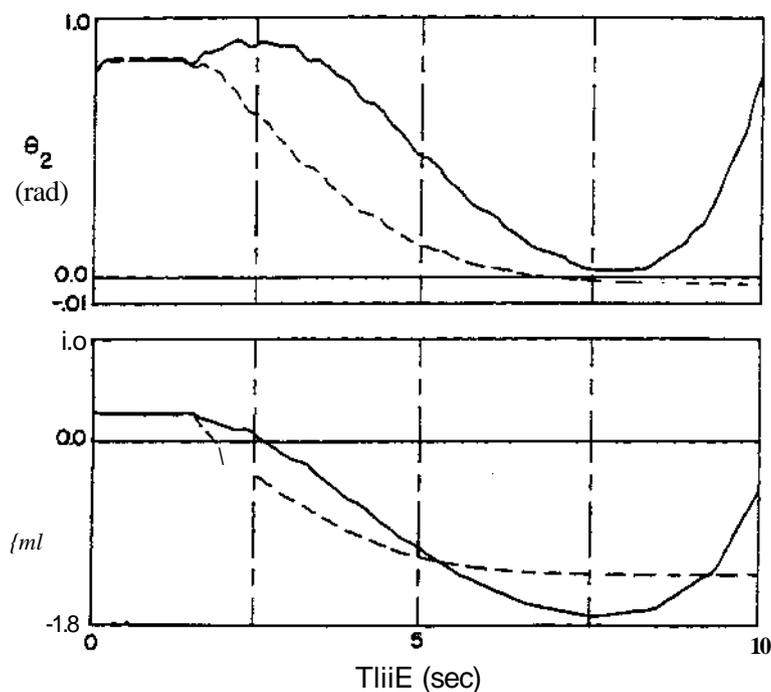
Figure 2-15: Drop balance test controlled by 86 term polynomial. Solid lines show results under control of polynomial approximation. Dotted lines are results using original tabular data.

Several polynomials have been tested:

- A 70 term polynomial consisting of those terms where the sum of the exponents of the four independent variables Is less than or equal to 4. The root mean square errors across the 5421 points in the state space memory were 0.961, 4.92 and 15.9 for 0_2, $\dot{6}_2$ and $\dot{x}_2$ respectively. The trajectory of the hopping machine diverged severely from that determined by use of the tabular data.
- The 70 term polynomial was augmented with 16 additional terms consisting of a limited selection of fifth order terms. The RMS errors decreased somewhat to 0.9O5, 4.83 and

12.9 for the 86 term polynomial. This change did not significantly improve the control system's performance in the drop balance test. The results are illustrated by the trajectory plotted in Figure 2-15 where it is shown with the comparable trajectory for the table-based system.

- The state space was divided into 16 subregions according to combinations of the signs of the four independent variables. The same 86 terms were fit to the data in each of these subregions. The RMS errors were reduced further to 0.283, 3.86 and 9.42. The resulting performance of the control system was disappointing, however, since storage requirements for the coefficients were increased by a factor of 16 over the previous case. Among other problems, this piecewise 86 term polynomial had discontinuities near the inter-region boundaries. Various other divisions of the state space were tested using this approach, including overlapping regions. Problems were encountered when systems were singular because of small numbers of data points in certain subregions. In no case was the fit adequate for stable control of the simulated hopping machine.

- A polynomial was constructed using all terms with exponents of the four independent variables between 0 and 4. This implies 625 terms and hence a linear system of order 625. Using Gaussian elimination the system was successfully solved in about 20 hours of CPU time on a DECSystem 20. The resulting fit was clearly superior to the earlier ones; the RMS errors were reduced to 0.246, 0.172 and 0.325. Figure 2-16 plots a cross section of the polynomial surface along with values from the state space table. The remaining errors still resulted in inadequate control, however. This can be seen in Figure 2-17. The polynomial-based control system approached the set points along a path similar to that determined by the table-based control system. However when the set points were reached, the polynomial system drifted and finally became unstable.

Keep in mind that the time required to determine the coefficients of the polynomial does not limit the usefulness of the approach in a strong way, provided *evaluation* of the polynomial can be accomplished with adequate speed. However, indefinitely increasing the number of terms is not an effective approach since it becomes very expensive to evaluate the resulting polynomials. Of course, as the number of terms increases the memory required to store the coefficients eventually approaches that required for the original state space tables.

Despite our lack of success with this method to date, we remain optimistic. Further work will include sensitivity analysis, exploration of optimal partitions of the state space for piecewise polynomials, and testing of spline techniques for smooth approximation across subregions.

2.2.2.5 Optimal Control

The feedback gains used in the linear methods describe above were chosen and adjusted by hand. Without a more formal method for selecting gains we have no way to distinguishing between those performance deficits due to fundamental limitations and those due to poor choices of gain. The optimal control theory gives the control designer rational methods for choosing gains. However application of the theory to the present problem is complicated by the non-linear dynamics and the non-holonomic nature of the locomotion problem.
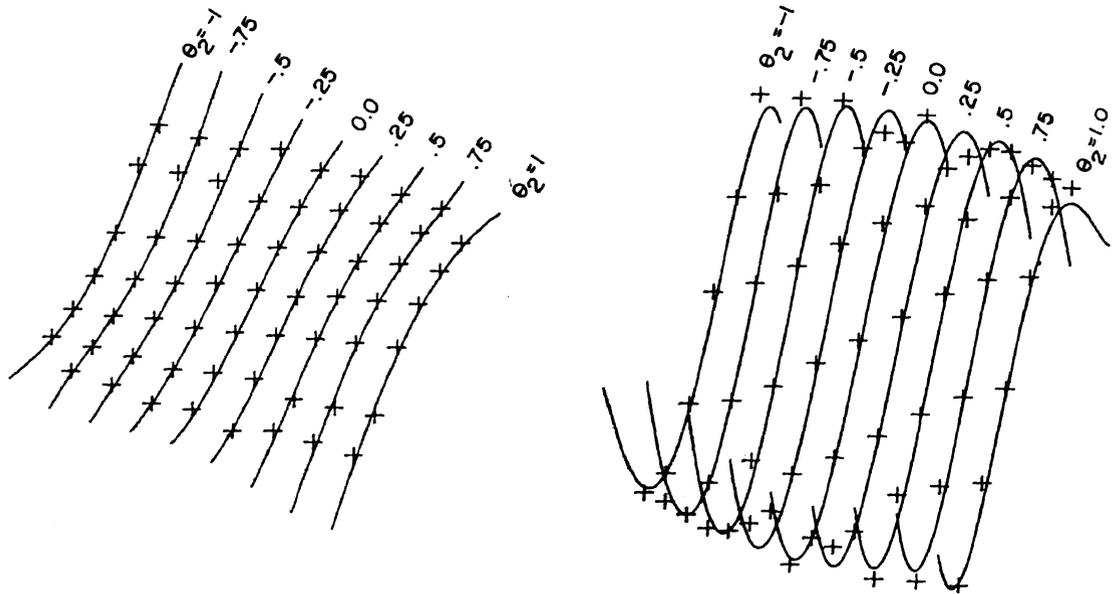
Figure 2-16: Cross section through state space that compares tabular data, shown by " + "s, to 625 term polynomial. *Slice* of state space specified by $\dot{\theta}_2 = \dot{x}_2 = 0$. Plotted on ordinate are: Left) θ_2, Right) $\dot{x}_2$.

As a first approximation to controlling the one-legged hopper, we consider a model with massless leg and study its control by means of linearization about a nominal trajectory. The linearized model will be posed in discrete time with a quadratic performance index, and this will be controlled by the standard theory involving a Riccati equation.

As one example of this method, consider the problem of applying hip torque while the hopper is on the ground in an attempt to bring it to an upright posture before TAKE-OFF. We assume that the foot strikes the ground and remains fixed at the origin. The assumption of a massless leg allows us to model the hopper as a system without energy loss, so we can take u_2 to be identically zero. When $w = w_0$, the main spring is at rest, which we assume to be the case at TOUCH-DOWN and LIFT-OFF.

Let (x,y) denote the coordinates of the center of mass of the body. When the hopper is hopping in place, we have (with $u_1 \equiv 0$):

$$\vec{x}(t) \equiv 0 \tag{2.30}$$

$$\vec{\theta}_2(t) \equiv 0 \tag{2.31}$$

$$\vec{y}(t) = -a \operatorname{Sin}\left(t\sqrt{\frac{k}{M}} - c\right) + w_0 + r - \frac{gM}{k} \tag{2.32}$$
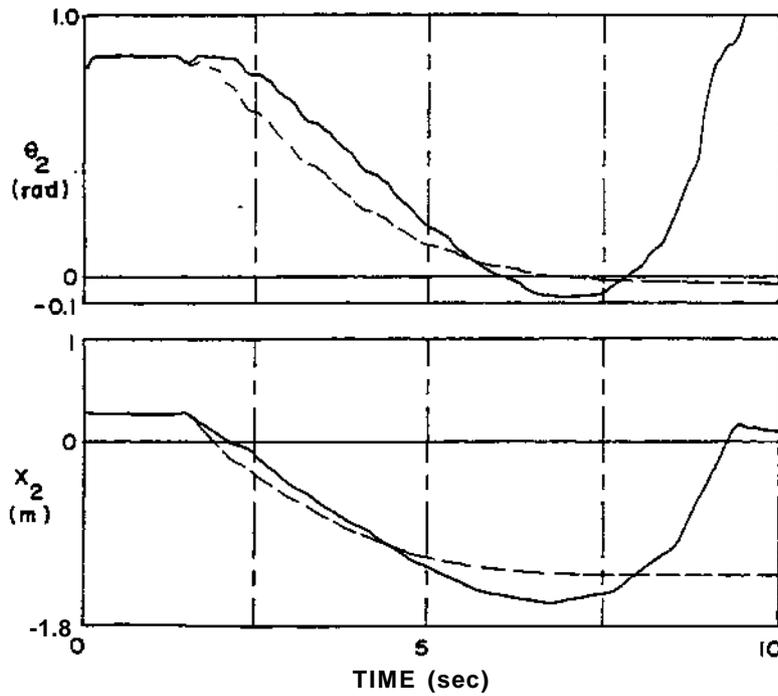
Figure 2-17: Drop balance test under control of 625 term polynomial.

where

$$a = \sqrt{\frac{g^2 M^2}{k^2} + \dot{y}^2(0)\,\frac{M}{k}}$$

$$c = \text{Arcsin}\left[\frac{gM}{ak}\right]$$

M is the borjy mass,
k is the main spring constant,
t = 0 is the time of TOUCH-DOWN $(y(0) = w_Q + r$, if $\dot{y}(0) < 0)$
r is the distance from hip to center of gravity of body.

The time of LIFT-OFF is

$$T = 2(\pi - c)\sqrt{\frac{M}{k}}.$$ (2.33)

We take this trajectory as nominal and study deviations from it. Suppose $\bar{x}(t)$, $\bar{y}(t)$, $\vec{0}_2(t)$ is another trajectory, and define

$Ax(t) = x(t) - \bar{x}^*(t),$ (2-34)
$Ay(t) = y(t) - \bar{y}(t),$ (2-35)

$$\Delta\theta_2(t) = \theta_2(t) - \vec{\theta}_2(t). \tag{2.36}$$

Linearization around the nominal trajectory yields

$$\Delta\ddot{x} \sim \frac{k(w_0 + r - \vec{y})}{M(\vec{y}- r)}\Delta x - \frac{kr(w_0 + r - \vec{y})}{m(\vec{y}- r)}\Delta\theta_2 \tag{2.37}$$

$$\Delta\ddot{y} \sim -\frac{k}{M}\Delta y \tag{2.38}$$

$$\Delta\ddot{\theta}_2 \sim -\frac{kr(w_0 + r - \vec{y})}{l(\vec{y}- r)}\Delta x + \frac{kr\vec{y}(w_0 + r - \vec{y})}{l(\vec{y}- r)}\Delta\theta_2 + u_1 \tag{2.39}$$

Let

$$X(t) = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta_2 \\ \Delta\dot{x} \\ \Delta\dot{y} \\ \Delta\dot{\theta}_2 \end{bmatrix} \qquad A(t) = \begin{bmatrix} 0 & I \\ S(t) & 0 \end{bmatrix} \tag{2.40}$$

where each submatrix in A(t) is 3x3 and

$$S(t) = \begin{bmatrix} \dfrac{k(w_0 + r - \vec{y})}{Mw} & 0 & -\dfrac{kr(w_0 + r - \vec{y})}{M(\vec{y}-r)} \\[3ex] 0 & -\dfrac{k}{M} & 0 \\[3ex] -\dfrac{kr(w_0 + r - \vec{y})}{l(\vec{y}-r)} & 0 & \dfrac{kr\vec{y}(w_0 + r - \vec{y})}{l(\vec{y}-r)} \end{bmatrix}$$

Let

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Then:

$$\frac{d}{dt} X(t) = A(t) X(t) + B u_1(t). \tag{2.41}$$

We want to control this system from $t = 0$ to $t = T$. Divide $[0,T]$ into subintervals of length $\frac{T}{n}$ and then let

$t_k = \frac{kT}{n}, k = 0, \cdots, n.$ Let:

$$X_k = X(t_k), A_k = A(t_k), u_k = u_1(t_k). \tag{2.42}$$

We have:

$$X_{k+1} \sim (I + \frac{1}{n} A_k) X_k + \frac{1}{n} B U_k. \tag{2.43}$$

Define:

$$Q_n = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6) \tag{2.44}$$

$$R_k = 1, k = 0, \cdots, n-1. \tag{2.45}$$

To control the system so as to minimize $X_n^1 Q_n X_n + \sum_{k=1}^{n-1} u_k$, we let P_k be given by the Riccati equation:

$$P_n = Q_n \tag{2.46}$$

$$P_k = (I + \frac{1}{n} A_k)^T [P_{k+1} - \frac{1}{n^2 + p_{k+1}(6,6)} P_{k+1} B B^T P_{k+1}] (I + \frac{1}{n} A_k), \quad k = N-1, \cdots, 0, \tag{2.47}$$

where
$p_{k+1}(6,6)$ is the $(6,6)$ element of P_{k+1}.

Let:

$$L_k = -\frac{n}{n^2 + p_{k+1}(6,6)} B P_{k+1} (I + \frac{1}{n} A_k). \tag{2.48}$$

Then the optimal feedback control law is given by:

$$u_k = L_k X_k. \tag{2.49}$$

This control requires a minimum of calculation and so can be implemented in real time. More importantly, it will allow for simulation which should help determine the main features of the controlled system. It is also possible to apply this control law to models with legs of non-zero mass to determine its robustness.

3. Physical Experiments

This section describes laboratory work aimed at building and testing physical legged machines that locomote with dynamic stability. Our first project is the construction and control of the Planar Hopper, a machine with one leg similar to the ideal device described earlier. At the time of this writing the planar hopper is able to produce high quality vertical hopping, while our current effort focuses on achieving balance. Our second project is aimed at moving our work mto the third dimension. Two designs for 3-D hoppers are presented, a short term design that is simple, and a more ambitious long range design.

3.1 Planar Hopper

Here work on the physical Planar Hopper is described. We include a description of the mechanism, its sensors, and progress in controlling it.

3.1.1 Mechanism

Figure 3-1 is a diagram of the apparatus. The important features of this mechanism are a body, a leg, two internal bearings, a main spring, two pneumatic cylinders, and three air bearings.

Body	Most parts of the planar hopper are mounted on the body, or connected to it. The body is a set of parts machined from aluminum stock, with manifolding for distribution of compressed air, supports for the actuators, a place for sensors and electronics, and pillow blocks that support bearings for the leg.
Leg	The leg is a 0.5 m length of machined square aluminum tubing, with a rubber tip at the foot. The leg is connected to the body with a coupling that provides two degrees of freedom: the leg can translate along its long axis through the body, and rotate about one of its short axis with respect to the body.
Cylinders	Two double acting air cylinders of 1.0625" bore actuate the hopper. Each cylinder housing is attached to the body with trunion mounts, while the rods connect to the leg through a sliding collar. These cylinders are mounted in such a way that the sum of their lengths determines the distance between the sliding collar and the body, and the difference of their lengths determines the angle between leg and body.
Main spring	A metal compression spring fits over the lower leg. Its lower end is held in place by a bracket fixed to the leg, while its upper end indirectly supports the body through the sliding collar and cylinders. In parallel with the metal compression spring are two feather tension straps. They limit the maximum length of the metal spring.
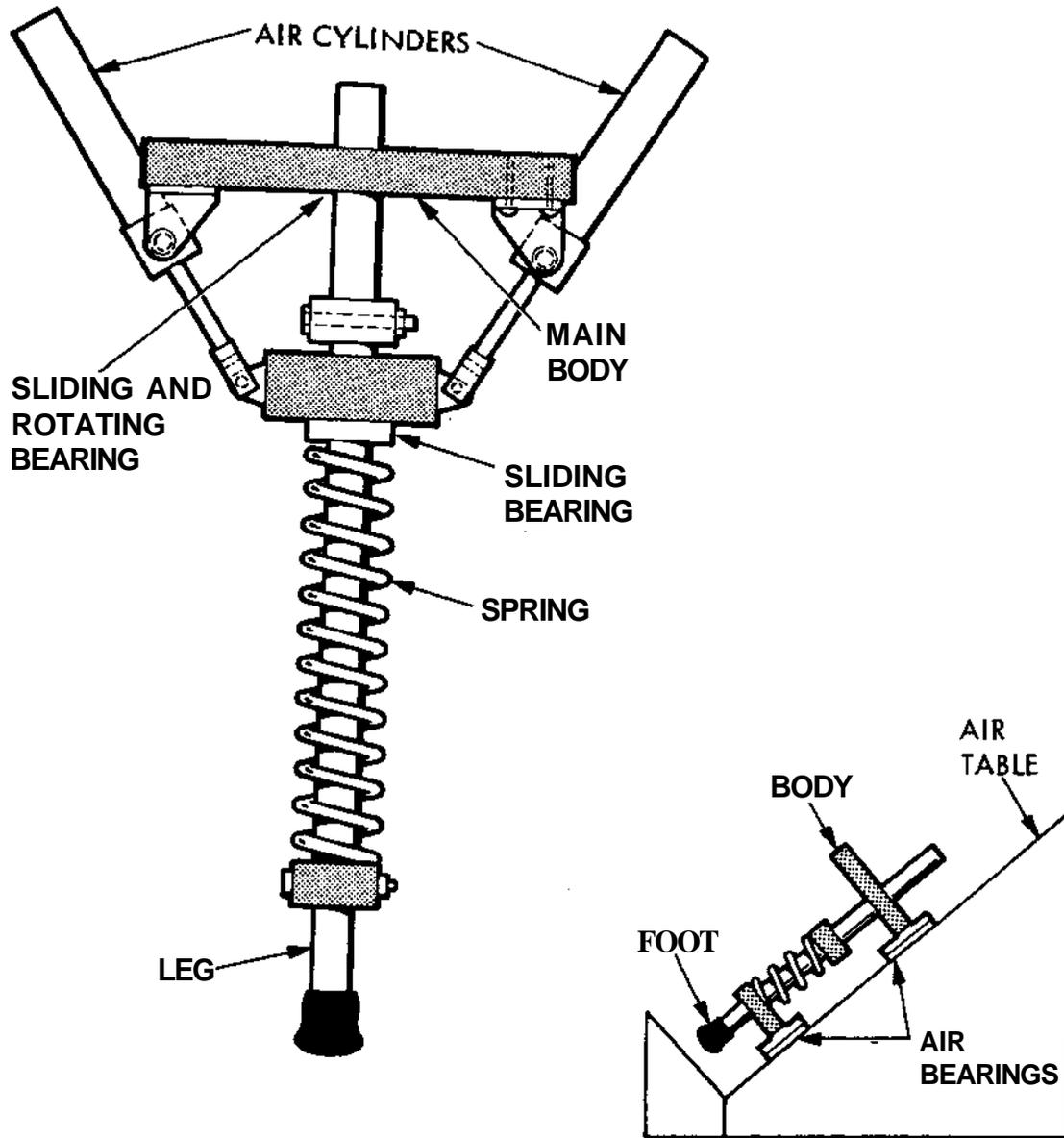
Figu re 3-1: Physical Planar Hopper shown in front and side view.

Manifold and valves

Between two plates that form the body are grooves and passages that form a manifold for distribution of compressed gas. Eight electrically operated solenoid valves mounted to the underside of the body control air flow to the cylinders. Each end of each cylinder is controlled by a pair of valves: one valve connects the cylinder to high pressure gas, the other connects it to atmosphere. When both valves of a pair are off no gas enters or leaves that end of the cylinder. Each valve is separately controlled by one bit from the computer and by a transistor switch.

Air bearings and table

Motion of the entire mechanism is constrained to a plane by three air bearings and a table inclined to 45°, as shown in Figure 3-1b. Each air bearing is attached to the hopper by a self centering bellows coupling through which air flows. Two air bearings are attached to the body, and one to the lower leg. The hopper is free to translate vertically and horizontally in the plane of the table, and to rotate about an axis perpendicular to the table. A second part of the table, perpendicular to and below the first part, acts as the support surface upon which hopping takes place.
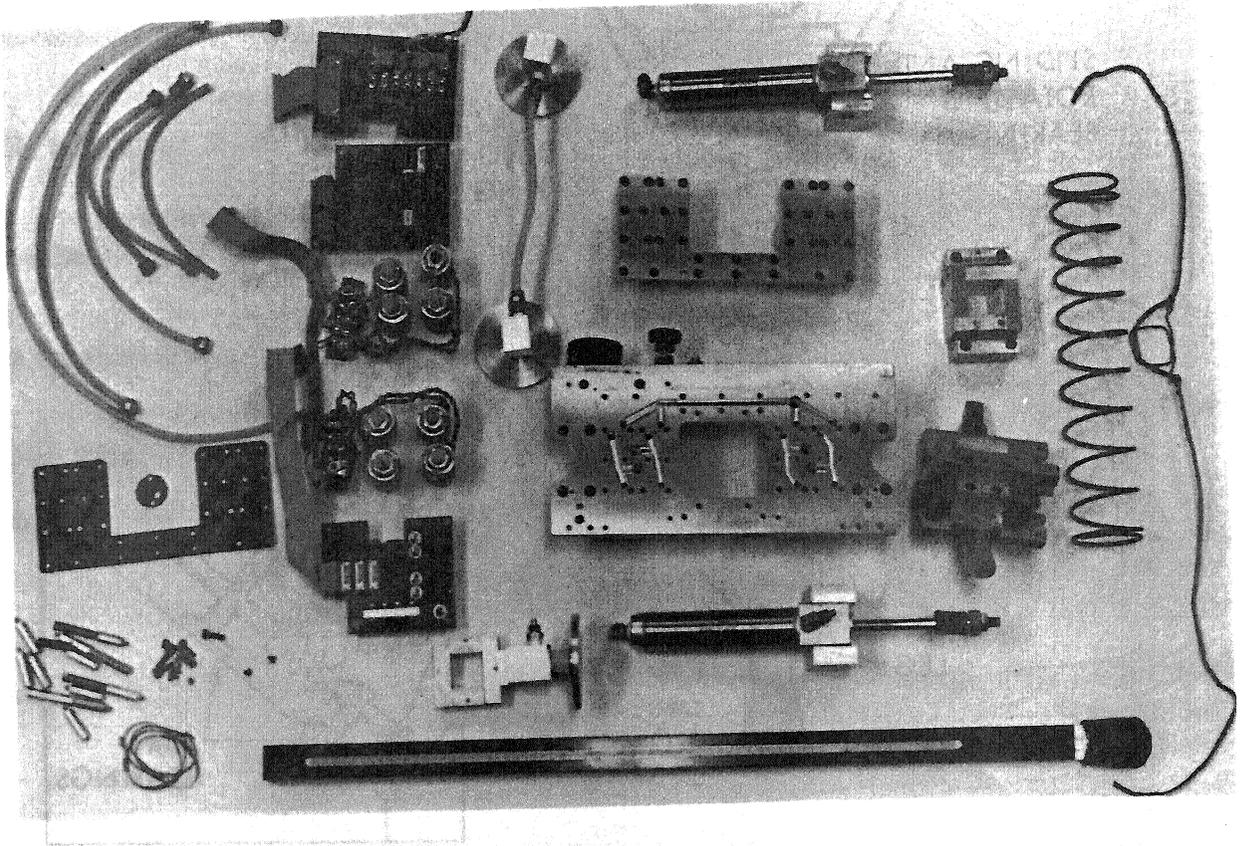
Figure 3-2: Parts that make up Planar Hopper: body, leg, spring, cylinders, valves, manifold, air bearings, and assorted miscellany.

The hopper operates as follows: Simultaneous in-phase rhythmic activation of the two air cylinders causes resonant excitation of the spring-body system. Hopping results when the leather straps restrict extension of the spring during oscillation. While the device is in flight differential control of the cylinders causes the leg to rotate relative to the body, thereby enabling selection of a new point for foot placement on subsequent landing.

3.1.2 Sensors

In the long run, the hardest problems in locomotion may be sensing problems. How can a system know where it is in space and where it is with respect to other objects without touching them? How can solid support points be chosen and dangerous ones avoided? And how can all this be accomplished in a system that is bobbing and weaving, oscillating and undulating at more than 40 km/hr? In biological systems these problems are solved by a variety of sensory systems that work together, vision being an important contributor.

We characterize the functions that sensors perform in controlling locomotion systems and other forms of robots:

- They provide information that permits control of actuators.
- They provide information that gives control over a system's state variables.
- They provide information about a system's interaction with its environment.

In order to control the planar hopper a number of measurements need to be made. At the lowest level the controller must know what each actuator is doing, the angle of the leg, and the length of the main spring -- in our design this information is provided by two encoders and a potentiometer. At the next level the vehicle's dynamic state must be assessable. Horizontal vertical and angular position of the body must be known, along with velocity. A rate gyro and sonar provide some of this information, a Selspot motion recording system fills in the gaps. We have not dealt with obstacle detection or avoidance, nor with irregular terrain. The following is a list of sensors that currently work, or that are being implemented.

Encoders   A linear encoder is mounted on each of the bearings that move along the leg. Number 1 is mounted on the leg-body coupling, number 2 is on the sliding collar at the top of the spring. These electro-optical devices and their support circuitry count bars of a 1. mm/cycle square wave pattern affixed to the leg as the leg passes by. Encoder 2 gives information about compression of the main spring. Together the encoders give the separation between the body and sliding collar.

Body potentiometer

The coupling between leg and body is instrumented with a potentiometer that measures hip angle. The body potentiometer and linear encoders provide measurements from which the length of each air cylinder can be calculated:

$$c_R = \{l_1 - w(t)\cos(\varphi) - l_4\sin(\varphi)\}^2 + \{-l_2 + w(t)\sin(\varphi) - l_4\cos(\varphi)\}^2 \quad (3.1)$$

$$c_L = \{-l_1 - w(t)\cos(\varphi) + l_4\sin(\varphi)\}^2 + \{-l_2 + w(t)\sin(\varphi) + l_4\cos(\varphi)\}^2 \quad (3.2)$$

where
  Figure 3-3 defines all variables
  $w(t)$ is obtained from the difference of the encoders
  φ is obtained from the potentiometer.

Rate gyro We have purchased an oscillating beam rate gyroscope with 0.04 deg/sec resolution to provide measurements of body angle, θ_2, and body angular rate $\dot{\theta}_2$. It will mount on the main body along with support electronics.

Foot potentiometer

During stance on level terrain it is possible to measure leg angle θ_1 with an instrumented foot. This is done with a potentiometer in the ankle. Eric Saund of Caltech constructed such a foot that serves as a prototype.

Sonar A way to measure altitude y_2 and get additional body angle data θ_2 is to use a pair of down looking sonar sensors. Figure 3-4 shows the scheme we are implementing. It combines Polaroid sensors and drive electronics with custom electronics of our own design.
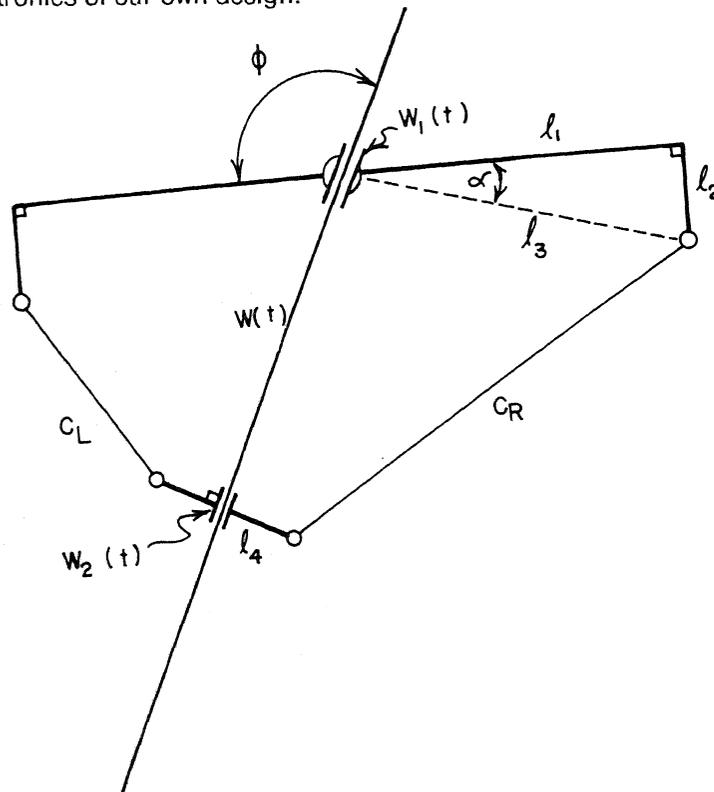
Figure 3-3: Kinematics of Planar Hopper.

It will be some time before this array of sensors is completely functional. In the interim a commercially available measurement system, Selspot, provides the information needed for control. Selspot is an electro-optical system that records the paths of electronic markers as they move through space. The markers are infrared LEDs that are attached to the subject. A transducer generates x and y analog signals that specify the average location of light focused on its surface. The entire system is comprised of cameras containing the solid state sensors, a controller that turns on each of 30 LEDs in sequence every 3 msec, a multiplexer that separates the digitized signals into 30
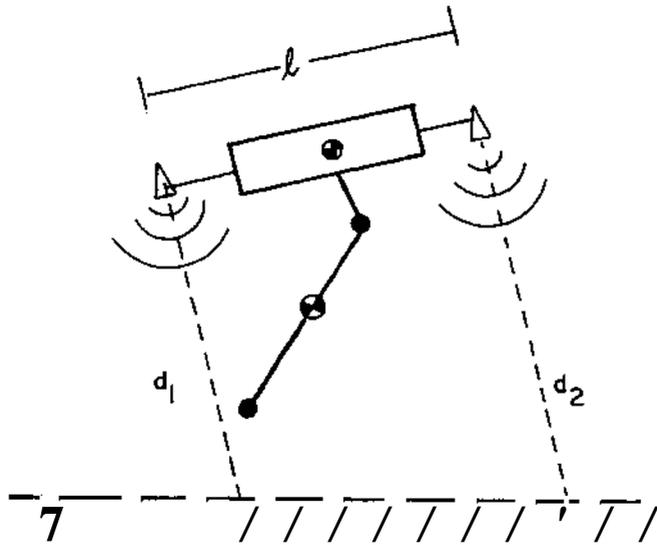
Figu re 3-4: How sonar will be used to measure body altitude and body angle.

coordinate pairs, and a computer interface. Use of two cameras permits 3-dimensional localization of the LEDs.

As we said, Selspot provides feedback for control experiments while other types of sensors are under development. In addition Selspot is a means of independently measuring and analyzing the performance of an experimental system. In planar hopping experiments where one camera is adequate, a Selspot camera sits on a nearby tripod overlooking the inclined air table. See Figure 3-5a. LEDs are attached to the hopper in five locations as shown in Figure 3-5b.

We have made three improvements to the Selspot system as sold to us by Selective Electronics:

- A special LED scanner permits five LEDs to be scanned at 2 KHz, six times the normal rate. These higher rates mean more precise measurements with less delay, and ultimately, better control.
- A microcoded digital filter smooths each LED position value and calculates a smoothed velocity. This special purpose processor frees the control computer for other activity.
- A two port memory interface between Selspot and our computer's UNIBUS facilitates communication.

3.1.3 Results

After shortening some hose and increasing its diameter, after replacing a sliding linear bearing with a rolling one, after replacing a narrow spring that buckles with a wider one that does not buckle, and after replacing fixed air bearings with self-centering bearings, the hopper is hopping vertically. Recorded data for one test run are plotted in Figures 3-6 and 3-7. These plots look very
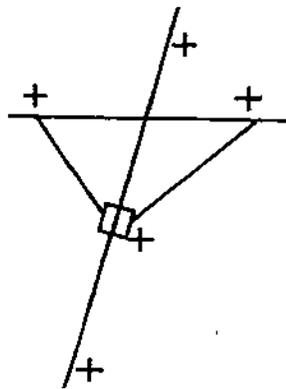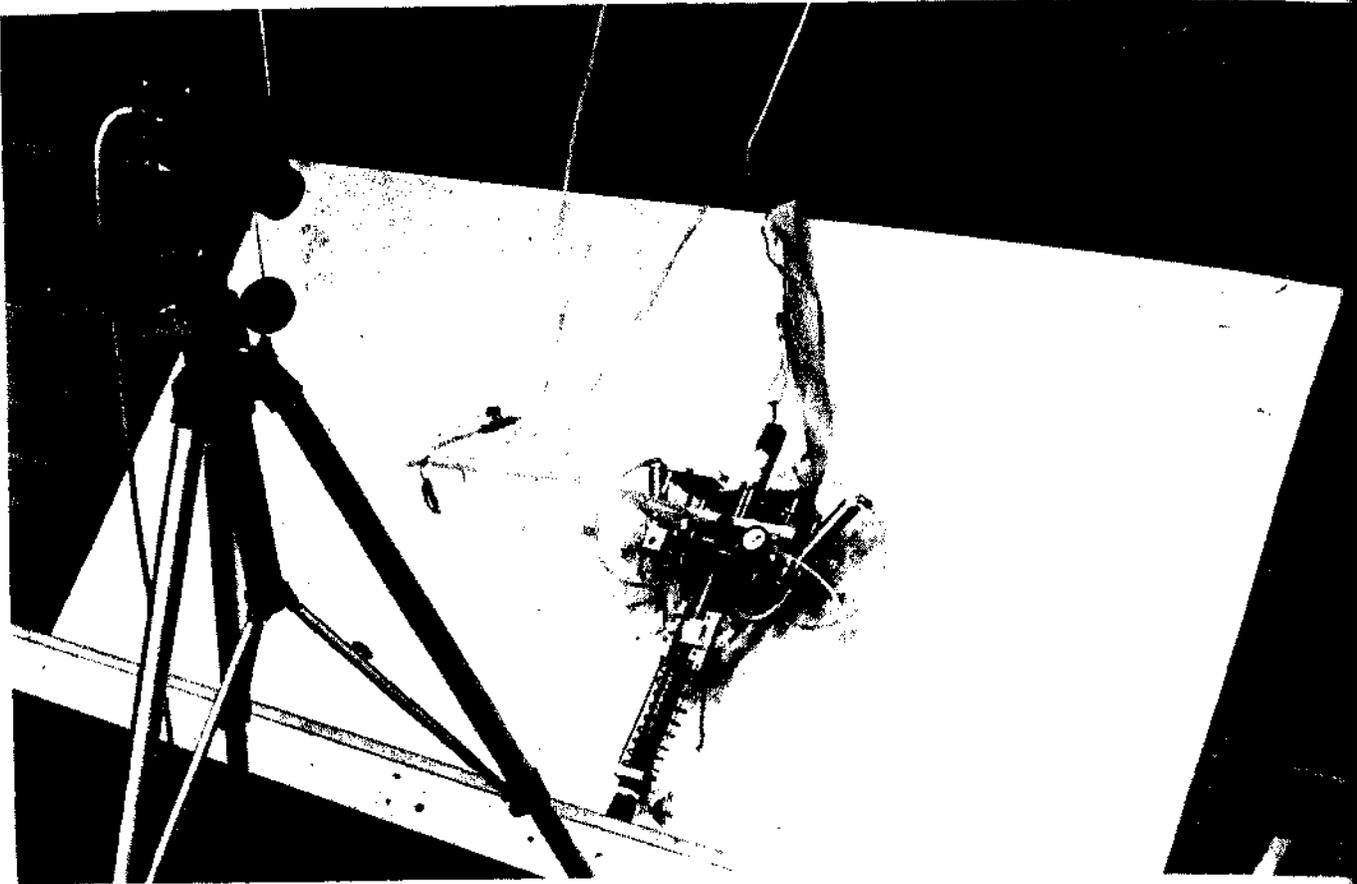
Figure 3-5: Top) Selspot camera overtooking planar hopper. Bottom) Pattern
of tEDs on hopper.
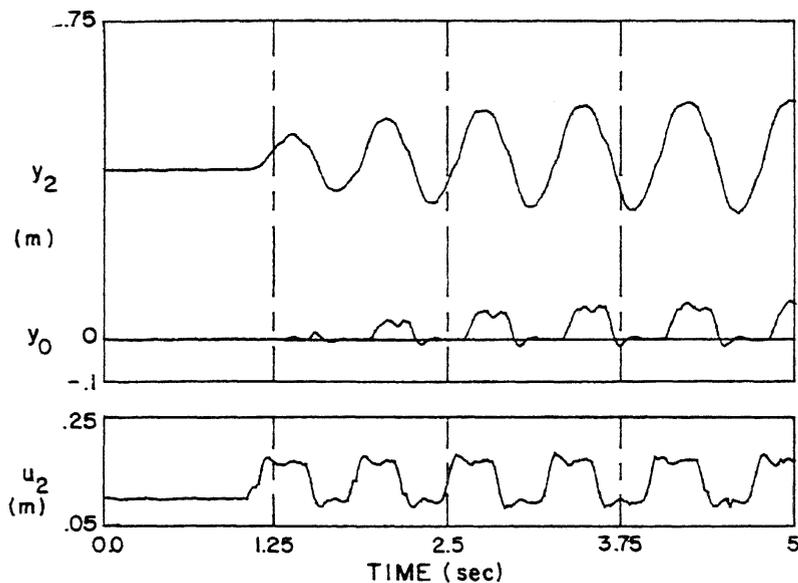
Figure 3-6: Vertical behavior of physical hopper. Left) As recorded by Selspot.
Right) As recorded by camera with low shutter speed.

similar to those obtained in simulation, eg. Figure 2-2. Though ever increasing hopping altitudes are not obtained, the foot clears the ground by more than 3 inches at PEAK.

Balance during hopping has not yet been accomplished, though our progress in this area is good. We are now experimenting with a linear balance algorithm similar to the one explored in simulation. Completion of the Selspot filter has permitted us to try this even though the sonar and gyro sensors are still being developed. The actual construction and testing of this physical system has taught us basic lessons we will remember in the future:

- Unsprung mass in a leg must be minimized. If not, there are energy losses, large forces, and lots of noise.
- Springs in compression buckle.
- High performance linear bearings require great care. Light weight, speed, and efficiency are not easy to combine in a linear bearing.
- The importance of adequate gas flow is easy to forget. We replaced small fittings with larger ones, 1/4 inch hose with 3/8, small valves with larger valves, and larger valves with still larger ones.
- When two actuators are mechanically coupled to control motions in two degrees of
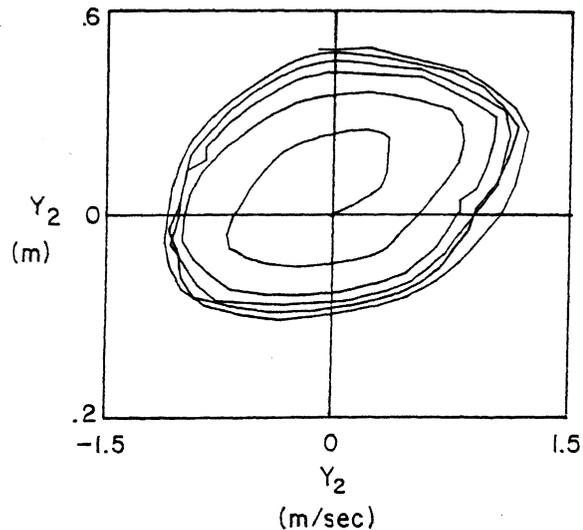
Figure 3-7: Phase plot of vertical hopping data recorded from physical hopper with Selspot.

freedom, precise trajectory control is essential. Bang bang control is not appropriate in such systems.

We made some bad design choices early on that lead to this last lesson. We have had difficulty in separating the actions of the actuators into a vertical part and a horizontal part. In our planar design, linear extension of the leg and torque about the hip are achieved by coordinating the actions of both actuators. They must move in precise synchrony in order to produce extension without hip torque, and hip torque without extension. Bang-bang pneumatic control of coupled actuators is not a good method for achieving such motions. Three solutions present themselves: find a better algorithm for using temporal modulation of on-off valves to approximate proportional control, replace the on-off valves with proportional servo valves, decouple the cylinders. Each of these options is being explored.

3.2 Design of 3-Dimensional Hoppers

Balance as it occurs in advanced legged systems is a 3-dimensional phenomenon. Therefore, if we are to fully understand balance and take advantage of it, we must extend our physical experiments into the third dimension where there are six degrees of freedom. We have begun to do so by designing experimental walking machines that embrace balance in 3-D, and that incorporate what we have learned from building and experimenting with the planar hopper. The goal of these designs is to create a vehicle for learning about balancing in 3-D in the coming year. A secondary goal is to generate preliminary ideas on what one leg of a multi-legged balancing vehicle should be like.

The objectives of the design are:

- To embody dynamic balance capability. The machine should be able to balance about a point on the floor while hopping in place, to translate in a preferred direction with high speed and efficiency, and to translate in any direction with less speed and efficiency.
- To achieve mechanical efficiency through proper tuning of the components of a spring-mass system. Hopping should be efficient.

The present intent is to leave for later the control of rotation about the vertical axis, and simply design with a large polar moment of inertia to minimize rotation.

The results are designs for three 3-D machines. Two are simple machines that function with circular symmetry. They are designed to hop in place or translate in any direction over relatively flat terrain. We will build at least one of them in the coming year. The third design is more advanced, reflecting our initial thinking about how to optimize a one-legged balancer for use in a multi-legged system. We believe that the basic role played by each leg in such a system is similar to its role in a monoped, but that the design must be biased for forward motion and for coupling between the legs.

3.2.1 Simple 3-D Hoppers

We have designed two simple 3-D hoppers. The first, shown in Figure 3-8, has a linear sliding leg that is gimbal mounted to a wide body and frame. The leg is composed of a long double ended pneumatic cylinder with an extension at one end. The cylinder acts both as an actuator that changes length of the leg under control of an on-off valve, and as a spring that stores energy in gas compression during stance. Two hydraulic cylinders angle the leg with respect to the body in two degrees of freedom. These cylinders are controlled by flapper type proportional servo valves. The basic design has strong similarities to the planar hopper with the following important improvements:

- Motion that changes leg length is decoupled from motion that changes leg angle.
- Bang-bang pneumatic control is no longer responsible for precise positioning.
- Energy is stored efficiently in compressed gas rather than in a metal compression spring that buckles and binds.
- The only sliding bearing is that of the vertical actuator -- since there are three points of support there should be little binding.

The second design, shown in Figure 3-9, features a leg with all rotary joints. Two hydraulic cylinders with proportional servo valves control hip rotation in two directions. The angle of the entire leg is changed by these actuators. The knee incorporates two gears that act as an angle splitter so that the upper and lower legs *fold* at the same rate. Acting with the parallel-bar mechanism of the thigh, this angle splitter causes upper and lower legs to maintain equal angles with respect to the rotating portion of the hip. This linkage structure permits a single actuator to produce linear foot motion. The third actuator is a hydraulic cylinder working through a tension spring to lengthen and shorten the leg.
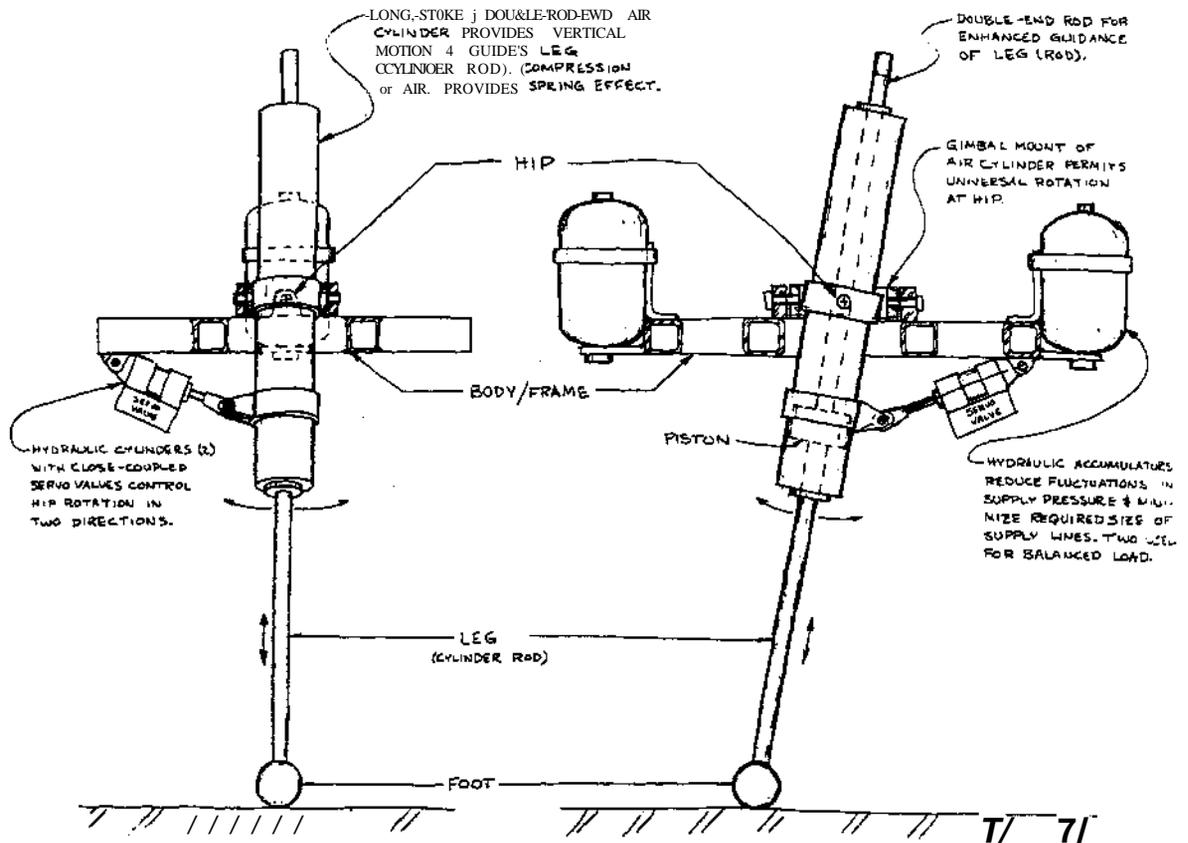
Figure 3-8: Simple 3-dimensiona! hopper with linear acting leg.

The basic difference between these simple 3-D machines is the method for obtaining linear leg motion: one uses the linear bearing of a pneumatic cylinder, the other uses gear and cable linkages. Both designs use a pair of precisely controlled hydraulic actuators to control leg angle, and both use a third actuator to change leg length. Neither will carry its own power supply nor its own computer - they will connect with hoses to a stationary hydraulic power supply, and with wires to an off board computer. An on board accumulator will guarantee adequate control response and minimize peak flow rates through the umbilical. Some analog hardware will be present on board to support joint sensors, a set of rate gyros, and force sensors.

It is anticipated that a large amount of tuning of the mechanism and controls will be required before effective locomotion, is achieved. To facilitate this, it is planned to build an arm, or other restraint device, to which the machine will be attached. This will enable development of hopping in stages: hop in place while externally stabilized, balance in two dimensions, balance in three dimensions. This approach will also help us to observe, measure, and understand the various phenomena occurring during each behavior.
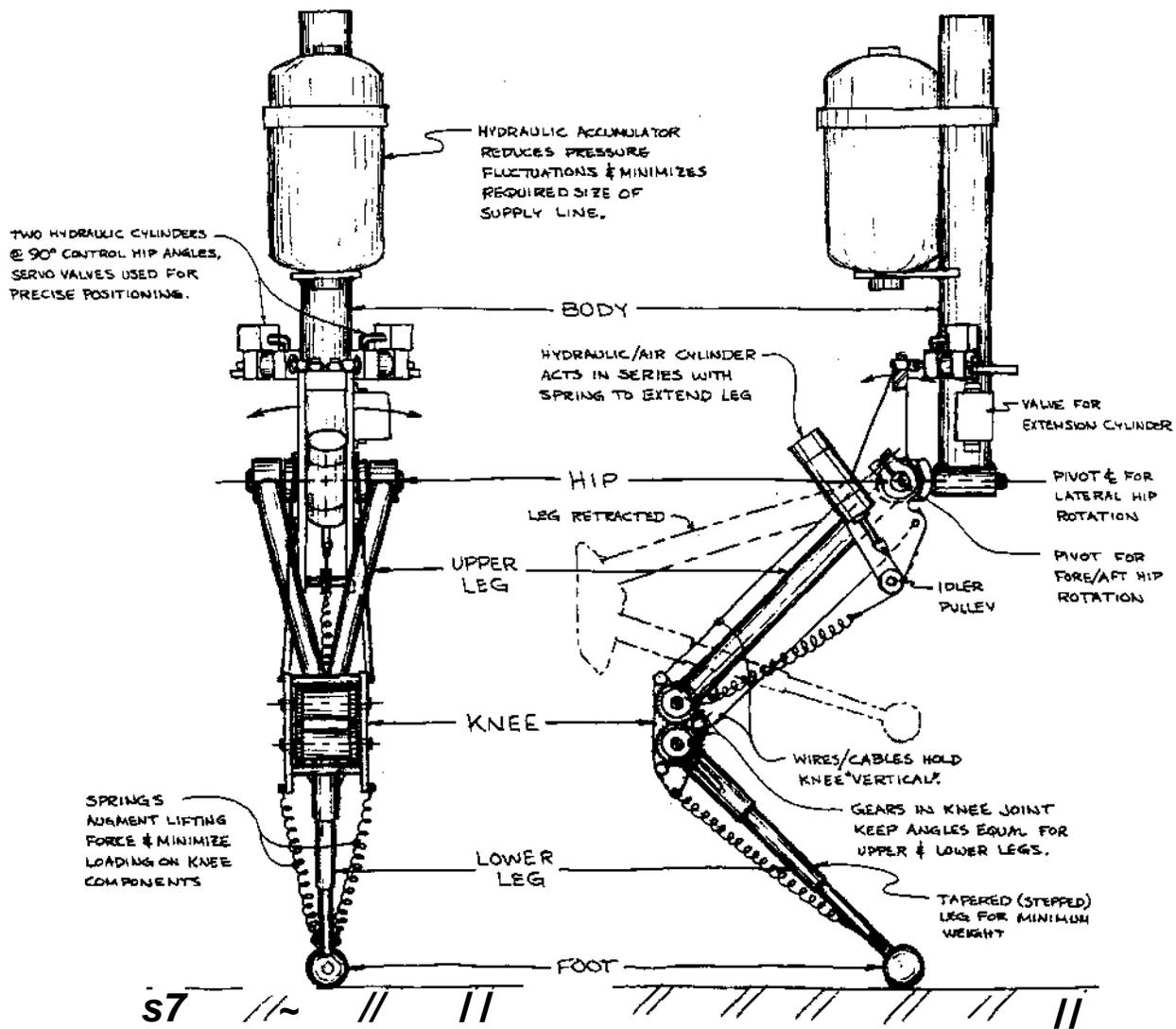
Figure 3-9: Simple S-dimensional hopper with articulated leg.

Construction of the 3-D hopper will be from both standard components and parts we manufacture in the CMU machine shop. Aluminum, particularly tubing, will be used to minimize weight. Standard hydraulic and pneumatic actuators and valving are planned. Rubber band stock will be used for springs due to its light weight and ease of adjustment.

3.2.2 Advanced 3-D Hopper

The idea of a one legged hopping machine is important for legged locomotion not because one legged vehicles will come into wide spread use, but because dynamically balanced multi-legged vehicles will incorporate principles revealed by studying the extreme one-legged case. Therefore it is useful, even at this early point in balance research, to contemplate what the legs of a running machine might look like. We ask this question by proposing a specific leg design that will provoke thought and discussion.
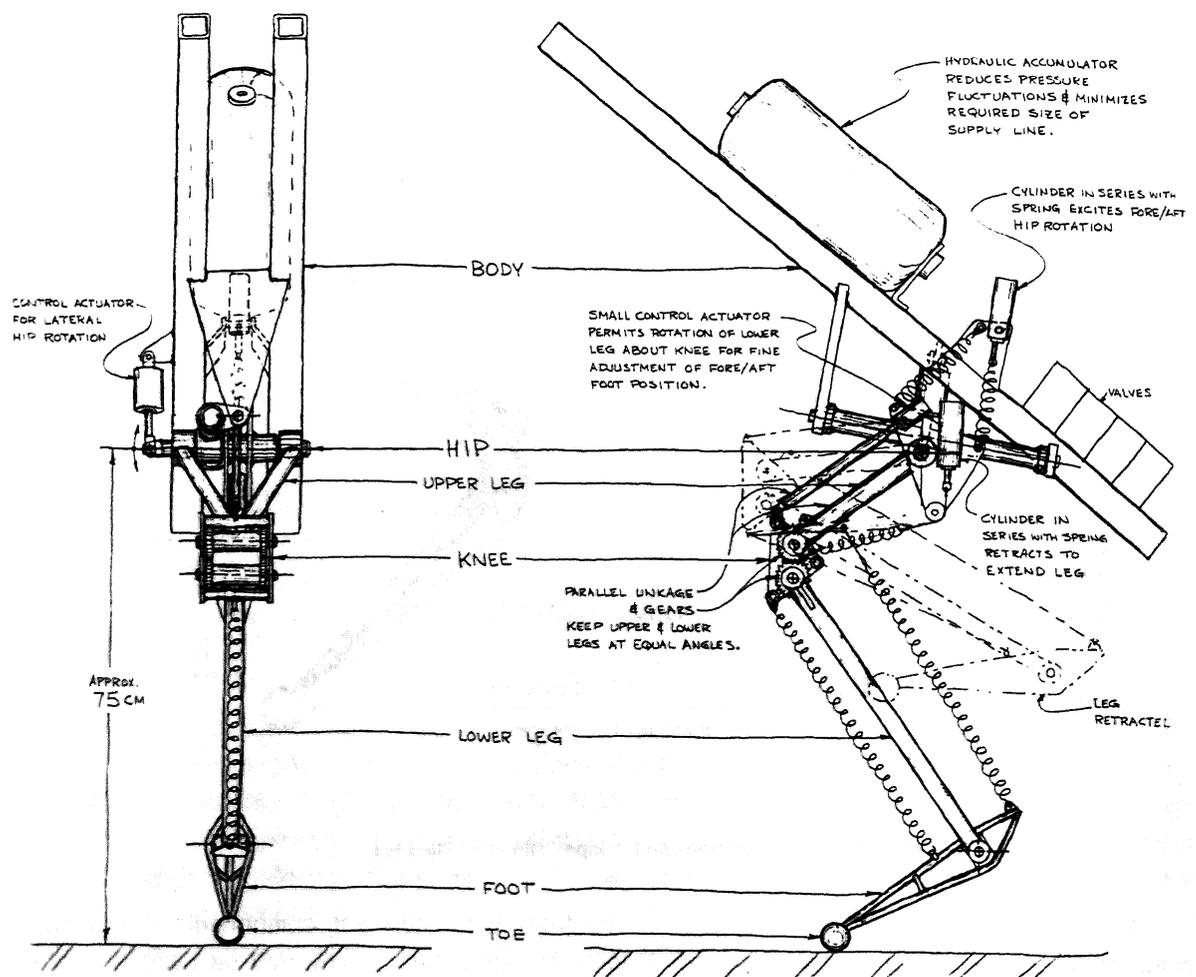
Figure 3-10: Preliminary leg design for multi-legged balancing vehicle.

Figure 3-10 embodies the basic concept and more significant features of an advanced 3-D leg as presently envisioned. The design begins with the basic structure of the simple articulated hopper: hip rotation is controlled about two axes, angle splitter in the knee, and parallel-bar linkage between

hip and lower leg. To this has been added a foot and increased range for fore and aft him motions. The lower leg and spring form a linkage that tends to keep the foot parallel to the upper leg. Thus, extension and contraction of the three elements of the leg can be controlled by a single actuator. The considerable elasticity of this spring will be adjustable in order to make testing easier.

The design incorporates four hydraulic actuators, two that balance and two that supply propulsive energy. Fore and aft rotation of the hip is controlled by an actuator in series with a spring. During steady state hopping, the leg will swing at its resonant frequency, requiring only small amounts of energy from the actuator to maintain the motion. This arrangement of spring and actuator is based on Ivan Sutherland's idea for a *tuning fork* quadruped. A second actuator produces fine fore and aft motions by rotating the knee. A third actuator, also in series with a spring, acts on the upper leg to cause the leg to lengthen and shorten. The angle splitter and parallelogram linkages cause the action of this cylinder to move the toe along a straight line that passes through the hip. Operating directly on the hip, a fourth actuator causes lateral displacement of the entire leg.

An articulated, three element leg has been chosen as having distinct performance advantages over other arrangements. Compared to linear sliding *pogo stick* designs, the articulated leg has the following advantages:

- Sliding joints are eliminated, thereby reducing inefficiency and fabrication problems.
- Unsprung mass is minimized by using a series of jointed masses in the leg -- at TOUCH-DOWN only the toe is unsprung.
- An articulated leg can reduce its vertical dimension, thereby requiring a smaller vertical clearance than a comparable linear leg. By the same mechanism, the moment of inertia of an articulated leg can be reduced in order to swing it forward more easily during flight.
- It is easier to match characteristics of springs and actuators by means of adjustable lever arm lengths.
- All springs and actuators act in tension, eliminating the buckling limitations associated with compression springs.
- The design can be optimized for unidirectional travel.

It is significant that most creatures in nature that are capable of reasonable speed on land and over uneven terrain employ the same basic three-element articulated leg design. Even though an articulated leg requires only three degrees of freedom, as provided by the simple hoppers of the last section, the addition of a lower element improves performance in three ways:

1. It reduces the unsprung mass.
2. It causes the ground force to act nearly through the knee, minimizing the moment at this generally fragile joint.
3. It permits energy storage in the Achilles tendon, which has been found significant in locomotion of many animals, most notably the kangaroo [3, 4].

I. Equations of Motion Planar Hopper

Equations for model shown in Figure 2-1.

$$\ddot{y}_x = \ddot{y}_0 - r^{\wedge\wedge}Sintfj) + {}^{\wedge}Cos^{\wedge})) \tag{3.3}$$

$$\ddot{x}j = \dot{\wedge} - r^{\wedge} \quad Cos(\theta J - \dot{\theta} \, Sin(\theta \,) \tag{3.4}$$

$$\ddot{y}_2 = \ddot{y}_0 + \ddot{w} \, Costfj) - w^{\wedge}Sin^{\wedge}) - wfilJCostfj) \tag{3.5}$$

$$\qquad - r_2(\ddot{0}_2 Sin(0_2> + \dot{\theta}_2^2 Cos(\theta_2)) - 2\dot{w}\dot{\theta}_1 Sin(\theta_1)$$

$$\ddot{x}_2 = \ddot{x}_0 + \ddot{w} \, Sint^{\wedge}j) + \ddot{w}tfjCos^{\wedge}) - w^{\wedge}Sin^{\wedge}) \tag{3.6}$$

$$\qquad + r_2(\ddot{\theta}_2 Cos(\theta_2) - \dot{\theta}_2^2 Sin(\theta_2)) + 2\dot{w}\dot{\theta}_1 Cos(\theta_1)$$

$$M_1\ddot{y}_1 = {}_{F_y} \; _- F_T Cos(\theta_1) + F_N Sin(\theta_1) - M_1 g \tag{3.7}$$

$$M_1\ddot{x}_1 = F_x - F_T Sin(tf_x) - F^{\wedge}tfj) \tag{3.8}$$

$$I_1\ddot{\theta}_1 = -F_x r_1 Cos(\theta_1) + F_y r_1 Sin(\theta_1) - F_N(w - r_1) - u_1(t) \tag{3.9}$$

$$M_2\ddot{y}_2 = F_T Cos(\theta_1) - F_N Sin(\theta_1) - M_2 g \tag{3.10}$$

$$M_2\ddot{x}_2 = F^{\wedge}iiK^{\wedge}i) + F^{\wedge}os^{\wedge}) \tag{3,11}$$

$$I_2\ddot{tf}_2 = F/jSintfj - tfj) - F_N r_2 Cos(^{\wedge}_2 - tf_x) + a^{\wedge}t) \tag{3.12}$$

where:

(x_Q, y_0) are the coordinates of the **foot.**
(x_1, y_r) are the coordinates of the leg's CG.
(x_2, y_2) are the coordinates of the body's CG.
F_x, F_y are the vertical and horizontal forces on the **foot.**
F_T, F_M are tangent and normal forces between the leg and body.

Eliminating x_1, y_1, x_2, y_2, and F_N:

$$\cos(\theta_1)(M_2Ww + I_1)\theta_1 + M_2r_2W\cos(\theta_2)\theta_2 + M_2Wx_0 + M_2W\sin(\theta_1)w = \qquad (3.13)$$

$$WM_2(\theta_1^2W\sin(\theta_1) - 2\theta_1w\cos(\theta_1) + r_2\theta_2^2\sin(\theta_2) + r_1\theta_1^2\sin(\theta_1)) -$$

$$r_1F_x\cos(\theta_1)2 + \cos(\theta_1)(r_1F_y\sin(\theta_1) - u_1(t)) + F_KW\sin(\theta_1)$$

$$-\sin(\theta_1)(M_2Ww + I_1)\theta_1 - M_2r_2W\sin(\theta_2)\theta_2 + M_2Wy_0 + M_2W\cos(\theta_1)w = \qquad (3.14)$$

$$WM_2(\theta_1^2W\cos(\theta_1) + 2\theta_1w\sin(\theta_1) + r_2\theta_2^2\cos(\theta_2) + r_1\theta_1^2\cos(\theta_1) - g)$$

$$+ r_1F_x\cos(\theta_1)\sin(\theta_1) - \sin(\theta_1)(r_1F_y\sin(\theta_1) - u_1(t)) + F_KW\cos(\theta_1)$$

$$\cos(\theta_1)(M_1r_1W - I_1)\theta_1 + M_1Wx_0 = \qquad (3.15)$$

$$W(M_1r_1\theta_1^2\sin(\theta_1) - F_K\sin(\theta_1) + F_x) - \cos(\theta_1)(F_yr_1\sin(\theta_1) -$$

$$F_xr_1\cos(\theta_1) - u_1(t))$$

$$-\sin(\theta_1)(M_1r_1W - I_1)\theta_1 + M_1Wy_0 = \qquad (3.16)$$

$$W(M_1r_1\theta_1^2\cos(\theta_1) - F_K\cos(\theta_1) + F_y - M_1g) -$$

$$\sin(\theta_1)(F_yr_1\sin(\theta_1) - F_xr_1\cos(\theta_1) - u_1(t))$$

$$-\cos(\theta_2 - \theta_1)I_1r_2\theta_1 + I_2w\theta_2 = \qquad (3.17)$$

$$W(F_Kr_2\sin(\theta_2 - \theta_1) + u_1(t)) - r_2\cos(\theta_2 - \theta_1)(r_1F_y\sin(\theta_1) - r_1F_x\cos(\theta_1) - u_1(t))$$

where

$$W = \quad w - r_1$$

$$F_K = \quad \begin{cases} K_s(w_0 - w + u_2) & \text{for } (w_0 - w + u_2) > 0 \\ K_{s2}(w_0 - w + u_2) - B_{s2}w & \text{otherwise} \end{cases} \qquad (3.18)$$

$$F_x = \quad \begin{cases} K_G(x_0 - x_{y_0 = 0}) - B_Gx_0 & \text{for } y_0 < 0 \\ 0 & \text{otherwise} \end{cases} \qquad (3.19)$$

$$F_y = \quad \begin{cases} K_Gy_0 - B_Gy_0 & \text{for } y_0 < 0 \\ 0 & \text{otherwise} \end{cases} \qquad (3.20)$$

II. Simulation Parameters

$M_1 = 1\,\text{kg}$

$I_1 = 1\,\text{kg-m}^2$

$r_1 = .5\,\text{m}$

$w_0 = 1\,\text{m}$

$K_P = 900$

$K_S = 10^3\,\text{Nt/m}$

$K_{S2} = 10^5\,\text{Nt/m}$

$K_G = 10^4\,\text{Nt/m}$

$Q_1 = 5.$

$M_2 = 10\,\text{kg}$

$I_2 = 10\,\text{kg}$

$r_2 = .4\,\text{m}$

$K_v = 60$

$B_{S2} = 125\,\text{Nt-sec/m}$

$B_G = 75\,\text{Nt-sec/m}$

$Q_2 = Q_3 = 1$

III. Legged Locomotion Bibliography

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